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# A new regularisation method of magnetostatic inverse problems applied to thin-shells

Olivier Chadebec\*, Jean-Louis Coulomb<sup>^</sup>, Mauricio Caldora Costa<sup>^</sup>,  
Jean-Paul Bongiraud\*, Gilles Cauffet\*, Philippe Le Thiec\*

\*Laboratoire du magnétisme du navire.

LMN/ENSIEG, BP 46, 38402 Saint Martin d'Hères, France.

<sup>^</sup>Laboratoire d'electrotechnique de Grenoble

LEG/ENSIEG, BP 46, 38402 Saint Martin d'Hères, INPG/UJF CNRS UMR 5529-France

Abstract: This paper proposes an original approach for modelling ferromagnetic sheet in inverse problem. It is realised with layers of tangential dipoles. This distribution is built thanks to measurements on sensors located in the interior of the device. The problem being ill-posed, we propose a new regularisation method to obtain a physical solution by injection of direct problem (DPI). It's then easy to compute the field everywhere around the device where sensors can't be placed. Moreover, the permanent magnetisation of the shell can be identified and systems with fewer equations than unknowns can be solved.

## GENERAL FORMULATION OF THE DIRECT PROBLEM

Let's consider a ferromagnetic shell placed in an external inductor field  $\mathbf{H}_0$ , the earth magnetic field for example. The magnetisation  $\mathbf{M}$  of the sheet can be divided in two parts: An induced one, due to the reaction of the material to the inductor field and a permanent one due to the magnetic history of the material. This device creates a local perturbation of the field, which can be expressed in terms of reduced scalar potential.

$$\mathbf{H} = \mathbf{H}_0 + \mathbf{H}_{\text{red}} = \mathbf{H}_0 - \text{grad } \varphi_{\text{red}} \quad (1)$$

If the thickness  $e$  of the sheet is small and  $\mu_r$  its relative permeability high, we can consider that induced the magnetisation  $\mathbf{M}_i$  is parallel to the shell and constant through it [1]. We can assume that it's the case for the permanent magnetisation  $\mathbf{M}_p$  too. We can then model the plate by a median surface  $S$ , where tangential dipoles  $\mathbf{p}$  are located. They are directly linked to the magnetisation by the following equation [2]:

$$\mathbf{p} = \mathbf{p}^{\text{ind}} + \mathbf{p}^{\text{per}} = e \mathbf{M} = e (\mathbf{M}^{\text{per}} + \mathbf{M}^{\text{ind}}) \quad (2)$$

Where  $\mathbf{M}^{\text{per}}$  and  $\mathbf{M}^{\text{ind}}$  are respectively the permanent and the induced magnetisations. The advantage of this formulation is its validity everywhere in the air region (i.e. outside and inside the device). The local field perturbation can then be expressed by the following expression:

$$\mathbf{H}_{\text{red}}(\mathbf{Q}) = \frac{1}{4\pi} \iint_S \frac{1}{QS^5} (3(\mathbf{p} \cdot \mathbf{Q}\mathbf{S})\mathbf{Q}\mathbf{S} - QS^2\mathbf{p}) dS \quad (3)$$

Where  $\mathbf{Q}$  is a point of the air-region and  $\mathbf{S}$  a point of the median surface  $S$ .

## CALCULATION OF THE INDUCED MAGNETISATION

In this section, we are interested by the calculation of unknown  $\mathbf{p}^{\text{ind}}$  by knowing the induced field, the permeability of the material and geometry of the device. We have developed a 3D formulation based on [3]. Let's assume that the structure is meshed in  $N$  elements. The distribution is reduced to discrete dipoles by condensing the magnetisation at the center of each element. By applying a collocation method, we obtain the following equations:

$$\frac{\mathbf{p}_i^{\text{ind}}}{e(\mu_r - 1)} - \sum_k^N \int_{L_i} \mathbf{p}_k^{\text{ind}} \cdot \text{grad } G_i \cdot \mathbf{n}_{L_i} dL_i = \iint_{S_i} \mathbf{H}_0 dS_i \quad (4)$$

Where  $G$  is the  $1/4\pi r$  Green's function,  $L_i$  is the line contour of element  $i$  and  $\mathbf{n}_{L_i}$  the external normal to  $L$ . By writing this equation on each element, we obtain, by projecting it on a basis of vectors, a square system of  $2N$  unknowns.

$$\mathbf{C}\mathbf{x} = \mathbf{d} \quad (5)$$

The expressions for field in the air region becomes then:

$$\mathbf{H}_{\text{red}}(\mathbf{Q}) = \frac{1}{4\pi} \sum_k^N \frac{1}{MS^5} (3(\mathbf{p}_k^{\text{ind}} \cdot \mathbf{Q}\mathbf{S})\mathbf{Q}\mathbf{S} - QS^2\mathbf{p}_k^{\text{ind}}) \quad (6)$$

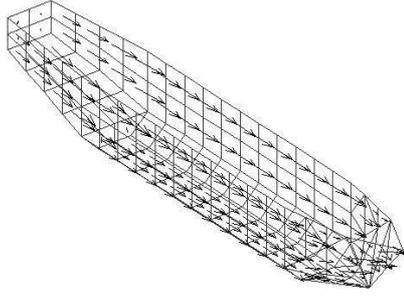


Figure 1: Example of a discrete distribution of tangential induced dipoles for a hull of a ship placed in a longitudinal inductor field.

### INVERSE PROBLEM

The aim of our work is, from measurements of the field in the air, to determine the dipoles distribution. There are  $M$  tri-axis sensors located inside the structure. Thanks to an equivalent equation of (6), we can build a system where the unknowns are the sum of permanent and the induced ones.

$$\mathbf{A} \mathbf{x} = \mathbf{A} (\mathbf{x}^{\text{ind}} + \mathbf{x}^{\text{per}}) = \mathbf{b} \quad (7)$$

Where  $\mathbf{A}$  is a  $(3M \times 2N)$  matrix. Unfortunately, small measurements inaccuracies give us contradictory equations, such as  $\mathbf{A}$  is very close to singularity. The problem is then ill posed and need to be regularised. Classical approaches like Tikhonov's regularisation or truncation of the single value decomposition have failed and the solutions obtained were divergent. The main idea of the regularisation technique is to add to the system to solve some known information. The unknown  $\mathbf{x}$  is composed by  $\mathbf{x}^{\text{per}}$ , that we try to identify and  $\mathbf{x}^{\text{ind}}$  that we are able to express. Our new approach called *Direct Problem Injection (DPI)* is to solve both the inverse problem and a part of the direct problem. We are going then to minimise the following norm:

$$\| \mathbf{A} \mathbf{x} - \mathbf{b} \| + \alpha \| \mathbf{C} \mathbf{x} - \mathbf{d} \| \quad (8)$$

The system to solve becomes, in the least-square sense:

$$\mathbf{x} = (\mathbf{A}^T \mathbf{A} + \alpha \mathbf{C}^T \mathbf{C})^{-1} (\mathbf{A}^T \mathbf{b} + \alpha \mathbf{C}^T \mathbf{d}) \quad (9)$$

Notice that if  $\alpha=0$ , we only solve the system built thanks to measurements and if  $\alpha$  is high, we only solve the problem of the induced magnetisation. The good parameter is between these two limits and its choice is made with the L-curve [4]. To validate our approach, we have realised inversions with measurements simulated by finite elements method. The device is placed in an external field and some parts of it have a permanent state magnetisation. These numerical tests lead to very promising results (figure 2).

### CONCLUSION

We are especially interested by the magnetic the magnetic anomaly created by ferromagnetic ship. Once the distribution have been obtained, it's easy to compute the field everywhere in the air and in peculiar outside of the hull where sensors can be placed. Moreover, our new technique of regularisation allows us to inverse dipoles system (where classical method have failed) and to solve systems with fewer equations than unknowns. Our method isn't restricted to ferromagnetic plates and can be applied easily to volume device.

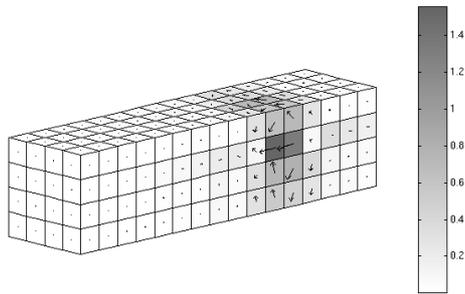


Figure 2: Identification of a permanent magnetisation (60 sensors simulated by FEM-288 dipoles)

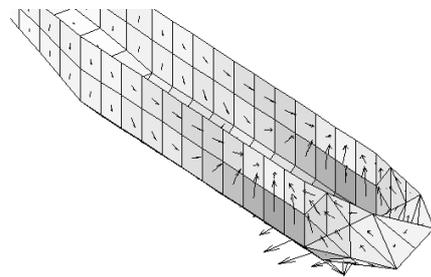


Figure 3: Identification of a permanent magnetisation by measurements on a real mok-up of a ship (4.6m length) regularised by the induced magnetisation created by the earth magnetic field.

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