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Quantum vacuum properties of the intersubband cavity polariton field

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We present a quantum description of a planar microcavity photon mode strongly coupled to a semiconductor intersubband transition in presence of a two-dimensional electron gas. We show that, in this kind of system, the vacuum Rabi frequency Ω_R can be a significant fraction of the intersubband transition frequency ω_{12} . This regime of ultra-strong light-matter coupling is enhanced for long wavelength transitions, because for a given doping density, effective mass and number of quantum wells, the ratio Ω_R/ω_{12} increases as the square root of the intersubband emission wavelength. We characterize the quantum properties of the ground state (a two-mode squeezed vacuum), which can be tuned *in-situ* by changing the value of Ω_R , e.g., through an electrostatic gate. We finally point out how the tunability of the polariton quantum vacuum can be exploited to generate correlated photon pairs out of the vacuum via quantum electrodynamics phenomena reminiscent of the dynamical Casimir effect.

In the last decade, the study of intersubband electronic transitions¹ in semiconductor quantum wells has enjoyed a considerable success, leading to remarkable opto-electronic devices such as the quantum cascade lasers^{2,3,4}. In contrast to the more conventional interband transitions between conduction and valence bands, the frequency of intersubband transitions is not determined by the energy gap of the semiconductor material system used, but rather can be chosen via the thickness of the quantum wells in the active region, providing tunable sources emitting in the mid and far infrared.

One of the most fascinating aspects of light-matter interaction is the so-called strong light-matter coupling regime, which is achieved when a cavity mode is resonant with an electronic transition of frequency ω_{12} , and the so-called vacuum Rabi frequency Ω_R exceeds the cavity mode and electronic transition linewidths. The strong coupling regime has been first observed in the late '80s using atoms in metallic cavities^{5,6}, and a few years later in solid-state systems using excitonic transitions in quantum wells embedded in semiconductor microcavities⁷. In this regime, the normal modes of the system consist of linear superpositions of electronic and photonic excitations, which, in the case of semiconductor materials, are the so-called *polaritons*. In both these systems, the vacuum Rabi frequency Ω_R does not exceed a very small fraction of the transition frequency ω_{12} .

Recently, Dini *et al.*⁸ have reported the first demonstration of strong coupling regime between a cavity photon mode and a mid-infrared intersubband transition, in agreement with earlier semiclassical theoretical predictions by Liu⁹. The dielectric Fabry-Perot structure realized by Dini *et al.*⁸ consists of a modulation doped multiple quantum well structure embedded in a microcavity, whose mirrors work thanks to the principle of total internal reflection. The strong coupling regime has been also observed in quantum well infra-red detectors¹⁰. As we will show in detail, an important advantage of using intersubband transitions is the possibility of exploring a regime where the normal-mode polariton splitting is

a significant fraction of the intersubband transition (in the pioneering experiments by Dini *et al.*⁸, $2\hbar\Omega_R = 14$ meV compared to $\hbar\omega_{12} = 140$ meV). Furthermore, recent experiments have also demonstrated the possibility of a dramatic tuning of the strong light-matter coupling through application of a gate voltage¹¹ which is able to deplete the density of the two-dimensional electron gas.

Although the quest for quantum optical squeezing effects in the emission from atoms strongly coupled to a cavity mode has been an active field of research¹², all systems realized up to now show a vacuum Rabi frequency Ω_R much smaller than the frequency of the optical transition. In this parameter regime, the relative importance of the anti-resonant terms in the light-matter coupling is small and, as far as no strong driving field is present, they can be safely neglected under the so-called rotating-wave approximation. In the presence of a strong driving field, however, anti-resonant terms are known to play a significant role, giving, e.g., the so-called Bloch-Siegert shift in magnetic resonance experiments¹³, or determining the quantum statistical properties of the emission from dressed-state lasers¹⁴.

A few theoretical studies have pointed out the intrinsic non-classical properties of exciton-polaritons in solid-state systems^{15,16,17}, but the small value of the ratio Ω_R/ω_{exc} , typically less than 0.01, has so far prevented the observation of quantum effects due to the anti-resonant terms of the light-matter coupling. All the squeezing experiments that have been performed so far in fact required the presence of a strong coherent optical pump beam in order to inject polaritons and take advantage of nonlinear polariton parametric processes^{18,19,20,21,22}.

In this paper, we show that in the case of *intersubband* cavity polaritons, it is instead possible to achieve an unprecedented *ultra-strong coupling* regime, in which the vacuum Rabi frequency Ω_R is a large fraction of the intersubband transition frequency ω_{12} . To this purpose, transitions in the far infrared are most favorable, because the ratio Ω_R/ω_{12} scales as the square root of the intersubband emission wavelength. Within a second quanti-

zation formalism, we characterize the polaritonic normal modes of the system in the weak excitation limit, in which the density of intersubband excitations is much smaller than the density of the two-dimensional electron gas in each quantum well (in this very dilute limit, the intersubband excitations behave as bosons). We point out the non-classical properties of the ground state, which consists of a two-mode squeezed vacuum. As its properties can be modulated by applying an external electrostatic bias, we suggest the possibility of observing quantum electrodynamics effects, such as the generation of correlated photon pairs from the initial vacuum state. Such an effect closely reminds the so-called dynamical Casimir effect^{23,24,25}, whose observation is still an open challenge and is actually the subject of intense effort. Many theoretical works have in fact predicted the generation of photons in an optical cavity when its properties, e.g. the length or the dielectric permittivity of the cavity spacer material, are modulated in a rapid, non-adiabatic way^{26,27,28}.

The present paper is organized as follows. In Sec. I we describe the system under examination and in Sec. II we introduce its Hamiltonian. The scaling of the coupling intensity with the material parameters is discussed in Sec. III, while Sec. IV is devoted to the diagonalization of the Hamiltonian and the discussion of the polaritonic normal modes of the system in the different regimes. The quantum ground state is characterized in Sec. V and its quantum properties are pointed out. Two possible schemes for the generation of photon pairs from the initial vacuum by modulating the properties of the ground state are suggested in Sec. VI. Conclusions are finally drawn in Sec. VII.

I. DESCRIPTION OF THE SYSTEM

In the following, we will consider a planar Fabry-Perot resonator embedding a sequence of n_{QW} identical quantum wells (see the sketch in Fig. 1). Each quantum well is assumed to be doped with a two-dimensional density N_{2DEG} of electrons, which, at low temperatures, populate the first quantum well subband. Due to the presence of the two-dimensional electron gas, it is possible to have transitions from the first to the second subband of the quantum well. We will call $\hbar\omega_{12}$ the considered intersubband transition energy. If we denote with z the growth direction of the multiple quantum well structure, then the dipole moment of the transition is aligned along z , i.e., $\mathbf{d}_{12} = d_{12}\hat{z}$. This property imposes the well known polarization selection rule for intersubband transitions in quantum wells, i.e., the electric field must have a component along the growth direction. We point out that in the case of a perfect planar structure, the in-plane wavevector is a conserved quantity, unlike the wave-vector component along the z direction. Therefore, all wave-vectors \mathbf{k} will be meant as in-plane wave-vectors, unless differently stated.

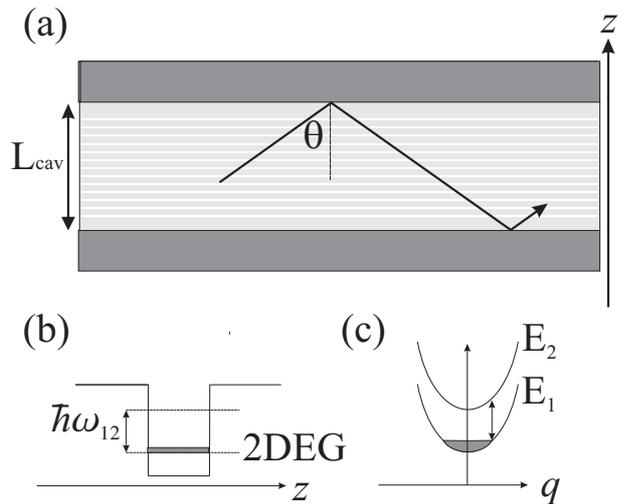


FIG. 1: (a) Sketch of the considered planar cavity geometry, whose growth direction is called z . The cavity spacer of thickness L_{cav} embeds a sequence of n_{QW} identical quantum wells. The energy of the cavity mode depends on the cavity photon propagation angle θ . (b) Each quantum well contains a two-dimensional electron gas in the lowest subband (obtained through doping or electrical injection). The transition energy between the first two subbands is $\hbar\omega_{12}$. Only the TM-polarized photon mode is coupled to the intersubband transition and a finite angle θ is mandatory to have a finite dipole coupling. (c) Sketch of the energy dispersion $E_1(q)$ and $E_2(q) = E_1(q) + \hbar\omega_{12}$ of the first two subbands as a function of the in-plane wavevector q . The dispersion of the intersubband transition is negligible as compared to the one of the cavity mode. For a typical value of the cavity photon in-plane wavevector \mathbf{k} , one has in fact $E_2(|\mathbf{k} + \mathbf{q}|) - E_1(q) \simeq \hbar\omega_{12}$.

In the following, we will consider the fundamental cavity photon mode, whose frequency dispersion is given by $\omega_{cav,k} = \frac{c}{\sqrt{\epsilon_\infty}} \sqrt{k_z^2 + k^2}$, where ϵ_∞ is the dielectric constant of the cavity spacer and k_z is the quantized photon wavevector along the growth direction, which depends on the boundary conditions imposed by the specific mirror structures. In the simplest case of metallic mirrors, $k_z = \frac{\pi}{L_{cav}}$, with L_{cav} the cavity thickness.

II. SECOND QUANTIZATION HAMILTONIAN

In this Section, we introduce the system Hamiltonian in a second quantization formalism. In the following, we will call $a_{\mathbf{k}}^\dagger$ the creation operator of the fundamental cavity photon mode with in-plane wave-vector \mathbf{k} . Note that, in order to simplify the notation, we will omit the polarization index of the photon mode, which is meant to be Transverse Magnetic (TM)-polarized (also known as p-polarization). This photon polarization is necessary to have a finite value of the electric field component along the growth direction z of the multiple quan-

tum well structure, direction along which the transition dipole of the intersubband transition is aligned. $b_{\mathbf{k}}^{\dagger}$ will be instead the creation operator of the bright intersubband excitation mode of the doped multiple quantum well structure. In the simplified case of n_{QW} identical quantum wells that are identically coupled to the cavity photon mode, the only bright intersubband excitation is the totally symmetric one, with an oscillator strength n_{QW} times larger than the one of a single quantum well. The $n_{QW} - 1$ orthogonal excitations are instead dark and will be neglected in the following. The creation operator corresponding to the bright intersubband transition can be written as

$$b_{\mathbf{k}}^{\dagger} = \frac{1}{\sqrt{n_{QW} N_{2DEG} S}} \sum_{j=1}^{n_{QW}} \sum_{|\mathbf{q}| < k_F} c_{2,\mathbf{q}+\mathbf{k}}^{(j)\dagger} c_{1,\mathbf{q}}^{(j)}, \quad (1)$$

where N_{2DEG} is the density of the two-dimensional electron gas in each quantum well and S is the sample area. The fermionic operator $c_{1,\mathbf{q}}^{(j)}$ annihilates an electron belonging to the first subband and j -th quantum well, while $c_{2,\mathbf{q}+\mathbf{k}}^{(j)\dagger}$ creates an electron in the second subband of the same well. k_F is the Fermi wavevector of the two-dimensional electron gas, whose electronic ground state at low temperature is

$$|F\rangle = \prod_{j=1}^{n_{QW}} \prod_{|\mathbf{q}| < k_F} c_{1,\mathbf{q}}^{(j)\dagger} |0\rangle_{cond}, \quad (2)$$

where $|0\rangle_{cond}$ is the empty conduction band state.

In the following, we will consider the situation of a weakly excited intersubband transition, i.e.,

$$\frac{1}{S} \sum_{\mathbf{k}} \langle b_{\mathbf{k}}^{\dagger} b_{\mathbf{k}} \rangle \ll N_{2DEG}. \quad (3)$$

In this dilute limit, the intersubband excitation field is approximately bosonic, namely

$$[b_{\mathbf{k}}, b_{\mathbf{k}'}^{\dagger}] \simeq \delta_{\mathbf{k},\mathbf{k}'}. \quad (4)$$

Starting from the coupled light-matter Hamiltonian of the semiconductor and retaining only the considered cavity photon mode for the electromagnetic field and the considered intersubband transition for the electronic polarization field, one finds a standard Hopfield-like Hamiltonian²⁹

$$H = H_0 + H_{res} + H_{anti} \quad (5)$$

which consists of three qualitatively different contributions, namely

$$H_0 = \sum_{\mathbf{k}} \hbar \omega_{cav,k} \left(a_{\mathbf{k}}^{\dagger} a_{\mathbf{k}} + \frac{1}{2} \right) + \sum_{\mathbf{k}} \hbar \omega_{12} b_{\mathbf{k}}^{\dagger} b_{\mathbf{k}}, \quad (6)$$

$$H_{res} = \hbar \sum_{\mathbf{k}} \left\{ i \Omega_{R,k} \left(a_{\mathbf{k}}^{\dagger} b_{\mathbf{k}} - a_{\mathbf{k}} b_{\mathbf{k}}^{\dagger} \right) + D_k \left(a_{\mathbf{k}}^{\dagger} a_{\mathbf{k}} + a_{\mathbf{k}} a_{\mathbf{k}}^{\dagger} \right) \right\}, \quad (7)$$

$$H_{anti} = \hbar \sum_{\mathbf{k}} \left\{ i \Omega_{R,k} \left(a_{\mathbf{k}} b_{-\mathbf{k}} - a_{\mathbf{k}}^{\dagger} b_{-\mathbf{k}}^{\dagger} \right) + D_k \left(a_{\mathbf{k}} a_{-\mathbf{k}} + a_{\mathbf{k}}^{\dagger} a_{-\mathbf{k}}^{\dagger} \right) \right\}. \quad (8)$$

H_0 in Eq. (6) describes the energy of the bare cavity photon and intersubband polarization fields, which depend on the numbers $a_{\mathbf{k}}^{\dagger} a_{\mathbf{k}}$, $b_{\mathbf{k}}^{\dagger} b_{\mathbf{k}}$ of cavity photons and intersubband excitations, respectively.

H_{res} in Eq. (7) is the resonant part of the light-matter interaction, depending on the vacuum Rabi energy $\hbar \Omega_{R,k}$ and on the related coupling constant D_k . The terms proportional to $\Omega_{R,k}$ describe the creation (annihilation) of one photon and the annihilation (creation) of an intersubband excitation with the same in-plane wavevector. In contrast, the term proportional to D_k contains only photon operators, because it originates from the squared electromagnetic vector potential part of the light-matter interaction. Note that this term in H_{res} depends on the photon number operator $a_{\mathbf{k}}^{\dagger} a_{\mathbf{k}}$ as the bare cavity photon term in Eq. (6). Hence, it gives a mere blueshift ($D_k > 0$) of the bare cavity photon energy $\hbar \omega_{cav,k}$.

Finally, H_{anti} in Eq. (8) contains the usually neglected anti-resonant terms, which correspond to the simultaneous destruction or creation of two excitations with opposite in-plane wavevectors. The terms proportional to $\Omega_{R,k}$ describe the creation (or destruction) of a cavity photon and an intersubband excitation, while the terms proportional to D_k describe the corresponding process involving a pair of cavity photons.

Before continuing our treatment, we wish to point out that the considered Hamiltonian in Eq.(5) contains only the energy associated to the fundamental cavity mode (including the zero-point energy $\sum_{\mathbf{k}} \frac{1}{2} \hbar \omega_{cav,k}$), the energy associated to the creation of intersubband excitations and the full light-matter interaction between the considered modes. The energy terms associated to the other photon modes, the electronic energy of the filled electronic bands as well as the electrostatic energy associated to an applied bias have been here omitted for simplicity, as they do not take part in the dynamics discussed in the following of the paper.

III. SCALING OF THE INTERACTION

The specific values of the coupling constants $\Omega_{R,k}$ and D_k depend on the microscopic parameters of the intersubband microcavity system.

The so-called vacuum Rabi energy $\hbar \Omega_{R,k}$ is the Rabi energy obtained with the electric field corresponding to one photon^{5,30}. For the system under consideration^{8,9}, the polariton coupling frequency for the TM-polarized mode³¹ reads

$$\Omega_{R,k} = \left(\frac{2\pi e^2}{\epsilon_{\infty} m_0 L_{cav}^{eff}} N_{2DEG} n_{QW}^{eff} f_{12} \sin^2 \theta \right)^{1/2}, \quad (9)$$

where ϵ_{∞} is the dielectric constant of the cavity, L_{cav}^{eff} the

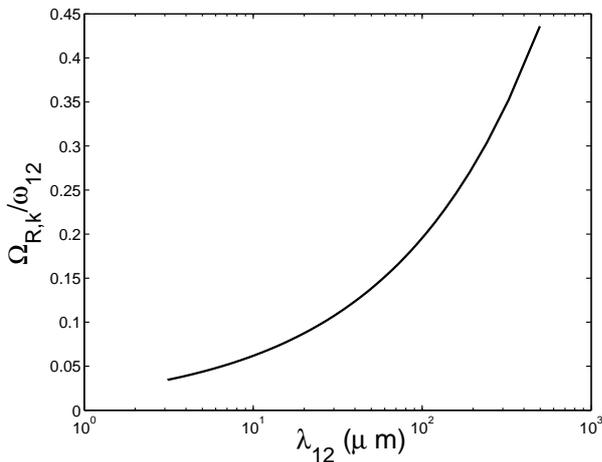


FIG. 2: Coupling ratio $\Omega_{R,k}/\omega_{12}$ as a function of the intersubband emission wavelength λ_{12} (μm). Parameters: $f_{12} = 12.9$ (GaAs quantum well), cavity spacer refraction index $\sqrt{\epsilon_{\infty}} = 3$, $n_{QW}^{\text{eff}} = 50$, $N_{2DEG} = 5 \times 10^{11} \text{ cm}^{-2}$ and $\theta = 60^\circ$. The Fabry-Perot resonator is a $\lambda/2$ -microcavity. Results obtained from the analytical expressions in Eqs. (14) and (15).

effective thickness of the cavity photon mode (which depends non-trivially on the boundary conditions imposed by the specific mirror structures), and n_{QW}^{eff} the effective number of quantum wells ($n_{QW}^{\text{eff}} = n_{QW}$ in the case of quantum wells which are identically coupled to the cavity photon field and which are located at the antinodes of the cavity mode electric field). The oscillator strength of the considered intersubband transition reads

$$f_{12} = 2m_0\omega_{12}d_{12}^2/\hbar, \quad (10)$$

where m_0 is the free electron mass and d_{12} is the electric dipole moment of the transition. Under the approximation of a parabolic energy dispersion of the quantum well subbands, the oscillator strengths of the different intersubband transitions satisfy the f -sum rule³²

$$\sum_j f_{1j} = m_0/m^*, \quad (11)$$

where m^* is the effective electron mass of the conduction band. In particular, for our case of a deep rectangular well, the sum rule is almost saturated by the first intersubband transition $f_{12} \simeq m_0/m^*$. Finally, θ is the propagation angle *inside* the cavity (which is different from the propagation angle in the substrate), and is related to the in-plane wavevector k by $k/k_z = \sin\theta/\cos\theta$.

As we will see in the next section, the relevant parameter quantifying the importance of the quantum effects considered in this paper is the dimensionless ratio $\Omega_{R,k_{\text{res}}}/\omega_{12}$, where k_{res} is the resonance in-plane wavevector such as $\hbar\omega_{\text{cav},k_{\text{res}}} = \hbar\omega_{12}$. In the system studied by Dini *et al.*⁸, this ratio is already significant, namely $\Omega_{R,k}/\omega_{12} = 0.05$. Here, we show that the ratio $\Omega_{R,k}/\omega_{12}$ can be largely increased designing structures in the far

infra-red, by increasing the number of quantum wells and by choosing semiconductors with smaller effective mass.

Let be θ_{res} the cavity propagation angle corresponding to k_{res} . From the relation

$$k_{\text{res}} = \frac{\omega_{12}}{c} \sqrt{\epsilon_{\infty}} \sin\theta_{\text{res}}, \quad (12)$$

we get that for metallic mirrors

$$L_{\text{cav}} = \frac{\lambda_{12}}{2\sqrt{\epsilon_{\infty}}\cos\theta_{\text{res}}}, \quad (13)$$

where $2\pi/\lambda_{12} = \omega_{12}/c$. Under these conditions, the light-matter coupling ratio at the resonance angle is

$$\frac{\Omega_{R,k_{\text{res}}}}{\omega_{12}} = \eta\sqrt{\lambda_{12}}, \quad (14)$$

with

$$\eta = \sqrt{\frac{e^2 f_{12} \sin^2\theta_{\text{res}} \cos\theta_{\text{res}} N_{2DEG} n_{QW}}{\pi m_0 c^2 \sqrt{\epsilon_{\infty}}}}. \quad (15)$$

Note that the prefactor given in Eq.(15) has a weak dependence on λ_{12} . In fact, in the limit case of a rectangular quantum well with high potential barriers, $f_{12} = 0.96 m_0/m^*$ and does not depend at all on λ_{12} . More refined calculations³² including the non-parabolicity of the semiconductor band and the finite depth of the potential well show that f_{12} has a moderate dependence on the emission wavelength λ_{12} (it actually increases with λ_{12}). Hence, the normalized vacuum Rabi frequency $\Omega_{R,k_{\text{res}}}/\omega_{12}$ increases at least as $\sqrt{\lambda_{12}}$. The predictions of Eqs. (14) and (15) are reported in Fig. 2 for a system of 50 GaAs quantum wells and a doping density $N_{2DEG} = 5 \times 10^{11} \text{ cm}^{-2}$. For an intersubband emission wavelength of $100\mu\text{m}$, the ratio $\Omega_{R,k}/\omega_{12}$ can be as high as 0.2. The values in Fig. 2 can be significantly increased using semiconductors with smaller effective mass, such as InGaAs/AlInAs-on-InP³³.

To complete our description, we need to provide the explicit expression for the coefficient D_k , which quantifies the effect of the squared electromagnetic vector potential in the light-matter interaction. Generalizing Hopfield's procedure²⁹ to the case of intersubband transitions, we find that all the intersubband transitions give a contribution to D_k , namely

$$D_k = \frac{\sum_j f_{1j} \Omega_{R,k}^2}{f_{12} \omega_{12}}. \quad (16)$$

However, as the oscillator strength of a deep rectangular well is concentrated in the lowest transition at ω_{12} , the effect of the higher transitions is a minor correction, namely

$$D_k \simeq 1.04 \frac{\Omega_{R,k}^2}{\omega_{12}} \approx \frac{\Omega_{R,k}^2}{\omega_{12}}. \quad (17)$$

Note that for a quantum well with a parabolic confinement potential $V(z) = (1/2)m^*\omega_{12}^2 z^2$, the expression $D_k = \Omega_{R,k}^2/\omega_{12}$ would be exact, since in this case all the intersubband oscillator strength is exactly concentrated in the lowest transition ω_{12} .

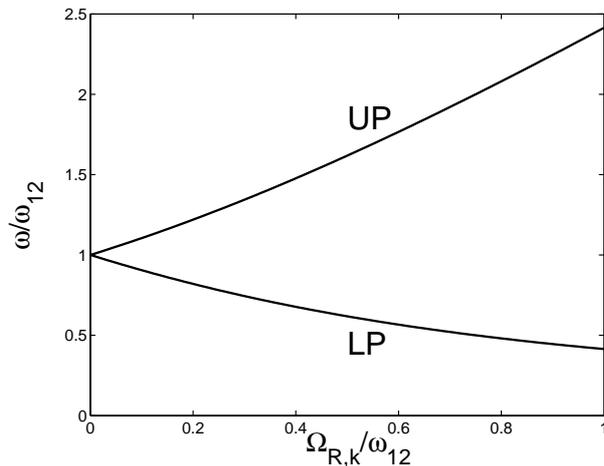


FIG. 3: Normalized polariton frequencies $\omega_{LP,k}/\omega_{12}$ and $\omega_{UP,k}/\omega_{12}$ as a function of $\Omega_{R,k}/\omega_{12}$ for $D_k = \Omega_{R,k}^2/\omega_{12}$. The calculation has been performed with $\omega_{cav,k} = \omega_{12}$. Note that for a given microcavity system, $\Omega_{R,k}/\omega_{12}$ can be tuned *in-situ* by an electrostatic bias, which is able to change the density of the two-dimensional electron gas.

IV. INTERSUBBAND POLARITONS

As all the terms in the Hamiltonian $H = H_0 + H_{res} + H_{anti}$ are bilinear in the field operators, H can be exactly diagonalized through a Bogoliubov transformation. Following the pioneering work by Hopfield²⁹, we introduce the Lower Polariton (LP) and Upper Polariton (UP) annihilation operators

$$p_{j,\mathbf{k}} = w_{j,\mathbf{k}} a_{\mathbf{k}} + x_{j,\mathbf{k}} b_{\mathbf{k}} + y_{j,\mathbf{k}} a_{-\mathbf{k}}^\dagger + z_{j,\mathbf{k}} b_{-\mathbf{k}}^\dagger, \quad (18)$$

where $j \in \{LP, UP\}$. The Hamiltonian of the system can be cast in the diagonal form

$$H = E_G + \sum_{j \in \{LP, UP\}} \sum_{\mathbf{k}} \hbar \omega_{j,\mathbf{k}} p_{j,\mathbf{k}}^\dagger p_{j,\mathbf{k}}, \quad (19)$$

where the constant term E_G will be given explicitly later. The Hamiltonian form in Eq. (19) is obtained, provided that the vectors

$$\vec{v}_{j,\mathbf{k}} = (w_{j,\mathbf{k}}, x_{j,\mathbf{k}}, y_{j,\mathbf{k}}, z_{j,\mathbf{k}})^T \quad (20)$$

satisfy the eigenvalues equation

$$M_k \vec{v}_{j,\mathbf{k}} = \omega_{j,\mathbf{k}} \vec{v}_{j,\mathbf{k}} \quad (21)$$

with $\omega_{j,\mathbf{k}} > 0$. The Bose commutation rule

$$[p_{j,\mathbf{k}}, p_{j',\mathbf{k}'}^\dagger] = \delta_{j,j'} \delta_{\mathbf{k},\mathbf{k}'} \quad (22)$$

imposes the normalization condition

$$w_{j,\mathbf{k}}^* w_{j',\mathbf{k}} + x_{j,\mathbf{k}}^* x_{j',\mathbf{k}} - y_{j,\mathbf{k}}^* y_{j',\mathbf{k}} - z_{j,\mathbf{k}}^* z_{j',\mathbf{k}} = \delta_{j,j'}. \quad (23)$$

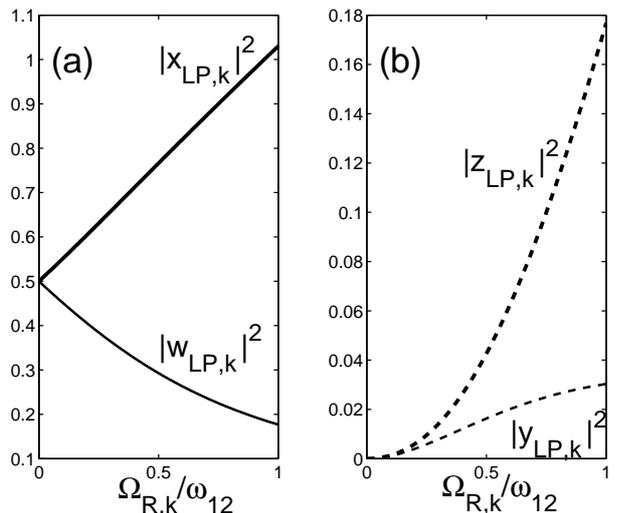


FIG. 4: Mixing fractions for the Lower Polariton (LP) mode as a function of $\Omega_{R,k}/\omega_{12}$ (see Eq. (18) in the text). The calculation has been performed for the resonant case $\omega_{cav,k} = \omega_{12}$, as in the previous figure. Panel (a): $|w_{LP,k}|^2$ (thin solid line), $|x_{LP,k}|^2$ (thick solid line). Note that for $\Omega_{R,k}/\omega_{12} \ll 1$, $|w_{LP,k}|^2 \simeq |x_{LP,k}|^2 \simeq 1/2$. Panel (b): $|y_{LP,k}|^2$ (thin dashed line), $|z_{LP,k}|^2$ (thick dashed line). For $\Omega_{R,k}/\omega_{12} \ll 1$, $|y_{LP,k}|^2 \simeq |z_{LP,k}|^2 \simeq 0$. The Upper Polariton (UP) fractions (not shown) are simply $|w_{UP,k}|^2 = |x_{LP,k}|^2$, $|x_{UP,k}|^2 = |w_{LP,k}|^2$, $|y_{UP,k}|^2 = |z_{LP,k}|^2$, $|z_{UP,k}|^2 = |y_{LP,k}|^2$.

The Hopfield-like matrix for our system reads

$$M_k = \begin{pmatrix} \omega_{cav,k} + 2D_k & -i\Omega_{R,k} & -2D_k & -i\Omega_{R,k} \\ i\Omega_{R,k} & \omega_{12} & -i\Omega_{R,k} & 0 \\ 2D_k & -i\Omega_{R,k} & -\omega_{cav,k} - 2D_k & -i\Omega_{R,k} \\ -i\Omega_{R,k} & 0 & i\Omega_{R,k} & -\omega_{12} \end{pmatrix}. \quad (24)$$

The four eigenvalues of M_k are $\{\pm\omega_{LP,k}, \pm\omega_{UP,k}\}$. Under the approximation $D_k = \Omega_{R,k}^2/\omega_{12}$ (i.e., all the oscillator strength concentrated on the ω_{12} transition), $\det M_k = (\omega_{cav,k} \omega_{12})^2$, giving the simple relation

$$\omega_{LP,k} \omega_{UP,k} = \omega_{12} \omega_{cav,k}, \quad (25)$$

i.e., the geometric mean of the energies of the two polariton branches is equal to the geometric mean of the bare intersubband and cavity mode energies. The dependence of the exact polariton eigenvalues as a function of $\Omega_{R,k}/\omega_{12}$ is reported in Fig. 3, for the resonant case $\omega_{cav,k} = \omega_{12}$.

A. Ordinary properties in the limit $\Omega_{R,k}/\omega_{12} \ll 1$

In the standard case $\Omega_{R,k}/\omega_{12} \ll 1$, the polariton operator can be approximated as

$$p_{j,\mathbf{k}} \simeq w_{j,\mathbf{k}} a_{\mathbf{k}} + x_{j,\mathbf{k}} b_{\mathbf{k}}, \quad (26)$$

with $|w_{j,k}|^2 + |x_{j,k}|^2 \simeq 1$. This means that the annihilation operator for a polariton mode with in-plane wavevector \mathbf{k} is given by a linear superposition of the photon and intersubband excitation annihilation operators with the same in-plane wavevector, while mixing with the creation operators (represented by the coefficients $y_{j,k}$ and $z_{j,k}$) is instead negligible [see Fig. 4]. In this limit, the geometric mean can be approximated by the arithmetic mean and Eq. (25) can be written in the more usual form:

$$\omega_{LP,k} + \omega_{UP,k} \simeq \omega_{cav,k} + \omega_{12} . \quad (27)$$

For the specific resonant wavevector k_{res} such that $\omega_{cav,k_{res}} = \omega_{12}$, the polariton eigenvalues are

$$\omega_{LP(UP),k_{res}} \simeq \omega_{12} \mp \Omega_{R,k_{res}} , \quad (28)$$

and the mixing fractions are $|w_{LP,k_{res}}|^2 \simeq |x_{LP,k_{res}}|^2 \simeq 1/2$.

B. Ultra-strong coupling regime

When the ratio $\Omega_{R,k}/\omega_{12}$ is not negligible compared to 1, then the anomalous features due to the anti-resonant terms of the light-matter coupling becomes truly relevant.

In the resonant $\omega_{cav,k_{res}} = \omega_{12}$ case and under the approximation $D_k = \Omega_{R,k}^2/\omega_{12}$, the polariton frequencies are given by

$$\omega_{LP(UP),k_{res}} = \sqrt{\omega_{12}^2 + (\Omega_{R,k_{res}})^2} \mp \Omega_{R,k_{res}} , \quad (29)$$

which, as it is apparent in Fig. 3, corresponds to a strongly asymmetric anti-crossing as a function of $\Omega_{R,k_{res}}/\omega_{12}$. This is due to the combined effect of the blue-shift of the cavity mode frequency due to the terms proportional to D_k in Eq. (7), and of the anomalous coupling terms in Eq.(8).

These same effects contribute to the non-trivial evolution of the Hopfield coefficients shown in Fig. 4. The anomalous Hopfield fractions $|y_{LP,k}|^2$ and $|z_{LP,k}|^2$ significantly increase because of the anomalous coupling, and eventually become of the same order as the normal ones $|x_{LP,k}|^2$ and $|w_{LP,k}|^2$. Due to the normalization condition

$$|w_{j,k}|^2 + |x_{j,k}|^2 - |y_{j,k}|^2 - |z_{j,k}|^2 = 1 , \quad (30)$$

this affects the ordinary fractions $|w_{LP,k}|^2$, $|x_{LP,k}|^2$ as well. Owing to the blue-shift of the cavity photon frequency induced by the light-matter coupling, at the resonance wavevector $k = k_{res}$ the lower polariton becomes more matter-like (i.e., $|x_{LP,k_{res}}|^2 > |w_{LP,k_{res}}|^2$ and $|z_{LP,k_{res}}|^2 > |y_{LP,k_{res}}|^2$), while the upper polariton more photon-like. In this resonant case, the UP Hopfield coefficients (not shown) are simply related to the LP ones by: $|w_{UP,k_{res}}|^2 = |x_{LP,k_{res}}|^2$, $|x_{UP,k_{res}}|^2 = |w_{LP,k_{res}}|^2$, $|y_{UP,k_{res}}|^2 = |z_{LP,k_{res}}|^2$, $|z_{UP,k_{res}}|^2 = |y_{LP,k_{res}}|^2$.

V. THE QUANTUM GROUND STATE

A. The normal vacuum state $|0\rangle$ for $\Omega_{R,k} = 0$

In the case $\Omega_R = 0$ (negligible light-matter interaction), the quantum ground state $|G\rangle$ of the considered system is the ordinary vacuum $|0\rangle$ for the cavity photon and intersubband excitation fields. Such ordinary vacuum satisfies the relation

$$a_{\mathbf{k}}|0\rangle = b_{\mathbf{k}}|0\rangle = 0 , \quad (31)$$

which means a vanishing number of photons and intersubband excitations:

$$\langle 0|a_{\mathbf{k}}^\dagger a_{\mathbf{k}}|0\rangle = \langle 0|b_{\mathbf{k}}^\dagger b_{\mathbf{k}}|0\rangle = \langle 0|a_{\mathbf{k}}^\dagger b_{\mathbf{k}}|0\rangle = 0 \quad (32)$$

and no anomalous correlations, i.e.,

$$\langle 0|a_{\mathbf{k}} a_{\mathbf{k}'}|0\rangle = \langle 0|b_{\mathbf{k}} b_{\mathbf{k}'}|0\rangle = \langle 0|a_{\mathbf{k}} b_{\mathbf{k}'}|0\rangle = 0 . \quad (33)$$

B. The squeezed vacuum state

With a finite $\Omega_{R,k}$, the ground state of the system $|G\rangle$ is no longer the ordinary vacuum $|0\rangle$ such that:

$$a_{\mathbf{k}}|0\rangle = b_{\mathbf{k}}|0\rangle = 0 , \quad (34)$$

but rather the vacuum of polariton excitations:

$$p_{j,\mathbf{k}}|G\rangle = 0 . \quad (35)$$

As the polariton annihilation operators are linear superpositions of annihilation and *creation* operators for the photon and the intersubband excitation modes, the ground state $|G\rangle$ is, in quantum optical terms, a squeezed state^{34,35}. By inverting Eq.(18), one gets

$$\begin{pmatrix} a_{\mathbf{k}} \\ b_{\mathbf{k}} \\ a_{-\mathbf{k}}^\dagger \\ b_{-\mathbf{k}}^\dagger \end{pmatrix} = \begin{pmatrix} w_{LP,k}^* & w_{UP,k}^* & -y_{LP,k} & -y_{UP,k} \\ x_{LP,k}^* & x_{UP,k}^* & -z_{LP,k} & -z_{UP,k} \\ -y_{LP,k}^* & -y_{UP,k}^* & w_{LP,k} & w_{UP,k} \\ -z_{LP,k}^* & -z_{UP,k}^* & x_{LP,k} & x_{UP,k} \end{pmatrix} \begin{pmatrix} p_{LP,\mathbf{k}} \\ p_{UP,\mathbf{k}} \\ p_{LP,-\mathbf{k}}^\dagger \\ p_{UP,-\mathbf{k}}^\dagger \end{pmatrix} , \quad (36)$$

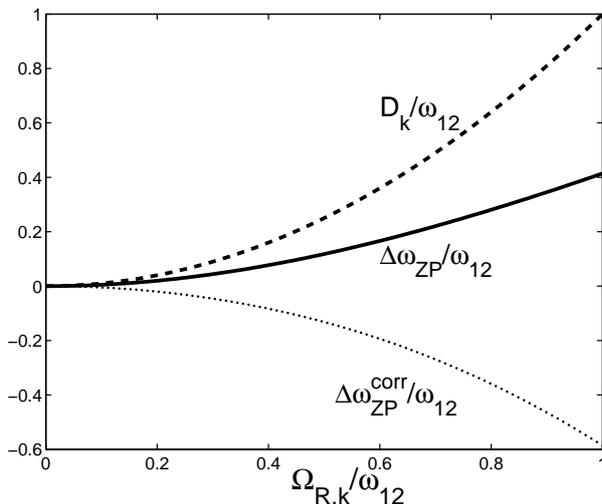


FIG. 5: Solid line: normalized differential zero-point energy (per mode) $\Delta\omega_{ZP}(k)/\omega_{12}$ as a function of $\Omega_{R,k}/\omega_{12}$. Dashed line: D_k/ω_{12} . Dotted line: normalized correlation contribution $\Delta\omega_{ZP}^{corr}(k)/\omega_{12}$. The calculation has been performed with $\omega_{cav,k} = \omega_{12}$.

from which, using Eq. (35) and the boson commutation rules, we obtain that the ground state contains a finite number (per mode) of cavity photons and intersubband excitations:

$$\langle G|a_{\mathbf{k}}^\dagger a_{\mathbf{k}}|G\rangle = |y_{LP,k}|^2 + |y_{UP,k}|^2 \quad (37)$$

$$\langle G|b_{\mathbf{k}}^\dagger b_{\mathbf{k}}|G\rangle = |z_{LP,k}|^2 + |z_{UP,k}|^2, \quad (38)$$

as well as some correlation between the photon and intersubband fields:

$$\langle G|a_{\mathbf{k}}^\dagger b_{\mathbf{k}}|G\rangle = y_{LP,k}^* z_{LP,k} + y_{UP,k}^* z_{UP,k}. \quad (39)$$

Moreover, significant anomalous correlation exist between opposite momentum components of the fields:

$$\langle G|a_{\mathbf{k}} a_{-\mathbf{k}}|G\rangle = -w_{LP,k}^* y_{LP,k} - w_{UP,k}^* y_{UP,k} \quad (40)$$

$$\langle G|b_{\mathbf{k}} b_{-\mathbf{k}}|G\rangle = -x_{LP,k}^* z_{LP,k} - x_{UP,k}^* z_{UP,k} \quad (41)$$

$$\langle G|b_{\mathbf{k}} a_{-\mathbf{k}}|G\rangle = -x_{LP,k}^* y_{LP,k} - x_{UP,k}^* y_{UP,k}. \quad (42)$$

Note that the finite photonic population which is present in the ground state $|G\rangle$ of our system is composed of "virtual" photons. In the absence of any perturbation or modulation of the parameters of the quantum Hamiltonian, these virtual photons can not escape from the cavity and therefore do not result in any observable emitted radiation (indeed, energy would not be conserved in such a process).

As it has been shown in Fig. 4, the Hopfield coefficients $x_{j,k}, y_{j,k}, w_{j,k}, z_{j,k}$ as well as the ground state $|G\rangle$ of the system strongly depend on the vacuum Rabi energy $\Omega_{R,k}$, which in our case can be dramatically modulated *in situ*, e.g. by changing the electron density N_{2DEG} via

a time-dependent external electrostatic bias. In particular, we shall discuss how this remarkable tunability of the system can be used to "unbind" the virtual photons by modulating the parameters of the system in a time-dependent way, and generate some radiation which can be actually detected outside the cavity. These issues will be the subject of sec. VI.

C. The ground state energy

Also the energy E_G of the quantum ground state has a significant dependence on the coupling $\Omega_{R,k}$. Defining E_0 as the ground state energy of the uncoupled ($\Omega_{R,k} = 0$) system, we have that:

$$E_G - E_0 = \sum_{\mathbf{k}} [\hbar D_k - \sum_{j \in \{LP, UP\}} \hbar \omega_{j,k} (|y_{j,k}|^2 + |z_{j,k}|^2)]. \quad (43)$$

Note that this energy difference includes only the contribution of the zero-point fluctuations of the intersubband polariton field and does not take into account the other contributions coming, e.g. from the change of the electrostatic energy of the system (which is imposed by an applied bias), as already discussed at the end of Sec. II. The (always positive) term D_k in Eq.(43) is the zero-point energy change due to the mere blue-shift of the bare cavity mode frequency and does not correspond to any squeezing effect. The second term is instead due to the mixing of creation and annihilation operators into the polaritonic operators as described in Eq.(18) and is proportional to the number of virtual photons and intersubband excitations present in the ground state $|G\rangle$ of the system according to Eqs.(37-38). As it is usual for a correlation contribution, it tends to lower the ground state energy. It is interesting to study the differential "zero-point" energy per mode $\hbar \Delta\omega_{ZP}(k)$, whose sum over all the \mathbf{k} -modes gives the quantum ground state energy difference $E_G - E_0$. The differential "zero-point" frequency reads

$$\Delta\omega_{ZP}(k) = D_k + \Delta\omega_{ZP}^{corr}(k), \quad (44)$$

where the (negative) correlation contribution reads

$$\Delta\omega_{ZP}^{corr}(k) = - \sum_{j \in \{LP, UP\}} \omega_{j,k} (|y_{j,k}|^2 + |z_{j,k}|^2). \quad (45)$$

These quantities (normalized to ω_{12}) are plotted in Fig. 5 as a function of $\Omega_{R,k}/\omega_{12}$ for the resonant case $\omega_{cav,k} = \omega_{12}$. Although it is the diagonal blueshift which gives the dominant contribution to the ground state energy shift, the negative contribution due to the correlation effects is important, being as large as $-0.13\hbar\omega_{12}$ already for $\Omega_{R,k}/\omega_{12} = 0.5$.

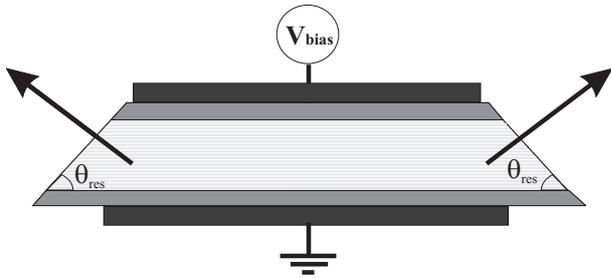


FIG. 6: Sketch of a possible set-up for the generation of correlated photon pairs in the intersubband cavity system. The vacuum Rabi frequency of the intersubband cavity system can be modulated through an electric gate, which changes the density of the two-dimensional electron gas or alternatively the dipole moment of the intersubband transition. A modulation of the bias is expected to induce the emission of correlated photon pairs with opposite in-plane wavevectors. This kind of radiation can be optimally guided out of the cavity through wedged lateral facets, with inclination equal to the resonance angle θ_{res} .

VI. TUNING THE QUANTUM VACUUM: QUANTUM RADIATION EFFECTS

The possibility of tuning in a dramatic way the properties (energy and squeezing) of the ground state of the system, as well as the significant number of (virtual) excitations already present in the ground state suggests that the present system could be a potential laboratory to study Quantum Electro-Dynamics (QED) phenomena, which are reminiscent of the dynamic Casimir effects^{23,24,25}. In particular, we shall discuss how a time-modulation of the ground-state properties of the system can parametrically produce *real* excitations above the ground state of our cavity, which then escape from the cavity as photons and propagate in the external free-space.

In the typical arrangement for the observation of the dynamical Casimir effect, one has to modulate in time the properties of an optical cavity and, in particular, its resonance frequencies. Several proposals have appeared in order to do this: in the simplest ones, one has to periodically move the mirrors so as to modify the boundary conditions of the field^{25,26}. Other proposals²⁷ deal with a time-dependence of the refractive index of a dielectric medium placed inside the cavity. A recent work proposes to vary the effective length of the cavity by changing the reflectivity of a composite mirror²⁸.

The main peculiarity of our system as compared to previous proposals is due to the possibility of modulating the properties of the ground state in a much stronger way, due to the ultra-strong and tunable light-matter coupling.

In the next subsection, we shall give a detailed analysis of a simple *gedanken* experiment, where the vacuum Rabi frequency is assumed to be switched off in an in-

stantaneous way. This scheme has the merit of allowing to grasp the essential physics of the problem, providing quantitative estimates without the need of embarking in complicate calculations.

A complete and quantitative calculation of the spectral shape and intensity of the emitted radiation for the most relevant case of a periodic modulation of $\Omega_{R,k}$ is beyond the scope of the present paper, as it would require a careful analysis of the coupling of the cavity system to the extra-cavity field as well as of the other non-radiative loss mechanisms of the electronic system³⁶. This is work actually in progress and here we shall restrict ourselves to a very qualitative discussion of its main features.

A. Abrupt switch off of the vacuum Rabi energy

Let us suppose that the considered intersubband cavity system is in the ground state $|G\rangle$. As we have already discussed, the squeezed vacuum $|G\rangle$ contains a finite number of cavity photons and intersubband excitations because of the correlations due to the anomalous coupling terms in Eq.(8).

If one switches off the vacuum Rabi frequency $\Omega_{R,k}$ of the system in an abrupt, non-adiabatic way by suddenly depleting the electron gas, the photon mode does not have the time to respond to the perturbation and will remain in the same squeezed vacuum state as before. As this state is now an excited state of the Hamiltonian for $\Omega_{R,k} = 0$, the system will relax towards its ground state, which now corresponds to the standard vacuum, by emitting the extra photons as propagating radiation.

One possible way to collect this *quantum vacuum radiation* is through the set-up sketched in Fig. 6, which allows one to collect the photons which are emitted with internal propagation angle θ around the resonance value θ_{res} . If one neglects the losses due to the background absorption by the dielectric material forming the micro-cavity, an estimate of the number of emitted photons can be obtained as follows. The number of photon states (per unit area) in the 2D momentum volume $d^2\mathbf{k}$ is simply $d^2\mathbf{k}/(2\pi)^2$. Hence, the differential density of photons (per unit area) in the 2D momentum volume $d^2\mathbf{k}$ is

$$d\rho_{phot} = \frac{d^2\mathbf{k}}{(2\pi)^2} \langle G | a_{\mathbf{k}}^\dagger a_{\mathbf{k}} | G \rangle, \quad (46)$$

where the photon number $\langle G | a_{\mathbf{k}}^\dagger a_{\mathbf{k}} | G \rangle$ in the quantum ground state is given by Eq. (37). Now, all the expectation values depend only on $|\mathbf{k}|$ and hence we can rewrite the momentum volume as $d^2\mathbf{k} = 2\pi k dk$. Knowing that the in-plane wavevector k is given by the relationship $k = k_z \tan(\theta)$ and using Eq. (12), we find the final result

$$\frac{d\rho_{phot}}{d\theta}(\theta_{res}) = \frac{1}{2\pi} \frac{\omega_{12}^2}{c^2} \epsilon_\infty \tan(\theta_{res}) \langle G | a_{\mathbf{k}}^\dagger a_{\mathbf{k}} | G \rangle. \quad (47)$$

To give a numerical application of Eq. (47), let us consider an intersubband cavity system with $\hbar\omega_{12} = 140$

meV, resonance angle $\theta_{\text{res}} = 65^\circ$ and $\hbar\Omega_{R,k_{\text{res}}} = 7$ meV (these are approximately the values in the sample measured by Dini *et al*⁸). For these parameters, Eq. (47) gives the differential photon density $d\rho_{\text{phot}}/d\theta \simeq 1 \times 10^5$ $\text{cm}^{-2} \text{rad}^{-1}$.

Note that the emission corresponding to the \mathbf{k} -mode is correlated to the emission corresponding to the mode with opposite in-plane wavevector, as shown in Eq. (40). Indeed, the "quantum vacuum radiation" here described consists in the emission of correlated photon pairs³⁷.

B. Periodic modulation of $\Omega_{R,k}$

The requirement of a abrupt, non-adiabatic, switch-off of the Rabi coupling $\Omega_{R,k}$ imposes very stringent limits on the time-scale τ_{sw} over which the electrostatic bias has to be applied. In particular, we expect that in order to maximize the quantum vacuum radiation generation, τ_{sw} can not be too much longer than the oscillation period of the lower polaritonic mode.

It is then perhaps more accessible from an experimental point of view to try to detect the vacuum radiation by periodically modulating the vacuum Rabi frequency at an angular frequency ω_{mod}

$$\Omega_{R,k}(t) = \bar{\Omega}_{R,k} + \Delta\Omega_{R,k} \sin(\omega_{mod}t). \quad (48)$$

Note that in principle this kind of modulation can be obtained not only through a gate-induced depletion of the two-dimensional electron gas¹¹, but also by modulating the dipole moment of the intersubband transition or alternatively the reflectivity of the mirrors. As all the relevant physical quantities in the present problem (polariton energies, Hopfield coefficients, ground state energy) depend in a nonlinear way on the vacuum Rabi frequency $\hbar\Omega_{R,k}$, we expect that for large modulation amplitudes high order harmonics of the fundamental modulation frequency ω_{mod} will play a significant role in the parametric process which is responsible for the vacuum radiation generation²⁵. In particular, emission will be enhanced if

$$\omega_{j,\mathbf{k}} + \omega_{j',-\mathbf{k}} = r \omega_{mod}, \quad (49)$$

with r being a generic positive integer number, and $j, j' = \{LP, UP\}$. This is the phase-matching condition for the parametric generation of two polaritons with opposite momentum. As usual, the narrower the polaritonic resonance, the stronger the resonant enhancement.

As it is generally the case for parametric processes in a cavity, the number of photons which are generated in the cavity and then emitted as radiation is determined by a dynamical equilibrium between the parametric processes generating them and the losses, the radiative as well as

the non-radiative ones³⁶. For a complete and quantitative treatment of these issues, further investigations are in progress.

VII. CONCLUSIONS

In conclusion, we have shown that in the intersubband cavity polariton system, a new regime of ultra-strong coupling can be achieved, where the vacuum Rabi frequency Ω_R is a large fraction of the intersubband transition frequency ω_{12} . This scenario appears to be easier to achieve in the far infrared, since the ratio Ω_R/ω_{12} scales as the square root of the intersubband transition wave-length. In the ultra-strong coupling regime, the usually neglected anti-resonant terms of the light-matter coupling start playing an important role. In particular, the ground state of system is no longer the ordinary vacuum of photons and electronic excitations, but rather a two-mode squeezed vacuum, whose properties strongly depend on the ratio Ω_R/ω_{12} . As this quantity can be dramatically tuned by applying an electrostatic bias, we have pointed out the possibility of observing interesting quantum electrodynamic effects reminiscent of the dynamical Casimir effect, i.e. the generation of correlated photon pairs out of the initial polariton vacuum state. A quantitative estimate of the number of emitted photons has been given for the simplest case of an instantaneous switch-off of the light-matter coupling, and the results look promising in view of experimental observations. Work is actually in progress in the direction of extending the analysis to the case of a periodic modulation of $\Omega_{R,k}$, case in which one should be able to enhance the emitted intensity via parametric resonance effects. From the theoretical point of view, this study requires a complete treatment of losses in order to describe the dynamical equilibrium between the parametric process generating the quantum radiation and the dissipation.

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¹ *Intersubband Transitions in Quantum Wells: Physics and Device Applications I*, edited by H. C. Liu and F. Capasso,

- Semiconductors and Semimetals Vol. 62 (Academic Press, San Diego, 2000).
- ² J. Faist, F. Capasso, D.L. Sivco, C. Sirtori, A. L. Hutchinson, A.Y. Cho, *Science* **264**, 5553 (1994).
 - ³ R. Köhler, A. Tredicucci, F. Beltram, H.E. Beere, E.H. Linfield, A.G. Davies, D.A. Ritchie, R.C. Iotti, F. Rossi, *Nature* **417**, 156 (2002).
 - ⁴ R. Colombelli, K. Srinivasan, M. Troccoli, O. Painter, C. F. Gmachl, D. M. Tennant, A. M. Sergent, D. L. Sivco, A. Y. Cho, F. Capasso, *Science* **302**, 1374 (2003).
 - ⁵ G. Rempe, H. Walther, and N. Klein, *Phys. Rev. Lett.* **58**, 353 (1987).
 - ⁶ For a recent review of cavity quantum electrodynamics for atoms, see J. M. Raimond, M. Brune, and S. Haroche, *Rev. Mod. Phys.* **73**, 565 (2001).
 - ⁷ C. Weisbuch, M. Nishioka, A. Ishikawa, Y. Arakawa, *Phys. Rev. Lett.* **69**, 3314 (1992).
 - ⁸ D. Dini, R. Kohler, A. Tredicucci, G. Biasol, and L. Sorba, *Phys. Rev. Lett.* **90**, 116401 (2003).
 - ⁹ A. Liu, *Phys. Rev. B* **55**, 7101 (1997) and references therein.
 - ¹⁰ E. Dupont, H. C. Liu, A. J. SpringThorpe, W. Lai, and M. Extavour, *Phys. Rev. B* **68**, 245320 (2003).
 - ¹¹ A. Tredicucci, private communication; A. A. Anappara, A. Tredicucci, F. Beltram, G. Biasiol, L. Sorba, submitted to *Appl. Phys. Lett.*
 - ¹² *Cavity quantum electrodynamics*, edited by P. R. Berman (Academic Press, Boston, 1994)
 - ¹³ F. Bloch and A. Siegert, *Phys. Rev.* **57**, 522 (1940)
 - ¹⁴ J. Zakrzewski, M. Lewenstein, and T. W. Mossberg, *Phys. Rev. A* **44**, 7717 (1991); *ibidem* **44**, 7732 (1991); *ibidem* **44**, 7746 (1991).
 - ¹⁵ M. Artoni and J. L. Birman, *Phys. Rev. B* **44**, 3736 (1991).
 - ¹⁶ P. Schwendimann and A. Quattropani, *Europhys. Lett.* **17**, 355 (1992).
 - ¹⁷ Z. Hradil, A. Quattropani, V. Savona, and P. Schwendimann, *Journ. of Stat. Phys.* **76**, 299 (1994).
 - ¹⁸ C. Ciuti, P. Schwendimann, A. Quattropani, *Phys. Rev. B* **63**, 041303(R)(2001).
 - ¹⁹ P. Schwendimann, C. Ciuti, A. Quattropani, *Phys. Rev. B* **68**, 165324 (2003).
 - ²⁰ J. Ph. Karr, A. Baas, R. Houdré, and E. Giacobino, *Phys. Rev. A* **69**, 031802 (R) (2004).
 - ²¹ J. Ph. Karr, A. Baas, and E. Giacobino, *Phys. Rev. A* **69**, 063807 (2004).
 - ²² C. Ciuti, *Phys. Rev. B* **69**, 245304 (2004).
 - ²³ G. T. Moore, *J. Math. Phys. (N.Y.)* **11**, 2679 (1970); S. A. Fulling and P. C. W. Davies, *Proc. R. Soc. London A* **348**, 393 (1976).
 - ²⁴ M. Kardar and R. Golestanian, *Rev. Mod. Phys.* **71**, 1233 (1999).
 - ²⁵ A. Lambrecht, *J. Opt. B: Quantum Semiclass. Opt.* **7**, S3 (2005).
 - ²⁶ A. Lambrecht, M. T. Jaekel, and S. Reynaud, *Phys. Rev. Lett.* **77**, 615 (1996).
 - ²⁷ C. K. Law, *Phys. Rev. A* **49**, 433 (1994); M. Artoni, A. Bulatov, and J. Birman, *Phys. Rev. A* **53**, 1031 (1996); M. Uhlmann, G. Plunien, R. Schützhold, and G. Soff, *Phys. Rev. Lett.* **93**, 193601 (2004); M. Crocce, D. A. R. Dalvit, F. C. Lombardo, and F. D. Mazzitelli, *Phys. Rev. A* **70**, 033811 (2004).
 - ²⁸ C. Braggio, G. Bressi, G. Carugno, C. Del Noce, G. Galeazzi, A. Lombardi, A. Palmieri, G. Ruoso, and D. Zanello, preprint quant-ph/0411085.
 - ²⁹ J. J. Hopfield, *Phys. Rev.* **112**, 1555 (1958).
 - ³⁰ C. Cohen-Tannoudji, J. Dupont-Roc, and G. Grynberg, *Atom-Photon Interactions. Basic Processes and Applications.*, (Wiley Science Paperback Series, New York, 1998).
 - ³¹ L. C. Andreani, *Exciton-polaritons in confined systems*, in *Proceedings of the International School of Physics "Enrico Fermi"*, Course CL, edited by B. Deveaud, A. Quattropani, and P. Schwendimann, Vol. 150 (IOS Press, Amsterdam, 2003) pp. 105-145. In this recent review, a comprehensive discussion of the (polarization-dependent) polariton radiative coupling in confined systems can be found.
 - ³² C. Sirtori, F. Capasso, J. Faist, S. Scandolo, *Phys. Rev. B* **50**, 8663 (1994).
 - ³³ R. Colombelli, C. Ciuti, Y. Chassagneux, C. Sirtori, unpublished.
 - ³⁴ R. E. Slusher, L. W. Hollberg, B. Yurke, J. C. Mertz, and J. F. Valley *Phys. Rev. Lett.* **55**, 2409 (1985)
 - ³⁵ D. F. Walls and G. J. Milburn, *Quantum Optics* (Springer-Verlag, Berlin, 1994).
 - ³⁶ R. Ferreira and G. Bastard, *Phys. Rev. B* **40**, 1074 (1989).
 - ³⁷ For a recent and comprehensive analysis of the criteria concerning photonic quantum correlation, see N. Treps and C. Fabre, *Laser Physics* **15**, 187 (2005).