Parameters Screening of TEAM Workshop Problem 25 by the Application of Experimental Design Method

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***Abstract*-- This paper presents the application of Experimental Design Method as a preliminary step for the optimization process of electromagnetic devices. This method allows us to identify the significant parameters of an optimization problem and reduce the number of evaluations of the objective function. To show the efficiency of this method, we apply it in the parameters screening of the Problem 25 of TEAM Workshop, whose solution is also presented in this paper.**

***Index Terms*-- Optimization, Experimental Design Method, Response Surface, Genetic Algorithms.**

I. Introduction

The application of stochastic methods in the optimization of electromagnetic devices analysed by Finite Element Method can require a high computational time, due to the high number of evaluations of the objective function. A solution for this problem consists in replacing the objective function by an approximation with fast evaluation, which can decrease considerably the number of analysis by finite element [1][2].

This approximation represents a response surface of the objective function, and the number of fem analysis used to obtain a satisfactory approximation depends on the number of parameters to be optimized. This means that a reduction in the number of parameters can give us a faster solution of our optimization problem. At the same time, we must optimize all parameters that have a significant contribution in the value of the objective function.

The application of Experimental Design Method (EDM) allows the identification of these parameters, as well as its more significant interactions using a reduced number of evaluations of the objective function [3].

II. Experimental Design Method

The Experimental Design Method is an analysis tool that allows us to quantify the contribution of each parameter in the objective function value. The method consists in to perform some evaluations of the objective function, modifying the value of each parameter in a reasonable way [4]. This set of evaluations represents a full factorial design, whose number of experiences to be performed is given by:

$$N_{exp} = n^k \qquad (1)$$

where
k = number of parameters
n = number of levels used for each parameter

We can see that the number of experiences becomes prohibitive when the number of parameters increases. To avoid this problem, we use a "fraction" of the full design. This type of design is known as fractional or incomplete factorial design [4].

The use of fractional factorial designs reduces the number of experiences, but at the same time it gives a great problem: the presence of confusions between the main effects of the parameters and their interactions. This means that if we consider, for example, a four-parameter problem and we use only eight experiences in the EDM analysis, we will obtain the following confusions presented in Table I.

Table I – List of confusions of a fractional factorial design

Contrast	Confusions	Contribution (%)
A	$p_1 + p_2p_3p_4$	C_A
B	$p_2 + p_1p_3p_4$	C_B
C	$p_3 + p_1p_2p_4$	C_C
D	$p_4 + p_1p_2p_3$	C_D
E	$p_1p_2 + p_3p_4$	C_E
F	$p_1p_3 + p_2p_4$	C_F
G	$p_2p_3 + p_1p_4$	C_G
H	$p_1p_2p_3p_4$	C_H

In this situation, we can not identify if the contribution C_A of the contrast (a set of confusions) *A* is due to the parameter p_1, the interaction $p_2p_3p_4$ or both.

To avoid the influence of these confusions in the analysis of EDM, we must consider the following hypotheses [4]:

a)- High order interactions (more than two) between parameters can be considered negligible.

b)- Once a contrast is negligible, all the effects that compose this contrast can be considered negligible too.

c)- Two significant factors (parameters) can also result in a significant interaction. However, two non-significant factors don't give significant interactions.

Manuscript received May 1, 2001.
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Based on these hypotheses, we will present the results obtained from the application of EDM in the analysis of the Problem 25 of TEAM Workshop.

III. Analysis of the Problem 25 of TEAM Workshop

A. Description and Parameters

The goal of this problem is to optimize the shape of a die mold used in the production of permanent magnets. The die mold is described by an internal circle of radius R_1 and by an external ellipse represented by L_2, L_3 and L_4. We must find the values of R_1, L_2, L_3 and L_4 to obtain a constant radial magnetic induction on ten different points defined on the arc *ef*, as shown in Fig. 1.

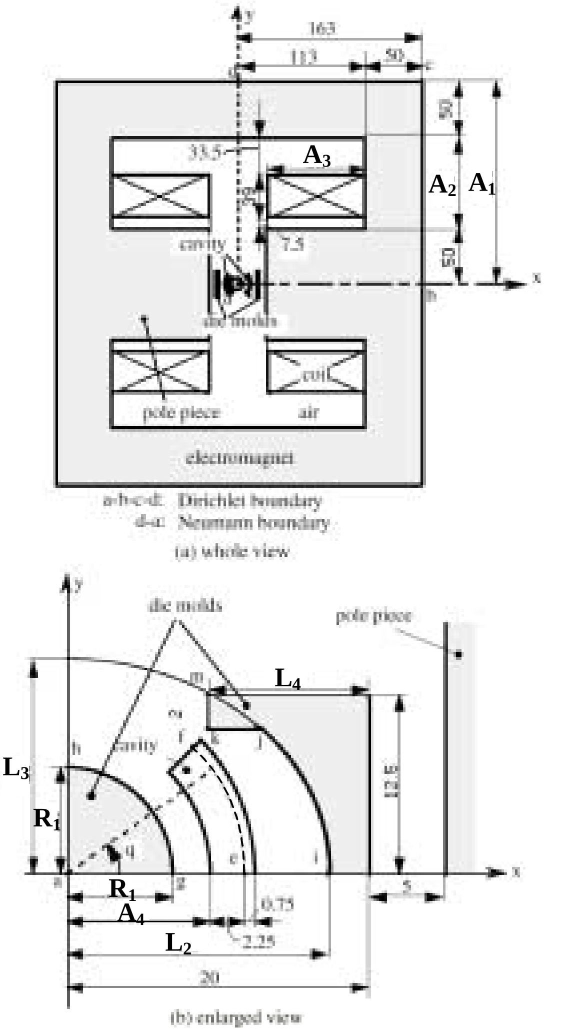

Fig. 1- Problem 25 of TEAM Workshop

To show the efficiency of the Experimental Design Method in the identification of significant parameters, we added 4 new parameters (A_1, A_2, A_3, and A_4) to the original problem. We can identify these parameters in Fig. 1, and their respective constraints in Table II.

We know that these additional parameters don't have a significant influence in the value of the objective function and we will try to verify this consideration using the EDM.

TABLE II – Defined parameters and their constraints

Parameter	Minimum Value	Maximum Value
R_1	5.0	9.4
L_2	12.6	18.0
L_3	14.0	45.0
L_4	4.0	19.0
A_1	170.0	190.0
A_2	70.0	90.0
A_3	86.0	88.0
A_4	9.5	11.0

B. Objective Function

The objective function that describes this problem is given by the following equation:

$$F = \sum_{i=1}^{10} \left\{ \left(B_{xp_i} - B_{xo_i}\right)^2 + \left(B_{yp_i} - B_{yo_i}\right)^2 \right\}, \qquad (2)$$

where:

$B_{xo} = 0.35\cos\theta$

$B_{yo} = 0.35\sin\theta$

C. Identification of Significant Parameters

In the first stage of our analysis, we will use the experimental design method to identify the parameters that have a significant influence on the value of the objective function.

1) Identification using a Fractional Factorial Design

As described in equation (1), the application of a two-level full factorial design in the parameters screening of this problem needs 256 (2^8) evaluations of the objective function.

To reduce the number of evaluations, we will use a fractional factorial design based on the Taguchi's L_{16} table [4], which requires only 16 experiences to identify the influence of each parameter in the value of the objective function.

The values of the parameters used in each experience, as well as the values of the objective function for each configuration, are shown in Table III.

TABLE III – Experiences of fractional factorial design

R_1	L_2	L_3	L_4	A_1	A_2	A_3	A_4	F_{obj}
5.0	12.6	14.0	4.0	170.0	70.0	86.0	9.5	0.0522
5.0	12.6	14.0	19.0	170.0	90.0	90.0	11.0	0.0710
5.0	12.6	45.0	4.0	190.0	90.0	90.0	9.5	0.2288
5.0	12.6	45.0	19.0	190.0	70.0	86.0	11.0	0.1712
5.0	18.0	14.0	4.0	190.0	90.0	86.0	11.0	0.1273
5.0	18.0	14.0	19.0	190.0	70.0	90.0	9.5	0.1756
5.0	18.0	45.0	4.0	170.0	70.0	90.0	11.0	0.1665
5.0	18.0	45.0	19.0	170.0	90.0	86.0	9.5	0.2646
9.4	12.6	14.0	4.0	190.0	70.0	90.0	11.0	1.2066
9.4	12.6	14.0	19.0	190.0	90.0	86.0	9.5	0.3466
9.4	12.6	45.0	4.0	170.0	90.0	86.0	11.0	1.1023
9.4	12.6	45.0	19.0	170.0	70.0	90.0	9.5	0.5924
9.4	18.0	14.0	4.0	170.0	90.0	90.0	9.5	0.1230
9.4	18.0	14.0	19.0	170.0	70.0	86.0	11.0	0.0040
9.4	18.0	45.0	4.0	190.0	70.0	86.0	9.5	0.0537
9.4	18.0	45.0	19.0	190.0	90.0	90.0	11.0	0.0188

The contributions of the significant contrasts are presented in Table IV. All the other contrasts had a contribution smaller than 5%, so they can be considered negligible.

TABLE IV – CONTRIBUTIONS OBTAINED FROM THE APPLICATION OF A FRACTIONAL FACTORIAL DESIGN

Contrast	Confusions	Contribution (%)
A	$R_1 + L_2L_3A_1 + L_3L_4A_3$ + (*)	14.91
B	$L_2 + R_1L_3A_1 + L_3L_4A_2$ + (*)	25.02
C	$L_4 + L_2L_3A_2 + R_1L_3A_3$ + (*)	6.23
D	$R_1L_2 + L_3A_1 + L_4A_4 + A_2A_3$ + (*)	33.01
E	$R_1L_4 + L_3A_3 + L_2A_4 + A_1A_2$ + (*)	8.27
F	$L_2L_4 + L_3A_2 + R_1A_4 + A_1A_3$ + (*)	6.10

(*) some high-order interactions

Once high-order interactions can be usually considered negligible and only interactions of significant parameters are also significant, we can conclude that the parameters R_1, L_2, L_4 and the interactions R_1L_2, R_1L_4 and L_2L_4 are the only significant factors for the objective function value.

So, we can consider only R_1, L_2 and L_4 as the parameters of our optimisation problem, which represents a reduction of more than 50% in the number of parameters.

2) Validation using a Full Factorial Design

To validate these conclusions, we will use the full factorial design in the analysis of the original problem, which has only 4 parameters (R_1, L_2, L_3 and L_4) to optimize. This step is just executed to verify if L_3 is really negligible and it is not normally used in the standard process of parameters screening.

The full factorial analysis of this problem gives us results without confusions between the parameters and their interactions using only 16 experiments, whose values are shown in Table V.

TABLE V – EXPERIMENTS OF FULL FACTORIAL DESIGN

R_1	L_2	L_3	L_4	F_{obj}
5.0	12.6	14.0	4.0	0.0543
5.0	12.6	14.0	19.0	0.0767
5.0	12.6	45.0	4.0	0.2197
5.0	12.6	45.0	19.0	0.1660
5.0	18.0	14.0	4.0	0.1187
5.0	18.0	14.0	19.0	0.1905
5.0	18.0	45.0	4.0	0.1737
5.0	18.0	45.0	19.0	0.2475
9.4	12.6	14.0	4.0	1.1039
9.4	12.6	14.0	19.0	0.4071
9.4	12.6	45.0	4.0	1.2488
9.4	12.6	45.0	19.0	0.5248
9.4	18.0	14.0	4.0	0.1068
9.4	18.0	14.0	19.0	0.0063
9.4	18.0	45.0	4.0	0.0622
9.4	18.0	45.0	19.0	0.0404

Table VI presents the parameters and the interactions whose contribution in the value of the objective function is higher than 5%. The parameter L_3 does not appear is this table, so it can be considered negligible, which verifies the conclusions obtained by the fractional factorial analysis and the hypotheses presented in section I.

TABLE VI – CONTRIBUTIONS OBTAINED FROM THE APPLICATION OF THE FULL FACTORIAL DESIGN

Factor or Interaction	Contribution (%)
R_1	15.13
L_2	25.01
L_4	6.34
R_1L_2	32.99
R_1L_4	8.50
L_2L_4	6.43

D. Optimisation of Significant Parameters

After identifying the parameters that have a significant influence in the value of the objective function, we will optimize their values using a new approach based on the construction of a response surface by the Diffuse Element Method [1][2].

The Diffuse Element Method is an approximation technique whose standard process contains the following steps:

a)- First of all, we create a n-dimension domain where n is the number of parameters to optimize and whose limits are 0.0 and 1.0 in each direction. Value 0.0 represents the minimal value of each parameter p_j, while value 1.0 represents its maximum value.

b)- After the creation of this domain, we carry out a discretization in each direction to obtain the points where we will evaluate the objective function. These points are the centers of each diffuse element represented by a "bubble" of radius r chosen in order to fill the domain, as we can see in Fig. 2.

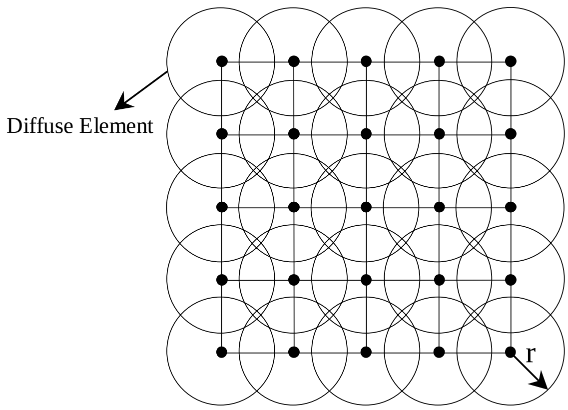

Fig. 2 - Standard process of Diffuse Element Method applied to a two-parameter function

c)- Once the diffuse elements have been created, we evaluate the objective function on the points given by their centers. We also determine the connectivity between each element and its neighbours and calculate the values of the shape functions N_i [2]. Using these values, we can create a matrix system, whose solution gives us the approximation coefficients [1][2].

We built the response surface of the objective function for the significant parameters R_1, L_2 and L_4, by using first order diffuse elements distributed on a regular grid discretized in 7 points by direction of the domain, which required 343 (7^3) objective function evaluations.

Fig. 3 presents the response surface obtained for the maximum value of the parameter L_4 (19.0 mm).

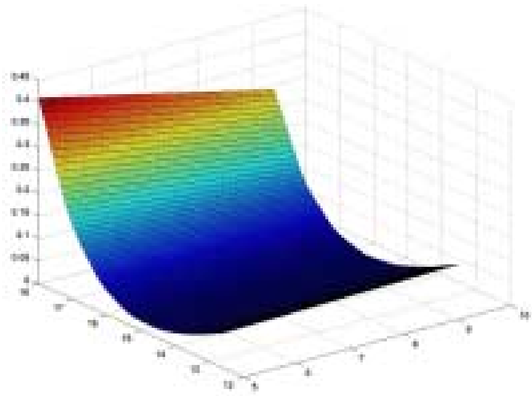

Fig. 3 - Response surface

Once the response surface has been generated, we applied a stochastic optimization method based on a genetic algorithm with a population of 30 individuals and 300 generations. The obtained value for each parameter at the end of the optimization process, as well as the evaluation of the objective function for these values of parameters are presented in Table VII.

TABLE VII – OPTIMIZED VALUES OF THE SIGNIFICANT PARAMETERS

Parameter	Value
R_1	7.1675
L_2	14.0804
L_4	14.3550
F_{app}	4.11E-4
F_{obj}	2.15E-4

We used the following values for the 5 other parameters of the problem to build the response surface:

– the value of parameter L_3 was defined in 14.0mm, because the application of experimental design method showed that the objective function presents a smaller value when L_3 is in its minimal value.

– parameters A_1, A_2, A_3 and A_4 were defined in their original values, presented in Table VIII. These values allow us to compare the obtained results with another reference of the optimization of the original problem [5].

TABLE VIII – VALUES OF NEGLIGIBLE PARAMETERS

Parameter	Value
L_3	14.0
A_1	180.0
A_2	80.0
A_3	88.0
A_4	9.5

Fig. 4 shows the equipotential lines of the optimized device, where we can visualise the radial induction in the cavity of the mold.

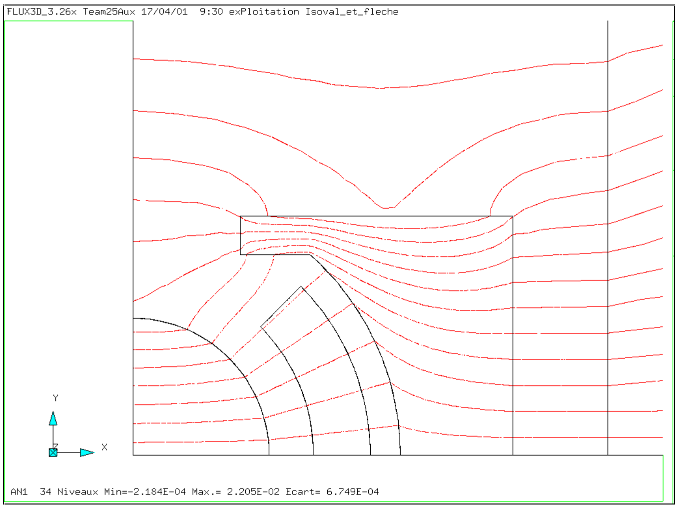

Fig. 4 - Optimized geometry

Table IX shows the comparison between the obtained results and the optimized values obtained by a direct application of genetic algorithms in the original problem [5]. We can see that the values of parameters are almost the same but the number of evaluations used by the approach “EDM + Response Surface” is very smaller.

TABLE IX – OPTIMIZED VALUES OF THE SIGNIFICANT PARAMETERS

Approach	R_1	L_2	L_3	L_4	F_{obj}	Evaluations
Direct	7.29	14.00	14.00	14.67	8,92E-4	2360
EDM + RS	7.17	14.08	14.00	14.36	2,15E-4	343

IV. CONCLUSIONS

We have verified that the Experimental Design Method is a useful tool to identify the significant parameters of optimization problems. The application of this method in the Problem 25 of TEAM Workshop allowed us a significant reduction in the number of parameters used in the description of this problem and a reduction in the number of evaluations of the objective function.

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