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Résumé.-Ce rapport discute le procès de neutronisation et de l'évasion du lepton pendant l'effondrement gravitationel des étoiles, et leurs conséquences pour les hydronamiques de l'effondrement.

1. The Process of Neutronization.-After silicon burning stellar matter has roughly equal numbers of neutrons and protons. Because neutron-star matter has a large excess of neutrons, there must be an intermediate stage of evolution involving copious electron capture; this stage is called neutronization (see Zel'dovich and Novikov 1971 and references therein). As Fowler and Hoyle (1964) has shown, with increasing temperature nuclear statistical equilibrium shifts toward free particles. In principle this process of nuclear disintegration can occur independently of neutronization; however the calculations reported below suggest that these two processes are simultaneous to some extent.

The thermal effects of neutronization and nuclear disintegration are complex and have been the source of some controversy. Consider the First Law of Thermodynamics:

$$dE/dt + P\ dV/dt = -\ dQ/dt \qquad (1)$$

where dE is the change in kinetic and binding energy, PdV the work done by pressure forces and dQ the net energy carried out of the system. This simply states that energy is conserved. With the possibility of composition changing in a nonequilibrium fashion it is not necessarily desirable to introduce a term involving entropy explicitly; instead we can solve the appropriate reaction networks.

It is useful to define specific quantities relative to a conserved quantity; we will use nucleon number rather than gratviational mass.

Let N be the number of nucleons per unit volume and A be Avagadro's number. Then define a density

$$\rho = N/A \qquad (2)$$

and specific volume $V = 1/\rho$. Let us also define some composition variables :

$$Y_i = N_i/N = N_i/\rho A, \qquad (3)$$

where N_i is the number of particles of any type i (nuclei, electrons, protons, etc.) per unit volume. Then the "mass-fraction" variable X_i commonly used in nuclear astrophysics is

$$X_i = Y_i A_i \qquad (4)$$

where A_i is the number of nucleons in a particle of species i. With these variables the E in eq. (1) becomes energy per mole of nucleons and the Q becomes energy lost per mole of nucleons.

Now (in those cases we can solve) we can represent E as a sum of terms:

$$E \simeq \sum_i E_i \, . \qquad (5)$$

Of particular importance here are the terms for electrons, nuclei and nucleons. Thus,

$$dE/dt = \left(\frac{\partial E}{\partial T}\right)_{V,Y_i} \dot{T} + \left(\frac{\partial E}{\partial V}\right)_{T,Y_i} \dot{V} + \left(\frac{\partial E}{\partial Y_i}\right)_{T,V} \dot{Y}_i \qquad (6)$$

where the implied-summation convention is used. In astrophysics, eq. 1 is usually written as

$$\left(\frac{\partial E}{\partial T}\right)_{V,Y_i} \dot{T} + \left[\left(\frac{\partial E}{\partial V}\right)_{T,Y_i} + P\right] \dot{V} = \varepsilon - \frac{\partial L}{\partial M} \qquad (7)$$

where ε is an "energy source" term. And for simplicity we have combined the part of dQ/dt due to convection, conduction and radiative diffusion into the usual ∂L/∂M term.

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Therefore we must require

$$\varepsilon = \frac{dQ}{dt}\nu - \dot{Y}_i \left(\frac{\partial E}{\partial Y_i}\right)_{T,V} \quad . \qquad (8)$$

The second term is <u>not</u> generally small, but can be the dominant one. It must include the kinetic <u>and</u> binding energies of electrons <u>and</u> nucleons <u>and</u> nuclei.

After silicon burning the matter approaches nuclear statistical equilibrium (Woosley, Arnett and Clayton 1973). Then E, P and $dY_i/dt = \dot{Y}_i$ are all functions of T, ρ and Y_i (and of certain nuclear properties of each of the i species). Epstein and Arnett (1975) have derived convenient algorithms for determining these functions numerically and critically reviewed the necessary input physics. At high degeneracy of electrons, neutronization can be a heating process. Some of the energy of the captured electron goes away with the neutrino but some is used to heat the matter which remains. Some can also be used to make more of the less-bound nuclei on the neutron-rich side of the valley of beta stability. All this containedin eq. (8). Finally, it should be noted that a useful parameter to monitor the progress of neutronization is Y_e, the ratio of electron number density to total nucleon number density. A useful expression is

$$\eta = 1 = 2Y_e \qquad (9)$$

where η is the neutron excess (per nucleon). For an assembly of nucleons, weak interactions have the important ability to change the net charge state ; that is, the total neutron/proton ratio. By charge conservation this corresponds to changing the mole number of electrons, Y_e. With the experimental discovery of neutral currents one feels more secure in the prediction of neutrino-antineutrino pair emission processes (real pair, plasmon, photoneutrino, neutrino-bremsstrahlung, etc.) wich dominate stellar cooling after helium burning. These do not change Y_e however, and do not contribute to neutronization. Increasing density gives higher electron Fermi energy,allowing electron capture to occur, which would not happen at lower densities. The dominant processes are capture on nuclei,

$$e^- + (Z,A) \rightarrow \nu + (Z-1,A)$$

and on free protons,

$$e^- + p \rightarrow \nu + n.$$

The rate of the former process depends upon which nuclei are present, and how abundant they are. The capture on free protons is the inverse of neutron decay, a superallowed process having a large matrix element. Nuclei resist electron capture due to large threshold energies required to make terrestrially-unstable nuclei. Capture on free protons is inhibited by the small abundance of free protons boiled out by nuclear statistical equilibrium.

Contraction of the core causes temperature and density to rise. This increases the neutronization rate and robs the core of electron pressure. It also increases nuclear photodisintegration. Both effects increase the rate of contraction, giving rise to a hydrodynamic collapse (Arnett 1977). A fundamental question : are neutrinos formed early enough to escape? At higher density ($\rho \gtrsim 10^{12}$ g/cc) the neutrinos formed by this neutronization are trapped and move with the matter : $\tau_{\text{diffusion}} \gg \tau_{\text{infall}}$ for the timescales τ. In this regime the neutrino distribution function approaches Fermi-Dirac form, which is specified by a temperature and a chemical potential for the neutrinos. In equilibrium $T_\nu \rightarrow T_{\text{matter}}$, so that T and Y_ν, the mole number of neutrinos, are adequate to specify the thermodynamic properties (T_ν= 0.5 Y_e if the neutrino chemical potential equals that for electrons). Thus to calculate the properties of the system we need only three quantities, for example, density ρ, entropy S, and Y_e (or lepton mole number $Y_\ell = Y_e + Y_\nu$).

Let us explicitly list the regimes.

<u>(1) 3×10^9 g/cc $\lesssim \rho \lesssim 10^{11}$ g/cc</u>. This is the regime at which nuclear statistical equilibrium (NSE) is obtained, due to the speed of reactions involving strong and electromagnetic (but not weak) interactions. By specifying ρ, T, and Y_e one can solve the coupled (very nonlinear) "Saha" equations (i.e., equate chemical potentials) to obtain pressure, entropy, internal energy, and nuclear abundances. The solution

is simple in principle but not trivial in practice.

(2) 10^{11} g/cc $\lesssim \rho \lesssim 10^{12}$ g/cc. In this regime neutronization and neutrino trapping occur. Thus specifying Y_e for a NSE calculation requires computation of neutrino production, absorption, and transfer, coupled to the hydrodynamic behavior, making this perhaps the most computationally-demanding regime.

(3) 10^{12} g/cc $\lesssim \rho \lesssim 3 \times 10^{13}$ g/cc. Neutrinos are trapped so that lepton number (i.e., $Y_\ell = Y_e + Y_\nu$) is a good state variable. This regime is technically no more difficult than (1) above.

(4) 3×10^{13} g/cc $\lesssim \rho \lesssim 4 \times 10^{14}$ g/cc. The density is so high that free nucleons and nuclei cannot be considered as "non-interacting". Nucleon-nucleon interaction and nuclear compressibility need to be included.

(5) $\rho \gtrsim 4 \times 10^{14}$ g/cc. At greater than nuclear density the nuclei and nucleons merge into a giant, warm nucleus. All the problems associated with neutron star equations of state arise.

In all of these regimes, low entropies are expected. Thus the nuclei do not dissociate much and the leptons are at least moderately degenerate. This simplifies the theoretical analysis. Note that knowledge of the two quantities entropy S and electron number Y_e, allows us to specify the equation of state for a density in any of these regions. See Bethe, Brown, Applegate and Lattimer (1979) for an excellent general discussion.

2. Neutrino Astrophysics of Collapse.- As collapse proceeds, the denser core evolves faster than the outer regions (the "mantle," which we define as the processed matter above the silicon-burning shell). This is described in detail by Arnett (1977), who shows "snapshots" of structure at several moments, from the initial instability until well after neutrino trapping, for an 8 $M_\odot$ helium star. The entire evolution shown takes one second, mostly spent in the early evolution away from hydrostatic equilibrium. The various burning shells are active.

The structure of the core can be described schematically. As we move inward we pass a neutrino "photosphere" (sometimes called a "nusphere" or "neutrinosphere"), which is about one mean free path in. Then there is a neutronization shell where most of the electron capture occurs, and a neutrino trapping region inside which the neutrino diffusion time exceeds the infall time. These regions overlap each other, allowing some loss of lepton number. The degree of this overlap determines the value of one of the state variables (lepton mole fraction, Y_ℓ) which we need to specify the equation of state at higher density. The inner core may have a core shock (see below).

In the inner zone the neutrino distribution function closely approximates a Fermi Dirac distribution at the local temperature, density, and mole number of neutrinos Y_ν. As we move outward the distribution function peaks at lower neutrino energy, and decouples from the matter with a broad maximum at a neutrino energy around 8 MeV. How does the other state variable, the entropy, change? The first law of thermodynamics is

$$\dot{E} + P\dot{V} = \dot{Q}$$
$$= T\dot{S} + \sum_i \dot{N}_i \mu_i$$

where μ_i is the chemical potential of the species i. Let $N_i = \rho a Y_i$ where a is Avogadro's number, $\sigma = S/ka$ is the dimensionless entropy per nucleon, and $\dot{q}$ is the added energy per nucleon due to heat flow. Now, following Arnett and Lamb,

$$\dot{q} = kT\dot{\sigma} + \Sigma\, \mu_i \dot{Y}_i$$
$$\simeq \langle\varepsilon_\nu\rangle_{escape}\, Y_\nu/\tau_{escape}$$

where $\langle\varepsilon_\nu\rangle_{escape}$ is the mean energy of escaping (non-trapped) neutrinos and τ_{escape} is the average escape time. The loss of leptons by neutrino diffusion is

$$\dot{Y}_\ell = \dot{Y}_e + \dot{Y}_\nu$$
$$\simeq -Y_\nu \tau_{escape}$$

Now the neutronization reactions have the form

$$e^- + p \rightleftarrows n + \nu$$

so $\dot{Y}_e = \dot{Y}_p = -\dot{Y}_n$ for these trapped species, and

$$\sum_i \mu_i \dot{Y}_i = \dot{Y}_e\, (\mu_e + \mu_p - \mu_n) + \mu_\nu\Big(-\dot{Y}_e - \frac{Y_\nu}{\tau_{escape}}\Big)$$

which can be written as

$$kT\dot{\sigma} = (-\dot{Y}_e)(\mu_\nu - \langle\varepsilon_\nu\rangle_{escape}) + (-\dot{Y}_e)(\mu_e + \mu_p - \mu_n - \mu_\nu).$$

The first term on the RHS is a "downscatter" term due to the fact that the escaping neutrinos have been degraded in energy. This may be the largest effect. The second term, of perhaps comparable size, is due to the lack of weak interaction (neutronization) equilibrium. C. Pethick calls this the "out-of-whackness" term. Both of these have a maximum in the neutronization region. Typical values for the total change in entropy are $\Delta\sigma \lesssim 1$, which are small.

To examine the effects of neutrino transport on explosion (?) consider an extreme case. We arbitrarily set the free proton abundance Y_p so large ($Y_p \simeq 10^{-3}$ not 10^{-6}) that the neutronization shell moves out toward the neutrinosphere. The results are shown in Arnett (1978). The electron mole fraction (in the core) goes from $Y_e \simeq 0.45$ to 0.08; most leptons escape! There is a large amount of neutrino transport; in fact a standing accretion shock is set up with neutrino stress stopping the incoming matter. However there is not the slightest hint of explosion. The neutrinos dissipate the infall kinetic energy, not reflect it. The failure of this highly favorable and extreme case illustrates the difficulty with the hypothesis that neutrino transport gives rise to an explosion.

3. Adiabatic Hydrodynamics of Collapse.-Let us consider what is probably a more realistic limit : strongly trapped neutrinos. In this limit we can neglect the effects of neutrino escape, to first order, and fudge our equation of state a bit to correct this neglect. The approximation is very good for $\rho \gtrsim 10^{12}$g/cc. Ken Van Riper (1978) examined the hydrodynamics of collapse in this approximation for spherical symmetry. In the Newtonian case he found that the results are sensitive to the effective adiabatic exponent Γ, i.e., to the equation of state. In the core (i.e., $\rho \gtrsim 4 \times 10^{14}$g/cc) Γ is sufficiently stiff to halt the collapse ($\Gamma_{core} \gtrsim \Gamma_{critical} = 4/3$ for the Newtonian case). In the mantle ($\rho < 4 \times 10^{14}$ g/cc) Γ is expected to be smaller than in the core. For $\Gamma_{mantle} < 1.27$ Van Riper found that after bounce an accretion shock formed, with no explosion. For a stiffer equation of state in the mantle, $\Gamma_{mantle} \geq 1.28$ a reflected shock propagates outward, ejecting mass.

Thus, if the mantle has too soft an equation of state, an explosion will not result. Even if the mantle is fairly soft ($\Gamma \geq 1.28$), the explosion can still occur if $\Gamma_{core} \simeq \Gamma_{critical}$. This occurs even if General Relativity is used (Van Riper and Arnett 1978). The core has a large amplitude bounce which gently pushes off the mantle. The rebound energy goes into ordered motion; a harder push causes dissipation and weakens the explosion. The stiffer the mantle, the harder a push it can transmit without dissipating. Thus a stiff mantle and a core close to critical favor explosions.

The Γ to be expected from a realistic equation of state has been calculated by Lamb, Lattimer, Pethick and Ravenhall (1979) and by El Eid and Hillebrandt (1979). Initially the entropy is $\sigma \simeq 1$ so that if $\Delta\sigma \simeq 1$ we should consider $\sigma = 1$ to 2. For the lower densities, Newtonian results apply and Γ is close to the value of 4/3 which allows explosion. At $\rho \gtrsim 10^{14}$ g/cc, Γ increases, increasing the pressure gradient force, but general relativistic effects tend to cancel this by increasing gravity. Depending upon the core mass and super-nuclear EOS, the core could be close to critical. Van Riper and Arnett (1978) have done hydrodynamic calculations including these effects. The explosion energy is a maximum for $M_{core} \lesssim M_{ov}$ and of the order of several times 10^{51}ergs. Note that lower masses can give modest explosion energies. For many choices of "neutron star" equation of state, a black hole results from a slightly larger M_{core}. It is interesting that for plausible equations of state and core masses determined by evolutionary sequences, explosions do result.

These results suggest :

(1) For such an explosion, we have a prediction of the mass range for the dense

remnant $M_{chandrasekhar} \lesssim M$ (collapsed remnant) $\lesssim M_{OV}$. Observations of x-ray binaries give $M_{rem} = 1$ to $2\ M_{\odot}$ and the binary pulsar PSR1913+16 gives masses of 1.39 ± 0.15 and 0.44 ± 0.15, which are consistent with this relation.

(2) The bounce occurs at greater than nuclear density. Either the effects of general relativity (Van Riper 1978) or the attractive part of the nucleon-nucleon interaction seems to be adequate to insure this result, which may be important for gravitional radiation.

A number of new and interesting questions have arisen.

(1) Does the Saenz-Shapiro (1978) effect, a growth of asymmetry after several core bounces, insure a large quadrapole moment and profuse gravitational radiation?

(2) What is the neutrino shock structure (Bruen, Buchler, Yueh 1978, Kazanas 1979) for a reflected shock propagating to $\rho \lesssim 10^{12}$ g/cc (which might be observable)?

(3) What effects will neutrino convection have (Epstein 1979)?

(4) What is the best equation of state, especially for $\rho \gtrsim 10^{14}$ g/cc?

(5) At the very high bounce densities, even a slowly rotating precollapse model may become non-pherical. What are the characteristics of the 2- and 3- dimensional hydrodynamics of core collapse?

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