



**HAL**  
open science

## Coopetition in a Mixed Oligopoly Market

Duc-De Ngo, Mahito Okura

► **To cite this version:**

| Duc-De Ngo, Mahito Okura. Coopetition in a Mixed Oligopoly Market. 2007. halshs-00258638

**HAL Id: halshs-00258638**

**<https://shs.hal.science/halshs-00258638>**

Preprint submitted on 22 Feb 2008

**HAL** is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

---

# Coopetition in a Mixed Oligopoly Market\*

**Duc De NGO**

LEO, Université d'Orléans

Postal Address: Rue De Blois, BP 6739, 45067 Orléans Cedex 2, France

E-mail: duc-de.ngo@laposte.net

**Mahito OKURA**

Nagasaki University

Postal Address: 4-2-1, Katafuchi, Nagasaki, 850-8506, Japan

E-mail: okura@nagasaki-u.ac.jp

**January 5, 2008**

## ABSTRACT

In this study, we aim to investigate the impact of privatization on the degree of cooperation and competition in a mixed oligopoly market. We consider a duopoly market that comprises one semipublic firm and one private firm. Each firm is assumed to determine the level of two types of effort: the cooperative effort made to enlarge the total market size and the competitive effort made to increase market share.

In a contest framework, our results show that the competitive effort level of the semipublic firm is smaller than that of the private firm. The more the semipublic firm is concerned for social welfare, the less it competes. On the basis of average costs, we then analyze the case in which only the semipublic firm undertakes cooperative effort. In this case, the private firm behaves as a free rider. Furthermore, we find that the semipublic firm expends more cooperative effort than does the private firm.

**Key words:** *Coopetition, Mixed oligopoly, Contests, Free rider.*

**JEL Classification:** C72, L13, L33

---

\* The authors appreciate helpful comments from Giovanni Battista Dagnino, Xavier Galiegue, and seminar participants at LEO (University of Orleans) and Koshiro Ota, Akira Maeda, Jun Nakabayashi, and participants at the 6th International Conference of the Japan Economic Policy Association. Mahito OKURA was supported in part by a Grant-in-Aid for Scientific Research from Nagasaki University, JAPAN.

## 1. Introduction

In reality, many firms choose to undertake not only competitive activities but also cooperative activities. For example, life insurance firms may divulge their private information to their rivals in order to reduce the incidence of insurance fraud, whilst competing vigorously over insurance premiums, quality and so on.

Studies such as those of Brandenburger and Nalebuff (1996) and Dagnino and Padula (2002) describe situations that contain both cooperative and competitive activities simultaneously in terms of cooptation. Studies on cooptation have developed rapidly, particularly in recent years, and the concept has been used to explain many economic and social phenomena in various industries and in different countries.

However, to the best of our knowledge, previous studies on cooptation have focused on the activities of private firms or public firms. In other words, they have not addressed the behavior of the semipublic firm that aims to mix profit maximization and welfare maximization. Examples are discussed in the next section.

The purpose of this study is to investigate a situation in which there are both semipublic and (purely) private firms in the market. Studies such as those of Merrill and Schneider (1966), Harris and Wiens (1980), Bös (1986, 1991), Vickers and Yarrow (1988), and De Fraja and Delbono (1990) describe such a market as a “mixed oligopoly market”. For a given set of market conditions, determined, for example, by the number of firms and the timing of decisions, the activities chosen by semipublic and private firms may differ because of differences in their respective objective functions. Specifically, if one firm is semipublic rather than private, how does this affect market equilibrium?

To answer this question, we consider the simplest duopoly market that contains one semipublic firm and one private firm. Each firm chooses two types of effort level: cooperative effort made to enlarge the market size and competitive effort made to increase market share. In other words, we develop a cooptation model of effort levels in a mixed duopoly (oligopoly) market.

One specific aim of this research is to analyze cooptation by using game theory. Although researchers such as Brandenburger and Nalebuff (1996) and Lado et al. (1997) have insisted on the usefulness of game theory for analyzing cooptation, few studies use game theory to analyze this issue (see, for example, Ngo (2006) and Okura (2007)). However, game theory represents a simple way of formalizing complicated situations that incorporate both cooperative and competitive behavior (Okura (2007)). Thus, it is rational to develop an economic model to analyze cooptation.

---

The remainder of this study is organized as follows. The next section briefly describes actual examples of mixed oligopoly. In Section 3, we review the related literature in order to shed light on the contribution made by our research. In Section 4, we develop a model of cooperative and competitive effort in a mixed oligopoly market that contains both private and semipublic firms. The model is used to derive several important and interesting results. Concluding remarks are given in Section 5.

## 2. Examples of cooptition

Nowadays, with privatization and deregulation waves in both developed and developing countries, we can find many examples of cooptition in mixed oligopoly markets.

For instance, the three mobile phone operators in the French market, Orange, SFR and Bouygues Telecom, initiated a cooperation project. Orange is a semipublic firm in the sense that 18.17% of its holding company, France Telecom, is controlled by the French government. Motivated to enhance the telecommunications services offered to their clients and thus increase their profits in the “saturated” French market, these rivals decided to cooperate to offer the best homogenous public wifi service covering the whole country. To do so, in October 2003, they created the Wireless Link Association and have since attracted nine members of varying size and some start-ups. Because establishing a high quality nationwide wifi network is a complex task and requires enormous investment in infrastructure and technology research, no operator can meet this challenge alone. Hence, cooperation between these rival operators unifies the complementary innovative techniques and the different experiences that each member has in different processes, toward providing a public wifi service for the common good.

Chung (1996) argues that in R&D tournaments, the knowledge generated by all agents may have positive spillover effects on the patentee. In other words, the winner’s profit increases with the total investments of all players. Consequently, R&D tournaments among public and private firms can lead to cooptition in a mixed oligopoly market.

Another example is the case of the Japanese life insurance market<sup>1</sup>. In this market, there are two types of life insurance providers, private and semipublic; the latter is Japan Post. In 2005, the aggregate insurance premiums of Japan Post amounted to ¥16,672 billion (about US\$142 billion). Its premium revenue exceeded that of Japan’s largest private life insurer Nippon, which amounted to ¥4,842 billion (about US\$41 billion).

---

<sup>1</sup> A more detailed explanation of this is given by Okura and Kasuga (2007).

Thus, one must consider both private and semipublic firms in order to understand the Japanese life insurance market.

Japan Post was originally a completely public entity, but in 2003 it reformed as a semipublic organization to improve its efficiency<sup>2</sup>. Japan Post sells low-amount (less than ¥10 million (about US\$87 thousand)) life insurance products termed “Kampo”. Some types of Kampo are similar to the insurance products sold by private life insurers. For example, in 2004, Japan Post started selling blended life insurance products (incorporating full-life and term insurance), which is one of the main life insurance products sold by private Japanese life insurers.

Both insurers cooperate to develop a healthier and more disciplined life insurance market. For example, Japan Post was admitted to the Life Insurance Association of Japan since October, 2007. This association coordinates services for policyholders, evaluates moral hazard and so on<sup>3</sup>. Japan Post and private life insurers are expected to adopt cooperative strategies to develop their common interests. However, both insurers also compete on premiums and quantities to expand their market shares. Thus, the Japanese life insurance market can be considered as a cooperative market.

### **3. Related literature**

In general, economists model the objectives of public firms in two ways. One way is to specify that a public firm’s objective is pure social welfare maximization (see, for example, De Fraja and Delbono (1990), and more recently, White (1996), Fjell and Pal (1996), Mujumdar and Pal (1998), and Pal (1998)). As an extension of the first, the second approach is to specify that the public firm is only a partial social welfare maximizer; see, for example, Bös (1991), and more recently, Matsumura (1998), Bárcena-Ruiz and Garzón (2003), and Matsumura and Kanda (2005). In this paper, we adopt the second approach. This framework is more general than the first because the latter can be treated as a special case of the former. In many cases in the past, the state has held, and sometimes still holds, a nonnegligible proportion of shares in privatized firms, and there are a number of firms that have a mixture of private and public ownership. Because privatized

---

<sup>2</sup> Japan Post will be privatized in October 2007 and will change its name to the Japan Post Co., Ltd. Japan Post will be divided into four stock companies (mail, deposits, life insurance, and networks). However, this will not mean complete privatization because the Japanese government holds all the stocks of the holding company that, in turn, holds all the stocks of the four new stock companies. In other words, after October 2007, these companies will consider not only private but also public benefits.

<sup>3</sup> For more details, see the webpage, <http://www.seiho.or.jp/english/about/activities/index.html> [Accessed: December 19, 2007]

---

firms with mixed ownership must also respect the interests of private shareholders, they cannot behave as pure welfare maximizers.

Moreover, our paper is closely related to the interesting work of Chung (1996) on the endogenization of prizes in contests. Chung (1996) analyzes the effort levels expended by players and focuses on social waste in rent-seeking contests, an important issue in the rent-seeking contest literature, in which the prize increases with the efforts of all players.

Our research is also inspired by Krishnamurthy (1999) and Dearden and Lilien (2001), who model cooperation in a contest framework. Both study firm behavior under the assumption that firms collaborate on advertising to increase the total market, so representing the cooperation aspect, at the same time as attempting to increase their shares of that demand, representing the competition aspect.

Our work contributes to this literature in two ways. First, while previous work assumes that payoff functions of players are similar, we consider the general case in which players pursue different objectives and thus have distinct payoff functions. Second, following Ngo (2006), we employ a two-stage model. As pointed out by Dumez and Jeunemaitre (2006), there are two types of cooperation. In the one-stage game, cooperation and competition occur simultaneously in a multi-dimension framework, and in a two-stage or multi-stage game, cooperation and competition take place sequentially.

The merit of modeling cooperation in a contest framework is that it enables one to take into account agents' competitive efforts, which are neglected in the traditional Cournot and Bertrand models. Ngo (2006) argues that many economic and social phenomena can be viewed as cooperation contests in which agents spend resources in order to win one or more prizes. Many examples of cooperation contests can be found in real life: employees compete with each other for promotion in organizational hierarchies but also work collectively to develop their firms; domestic firms compete for market share but also join together against foreign firms; athletes compete for prizes but are mutually responsible for attracting a crowd.

#### **4. The model**

For simplicity, we consider a model of two firms. Suppose that these firms play a two-stage game. Firm  $A$  is semipublic, while firm  $B$  is private.

In the first stage, both firms choose their cooperative effort levels simultaneously. The cooperative effort level is denoted by  $y_i$  for  $i \in \{A, B\}$ , which increases the total market. The overall market demand function is  $(a + y_A + y_B)$ , where  $a$  represents initial demand without any cooperative effort.

In the second stage, both firms simultaneously choose their competitive effort levels, which are denoted by  $x_i$ . Competitive effort levels can enhance individual firms' competitive power and market shares. That is, the market share of firm  $i$  denoted by  $s_i$  is

$$s_i = \begin{cases} 1/2 & \text{where } x_A = 0, x_B = 0 \\ x_i / (x_A + x_B) & \text{otherwise} \end{cases} .$$

If there is no competitive effort, the market is divided equally to two firms. Otherwise, the pie share of firm  $i$  is determined by the ratio  $x_i / (x_A + x_B)$ .

Let the inverse demand function be  $p(\bullet)$ . Assume that  $p'(\bullet) < 0$ . Average cost denoted by  $c$  is assumed to be constant and the same for both firms.

Then, the objective function of firm  $B$  is<sup>4</sup>

$$U_B = \Pi_B = (p - c)(a + y_A + y_B) \frac{x_B}{x_A + x_B} - k_x x_B - k_y y_B^2 \quad (1)$$

where  $p \equiv p(y_A + y_B)$ ,  $k_x x_B$  and  $k_y y_B^2$  represent the costs of expending competitive and cooperative efforts respectively.

Because firm  $A$  is a semipublic firm, its objective function is

$$U_A = \alpha W + (1 - \alpha) \Pi_A \quad (2)$$

where  $W$ , which represents the social surplus, is the sum of the producer's profit and the consumer's surplus; the parameter  $\alpha \in [0,1]$  can be interpreted at two levels. At one level, it represents the weight of the government's participation in the firm  $A$ . At the other level, it can be regarded as the importance level

---

<sup>4</sup>Here we consider a deterministic outcome competition in which each party receives a share of what is under dispute. The equivalent results under a probabilistic competition, i.e. a winner-take-all competition, can be derived under the assumption of risk neutrality.

attributed to the government's objective, i.e. the social welfare, in contrast with the profit objective.  $\alpha = 0$  signifies that the firm  $A$  is solely concerned about its profit.  $\alpha = 1$  means that the firm  $A$  aims to maximize the social welfare irregardless of its profit.

It follows that

$$\Pi_A = (p - c)(a + y_A + y_B) \frac{x_A}{x_A + x_B} - k_x x_A - k_y y_A^2. \quad (3)$$

Thus, the social surplus can be written as

$$W = \int_0^{a+y_A+y_B} p(q) dq - c(a + y_A + y_B) - k_x(x_A + x_B) - k_y(y_A^2 + y_B^2). \quad (4)$$

To derive the extensive form game, we solve the game by backward induction. That is, the equilibrium in the second stage is derived on the basis of the first stage before the first stage has been played. Once the equilibrium in the second stage is determined, the equilibrium in the first stage is derived by using the results from the second stage.

The second stage is described below.

The first-order conditions with respect to  $x_i$  are

$$\frac{\partial U_A}{\partial x_A} = \frac{(1 - \alpha)(p - c)x_B(a + y_A + y_B) - k_x(x_A + x_B)^2}{(x_A + x_B)^2} = 0, \quad (5)$$

$$\frac{\partial U_B}{\partial x_B} = \frac{(p - c)x_A(a + y_A + y_B) - k_x(x_A + x_B)^2}{(x_A + x_B)^2} = 0. \quad (6)$$

Given  $\alpha \in [0,1)$ , the equilibrium competitive effort levels are

$$x_A = \frac{(1 - \alpha)^2 (p - c)(a + y_A + y_B)}{(2 - \alpha)^2 k_x}, \quad (7)$$

$$x_B = \frac{(1 - \alpha)(p - c)(a + y_A + y_B)}{(2 - \alpha)^2 k_x}. \quad (8)$$

and the market shares of public and private firm are

$$s_A = \frac{1 - \alpha}{2 - \alpha},$$

$$s_B = \frac{1}{2 - \alpha}.$$

When  $\alpha \rightarrow 1$ , it is reasonable to assume that  $x_A \rightarrow 0, x_B = \varepsilon$  (where  $\varepsilon$  represents a very small positive number) and the market shares are  $s_A \rightarrow 0, s_B \rightarrow 1$ .

These results are used to state the following lemma.

**Lemma 1 (competitive effort levels):**

Both competitive effort levels satisfy

$$x_A = (1 - \alpha)x_B \tag{9}$$

Furthermore, the relationship between market shares of firm *A* and *B* :

$$s_A = (1 - \alpha)s_B$$

*Proof:*

From equations (7) and (8),

$$\frac{x_A}{x_B} = 1 - \alpha \Rightarrow x_A = (1 - \alpha)x_B. \tag{10}$$

Q. E. D.

This implies that the competitive effort level of the semipublic firm is below that of the private firm. The more concerned is the semipublic firm for social welfare (the closer is  $\alpha$  to 1), the less it competes. As a result of it, the market share of the public firm is never bigger than the private firm's one.

Several comments on the equilibrium cooperative effort levels shown in equations (7) and (8) are warranted. In this context, consider the case in which only one variable changes.

First,  $\partial x_i / \partial k_x < 0$  and  $\partial x_j / \partial k_x < 0$  have the simple and intuitive implication that the higher is the cost level, the lower is competitive effort.

Second, consider the relationship between competitive and cooperative effort levels. From equations (7) and (8), the following derivatives are obtained.

$$\frac{\partial x_A}{\partial y_A} = \frac{\partial x_A}{\partial y_B} = \frac{(1-\alpha)^2 \{p'(a+y_A+y_B) + p - c\}}{(2-\alpha)^2 k_x} = \frac{(1-\alpha)^2 \left\{ p \left( 1 - \frac{1}{e} \right) - c \right\}}{(2-\alpha)^2 k_x}, \quad (11)$$

$$\frac{\partial x_B}{\partial y_B} = \frac{\partial x_B}{\partial y_A} = \frac{(1-\alpha) \{p'(a+y_A+y_B) + p - c\}}{(2-\alpha)^2 k_x} = \frac{(1-\alpha) \left\{ p \left( 1 - \frac{1}{e} \right) - c \right\}}{(2-\alpha)^2 k_x} \quad (12)$$

where  $e \equiv -\frac{\partial(a+y_A+y_B)}{\partial p} \cdot \frac{p}{a+y_A+y_B}$  is the price elasticity of demand.

Thus, if the demand function is sufficiently price elastic, i.e.  $e > p/(p-c)$  then we have the following results: (i)  $\partial x_i / \partial y_i > 0$  implying that both types of effort spent by a firm are complements; (ii)  $\partial x_i / \partial y_j > 0$  implying that both types of effort spent by two different firms are also complements. By contrast, if the demand is sufficiently price inelastic, i.e.  $e < p/(p-c)$  then we have the following results: (i)  $\partial x_i / \partial y_i < 0$  implying that both types of effort spent by a firm are substitutes; (ii)  $\partial x_i / \partial y_j < 0$  implying that both types of effort spent by two different firms are also substitutes.

In general, competition and cooperation are considered as two polar opposites, that is, a higher level of cooperation naturally leads to a lower level of competition and *vice versa*. On the contrary, in the cooperative game, the relation between competition and cooperation can be positive or negative depending on the price elasticity level of the demand. The following lemma summarizes these results.

**Lemma 2 (relationship between competitive and cooperative efforts):**

If  $e$  is sufficiently large ( $e > p/(p-c)$ ), then both types of effort are complements. In contrast, if  $e$  is sufficiently small ( $e < p/(p-c)$ ), then both types of effort are substitutes.

Third, we investigate the effect of  $\alpha$ . Differentiating equations (7) and (8) with respect to  $\alpha$  yields

$$\frac{\partial x_A}{\partial \alpha} = -\frac{2(1-\alpha)(p-c)(a+y_A+y_B)}{(2-\alpha)^3 k_x} < 0, \quad (13)$$

$$\frac{\partial x_B}{\partial \alpha} = -\frac{\alpha(p-c)(a+y_A+y_B)}{(2-\alpha)^3 k_x} < 0. \quad (14)$$

This result is plausible. When  $\alpha$  increases, firm A has less incentive to expand its market share because it downweights its own payoff. This situation is similar to the competition *à la* Bertrand since the competition efforts of firms A and B are complements, that is  $\partial x_A / \partial x_B = (1-\alpha) > 0$ . As a result, both firms reduce their competitive effort level.

At present, we analyze the first stage of the game.

Substituting equations (7) and (8) into equations (1) and (2) yields

$$U_A = \frac{1}{(2-\alpha)^2} (p-c)(a+y_A+y_B) - \alpha p(a+y_A+y_B) - k_y (y_A^2 + \alpha y_B^2) + \alpha \int_0^{a+y_A+y_B} p(t) dt \quad (15)$$

$$U_B = \frac{1}{(2-\alpha)^2} (p-c)(a+y_A+y_B) - k_y y_B^2. \quad (16)$$

The first-order conditions with respect to  $y_i$  are

$$\frac{\partial U_A}{\partial y_A} = \frac{1}{(2-\alpha)^2} p'(a+y_A+y_B) + \frac{1}{(2-\alpha)^2} (p-c) - \alpha p'(a+y_A+y_B) - 2k_y y_A = 0, \quad (17)$$

$$\frac{\partial U_B}{\partial y_B} = \frac{1}{(2-\alpha)^2} p'(a+y_A+y_B) + \frac{1}{(2-\alpha)^2} (p-c) - 2k_y y_B = 0. \quad (18)$$

To obtain interior solutions from equations (17) and (18), the following condition must be satisfied:

$$p'(a+y_A+y_B) + p-c = p\left(1-\frac{1}{e}\right) - c \geq 0. \quad (19)$$

From equations (17) and (18), the following proposition can be derived.

**Proposition 1 (the degree of cooperation):**

Define that  $\bar{\Omega} \equiv p \left[ 1 - \frac{1}{e} (1 - \alpha (2 - \alpha)^2) \right]$  and  $\underline{\Omega} \equiv p \left( 1 - \frac{1}{e} \right)$ . Then, there are three outcomes for cooperative effort levels corresponding to three different average cost levels.

Case 1: If average cost is high, i.e.  $c > \bar{\Omega}$ , both firms expend no cooperative effort. Moreover, equilibrium competitive effort levels are  $x_A = \frac{(1 - \alpha)^2 (p - c) a}{(2 - \alpha)^2 k_x}$  and  $x_B = \frac{(1 - \alpha)(p - c) a}{(2 - \alpha)^2 k_x}$ .

Case 2: If average cost is moderate, i.e.  $\underline{\Omega} < c < \bar{\Omega}$ , only the semipublic firm expends cooperative effort. The private firm free rides.

Case 3: If average cost is low, i.e.  $c < \underline{\Omega}$  then, both firms expend cooperative effort.

*Proof:*

From equation (17), the following condition is necessary for  $x_A^* > 0$ .

$$\bar{\Omega} - c > 0. \quad (20)$$

From equation (18), the following condition is necessary for  $x_B^* > 0$ :

$$\underline{\Omega} - c > 0. \quad (21)$$

It is easy to verify the following inequality because  $\alpha(2 - \alpha)^2 \geq 0$ :

$$\underline{\Omega} < \bar{\Omega}. \quad (22)$$

From equations (20) to (22), all three cases in relation to average costs can be derived.

If equation (20) is not satisfied, then the best strategy for both firms is to produce no output ( $y_A = 0$  and  $y_B = 0$ ). The equilibrium competitive effort levels can be derived by substituting  $y_A = 0$  and  $y_B = 0$  into equations (7) and (8).

Q. E. D.

Proposition 1 has interesting implications. When  $\alpha$  rises, case 2 is more likely to arise because  $\frac{\partial \bar{\Omega}}{\partial \alpha} > 0$ . This property implies that the semipublic firm behaves more like a public firm and the private firm is more likely to free ride. In the context of a quantity-setting oligopoly, in their pioneering work, De Fraja and Delbono (1990) show that welfare may be higher when a public firm maximizes profits rather than welfare. Their result suggests that, in some cases, a public firm should be privatized so that it maximizes profits rather than welfare. In our cooperative framework, privatizing public firms would mitigate the free-rider problem. Our finding is consistent with that of Matsumura (1998) who shows that, under normal conditions, governments should not hold all the shares. Because welfare-maximizing behavior by the public firm in a mixed duopoly is detrimental, public firms that are not natural monopolies should be (at least partially) privatized.

The smaller is  $e$ , the more likely is case 2 to arise because  $\frac{\partial \bar{\Omega}}{\partial e} = \frac{p(1-\alpha(2-\alpha)^2)}{e^2} < \frac{p}{e^2} = \frac{\partial \Omega}{\partial e}$ .

This implies that the lower is the price elasticity of demand, the more likely is the private firm to free ride.

Next, we consider the case in which equation (24) is satisfied (that is, both equilibrium cooperative effort levels are strictly positive). In this case, following lemma is satisfied.

**Lemma 3 (the cooperative effort levels in case 3):**

If  $\alpha = 0$ , then  $y_A = y_B$ .

If  $\alpha > 0$ , then  $y_A > y_B$ .

*Proof:*

We derive the following equation by combining equations (17) and (18):

$$\frac{(1-\alpha)\{1-\alpha(3-\alpha)\}}{(2-\alpha)^2} p'(a+y_A+y_B) - 2k_y y_A = \frac{1}{(2-\alpha)^2} p'(a+y_A+y_B) - 2k_y y_B. \quad (23)$$

Equation (23) reduces to

$$\left[ \frac{(1-\alpha)\{1-\alpha(3-\alpha)\}}{(2-\alpha)^2} - \frac{1}{(2-\alpha)^2} \right] p'(a+y_A+y_B) = 2k_y (y_A - y_B)$$

---


$$\Rightarrow -\alpha p'(a + y_A + y_B) = 2k_y (y_A - y_B)$$

$$\Rightarrow y_A - y_B = \frac{-\alpha p'(a + y_A + y_B)}{2k_y} \geq 0. \quad (24)$$

This equation is satisfied with strict equality only if  $\alpha = 0$ .

Q. E. D.

Using the lemma 3 to make a general comparison of cooperative effort levels yields the following proposition.

**Proposition 2 (cooperative effort levels):**

The private firm has no strategic incentive to spend more cooperative effort than the semipublic firm.

That is,  $y_A^* \geq y_B^*$ .

*Proof:*

In case 1, both cooperative effort levels are the same (zero). In case 2, the cooperative effort level of the semipublic firm exceeds that of the private firm because  $y_A^* > 0$  and  $y_B^* = 0$ . In case 3, from the lemma 3,

$y_A^* \geq y_B^*$  whatever the value of  $\alpha$ .

Q. E. D.

## 5. Concluding remarks

In this study, we developed a coepetition model of a mixed oligopoly market. In particular, we applied game theory to analyze a duopoly market shared by one semipublic firm and one private firm. We built the model to describe a coepetitive mixed oligopoly market in which both public and private firms determine their levels of competitive effort to expand their market shares having chosen their cooperative effort levels to maximize the total market size.

Our results show that the competitive effort level of the semipublic firm is below that of the private firm. The more concerned is the semipublic firm about social welfare, the less it competes. On the basis of average costs, we then analyzed the case in which only the semipublic firm expends cooperative effort. In this case, the private firm behaves as a free rider. Furthermore, we found that the semipublic firm expends more cooperative effort than does the private firm. Our analysis generates many insights of interest to the government, public and private firms. Regarding the government, it can use the privatization level as a mean to regulate the competitiveness of public firms and to mitigate free-rider problem. Our model offers an explanation of why public firms almost always contribute the most in cooperative associations with private firms but the former often have difficulties in competing with the latter.

However, our research is incomplete in several respects. Following two out of such several aspects are the most interesting and important.

First, we assumed that both firms choose their effort levels simultaneously. However, in reality, semipublic firms may choose their cooperative effort levels before private firms do. For example, to increase social benefits, Japan Post developed and enlarged the potential demand for ordinary civilians who had little money. To represent this, a Stackelberg model may be more appropriate than a model *à la* Nash.

Second, the extent to which the semipublic firm cares about social welfare is implicitly assumed to be common knowledge. Thus, the private firm knows the extent to which the semipublic firm considers the social surplus. If there is some uncertainty about this and the private firm is risk averse, the private firm may change its cooperative and competitive effort levels.

These issues remain open to discussion. Much additional work is required by future researchers to improve our model. However, our results have important implications for competition in mixed oligopoly markets.

---

## REFERENCES

- Brandenburger, A. M. and B. J. Nalebuff (1996), *Co-opetition*. Doubleday, New York.
- Bös, D. (1986), *Public Enterprise Economics*. North-Holland, Amsterdam.
- Bös, D. (1991), *Privatization: A Theoretical Treatment*. Clarendon Press, Oxford.
- Bárcena-Ruiz, J. C. and M. B. Garzón (2003), Mixed Duopoly, Merger and Multiproduct Firms. *Journal of Economics* 80: 27–42.
- Chung, T. Y. (1996), Rent-seeking Contests when the Prize Increases with Aggregate Efforts. *Public Choice* 87: 55–66.
- Dagnino, G. B. and G. Padula (2002), Coopetition Strategy: A New Kind of Interfirm Dynamics for Value Creation. Paper presented at the Innovative Research in Management, European Academy of Management (EURAM), Second Annual Conference, Stockholm, May 9–11.
- De Fraja, G. and F. Delbono (1989), Alternative Strategies of a Public Enterprise in Oligopoly. *Oxford Economic Papers* 41: 302–311.
- De Fraja, G. and F. Delbono (1990), Game Theoretic Models of Mixed Oligopoly. *Journal of Economic Surveys* 4: 1–17.
- Dearden, J. A. and G. L. Lilien (2001), Advertising Coopetition: Who Pays? Who Gains? (in: Baye M. R. and J. P. Nelson eds., *Advertising and Differentiated Products*), JAI Press, London, 203–219.
- Dumez, H. and A. Jeunemaitre (2006), Multidimensional Strategic Sequences: A Research Programme Proposal on Coopetition. Paper presented at the 2nd Workshop on Coopetition Strategy, European Institute for Advanced Studies in Management (ELIASM), Milan (Italy), September, 14–15.
- Fjell, K. and D. Pal (1996), A Mixed Oligopoly in the Presence of Foreign Private Firms. *Canadian Journal of Economics* 29: 737–743.
- Harris, R. G. and E. Wiens (1980), Government Enterprise: An Instrument for the Internal Regulation of Industry. *Canadian Journal of Economics* 13: 125–132.
- Krishnamurthy S. (1999), Enlarging the Pie vs Increasing One's Own Slice: An Analysis of the Relationship between Generic and Brand Advertising. *Marketing Letters* 11: 37–48.

- Lado, A, Boyd, N., and S. Hanlon (1997), Competition, Cooperation, and the Search for Economic Rents: A Syncretic Model. *Academy of Management Review* 22: 110–141.
- Matsumura, T. (1998), Partial Privatization in Mixed Duopoly. *Journal of Public Economics* 70: 473–483.
- Matsumura, T. and O. Kanda (2005), Mixed Oligopoly at Free Entry Markets. *Journal of Economics* 84: 27–48.
- Merrill, W. and N. Schneider (1966), Government Firms in Oligopoly Industries: A Shortrun Analysis. *Quarterly Journal of Economics* 80: 400–12.
- Mujumdar, S. and D. Pal (1998), Effects of Indirect Taxation in a Mixed Oligopoly. *Economics Letters* 58: 199–204.
- Ngo, D. D. (2006), Coepetition Contest Model. Paper presented at the 2nd Workshop on Coepetition Strategy, European Institute for Advanced Studies in Management (EIASM), Milan (Italy), September, 14–15.
- Okura, M. (2007), Coepetitive Strategies of Japanese Insurance Firms: A Game-theory Approach. *International Studies of Management and Organization* 37: 53–69.
- Okura, M. and N. Kasuga (2007), Financial Instability and Life Insurance Demand. *Asia-Pacific Journal of Risk and Insurance* 2 (1): 63–75.
- Pal, D. (1998), Endogenous Timing in a Mixed Oligopoly. *Economics Letters* 61: 181–185.
- Vickers, J. and G. Yarrow (1988), *Privatization: An Economic Analysis*. MIT Press, Cambridge, MA.
- White, M. D. (1996), Mixed Oligopoly, Privatization and Subsidization. *Economics Letters* 53: 189–195.