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Patrice Ossona de Mendez

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## 3-colorability and contacts of segments <sup>★</sup>

P. Ossona de Mendez

CNRS UMR 0017, EHESS, 54 Boulevard Raspail, 75006, Paris, France.

**Abstract.** Using a result on planar graph representation [1], we shall prove that vertex 3-colorability is NP-complete for simple contact graphs of segments in 4 directions.

Let  $V$  be a finite set of segments in the plane. Associate a graph  $G(V)$  called the *intersection graph* of  $V$  as follows :  $V$  is the vertex set of  $G$  and there is an edge connecting  $v_1$  and  $v_2$  if and only if the segments  $v_1$  and  $v_2$  intersect.

G. Ehrlich, S. Even and R. Tarjano [3] proved that vertex 3-colorability of intersection graphs of segments is NP-complete.

It has been conjectured that any planar graph is an intersection graph of segments and this has been proved for 3-colorable graphs [2], what gives another proof of the previous theorem.

In case where the segments in  $V$  never cross, but may only touch, the intersection graph of  $V$  is a *contact graph of segments*.

Contact graphs of segments are precisely those graphs  $G$  that satisfy :

- $G$  is planar
- Any subgraph  $H$  of  $G$ , induced by  $n_H \geq 2$  vertices, has a number of edges at most equal to  $2.n_H - 3$

This class includes the planar graphs that are triangle free (3-colorable, according to Grötsch theorem), the outerplanar graphs (3 colorable, as proved by König), 3-regular graphs different from  $K_4$  (3 colorable, according to Brooks' theorem in the cubic case) but also 4 chromatic graphs.

If the segments of  $V$  belongs to  $k$  directions,  $G(V)$  is a  *$k$ -dir contact graph of segments*. Obviously, a 3-dir contact graph of segments is 3-colorable.

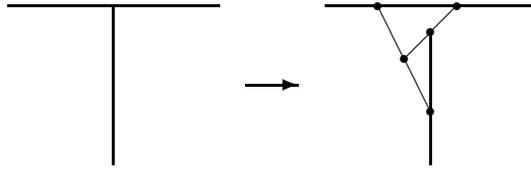
One may ask if the problem of finding the chromatic number is still NP-complete for contact graphs of segments, or even for 4-dir contact graphs of segments.

**Theorem 1.** *Vertex 3-colorability is NP-complete for 4-dir contact graphs of segments.*

*Proof.* As notice in [1], any planar graph  $G$  is the contact graph of T-shape. Moreover, the vertex 3-colorability of  $G$  is obviously equivalent to the vertex 3-colorability of the 4-dir contact graph of segments  $G'$  obtained by transforming each T-shape as shown in the figure.  $\square$

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**Fig. 1.** Transformation of a T-shape contact graph into a 4-dir contact graph of segments, while preserving vertex 3-colorability

## References

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