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We propose two interferometric schemes to experimentally detect the onset of pair condensation in a two spin-component Fermi gas. Two atomic wave-packets are coherently extracted from the gas at different positions and are mixed by a matter-wave beam splitter: we show that the spatial long range order of the atomic pairs in the gas then reflects in the atom counting statistics in the output channels of the beam splitter. Alternatively, the same long range order is also shown to create a matter-wave grating in the overlapping region of the two extracted wave-packets, grating that can be revealed by a light scattering experiment.

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The experimental possibility of controlling at will the scattering length a between two spin components of fermionic atoms *via* a Feshbach resonance has opened the way to a comprehensive study of the pairing transition in a degenerate Fermi gas [1, 2, 3]. The weakly interacting limits are well understood theoretically: the phase transition is the Bose-Einstein condensation (BEC) of diatomic molecules ($a = 0^+$) or the BCS transition due to pairing in momentum space ($a = 0^-$). But one can now investigate experimentally the theoretically challenging crossover region, including the unitary limit $|a| = \infty$ [4].

While the standard techniques used for atomic BECs have allowed to detect and characterize a molecular BEC [2], a debate is still in progress about experimental signatures of pair condensation for a negative scattering length. Several proposals have been put forward [5]; none of them was proved to demonstrate the existence of long range order in the pairing parameter. First experimental evidences of a condensation of fermionic pairs in the crossover regime have been recently presented [3], based on a fast ramping of the magnetic field to convert pairs on the $a < 0$ side into bound molecules on the $a > 0$ side, and on the observation of the Bose condensed fraction of the resulting gas of dimers. This method is expected to work only when the fermionic pairs are small enough, that is in the vicinity of the Feshbach resonance, $k_F|a| > 1$ where k_F is the Fermi momentum.

In this paper, we propose a more direct and general way of proving the condensation of pairs, by a measurement of the pairing order parameter, which is not restricted to the small pair regime $k_F|a| > 1$. This proposal is the fermionic analog of the atom interferometric measurement of the first order coherence function $G^{(1)}$ of a Bose gas [6]. More subtle schemes than the observation of the mean atomic density have however to be introduced as there is no long range first order coherence for fermions. Their experimental implementation would constitute a remarkable transposition of quantum optics techniques to a fermionic matter field.

In the current theories of the superfluid state in fermionic systems [4, 7, 8], the onset of pair condensation is defined by a non-zero long-distance limit $x_{AB} \equiv |\mathbf{x}_A - \mathbf{x}_B| \to +\infty$ of the pair coherence function

$$G^{(1)}_{\text{pair}}(\mathbf{x}_A, \mathbf{x}_B) = \left\langle \hat{\Psi}^\dagger_\uparrow(\mathbf{x}_A)\, \hat{\Psi}^\dagger_\downarrow(\mathbf{x}_A) \hat{\Psi}_\downarrow(\mathbf{x}_B)\, \hat{\Psi}_\uparrow(\mathbf{x}_B) \right\rangle, \tag{1}$$

this function then factorizing in the product of the order parameter in $\mathbf{x}_B$ and the complex conjugate of the order parameter in $\mathbf{x}_A$. In the following, we shall propose two distinct methods to measure $G^{(1)}_{\text{pair}}(\mathbf{x}_A, \mathbf{x}_B)$, both relying on the coherent extraction of two atomic wave-packets in $\mathbf{x}_{A,B}$ and their subsequent beating. The first method is based on a two-atom interferometric technique inspired by two-photon techniques [9]: it relies on atom counting in the two output channels of a matter-wave beam splitter. The second method is based on the coherence properties of light elastically scattered off the matter-wave interference pattern of the two overlapping wave-packets.

Consider a gas of spin-1/2 fermionic atoms at thermal equilibrium in a trap. At the time $t = 0$, the trap potential is suddenly switched off and the atom-atom interactions brought to a negligible strength, so that the subsequent propagation is the one of a free atomic field. At the same time, a suitable short pulse of spin-independent optical potential is applied (Fig.1) to the atoms situated in regions of size ℓ_u around the points $\mathbf{x}_A$ and $\mathbf{x}_B$ so to impart them a momentum kick of respectively $\mathbf{k}_0 \pm \mathbf{k}_1$ by means of Bragg processes and to produce wave packets which are a coherent copy of the field in the trap, but for a shift in momentum space. $\mathbf{k}_0$ is taken orthogonal to $\mathbf{x}_A - \mathbf{x}_B$, while $\mathbf{k}_1$ is parallel to it. The magnitude $\hbar k_1$ of the counter-propagating momentum kicks is taken larger than the momentum width Δp of the gas, which is on the order of the Fermi wavevector k_F in the resonance region ($|a| = +\infty$) and in the weakly interacting BCS regime ($a < 0$), or on the order of $\hbar/a$ in the case of a molecular condensate ($a > 0$). The size of the extraction region ℓ_u is taken much smaller than the distance x_{AB} between the extraction points. This latter is taken as macroscopic, that is much larger than any other length scale of the problem, e.g. the Fermi distance $1/k_F$ and the Cooper-pair size $\ell_{\rm BCS}$.

In Heisenberg picture the field operator at the end of the optical pulse can be related to the initial one by [12]:

$$\hat{\Psi}_\sigma(\mathbf{x},\Delta t) = u(\mathbf{x}-\mathbf{x}_A)\, e^{i(\mathbf{k}_0+\mathbf{k}_1)\cdot(\mathbf{x}-\mathbf{x}_A)}\hat{\Psi}_\sigma(\mathbf{x}) + \\ + u(\mathbf{x}-\mathbf{x}_B)\, e^{i(\mathbf{k}_0-\mathbf{k}_1)\cdot(\mathbf{x}-\mathbf{x}_B)}\,\hat{\Psi}_\sigma(\mathbf{x}) + \hat{\Psi}^{\rm bg}_\sigma(\mathbf{x}). \quad (2)$$

The atoms which are left in their original momentum state as well as the ones having received a different momentum kick during the extraction process are included in the background field $\hat{\Psi}^{\rm bg}_\sigma$: as they spatially separate during the evolution, they will be omitted in the discussion [13]. We shall assume for simplicity that the extraction function $u(\boldsymbol{\xi})$ is a Gaussian, $u(\boldsymbol{\xi}) = u_0\, e^{-\boldsymbol{\xi}^2/2\ell_u^2}$ of size ℓ_u; its peak amplitude u_0 is of modulus less than one.

This out-coupling scheme produces two atomic wave-packets traveling with momentum $\mathbf{k}_0 \pm \mathbf{k}_1$ starting from respectively $\mathbf{x}_{A,B}$. At a time $t_1 = m x_{AB}/2\hbar\,|\mathbf{k}_1|$, the two wave-packets superimpose around $\mathbf{X}$. As mentioned in the introduction, the mean density profile in the overlap region does not show fringes so that more elaborate manipulations have to be performed onto the atoms in order to measure the pair coherence function $G^{(1)}_{\rm pair}(\mathbf{x}_A,\mathbf{x}_B)$.

Atom-number correlations: At $t = t_1$, the two overlapping wave-packets of momentum respectively $\mathbf{k}_0 \pm \mathbf{k}_1$ can be coherently mixed by a spin-insensitive 50-50 matter-wave beam splitter, with reflection and transmission amplitudes of momentum-independent phase difference ϕ. Such a beam splitter may be realized[10] by applying a pulse of sinusoidal optical potential $U(\mathbf{x},t) = 4\hbar\Omega(t)\,\sin^2(\mathbf{k}_1\cdot\mathbf{x}+\phi/2)$ [14]. At a time t_2 after the splitting procedure, the two emerging wave-packets of momentum $\mathbf{k}_0 \pm \mathbf{k}_1$ will be again spatially separated and centered at $\mathbf{X}_\pm = \mathbf{X} + \hbar(\mathbf{k}_0 \pm \mathbf{k}_1)(t_2 - t_1)/m$. The total field can be written as $\hat{\Psi}_\sigma(\mathbf{x},t_2) = \hat{\Psi}^+_\sigma(\mathbf{x},t_2) + \hat{\Psi}^-_\sigma(\mathbf{x},t_2) + \hat{\Psi}^{\rm bg}(\mathbf{x},t_2)$ where the contribution of each packet is

$$\hat{\Psi}^\pm_\sigma(\boldsymbol{\xi}+\mathbf{X}_\pm,t_2) = \frac{e^{i(\mathbf{k}_0\pm\mathbf{k}_1)\boldsymbol{\xi}}\,e^{i\theta}}{\sqrt{2}}\int d\boldsymbol{\xi}'\,\mathcal{R}(\boldsymbol{\xi},\boldsymbol{\xi}';t_2)\times \\ \times\ u(\boldsymbol{\xi}')\left[\hat{\Psi}_\sigma(\mathbf{x}_{A,B}+\boldsymbol{\xi}') + i\,e^{\pm i\phi}\,\hat{\Psi}_\sigma(\mathbf{x}_{B,A}+\boldsymbol{\xi}')\right]. \quad (3)$$

$\mathcal{R}(\boldsymbol{\xi},\boldsymbol{\xi}';t)$ is the free-particle propagator and θ is an irrelevant propagation phase which depends on the details of the beam splitting procedure. The unitarity of $\mathcal{R}$ ensures that the results to come do not depend on t_2.

The operator $\hat{N}^\pm_\sigma$ giving the number of atoms with spin σ in the wave-packet $\pm$ is obtained by integration of $\hat{\Psi}^\dagger_\sigma(\mathbf{x},t_2)\hat{\Psi}_\sigma(\mathbf{x},t_2)$ over the spatial extension of the packet $\pm$ at time t_2. The operator giving the atom number difference between the two wave-packets is then $\hat{D}_\sigma = \hat{N}^+_\sigma - \hat{N}^-_\sigma$. Its expectation value $\langle\hat{D}_\sigma\rangle$ involves the first-order coherence function $\langle\hat{\Psi}^\dagger_\sigma(\mathbf{x}_A+\boldsymbol{\xi})\,\hat{\Psi}_\sigma(\mathbf{x}_B+\boldsymbol{\xi})\rangle$ of the initially trapped atoms, and therefore vanishes for a macroscopic distance $x_{AB} \gg \hbar/\Delta p$: we shall now take $\langle\hat{N}^+_\sigma\rangle = \langle\hat{N}^-_\sigma\rangle = N_\sigma$. Information on the pair coherence function $G^{(1)}_{\rm pair}$ is obtained from the correlation between the two spin components:

$$C_{\uparrow\downarrow} = \langle\hat{D}_\uparrow\,\hat{D}_\downarrow\rangle = \int d\boldsymbol{\xi}\,d\boldsymbol{\xi}'\,|u(\boldsymbol{\xi})|^2\,|u(\boldsymbol{\xi}')|^2 \\ \Big[-e^{2i\phi}\Big\langle\,\hat{\Psi}^\dagger_\uparrow(\mathbf{x}_A{+}\boldsymbol{\xi})\,\hat{\Psi}^\dagger_\downarrow(\mathbf{x}_A{+}\boldsymbol{\xi}')\,\hat{\Psi}_\downarrow(\mathbf{x}_B{+}\boldsymbol{\xi}')\,\hat{\Psi}_\uparrow(\mathbf{x}_B{+}\boldsymbol{\xi})\,\Big\rangle \\ +\Big\langle\,\hat{\Psi}^\dagger_\uparrow(\mathbf{x}_A{+}\boldsymbol{\xi})\,\hat{\Psi}^\dagger_\downarrow(\mathbf{x}_B{+}\boldsymbol{\xi}')\,\hat{\Psi}_\downarrow(\mathbf{x}_A{+}\boldsymbol{\xi}')\,\hat{\Psi}_\uparrow(\mathbf{x}_B{+}\boldsymbol{\xi})\,\Big\rangle{+}\text{h.c.}\Big]. \quad (4)$$

From an experimental measurement of the ϕ dependence of $C_{\uparrow\downarrow}$, it is therefore possible to determine whether the system has long range order or not.

An explicit calculation of $C_{\uparrow\downarrow}$ as a function of the energy gap Δ can be performed by using the zero temperature BCS theory [7, 8], with predictions that are accurate in the weakly interacting limit only. In the large x_{AB} limit, only the ϕ dependent part of Eq.(4) has a non-zero value: $C_{\uparrow\downarrow} = C^{(0)}_{\uparrow\downarrow}\,\cos(2\phi)$. In the local density approximation, and assuming for simplicity that the mean densities are the same in the two extraction points and in the two spin states, we find an analytical expression for a wide extraction region $\ell_u \gg \ell_{\rm BCS}$, both in the weakly interacting BCS regime

$$C^{(0)}_{\uparrow\downarrow} = -\frac{3\pi}{8\sqrt{2}}\,|u_0|^2\,\frac{\Delta}{E_F}\,N_\sigma \quad (5)$$

and in the regime of a molecular condensate

$$C^{(0)}_{\uparrow\downarrow} = -\frac{1}{\sqrt{2}}\,|u_0|^2\,N_\sigma. \quad (6)$$

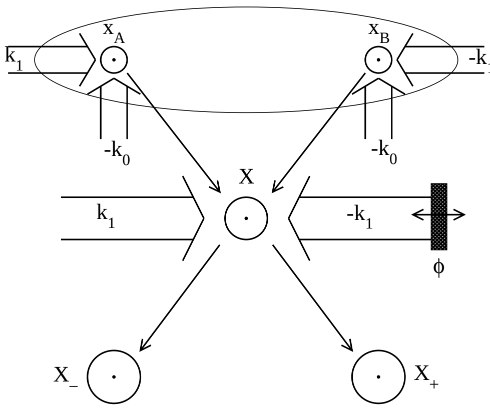

FIG. 1: First proposed set-up: atoms are extracted by a Bragg process from the gas at points $\mathbf{x}_{A,B}$, using pairs of laser beams of wavevectors $-\mathbf{k}_0$, $\mathbf{k}_1$ and $-\mathbf{k}_0$, $-\mathbf{k}_1$ respectively; at their overlap position $\mathbf{X}$, the two atomic wave-packets are coherently mixed by a laser standing wave acting as a 50-50 beam-splitter with adjustable phase shift ϕ; the number of atoms in each wave-packet is measured at the final positions $\mathbf{X}_\pm$.

$C_{\uparrow\downarrow}$ is obtained by averaging over many realizations of the whole experimental procedure starting from a trapped gas in the same initial conditions, so that a knowledge of the signal-to-noise ratio is relevant. We estimate the noise by the standard deviation of $\hat{D}_\sigma$: from Wick's theorem and to leading order in u_0, $\langle \hat{D}_\sigma^2 \rangle \simeq 2N_\sigma$ which shows that the shot-noise [11] in the initial extraction process is the dominant source of noise. The number of realizations over which to average therefore scales as $(N_\sigma/C^{(0)}_{\uparrow\downarrow})^2$, which is on the order of 1 in the BEC limit and on the order of $(E_F/\Delta)^2$ in the BCS limit.

Light scattering off the matter-wave grating: Information on the pairing coherence function $G^{(1)}_{\rm pair}$ of the trapped gas can also be obtained by means of light-scattering off the matter-wave interference pattern formed by the overlapping wave-packets at $t = t_1$ around point $\mathbf{X}$, which is taken in what follows as the origin of the coordinates. As already mentioned, the mean density does not show fringes. On the other hand, fringes appear in the opposite spin density-density correlation function $\mathcal{G}^{(2)}_{\uparrow\downarrow}(\mathbf{x},\mathbf{x}') = \langle \hat{\Psi}^\dagger_\uparrow(\mathbf{x},t_1)\,\hat{\Psi}^\dagger_\downarrow(\mathbf{x}',t_1)\hat{\Psi}_\downarrow(\mathbf{x}',t_1)\,\hat{\Psi}_\uparrow(\mathbf{x},t_1)\rangle$. As a guideline, one performs an explicit calculation for BCS theory: one finds that, as soon as a condensate of pairs is present, fringes appear as a function of the center of mass coordinates $(\mathbf{x}+\mathbf{x}')/2$ of a pair, with a sinusoidal oscillation of wavevector $4\mathbf{k}_1$. Their amplitude is proportional to the product of the in-trap anomalous averages in $\mathbf{x}_A$ and $\mathbf{x}_B$ and extends up to relative distances $|\mathbf{x}-\mathbf{x}'|$ of the order of the Cooper-pair size $\ell_{\rm BCS}$. This matter-wave grating is not easily detected in position space since its spatial period is smaller than the mean interatomic distance. We therefore switch to Fourier space:

$$\tilde{\mathcal{G}}^{(2)}_{\uparrow\downarrow}(\mathbf{q},\mathbf{q}') \equiv \int d\mathbf{x}\,d\mathbf{x}'\,e^{-i(\mathbf{q}\cdot\mathbf{x}+\mathbf{q}'\cdot\mathbf{x}')}\,\mathcal{G}^{(2)}_{\uparrow\downarrow}(\mathbf{x},\mathbf{x}'). \quad (7)$$

Since fringes show up on the center of mass coordinate with a wavevector $4\mathbf{k}_1$ we limit ourselves to the region $\mathbf{q}' = \mathbf{q} \simeq -2\mathbf{k}_1$ [15]. Taking into account the free expansion during t_1, one then obtains:

$$\begin{aligned}\tilde{\mathcal{G}}^{(2)}_{\uparrow\downarrow}(\mathbf{q},\mathbf{q}) = e^{i\hbar\,\Delta q^2 t_1/m}\int d\boldsymbol{\xi}\,d\boldsymbol{\xi}'\,e^{-i(\mathbf{q}+2\mathbf{k}_1)\cdot(\boldsymbol{\xi}+\boldsymbol{\xi}')}\\ \times\, u^*(\boldsymbol{\xi})u^*(\boldsymbol{\xi}')u(\boldsymbol{\xi}-\mathbf{b})u(\boldsymbol{\xi}'-\mathbf{b})\times\\ \left\langle \hat{\Psi}^\dagger_\uparrow(\mathbf{x}_A+\boldsymbol{\xi})\,\hat{\Psi}^\dagger_\downarrow(\mathbf{x}_A+\boldsymbol{\xi}')\,\hat{\Psi}_\downarrow(\mathbf{x}_B+\boldsymbol{\xi}'-\mathbf{b})\,\hat{\Psi}_\uparrow(\mathbf{x}_B+\boldsymbol{\xi}-\mathbf{b})\right\rangle\end{aligned} \quad (8)$$

where $\mathbf{b} = \hbar(\mathbf{q}+2\mathbf{k}_1)t_1/m$ and $\Delta q = |\mathbf{q}+2\mathbf{k}_1|$. Remarkably, for $\mathbf{q} = -2\mathbf{k}_1$ one recovers the factor in front of $e^{2i\phi}$ in Eq.(4).

This Fourier component of $\mathcal{G}^{(2)}_{\uparrow\downarrow}$ is detectable in the angular patterns of the elastic light scattering from the atomic cloud. The incoming laser intensity has to be weak enough to avoid saturation of the atomic transition. Optical pumping processes have to be negligible during the whole measurement time: the mean number of scattered photons per atom has to be much less than one not to wash out the information on the internal atomic state. The imaging sequence is assumed to take place in a short time so that the positions of the atoms can be safely considered as fixed. For each realization of the whole experiment, a different distribution of the atomic positions is obtained, and consequently a different angular pattern for the elastic scattering. Information on the density-density correlation function will be obtained by taking the average over many different realizations.

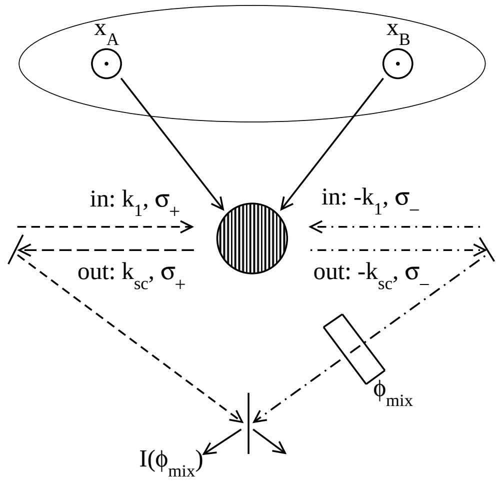

FIG. 2: Second proposed set-up: atoms are extracted from the cloud at $\mathbf{x}_{A,B}$ and create a matter-wave grating when the wave-packets overlap at $\mathbf{X}$; one detects this grating by shining a pair of counter-propagating, $\sigma_\pm$ polarized laser beams on the overlap region and by beating the two resulting backscattered light beams on a beam splitter: as function of the mixing phase $\phi_{\rm mix}$, the beating intensity $I(\phi_{\rm mix})$ averaged over many realizations presents fringes revealing the pair long range order. Dashed (dot dashed) lines: σ_+ (σ_-) polarization.

We consider here the simple case when the laser field frequency is close to resonance with the transition from the $F_g = 1/2$ ground state to a $F_e = 1/2$ hyperfine component of the excited state. In this case, $\sigma_\pm$ polarized light interacts only with atoms respectively in the $\downarrow,\uparrow$ spin state. The geometry adapted to get information on the condensation of pairs is shown in Fig.2: a pair of mutually coherent laser beams with a common intensity $I_{\rm inc}$ is sent on the atomic cloud with opposite circular polarizations $\sigma_\pm$ and opposite wavevectors $\pm\mathbf{k}_1$. We look at the mutual coherence of two back-scattered beams in opposite directions $\pm\mathbf{k}_{\rm sc}$ [16], with opposite circular polarizations. Within the Born approximation (valid if the cloud is optically dilute and optically thin), the amplitudes in Fourier space of the scattered light on the circular polarizations $\sigma_\pm$ are related to the ones of the incoming field by

$$E^{\rm sc}_\pm(\pm\mathbf{k}_{\rm sc}) = A\,\hat{\rho}_{\downarrow,\uparrow}(\pm\mathbf{q},t_1)\,E^{\rm inc}_\pm(\pm\mathbf{k}_1), \quad (9)$$

where A is a factor depending on the dipole moment of the transition and on the atom-laser detuning, and $\hat{\rho}_\sigma(\mathbf{q}, t_1)$ is the Fourier component at the transferred wavevector $\mathbf{q} = \mathbf{k}_{\rm sc} - \mathbf{k}_1$ of the density operator $\hat{\Psi}^\dagger_\sigma(\mathbf{x}, t_1)\hat{\Psi}_\sigma(\mathbf{x}, t_1)$ at time t_1. This mutual coherence is quantified by the correlation function

$$\begin{aligned} I_{-+}/I_{\rm inc} &= \left\langle \left[E^{\rm sc}_{-}(-\mathbf{k}_{\rm sc})\right]^\dagger \, E^{\rm sc}_{+}(\mathbf{k}_{\rm sc})\right\rangle/I_{\rm inc} \\ &= |A|^2 \left\langle \left[\hat{\rho}_\uparrow(-\mathbf{q}, t_1)\right]^\dagger \, \hat{\rho}_\downarrow(\mathbf{q}, t_1)\right\rangle = |A|^2 \tilde{\mathcal{G}}^{(2)}_{\uparrow\downarrow}(\mathbf{q}, \mathbf{q}). \end{aligned} \quad (10)$$

By using (8), one indeed sees that the correlation function I_{-+} can reveal the pair long range order. In the weakly interacting BCS regime as well as in the one of a molecular condensate, it has a simple expression for $\mathbf{q} \simeq -2\mathbf{k}_1$:

$$I_{-+} = -\frac{|A|^2}{2} \, C^{(0)}_{\uparrow\downarrow} \, e^{-\ell_I^2 \Delta q^2/2} I_{\rm inc}. \quad (11)$$

As a function of Δq, it has a narrow peak with a height proportional to the correlation function $C_{\uparrow\downarrow}$ of the first proposed set-up and with a width $1/\ell_I$ such that $\ell_I^2 = \ell_u^2 + (\hbar t_1/m\ell_u)^2$. Experimentally, this can be determined by beating the two scattered beams: as a function of the mixing phase $\phi_{\rm mix}$, the resulting intensity presents oscillations with an amplitude equal to $2\left|I_{-+}\right|$ on a background of value $\simeq 2|A|^2 N_\sigma I_{\rm inc}$.

In conclusion, we have proposed two possible ways of detecting a long-range pairing order in a degenerate Fermi gas by measuring the coherence function of the pairs via matter-wave interferometric techniques. This has the advantage over other techniques of directly measuring the order parameter without relying on a microscopic description of the many-body state, so that it applies in an unambiguous way both in the weakly interacting and the strongly interacting regimes. More generally, the proposed scheme is an application of quantum optics techniques to Fermi fields, a line of research expected to open new possibilities in the experimental manipulation and characterization of fermionic systems.

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[1] K.M. O'Hara, S.L. Hemmer, M.E. Gehm, S.R Granade, J.E. Thomas, Science **298**, 2179 (2002); T. Bourdel, J. Cubizolles, L. Khaykovich, K. M. F. Magalhães, S. J. J. M. F. Kokkelmans, G. V. Shlyapnikov, C. Salomon, Phys. Rev. Lett. **91**, 020402 (2003).

[2] M. Greiner, C. A. Regal, D. S. Jin, Nature **426**, 537 (2003); S. Jochim, M. Bartenstein, A. Altmeyer, G. Hendl, S. Riedl, C. Chin, J. Hecker Denschlag, R. Grimm, Science **302**, 2101 (2003); M. W. Zwierlein, C. A. Stan, C. H. Schunck, S. M. F. Raupach, S. Gupta, Z. Hadzibabic, W. Ketterle, Phys. Rev. Lett. **91**, 250401 (2003); T. Bourdel, L. Khaykovich, J. Cubizolles, J. Zhang, F. Chevy, M. Teichmann, L. Tarruell, S.J.J.M.F. Kokkelmans, C. Salomon, Phys. Rev. Lett. **93**, 050401 (2004).

[3] C. A. Regal, M. Greiner, and D. S. Jin, Phys. Rev. Lett. **92**, 040403 (2004); M. W. Zwierlein, C. A. Stan, C. H. Schunck, S. M. F. Raupach, A. J. Kerman, W. Ketterle, Phys. Rev. Lett. **92**, 120403 (2004).

[4] A. G. Leggett, J. Phys. (Paris) C **7**, 19 (1980); P. Nozières, S. Schmitt-Rink, J. Low Temp. Phys. **59**, 195 (1985); J. R. Engelbrecht, M. Randeria, and C. Sa de Melo, Phys. Rev. B **55**, 15153 (1997).

[5] J. Ruostekoski, Phys. Rev. A **60**, 1775 (1999); F. Weig and W. Zwerger, Europhys. Lett. **49**, 282 (2000); P. Törmä and P. Zoller, Phys. Rev. Lett. **85**, 487 (2000); A. Minguzzi, G. Ferrari, Y. Castin, Eur. Phys. J. D **17**, 49 (2001); Gh.-S. Paraoanu, M. Rodriguez, P. Törmä, J. Phys. B: At. Mol. Opt. Phys. **34**, 4763 (2001); C. Menotti, P. Pedri, S. Stringari, Phys. Rev. Lett. **89**, 250402 (2002); G. M. Bruun, Phys. Rev. Lett. **89**, 263002 (2002); C. P. Search, H. Pu, W. Zhang, and P. Meystre, Phys. Rev. Lett. **88**, 110401 (2002); E. Altman, E. Demler, M. D. Lukin, Phys. Rev. A **70**, 013603 (2004).

[6] T. Esslinger, I. Bloch, T. W. Hänsch, Jour. Modern Optics **47**, 2725 (2000). For a measurement of $G^{(1)}$ in Fourier space, see S. Richard, F. Gerbier, J. H. Thywissen, M. Hugbart, P. Bouyer, A. Aspect, Phys. Rev. Lett. **91**, 010405 (2003).

[7] J. R. Schrieffer and M. Tinkham, *Superconductivity*, Rev. Mod. Phys. **71**, S313 (1999).

[8] J. R. Schrieffer, *Theory of superconductivity*, Addison-Wesley, Redwood City, 1988; D. R. Tilley and J. Tilley, *Superfluidity and superconductivity*, Hilger, Bristol, 1990.

[9] C. K. Hong, Z. Y. Ou, and L. Mandel, Phys. Rev. Lett. **59**, 2044 (1987).

[10] P. J. Martin, B. G. Oldaker, A. H. Miklich, and D. E. Pritchard, Phys. Rev. Lett. **60**, 515 (1988); M. Kozuma, L. Deng, E. W. Hagley, J. Wen, R. Lutwak, K. Helmerson, S. L. Rolston, W. D. Phillips, Phys. Rev. Lett. **82**, 871 (1999).

[11] D. F. Walls, G. J. Milburn, *Quantum Optics*, Springer-Verlag. Berlin, 1994.

[12] We assume that $\Delta p|\mathbf{k}_0 \pm \mathbf{k}_1|\Delta t/m \ll 1$, where Δp is the initial momentum spread of the gas.

[13] Calculation of expectation values are performed by putting the observables in normal order and using the fact that $\hat{\Psi}^{\rm bg}_\sigma$, when acting on the state vector of the system, gives zero for $\mathbf{x}$ in one of the two considered wave-packets.

[14] In order to avoid scattering into higher, non-resonant, momentum states, the duration τ of the optical potential pulse has to be longer than the inverse of the atomic recoil frequency ω_R. In order for the reflection and transmission amplitude to have an equal modulus and a constant relative phase within the initial momentum width Δp of the gas, $\left|\mathbf{k} - (\mathbf{k}_0 \pm \mathbf{k}_1)\right| \lesssim \Delta p/\hbar$, τ has to be short enough for $\tau^{-1} \gg \hbar k_1 \Delta p/m$. The two conditions are compatible since we assumed that $\Delta p \ll \hbar k_1$.

[15] More precisely, one assumes $\hbar q t_1/m \gg \ell_u$ and $\hbar|\mathbf{q} - 2\mathbf{k}_1|t_1/m \gg \ell_u$.

[16] In practice, the scattering wavevectors $\mathbf{k}_{\rm sc}$ and $\mathbf{k}'_{\rm sc}$ can be considered as opposite if $|\mathbf{k}_{\rm sc} + \mathbf{k}'_{\rm sc}| < \ell^{-1}_{\rm BCS}, m\ell_{\rm BCS}/\hbar t_1$.