Modelling a ratchet with cold atoms in an optical lattice

Cécile Robilliard, D. Lucas, G. Grynberg

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C. Robilliard*, D. Lucas†, and G. Grynberg
Laboratoire Kastler Brossel,
Département de Physique de l'Ecole Normale Supérieure
24, rue Lhomond, F-75231 Paris cedex 05, France

Abstract : In this paper, we present an experimental study of a model ratchet consisting of laser-cooled rubidium atoms localized in an asymmetric periodic potential. This potential is obtained from a near-resonant light field and an additional static magnetic field. We show that there is a net motion of atoms with a mean velocity on the order of a few cm/s and we investigate the role of the asymmetry of the potential and of the dissipation in the atomic motion. A link with the stochastic resonance is mentioned.

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I. INTRODUCTION

Due to its application to molecular motors that convert chemical energy into mechanical energy, for example in muscles, the physics of ratchets has been increasingly studied in recent years [1]. From a general point of view, a ratchet is a non-linear system which is able to convert unbiased non-equilibrium fluctuations into mechanical work. A common model for ratchets consists of Brownian particles evolving in a periodic but spatially asymmetric potential in the presence of dissipation. Because both parity and time reversal symmetry are violated, a directional motion of particles is possible although there is apparently no net force acting on them [2].

More elaborate theoretical models consider the particles to exist in two states each of them experiencing a different potential. Both potentials are *periodic* but *asymmetric*. When the transition rates between the two states differ from their values at Boltzmann equilibrium, a directional average motion is predicted [3]. This paper presents an experimental realisation of a very similar model, using laser-cooled rubidium atoms.

In section II we explain how to obtain a ratchet-like system for cold atoms. The experimental set-up is presented in section III, and the next two sections are devoted to the experimental study of the influence of spatial asymmetry (section IV) and dissipation (section V), respectively. The role of dissipation here is very similar to the one found in stochastic resonance [4,5].

II. MODELLING A RATCHET WITH COLD ATOMS

Atoms cooled and manipulated by light provide a very versatile medium to model many phenomena usually observed in other systems. In the present case, we combine a near-resonant light field with a static magnetic field to tailor a potential which is periodic but asymmetric. In this system, dissipation originates from optical pumping between different Zeeman sublevels.

We first consider the interaction of an atom with an electromagnetic field, the atom having an electronic transition from a ground state with an angular momentum $F = 1$ to an excited state with the same angular momentum $F' = 1$. If the electromagnetic field is polarized in the xOy plane (no π component), it is well-known that all the physics can be described in terms of a Λ system which includes the two ground state Zeeman sublevels $m = -1$ and $m = 1$ and the excited sublevel $m' = 0$ [6]. This transition has been widely studied in the context of velocity selective cooling by population trapping (VSCPT) [7]. Furthermore, if the light intensity is sufficiently small or if the frequency detuning Δ from the atomic resonance is sufficiently large, it is possible to eliminate adiabatically the excited substate and to describe all the physical processes inside the two-level subspace $\{m = -1, m = 1\}$ [6]. The master equation then contains (i) a Hamiltonian evolution in the optical bipotential due to the light-shifts, (ii) damping processes originating from absorption-spontaneous emission cycles. Because such a Λ system accommodates a state $|NC\rangle$ which is not coupled to the excited state [8], the corresponding optical potential V_{NC} is flat ($V_{NC} = 0$) and the

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*Permanent address: Laboratoire Collisions, Agrégats, Réactivité (UMR 5589, CNRS and Université Paul Sabatier), Toulouse, France.

†Present address: Clarendon Laboratory, Parks Road, Oxford OX1 3PU, UK.

Hamiltonian appears as the sum of the kinetic energy $p^2/2M$ (where M is the atomic mass and p the momentum) and of the potential energy $V_C\,|C\rangle\langle C|$ (with $\langle NC|\,C\rangle = 0$).

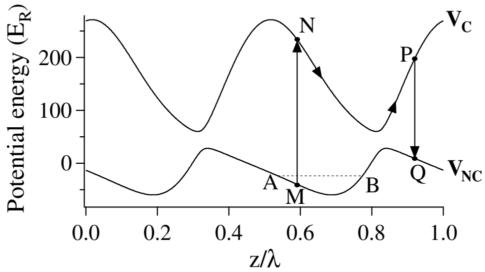


FIG. 1. Optical bipotential in the "lin θ lin" configuration with a longitudinal magnetic field $\mathbf{B}_0$. The parameters are $\theta = 45°$, $\Delta/\Gamma = 2$, $\Omega/\Gamma \simeq 0.65$ and $\Omega_0 = 60\omega_R$ (where $E_R = \hbar\omega_R = \hbar^2\omega^2/\left(2Mc^2\right)$ is the recoil energy). The curves represent the exact optical potential, obtained by diagonalizing the total Hamiltonian (light-shift and Zeeman-shift). An atom oscillating between A and B can shift to the neighbouring well because of a transition in V_C.

In the following, we consider a unidimensional (1D) situation with two counterpropagating light beams $\mathbf{E}_1$ and $\mathbf{E}_2$ of the same amplitude E_0, the same frequency $\omega = ck$ and linear polarizations $\mathbf{e}_1 = \cos(\theta/2)\,\mathbf{e}_x + \sin(\theta/2)\,\mathbf{e}_y$ and $\mathbf{e}_2 = \cos(\theta/2)\,\mathbf{e}_x - \sin(\theta/2)\,\mathbf{e}_y$ ("lin θ lin" configuration). The optical potential associated with $|C\rangle$ is $V_C = \hbar\Delta s D(z)/2$ where $\Delta = \omega - \omega_0$ (ω_0: atomic resonance frequency), $s = \frac{\Omega^2/2}{\Delta^2+\Gamma^2/4}$, $\Omega = dE_0/\hbar$ (d being the reduced matrix element of the electronic dipole moment), Γ the natural width of the excited state and $D(z) = 1 + \cos\theta\cos(2kz)$. In order to have a spatially modulated potential V_{NC}, a small magnetic field $\mathbf{B}_0$ along Oz can be added. The Zeeman shift then leads to

$$V_{NC} = -\hbar\Omega_0 \sin\theta \sin(2kz)/D(z) \tag{1}$$

with $\hbar\Omega_0 = g\mu_B B_0$ (g: Landé factor of the ground state, μ_B: Bohr magneton). The potentials V_{NC} and V_C (the latter now differs slightly from $\hbar\Delta s D(z)/2$ because of the Zeeman effect) are shown in Fig. 1. In this system, efficient Sisyphus cooling can occur for $\Delta > 0$ [9] through the following mechanism: an atom moving in $|NC\rangle$ can be transferred into $|C\rangle$ by motional coupling, which is most efficient near the anti-crossing points of the potential. It then climbs a potential hill of V_C before being optically pumped back into $|NC\rangle$. Because the optical pumping rate from $|C\rangle$ to $|NC\rangle$ is maximum near the top of the potential hill, the kinetic energy of the atom decreases. This process finally leads to cooling and trapping of a large fraction of the atoms in the asymmetric wells of the uncoupled potential. Such a system is called grey lattice, and the required light beams $\mathbf{E}_1$ and $\mathbf{E}_2$ are referred to as "grey lattice beams" in the following.

What happens to an atom oscillating in such a well (between A and B in Fig. 1) ? The transition probability from $|NC\rangle$ to $|C\rangle$ due to B_0 is maximum near A. Once in $|C\rangle$ (M $\rightarrow$ N in Fig. 1), the atom undergoes an oscillation that is interrupted by a spontaneous Raman process that brings the atom back into $|NC\rangle$ (P $\rightarrow$ Q in Fig. 1). The jumps between the potential curves thus tend to drag the atom to the right-hand side of Fig. 1. Note that because of the long lifetime of atoms in $|NC\rangle$, a jump from $|NC\rangle$ to $|C\rangle$ is a rather seldom event, so that the atoms undergo many oscillations in a well of the $|NC\rangle$ potential between two jumps[1]. One recognizes a mechanism similar to the stochastic resonance, where synchronisation of the noise (here optical pumping) induces the motion [5].

Inspecting Eq. 1, three basic properties of the potential V_{NC} can be noticed: (i) V_{NC} changes sign when $\mathbf{B}_0$ is reversed; (ii) V_{NC} has opposite asymmetries for $\theta = \theta_0$ and $\theta = -\theta_0$; (iii) V_{NC} is symmetrical for $\theta = 0, \pm\pi/2$. The experimental results are in good agreement with these features of the potential (see [10] and section IV).

III. EXPERIMENTAL SET-UP

To perform the experimental study, we start from rubidium atoms (^{87}Rb) trapped and cooled in a magneto-optical trap (MOT) [11]. At a certain time we switch off the MOT beams and inhomogeneous magnetic field and switch on

[1]There is another cause to the jumps which is the non-adiabatic coupling. This coupling is essential for the velocity damping.

the grey lattice beams operating on the blue side of the $5S_{\frac{1}{2}}(F=1) \rightarrow 5P_{\frac{3}{2}}(F'=1)$ transition as well as repumping beams detuned by 2Γ on the blue side of the $5S_{\frac{1}{2}}(F=2) \rightarrow 5P_{\frac{3}{2}}(F'=2)$ transition (to avoid a leakage of atoms into the $F=2$ hyperfine sublevel) and the static magnetic field $\mathbf{B}_0$. The counterpropagating grey lattice beams are vertical while the repumping beams lie in the horizontal plane. This arrangement allows a partial compensation of the effect of gravity while ensuring that the repumping beams do not contribute to the vertical motion.

The spatial displacement of cold atoms is measured using a CCD camera by direct imagery of the atomic cloud (whose initial radius is $\sim$1 mm). One technique is to use the faint image of the grey lattice itself. This requires the averaging of a significant number of images (~ 100) because atoms in $|NC\rangle$ scatter very few photons. An example of measurements done with this method is shown in [10]. Instead of tracking the faint image of the grey lattice, one can also apply after a grey lattice phase τ_L a flash of two beams nearly resonant on the $5S_{\frac{1}{2}}(F=1) \rightarrow 5P_{\frac{3}{2}}(F'=2)$ and $5S_{\frac{1}{2}}(F=2) \rightarrow 5P_{\frac{3}{2}}(F'=3)$ transitions. The results presented hereafter were obtained with this technique, which is easier and quicker as the atoms scatter many photons.

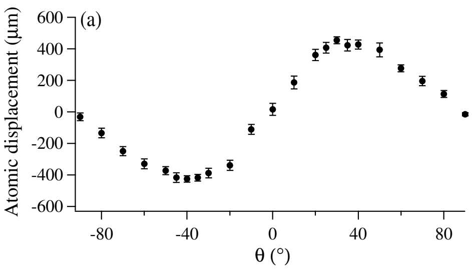
In fact, we can get more precise information than the average velocity. We can determine the whole velocity distribution using a ballistic method. For this purpose, we create a sheet of light 10 cm below the cloud of cold atoms. This sheet of light is resonant on the $5S_{\frac{1}{2}}(F=1) \rightarrow 5P_{\frac{3}{2}}(F'=2)$ transition. When the grey lattice beams and $\mathbf{B}_0$ are switched off, the rubidium atoms fall freely. The time-variation of the absorption when the atoms cross the sheet of light is directly related to the velocity distribution [11]. Such distributions are presented in [10].

IV. THE ROLE OF SPATIAL ASYMMETRY

One of the two essential features of a ratchet is the spatial asymmetry of the potential. In our system, we can easily modify its shape by changing the angle θ between the polarisations of the counterpropagating waves or changing the magnetic field.

We thus measured the displacement d of the atomic cloud between $t=0$ and $t=\tau_L$. In fact, to avoid the supplementary effect of gravity, we measured the displacements d_+ and d_- for opposite values of $\mathbf{B}_0$ and plotted the variation of $\overline{d} = (d_- - d_+)/2$. We show in Fig. 2.a the dependence of $\overline{d}$ versus θ for $\Omega_0 = -170\omega_R$, $\Delta/\Gamma = 2$, $s \simeq 0.2$ and $\tau_L = 6$ ms. As expected, $\overline{d} = 0$ when the potentials V_C and V_{NC} become symmetric, *i.e.* for $\theta = 0, \pm\pi/2$. We also find that $\overline{d}$ is an odd function of θ, in agreement with the change of asymmetry of the potentials in the transformation $\theta \rightarrow -\theta$.

We show in Fig. 2.b the variation of the displacement $\overline{d}$ as a function of the Zeeman shift Ω_0. At low Ω_0, the displacement exhibits a slow increase with Ω_0. This is because there are not so many trapped atoms when the potential wells of V_{NC} are too shallow[2] and because the rate of transitions from $|NC\rangle$ to $|C\rangle$ due to the magnetic coupling is roughly a quadratic function of Ω_0. When Ω_0 becomes too large, the perturbative expression of V_{NC} (Eq. 1) is no longer valid. A more accurate calculation shows that the potentials lose their asymmetry for high values of Ω_0. We find indeed a decrease of $\overline{d}$ when the Zeeman shift reaches a value that becomes comparable with the light-shift $\Delta s/2$.



[2]Because of the neighbouring transitions $F=1 \rightarrow F'=0$ and $F=1 \rightarrow F'=2$, the optical potential V_{NC} is not perfectly flat even when $\Omega_0 = 0$. The effect of Ω_0 is correctly described only when the Zeeman shift becomes larger than the light-shift due to the remote transitions.

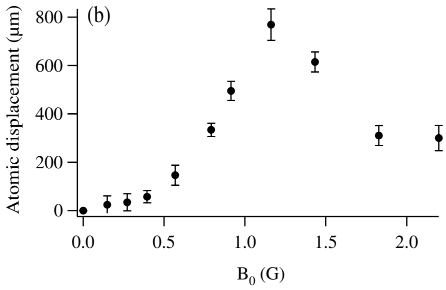


FIG. 2. (a) Atomic displacement $\overline{d} = (d_- - d_+)/2$ as a function of the angle θ between the linear polarizations of the two lattice beams, for $\Delta/\Gamma = 2$, $\Omega_0 = -170\omega_R$, $s \simeq 0.2$, $\tau_L = 6$ ms. (b) Atomic displacement $\overline{d} = (d_- - d_+)/2$ as a function of the magnetic field amplitude $\mathbf{B}_0 = \hbar\Omega_0/(g\mu_B)$, for $\Delta/\Gamma = 2$, $\theta \simeq 45°$, $s \simeq 0.15$, $\tau_L = 6$ ms. The largest displacement is obtained for $|\Omega_0| \sim \Delta s/2$.

V. THE ROLE OF DISSIPATION

In this section, we want to focus on the role of dissipation in the net motion of the atoms.

To do this, we measured the atomic displacement for different sets of laser intensity I and detuning Δ such that only the optical pumping rate varies: if we change I and Δ proportionally, then the potential curves are not affected since they scale as $\Delta s \propto I/\Delta$, while the optical pumping rate scales as I/Δ^2. The resulting curve is shown in Fig. 3: we find that the net atomic flux increases with dissipation, which is indeed expected in the oscillating regime, where the atom oscillates many times in a given potential well between two otical pumping processes. Such a behaviour is also expected in the framework of stochastic resonance: the displacement grows initially with noise [5] but decreases at large noise amplitude. The shape of the curve is in good agreement with semi-classical Monte-Carlo numerical calculations. Unfortunately, our experimental set-up does not allow us to explore the jumping regime (in this case the process can also be explained in terms of rectified forces [12]).

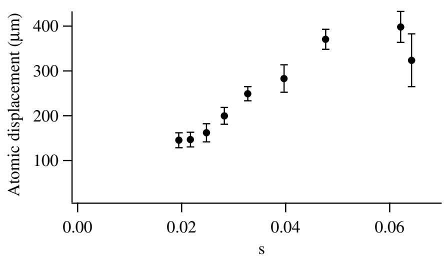


FIG. 3. Measured atomic displacement as a function of the saturation parameter s. Both the laser intensity and detuning are changed in order to keep the potentials constant while varying the optical pumping rate. The parameters are $\Omega_0 = -170\omega_R$, $\theta = 45°$, $\Delta s \simeq 150\omega_R$, $\tau_L = 6$ ms.

We also studied the role of dissipation through semi-classical numerical Monte-Carlo simulations of the atom dynamics [13], the results of which are in qualitative agreement with the experimental observations. For example, we show in Fig. 4 the time evolution of the spatial distribution function of an initially well-localized atomic sample (parameters: $\theta = 45°$, $\Delta/\Gamma = 2$, $\Omega/\Gamma = 1.8$, $\Omega_0 = 250\omega_R$). The initial sample appears in Fig. 4.a for reference, and we show in Fig. 4.b the spatial distribution of the atoms after $750\overline{\tau_C}$, where $\overline{\tau_C}$ is the atomic lifetime in $|C\rangle$ averaged over one spatial period. The directed motion and the spreading of the density are clearly visible in these data. The average velocity is $v = 1.5v_R$, which is 3 times smaller than the velocity found experimentally in the same conditions. Several explanations can be given for this discrepancy: first, it is well-known that in the case of a $F = 1 \rightarrow F' = 1$ transition the atom dynamics for small B_0 requires a full quantum treatment [7] as long as $\Omega_0 < s\Delta$ [14]. Indeed, there might be some relation between this discrepancy and the observation by Reimann *et al.* [15] that quantum tunnelling can significantly modify the flux in a similar problem. Furthermore, we neglect the neighbouring hyperfine transitions in the simulation.

If we now force the detailed balance to be verified, the situation is that of an “artificial thermal equilibrium”. This can be done easily in the simulation by replacing the actual transition rate Γ_{NC} from $|NC\rangle$ to $|C\rangle$ by one which is determined from Γ_C to obey detailed balance: $\Gamma_{NC}(z) = \Gamma_C(z) \exp\left[\frac{U_{NC}(z)-U_C(z)}{k_B T}\right]$ where T corresponds to the experimental temperature. The resulting spatial distribution is represented in Fig. 4.c: the atomic cloud spreads symmetrically and no net flux exists. In this case indeed, the transition rate Γ_{NC} is largest on the other side of the well (near B in Fig. 1), which inhibits the directed motion.

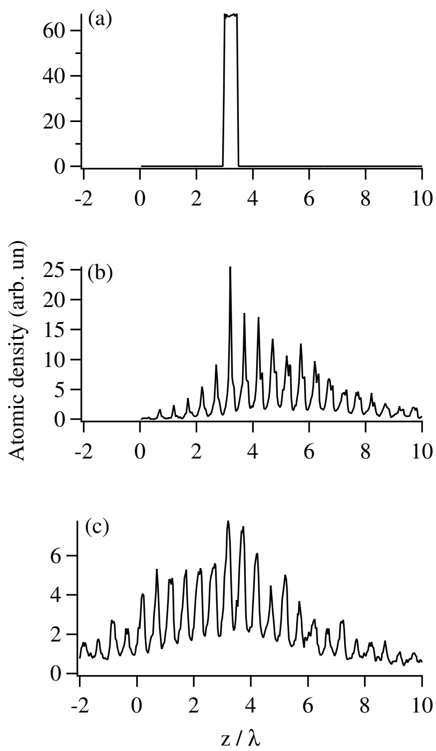


FIG. 4. Numerical results obtained by a semi-classical Monte-Carlo simulation, showing the temporal evolution of the spatial distribution of the atoms, for $\theta = 45°$, $\Delta/\Gamma = 2$, $\Omega/\Gamma = 1.8$, $\Omega_0 = 250\omega_R$. (a) Initial spatial distribution. (b) Spatial distribution after $750\overline{\tau_C}$, where $\overline{\tau_C}$ is the atomic lifetime in $|C\rangle$ averaged over one spatial period. (c) Spatial distribution after the same time of evolution, when we force the transition rates between the two states to obey detailed balance. Note the symmetry of the distribution.

VI. CONCLUSION

In this paper, we have used cold atoms in an optical lattice as a model system for a ratchet, which was originally considered in the context of biophysics. The next step to make our system even more versatile would consist of localizing the atoms in a far-off resonance optical lattice where there is virtually no optical pumping. Dissipation could then be added in a fully controlled manner, either with an additional light beam or by modulating for example the intensity of the lattice beams.

Another direction could be to pre-cool the atoms down to the quantum regime, in order to realize a quantum ratchet whose properties are known to be noticeably different from those of a classical ratchet [15].

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