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Search for optimal 2D and 3D wave launching configurations for the largest acceleration of charged particles in a magnetized plasma, Resonant Moments Method

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Abstract

Optimal two-dimensional (2D), three-dimensional (3D) wave launching configurations are proposed for enhanced acceleration of charged particles in magnetized plasmas. A primary wave is launched obliquely with respect to the magnetic field and a secondary, low amplitude, wave is launched perpendicularly. The effect of both the launching angle of the primary wave, and the presence of the secondary wave is investigated. Theoretical predictions of the highest performances of the three-dimensional (3D) configurations are proposed using a Resonance Moments Method (RMM) based on estimates for the moments of the velocity distribution function calculated inside the resonance layers (RL). They suggest the existence of an optimal angle corresponding to non parallel launching. Direct statistical simulations show that it is possible to rise the mean electron velocity up to the order of magnitude as compared to the primary wave launching alone. It is a quite promising result because the amplitude of the secondary wave is ten times lower the one of the first wave. The parameters used are related to magnetic plasma fusion experiments in electron cyclotron resonance heating and electron acceleration in planetary ionospheres and magnetospheres.

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Electron acceleration due to external radio frequency waves in a strong magnetic field has long been recognized as an important effect in a wide variety of problems ranging from plasma heating and current drive in fusion devices [1, 2, 3, 4] to electron acceleration in the Earth's radiation belts during geomagnetic storms [5, 19], active ionospheric and magnetospheric probing [6, 18].

It is well known that wave-particle interactions are most efficient when the particles are in resonance with the waves. The resonance conditions

$$k_{\parallel}v_{\parallel} - \omega = \frac{N\Omega}{\gamma} \quad (1)$$

define some regions 'of sensitivity' in wave-particle parameter space which can be described as Resonant Layers (RL). Here, N is the harmonic number, $\omega = 2\pi f$ is the wave frequency, k_{\parallel} and v_{\parallel} are the components of the wave vector and the electron velocity parallel to the constant magnetic field B_0 , $\gamma = (1 - v^2/c^2)^{-1/2}$ is the relativistic factor and $\Omega = eB_0/m_e$ the gyro-frequency. This view allows to develop significantly previous simulations of charged particle fluxes and plasma disturbances in ambient magnetic field in [11, 12, 13, 14, 15]. There is a special case when electrons permanently staying in the RL (1). Such a phenomenon has been referred to as autoresonance [20, 21, 22] and its conditions are known as the cyclotron auto-resonance maser (CARM) conditions [23]. Several mechanisms have been explored for maintaining the synchronization between electrons and waves not fully satisfying these CARM conditions such as changing the profile of the guide magnetic field or varying the wave phase velocity [24, 25]. Recently, the use of two parallel counter-propagating waves has been considered [26, 27]. Numerical tests [28] have shown that the two-wave scheme may lead to higher averaged parallel velocity. The stochastic acceleration mechanism for electrons in a plane monochromatic electromagnetic wave propagating obliquely to the external magnetic field has also been studied [29, 31, 32]. It was found that it is easier to accelerate electrons to high energies with increasing propagation angle when the electron motion becomes stochastic and the parallel phase velocity of wave is supraluminous ($n_{\parallel} = k_{\parallel}c/\omega < 1$). Furthermore, Karimabadi and Angelopoulos[33] studied the interaction of charged particles with a packet of obliquely propagating plane monochromatic electromagnetic waves under the special condition (of equal $n_i \cos \theta_i$ for all the waves, where for the i -wave n_i - refraction index, θ_i - the propagation angle). This condition allowed the system to be reduced to two degrees of freedom and the particles can be accelerated through a process of Arnol'd

diffusion [34] in two dimensions.

The majority of the existing works are based on the description of a single particle dynamics in one (or more but under condition of equal $n_i \cos \theta_i$) plane monochromatic radio frequency waves.

The coherent acceleration of nonrelativistic ions interacting with two and more electrostatic waves in a uniform magnetic field has been studied by [8], recently by [7], as a generalization of analysis of [9, 10] by including wavenumbers along the external magnetic field.

In this paper, a mechanism is discussed for the acceleration of electron populations resulting from the effect of crossing electromagnetic waves propagating in a dispersive medium according to the geometry represented in Figure 1 (the condition of equal $n_i \cos \theta_i$ for the two waves is thus clearly broken). To analyze this mechanism, the resonance moments (RM) of the distribution, i.e. velocity moments computed in the RL only, are evaluated. The first order RM suggests that a peculiar θ results in a maximal averaged parallel flux. Although the RM approach has to be considered as an approximation, this prediction is reasonably confirmed by direct statistical simulations. Moreover, the two-wave scheme allows to rise the mean electron velocity up to one order of magnitude when compared to the one-wave scheme, based on the primary wave only.

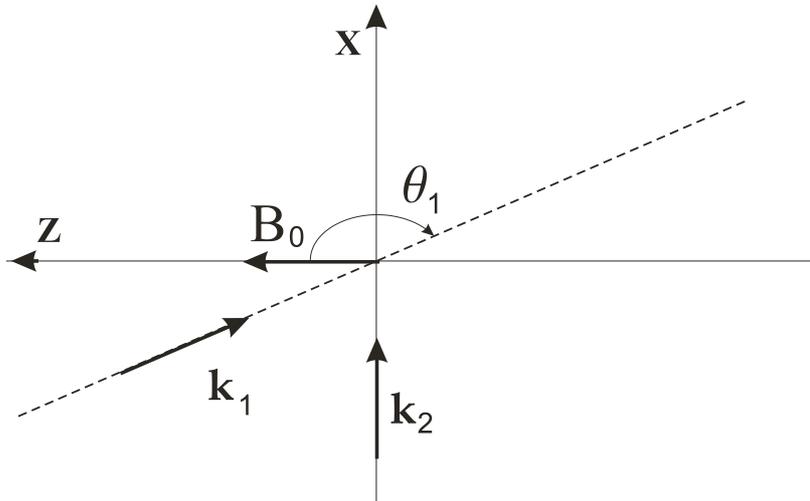


FIG. 1: Schematic picture of the electro-magnetic configuration.

The electromagnetic configuration that is considered (Figure 1) is the combination of a strong magnetic field (assumed to be along the z direction), a primary wave propagating

obliquely with respect to the magnetic field and a secondary wave propagating perpendicularly to the magnetic field. As a first step, to simplify direct particle simulations both the primary and the secondary waves are assumed to be in the plane (x, z) . This simplification can give not so impressive effect as compared to 3D launching. Nevertheless, it will give a first estimate of the secondary wave effect and motivation to develop more realistic 3D launching code that will be closer to real experimental setups.

This electromagnetic configuration is not an attempt to satisfy the resonance condition during a long time being close to the cyclotron auto-resonance maser (CARM) conditions [23]. Rather, a large population of electrons is considered and only the average effect of the waves on the population is considered. The fraction of electrons that are close to the condition (1), that corresponds to a resonance layer (RL) in the velocity space, becomes then as important as the time these electrons remain resonant. The secondary wave does not carry any parallel momentum and cannot induce any net parallel motion of the electrons. The purpose of this secondary wave is to maintain a pseudo-equilibrium velocity distribution in which the RL is continuously re-filled. Indeed, the combined effect of the two-waves and the magnetic field yields a stochastic motion during which the synchronization between the waves and the gyro-motion of the electron is, on average, more favorable for transferring momentum to the electrons.

The description of the electron trajectory (\mathbf{r}, \mathbf{p}) requires a relativistic treatment and is derived from the time dependent Hamiltonian:

$$H = \sqrt{m^2c^4 + c^2(\mathbf{p} + e\mathbf{A})^2}, \quad (2)$$

and the trajectories of the electrons are determined by the initial conditions and by the Hamilton equations. This picture corresponds to a test-electron in an external electromagnetic field. Assuming that the electrons interact with two monochromatic waves propagating in cold plasma at angles θ_1 and θ_2 with respect to the guide magnetic field, the total vector potential can be written as follows:

$$\begin{aligned} \mathbf{A} = & B_0 x \mathbf{e}_y + \frac{A_1}{2} e^{i(\mathbf{k}_1 \cdot \mathbf{r} - \omega_1 t)} (\cos \psi_1 \mathbf{e}_y - i \sin \psi_1 \mathbf{e}_1) \\ & + \frac{A_2}{2} e^{i(\mathbf{k}_2 \cdot \mathbf{r} - \omega_2 t)} (\cos \psi_2 \mathbf{e}_y - i \sin \psi_2 \mathbf{e}_2) + c.c., \end{aligned} \quad (3)$$

where \mathbf{e}_1 and \mathbf{e}_2 are two unit vectors in the plane $(x - z)$. These vectors, as well as the angles ψ_1 and ψ_2 and the refraction indices $n_1 = \|\mathbf{k}_1\|c/\omega_1$ and $n_2 = \|\mathbf{k}_2\|c/\omega_2$, are determined

by the Appleton-Hartree dispersion relation in the cold homogeneous plasma approximation [35]. Collisions with other particles, electrons or ions, are neglected. The dynamics of the electrons in the electromagnetic configuration (3) is chaotic and unpredictable analytically in general. Electrons with slightly different initial position and velocity may experience drastically different evolutions. The exact analytical prediction of the average effect of the waves is thus out of reach. It is however possible to anticipate the existence of a RL (1) in the velocity space to estimate the possible net effect of the waves by computing the RM defined by:

$$I^g = \int_{\mathbf{v} \in \text{res. layer}} d\mathbf{v} f(\mathbf{v}) g(\mathbf{v}) \quad (4)$$

for any function g of the velocity. It represents the density of electrons close to the RL for $g = 1$ and the mean parallel flux of the electrons on the layer for $g = v_{\parallel}$. These quantities should give some estimate on the efficiency of the electron-wave interaction. The RL corresponds to an ellipse in the velocity space and the integral I can be evaluated analytically using elliptical coordinates. For instance, assuming a Maxwellian distribution, $f(\mathbf{v}) = C \exp(-\beta \mathbf{v}^2)$, the evaluation of I for $g = v_{\parallel}$, which will be denoted I^* can be done explicitly. This quantity corresponds to the averaged parallel flux of the particles that belongs to the RL. It is, at least indirectly, related to the net averaged parallel current produced by the electron-wave interactions. Indeed, these interactions tend to remove electrons from the RL while the pseudo thermalization of the electrons due to the combined effect of the two waves is to refill constantly the layer. The thermalization is thus expected to add a net averaged parallel velocity proportional to I^* . Of course, I^* gives only a rough indication of the efficiency of electron-wave interactions and the final averaged velocity cannot be deduced directly from it. The exact expression for I^* is quite long. It is thus more illustrative to present the Figure 2 in which I^* is a function of θ_1 , the angle of the primary wave. The dimensionless wave amplitudes and the quadratic plasma frequency are defined by $\bar{A}_{1,2} = A_{1,2}\Omega/cB_0$, $e_0 = (\omega_{pe}/\Omega)^2$. The other parameters correspond to the values used in the simulations described below.

As expected, for perpendicular propagation $\theta_1 = 90^\circ$, no averaged parallel flux is observed. For two sets of parameters, there is a clear maximum of the averaged parallel flux of the particles in the RL for $\theta_1 \neq 0^\circ$. The explanation for such a phenomenon is that the averaged parallel velocity induced by the electron-wave interaction depends on both the angle of propagation of the primary wave and the number of electrons that are close to the RL.

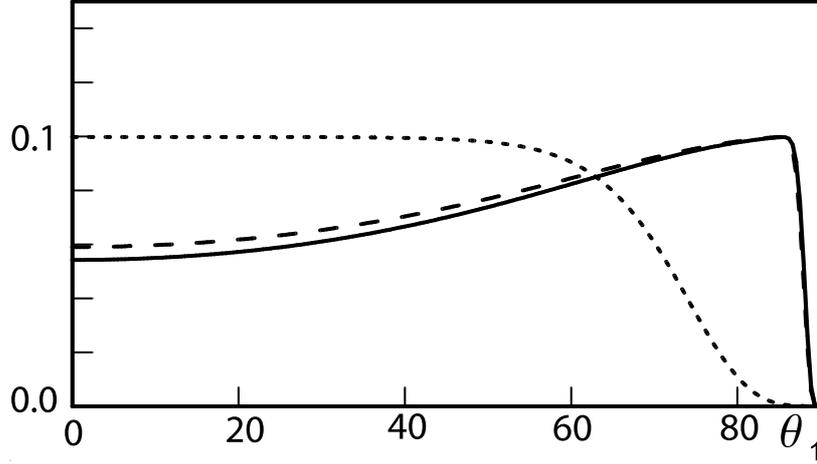


FIG. 2: Averaged parallel flux of particles in the RL in arbitrary units for a Maxwellian distribution for increasing densities, $e_0 = 0.3$ solid line, $e_0 = 0.6$ dashed line and $e_0 = 1.99$ dotted line.

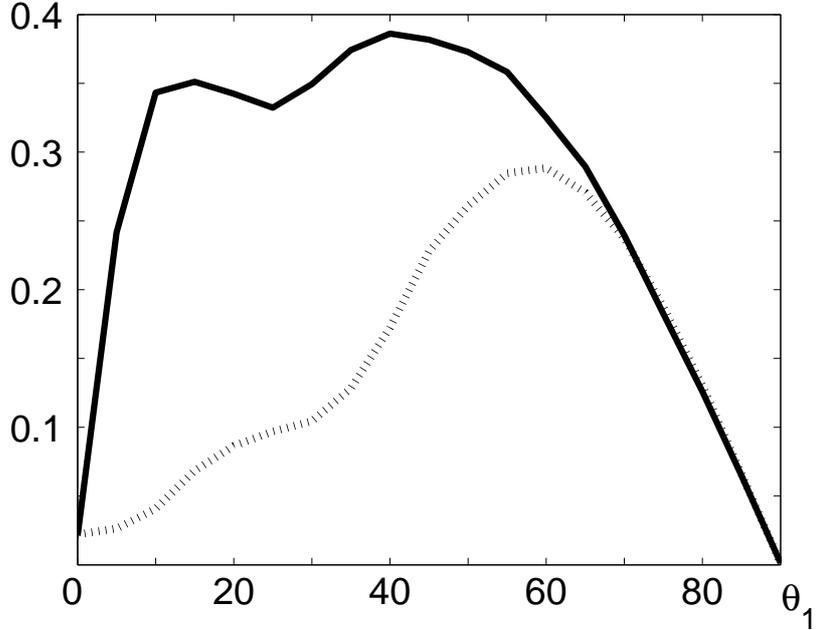


FIG. 3: Averaged electron velocity versus θ_1 (in degrees) for the one-wave scheme with $\bar{A}_1 = 0.1$, $\bar{A}_2 = 0$ (dotted lines) and the two-wave-scheme with $\bar{A}_1 = 0.1$, $\bar{A}_2 = 0.01$ (solid lines) at time $\Omega t = 7000$, for low-density runs $e_0 = 0.3$.

Smaller angles of propagation correspond to higher parallel momentum carried on by the wave. However, the RL condition is compatible with larger numbers of electrons for higher angles of propagation (at least assuming a Maxwellian distribution). It should be noted that here I^* has been computed using only the fundamental $N = 1$ RL. Contributions from the

higher harmonic layers decrease rapidly because these layers are more and more symmetric and because the absolute value of the resonant velocity increases towards high values for which the electron density is very small.

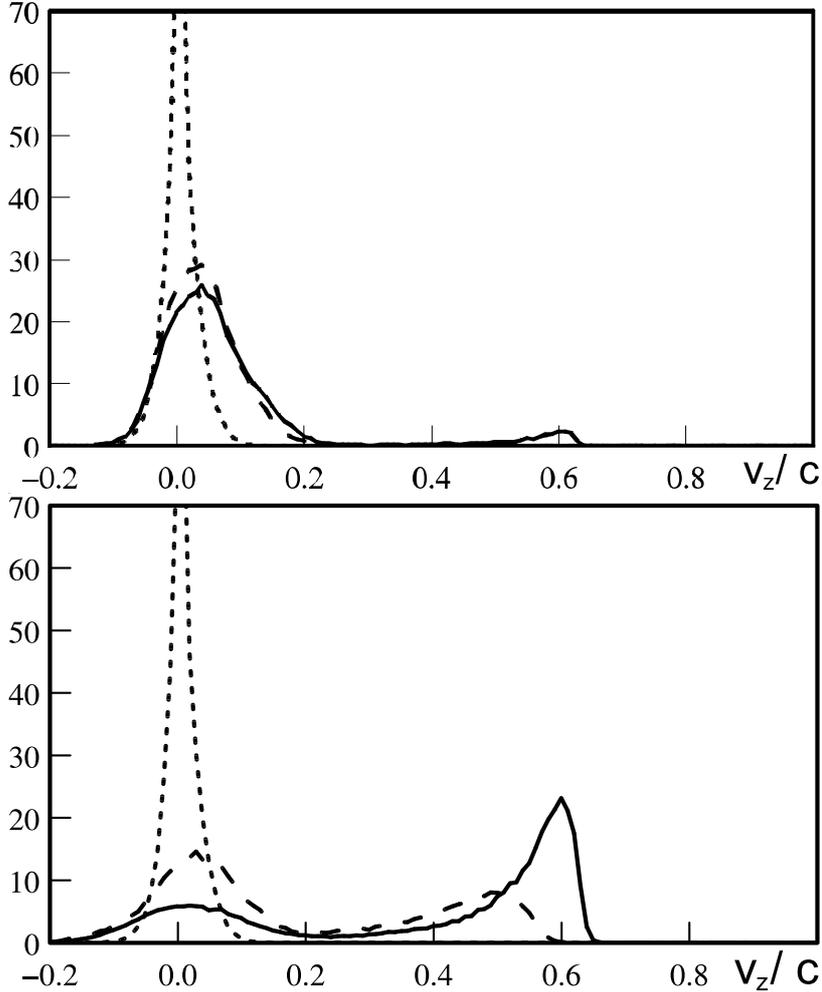


FIG. 4: Final probability distribution of the parallel velocity for $\theta = 15^\circ$ and $\bar{A}_1 = 0.1$ for the one-wave scheme with $\bar{A}_2 = 0$ (top) and the two-wave-scheme with $\bar{A}_2 = 0.01$ (bottom). The solid lines ($e_0 = 0.3$), dashed lines ($e_0 = 0.6$) and dotted lines ($e_0 = 1.99$) correspond to increasing densities.

The Hamiltonian equations for the \mathbf{r} and \mathbf{p} are solved using a 4th order Runge-Kutta method. The time step is adapted to ensure that the solution of a redundant evolution equation for H remains close to the expression (2). The initial velocity distribution has a temperature of the order of 1 keV. A population of $5 \cdot 10^4$ electrons is used in each simulation. Although running the code with larger populations is not an issue, no further information is derived from these larger runs, except of course more converged statistics.

Numerical results confirm that the angle dependency is not trivial and that parallel propagation ($\theta_1 = 0^\circ$) for the primary wave is not always optimal [29]. Three sets of simulations are presented hereafter. The parameters for these simulations are relevant in today tokamak plasma. In particular, the primary wave corresponds to the second harmonic of the cyclotron frequency and the secondary wave to the third harmonic Right Hand Polarized modes which are frequently used for instance in the TCV tokamak experiments [36]. The value of the constant magnetic field is $1.42 T$ for all simulations. Three electron densities have been considered: $n_e \approx 0.6 \cdot 10^{19} m^{-3}$ ($e_0 = 0.3$), $n_e \approx 1.2 \cdot 10^{19} m^{-3}$ ($e_0 = 0.6$) and $n_e \approx 3.9 \cdot 10^{19} m^{-3}$ ($e_0 = 1.99$). The wave amplitudes are $\bar{A}_1 = 0.1$ and $\bar{A}_2 = 0.01$ in the two-wave scheme and $\bar{A}_2 = 0$ in the one-wave scheme. They correspond to power fluxes which are by orders of magnitude higher than that achievable on gyrotron used in today tokamak. However, preliminary computations using the Resonant Moment Method suggest three-dimensional electromagnetic wave configurations are very promising for larger acceleration of charged particles in an external magnetic field with even lower wave amplitudes. In such a case the wave vectors and the magnetic field are not supposed to be co-planar and create a fully three dimensional system. On the other side, the required powers might be achievable by free electron maser [37] even for experiments with 2D launch configuration as predicted by our direct particle simulations.

Also, the parameters $e_0 = (\omega_{pe}/\Omega)^2 = 0.1 - 0.3$, correspond to the nighttime ionosphere at approximately 130km, $A=0.1$ - to a power flux $5W/cm^2$, and a frequency 2.6MHz of the primary wave. These parameters are close to ones considered in [29, 30] for single wave acceleration.

Figures 3 show a significant increase of the average parallel velocity for $e_0 = 0.3$ due to the secondary wave. Moreover, the angle dependency of the average parallel velocity appears to be maximal in the range $\theta_1 = 10^\circ - 60^\circ$. This range thus appears to significantly differ from the privileged value of the RM I^* (Fig.2). This is not too surprising since the RM have been computed assuming a Maxwellian distribution with zero mean. Thus, although the global picture from the RM description is reasonable, a more precise iterative approach, taking into account the averaged velocity suggested by the RM would be required for more accurate predictions. The probability distributions of parallel velocity (Figures 4) observed at the end of the simulations $\Omega t = 2 \cdot 10^4$ indicate that, in the two-wave scheme a much larger number of electrons have had the occasion to interact with the primary wave and

the distributions of velocity exhibit two well marked maxima. For $e_0 = 1.99$, the density is very close to the cut-off value of the wave propagation and almost no effect is observed in both the one-wave and the two-wave scheme. Also, if \bar{A}_1 is too small, no averaged parallel velocity is observed at all. Preliminary tests seem to reproduce the threshold previously observed [23, 32].

This paper presents a mechanism for enhancing acceleration of a population of electrons using crossing electromagnetic waves propagating at different angles with respect to an external magnetic field in a dispersive medium. The existence of optimal angles of propagation for the primary wave is suggested using the evaluation of resonant moments and is confirmed by direct numerical simulations of the electron trajectories. A secondary low amplitude perpendicular wave is used to induce a stochastization of the electron trajectories and, consequently, to maintain a pseudo-equilibrium. Although measures of the distributions (Figures 4) clearly show a departure from a thermal equilibrium, the stochastization effect of the secondary wave yields a clear increase of the average parallel electron velocity. It is a quite promising result since the amplitude of the secondary wave is ten times lower than the one of the first wave.

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