Identifying the magnetic part of the equivalent circuit of n-winding transformer

Xavier Margueron, Jean-Pierre Keradec

Identifying the Magnetic Part of the Equivalent Circuit of n-Winding Transformers

Xavier Margueron and Jean Pierre Keradec, *Member, IEEE*

***Abstract*—Representation of multiwinding transformers by equivalent circuits has been recently improved, and it is for the identification of components of these circuits. In this paper, the focus is on magnetic coupling with its related losses. A general method, based on external impedance measurements, is followed to determine inductances, coupling ratios, and resistances included in these equivalent circuits. Justification for impedance measurements, choice of measured impedances, and precautions regarding short-circuit compensation are discussed. For illustration, two components are tested, and their equivalent circuits are established.**

***Index Terms*—Equivalent circuit, identification, impedance measurement, multiwinding transformer, short-circuit compensation.**

I. Introduction

In POWER electronic converters, wound components (coils and transformers) play key roles: they provide temporary energy storage, voltage and current transformations, and electrical insulation. In medium and low-power converters, operating frequencies range from 20 kHz to some megahertz; magnetic cores are made of ferrite and, consequently, wound components behave almost linearly. Despite this, these components remain among the most difficult to represent by an equivalent circuit, especially when they own three, four, or more windings.

Our team has been working on the representation and the experimental characterization of these components for roughly 15 years, and successive refinements and extensions [1], [2] have been brought to our first published work on this subject [3]. Recently, we introduced a general method to represent the magnetic coupling of an n-winding transformer [4]. Because this coupling is the main property of a transformer, this approach provides the backbone of its equivalent circuit. Our purpose, in this paper, is to show how to deduce this circuit from a set of measurements acquired with an impedance analyzer. To complete, we have shown in the past that putting a linear electrostatic circuit in parallel with the magnetic one allows a large part of its high-frequency behavior to be described. For the determination of this capacitive part, please refer to [5].

At first sight, it is clear that measuring all of the self and mutual inductances leads to the full knowledge of the inductance matrix. Unfortunately, this leads to value leakage inductances with a poor accuracy because, doing so, they are deduced via differences of close quantities.

Adopting successively two distinct points of view (that of the experimenter and that of the circuit user), we will explain, in Section II, why we prefer impedance measurements to any other kind of measurements to characterize transformers.

In Section III, considering the representation of a two-winding transformer, we deepen the concept of leakage inductance localization. Measurable variations due to the move of these inductances from the primary to the secondary side are evaluated, and a criterion allowing the coupling to be neglected is given. Then, we summarize the method which gives equivalent circuits for n-winding transformers, and we briefly present two special cases that lead to simplifications.

The method intended for experimental identification is presented in Section IV, taking a 150-W, 100-kHz, three-winding transformer as an example. We address practical questions such as: what impedances to choose in order to fully and accurately characterize the magnetic coupling, how to connect the device to the test fixture, how to locate the compensation short circuit, and what is the final resolution for this kind of measurement. The presented experimental results are acquired with the 4294A Agilent impedance analyzer [6].

II. Why and What Impedance Measurements?

In order to characterize a transformer, magnetic coupling can be accessed by different methods, such as inductance matrix evaluation, impedance measurements, S-parameters, and so on. What is the most appropriate?

First, it must be underlined that impedances measured on a high-frequency transformer may range from 1 mΩ to 1 MΩ. Even if the frequency range to cover reaches 100 MHz, propagation-based measurements are more sensitive when impedances remain relatively close to 50 Ω. This leads to choosing an impedance-gain analyzer rather than a vector analyzer. Moreover, because wiring compensation is more efficient for impedance measurements than for gain measurements, we always try to use only impedance measurements.

Second, the inductance matrix may be fully determined by measuring self and mutual inductance. Unfortunately, this approach often leads to a very inaccurate determination of leakage inductances. Indeed, leakage inductances appear as the difference between two terms which are very close to each other, as follows:

$$L_f = L_{11} - L_{12}^2/L_{22} = L_{11} - (k_1)^2 L_{11} \qquad (1)$$

especially when coupling is strong (coupling factor k_1 close to 1) [7].

In switch-mode power supplies, transformers are included in electrical circuits in which their windings are periodically con-

Manuscript received June 15, 2005; revised September 14, 2006.

The authors are with the Laboratoire d'Electrotechnique de Grenoble, INPG/UJF UMR-CNRS 5529, ENSIEG, 38402 Saint-Martin-d'Hères Cedex France (e-mail: xavier.margueron@leg.ensieg.inpg.fr; keradec@leg.ensieg.inpg.fr).

Color versions of one or more figures are available online at http://ieeexplore.ieee.org.

Digital Object Identifier 10.1109/TIM.2006.887402

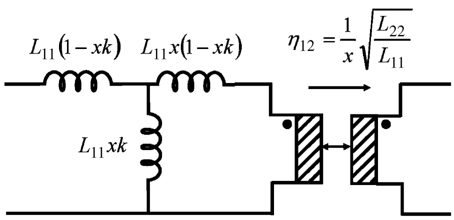


Fig. 1. Two-winding transformer.

nected to load or supplies with extreme impedances ($R = 0$ or $R \rightarrow \infty$). To provide accurate results, the equivalent circuit of the transformer must show, from any of its windings, the right impedance, whatever the number of its short-circuited windings is. As a consequence, our goal is now to identify the equivalent circuit on the basis of open circuit and short circuit impedance measurements.

It is easy to establish that, if the transformer has N windings, $N\ 2^{N-1}$ such impedances (defined with 0 to $N-1$ short circuits) are measurable. Of course, all of these impedances are not independent. For example, for any pair of ports of a linear passive circuit, the following relation applies:

$$Z_0 Z'_{cc} = Z'_0 Z_{cc} \tag{2}$$

where

Z measured from one port;
Z' measured from the other port;
0 open-circuit measurement;
cc short-circuit measurement.

In the following, we will select a reduced set of measurable impedances such that, when these impedances are known, all measurable impedances are accurately deducible.

III. Equivalent Circuit for N-Input Transformers

For a two-winding transformer, several representations do exist depending on the localization of leakage inductance. Owing to the arbitrary adjustable parameter x, the circuit of Fig. 1 leads to all of them. In that circuit, leakage inductance is split into two parts; the left one may be attributed to primary and the right one to secondary. When $x = 1$ is chosen, it is interesting to look at the variations of measurable inductances due to the move of magnetizing inductance from center to both left and right positions. It appears that, for $k > .995$, inductance variations are within 1%. Reciprocally, when the coupling factor is close to unity, leakage splitting does not matter.

Equation (1) shows that $L_{cc} = L_0(1-k^2)$. So, if $k < .2$, the input impedance keeps the same value when the secondary is open or shorted. According to the impedance criterion exposed before, the coupling can be neglected because its impact on all measurable inductances is very weak.

Choosing $x = 1/k$ and transferring series inductance to the secondary side of the coupler, a convenient circuit is drawn that is the first of a series we describe below.

To find the equivalent circuit of a three-winding transformer, we begin with (Fig. 2) one inductance (we call magnetizing inductance) and two couplers. These three components account for input inductance and for open-circuit voltage ratio. Then, we assume that the left input is shorted. In that situation, the two remaining windings can be looked at as a two-winding transformer we call the leakage transformer (surrounded by THE dotted line in Fig. 2). This approach is iterative: in a four-winding transformer [4], the leakage transformer is a three-winding one.

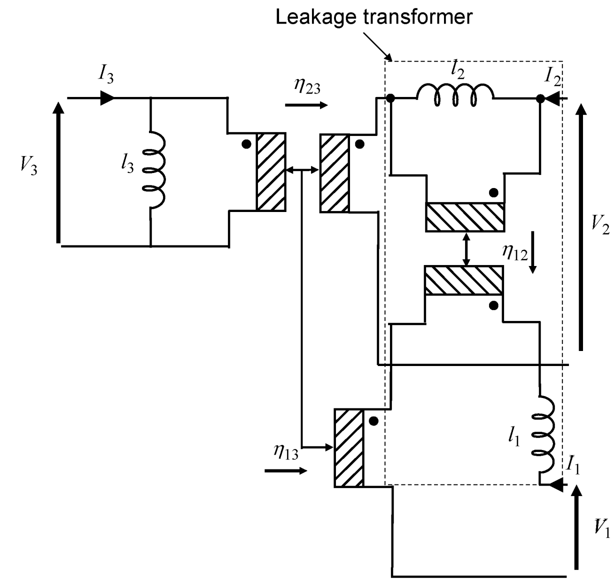


Fig. 2. Three-winding transformer.

Notice that, assuming inductors are replaced by impedance and couplers have complex ratios, this way of drawing an equivalent circuit is applicable to any passive linear circuit.

Despite the fact that this approach is general, some particular cases are interesting because they lead to simplifications. The first is related to identical windings. Windings are identical if they can be exchanged with no impact on electrical behavior. When an n-winding transformer owns two identical windings, its equivalent circuit is easily reached by drawing the circuit of an $(n-1)$-winding one and, then, splitting one winding into two [4]. The second case is connected to dominant coupling. Here, it is supposed that two windings are strongly coupled (k very close to 1) whereas coupling with other windings are smaller. A proposed circuit is presented in [4]. In addition, despite the fact that leakage inductances are generally coupled, owing to the above criterion, sometimes some couplings of the leakage transformer can be neglected.

As a conclusion, to find a convenient equivalent circuit, it is recommended to distinguish two stages. In the first one, every pair of windings is characterized, through open- and short-circuit measurements. This leads to the full knowledge of the inductance matrix. Looking at this matrix, appropriate simplifications are found, and a final representation is chosen. The second stage aims at refining evaluations of some coupling of the leakage transformer. It needs measurement of the inductance with, at least, two short circuits. This need is due to the fact that coupling factors of the leakage transformer are deduced from those of the initial transformer via differences of comparable quantities.

IV. Experimental Method

As explained above, to characterize an n-winding transformer, we begin by $n(n-1)/2$ characterizations of two-winding couplings. This needs the measurement of the n open-circuit inductances and of $n(n-1)/2$ inductances with one winding short-circuited. One can see that total number of chosen measurements equals that of independent elements of inductance matrix.

A. How to Choose the "Best Measurements" to Acquire?

Because we are dealing with the magnetic part of the equivalent circuit that represents the transformer, we focus on the low-frequency side of the impedances which roughly ends at the lowest resonance frequency of the transformer. In that frequency range, transformer impedances are never too high to be measured accurately but, sometimes, they are too low! For this reason, we choose to first measure all open-circuit impedances which are the higher ones. Then, we must measure some impedances with one short circuit. During these measurements and, more generally, for low-impedance measurements, care must be paid to the short-circuit compensation.

B. Unavoidable Limit of Inductance Resolution Due to Wiring

In order to value practical limits of the short-circuit compensation procedure used when impedances are very low, a 250-W, 250-kHz transformer having a low-voltage secondary has been tested. The goal was to measure the short-circuit impedance seen from its secondary when its primary was short-circuited. Roughly, this impedance is that of a 1-mΩ resistor in series with a 12-nH inductance.

The short-circuit compensation procedure is mandatory for such low-impedance measurements. To be efficient, series impedance due to wires connecting the device to the analyzer must be as stable as possible. With this in view, we first avoid disconnection on both sides of the wires. Moreover, in order to avoid tiny distortions of metal pieces and consecutive contact resistance variations that occur when round wires are grip by flat pliers of the test fixture (Agilent 16047E), wires are welded on small flat pieces of copper are introduced in the test fixture (Fig. 3).

Second, to insure dimensional stability of interwire space (that modifies series inductance), short bifilar wire (even coaxial) is preferred to separate wires. On the opposite side, wire is welded to component pins. Because short-circuit compensation roughly computes a difference, the smaller the series impedance is, the more accurate the result will be. So, large cross-section wires located close together perform better.

Now, let us look at the short-circuit introduction. Fig. 3 depicts the measurement setup.

Fig. 4 shows two possible locations to introduce the short circuit. In the first case [see Fig. 4(a)], the short-circuit impedance is measured before an extra wire is cut. This procedure overestimates the wire impedance because the added short-circuit wire (between component terminals) is not present during the measurement. As a result, the measurement is slightly overcompensated.

In the second case [see Fig. 4(b)], the short circuit is introduced by putting a drop of welding on both half loops of the wires which are as close as possible to the component. In this case, the two short parts of the connecting wires are not compensated: the measurement is undercompensated.

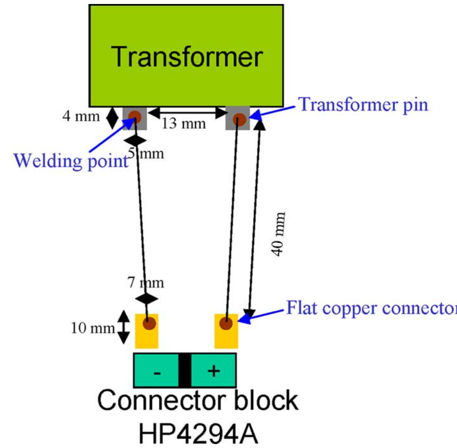


Fig. 3. Measurement setup.

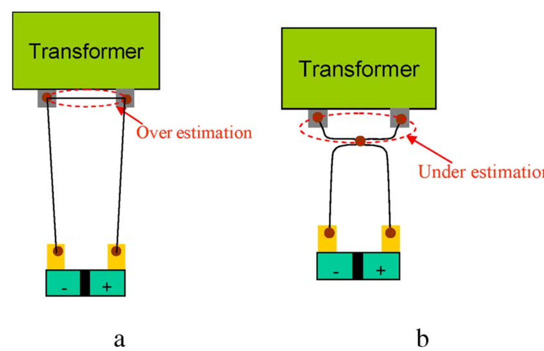


Fig. 4. Short-circuit compensation methods.

Fig. 5 shows the two resulting measurements. On the low-frequency side, the searched resistance is included between 0.5 and 1.5 mΩ (circle 1) that is consistent with 0.77 mΩ measured with an OM10 micro-ohmmeter [8]. On the high-frequency side, series inductance is between 4 and 20 nH. As a first conclusion, we can say that, using the mentioned apparatus and test fixture with plenty of precautions, the low-frequency series resistor is evaluated within ±0.5 mΩ and series inductance is known within ±8 nH. The low-frequency resistor limit is strongly linked to the resistance of the uncompensated wires: 1 mΩ is quite the resistance of a 26-mm-long, 0.7-mm-diameter cylindrical copper wire. Overcoming this limit is possible by using thicker wires or a more specialized measuring apparatus such as a micrometer.

To perform better in measuring small inductances, maybe a test fixture must be left and the four-wire connection of the impedance analyzer prolonged up to the component terminals. We have not yet verified this. Finally, let us underline that an overcompensated measurement exhibits negative resistances (circle 2) while the undercompensated does not. This is due to a parallel resonance of shortened wiring, which increases the serial resistance to 0.8 Ω at 10 MHz.

C. Identification of Parameters Describing a Pair of Windings

During the first stage of the identification, we characterize every pair of windings as independent two-winding transformers. The equivalent circuit of a two-winding transformer

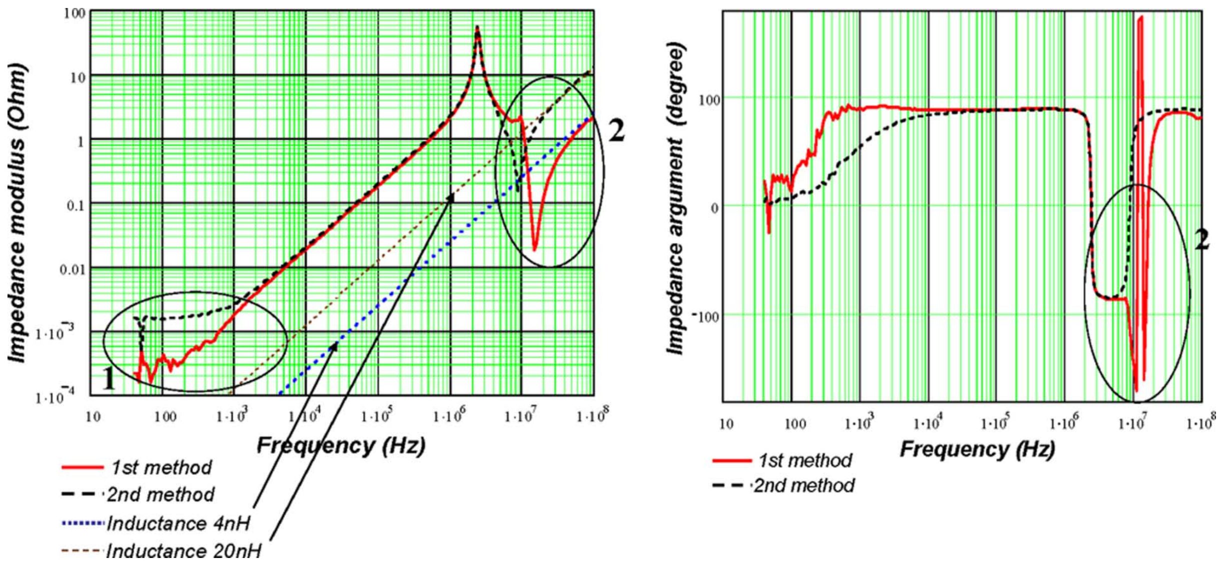


Fig. 5. Compensation method examples.

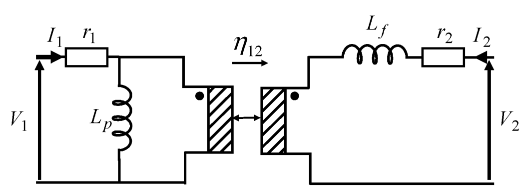


Fig. 6. Two-winding transformer.

TABLE I
MEASUREMENT POSSIBILITIES FOR A TWO-WINDING TRANSFORMER

Measurement	from winding	with
Z_0	1	$I_2 = 0$
Z_{cc}	1	$V_2 = 0$
Z_0'	2	$I_1 = 0$
Z_{cc}'	2	$V_1 = 0$

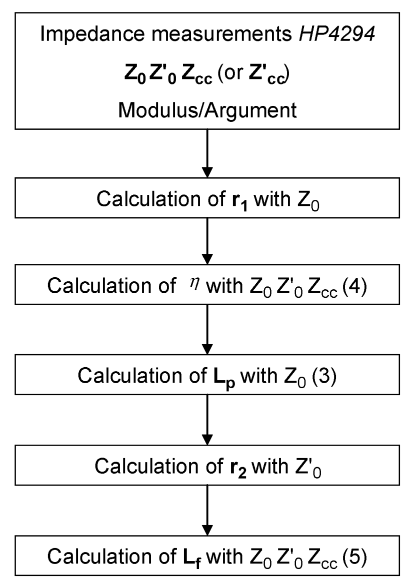


Fig. 7. Identification algorithm for a two-winding transformer.

is presented in Fig. 6. The circuit relative to magnetic coupling (Fig. 1) is completed by resistances located in series with both windings. These components represent dc resistances of each winding wire so the rest of the circuit must only account for dynamic losses.

With open and short circuit configurations, four impedances (Table I) can be measured.

Z_0 and Z_0' have the highest values, so generally they are easier to acquire. They are systematically measured, and r_1 and r_2 are found as their dc limits. Other impedances are smaller and, owing to (2), only one is still needed. Generally, the highest one is preferred because it is easier to measure.

Previous methods [1], [2] evaluated each inductance at one frequency only. Very often, open- and short-circuited inductances were measured at different frequencies, causing some inconsistencies. The method presented here leads to knowledge of each inductance over a wide frequency range. To reach this goal, we now consider that inductances are not perfect inductive components. They are studied as complex impedances (3)–(5) from which L_p and L_f are easily extracted. In the same way, the coupling ratio η [cf. (4)] is not supposed to be real:

$$Z_p = Z_0 - r_1 \tag{3}$$

$$\eta^2 = \frac{Z_0'(Z_0 - Z_{cc})}{(Z_0 - r_1)^2} = \frac{Z_0\,(Z_0' - Z_{cc}')}{(Z_0 - r_1)^2} \tag{4}$$

$$Z_f = \frac{Z_0'(Z_{cc} - r_1)}{Z_0 - r_1}. \tag{5}$$

In addition, we acquire the two impedances seen from the same pair of terminals without any dismounting. This warrants contact impedances of wiring to be the same during both measures so that their difference, which appears in (4), is almost exact.

Fig. 7 shows the algorithm which leads to the five parameters of a two-winding transformer (Fig. 6). For transformers

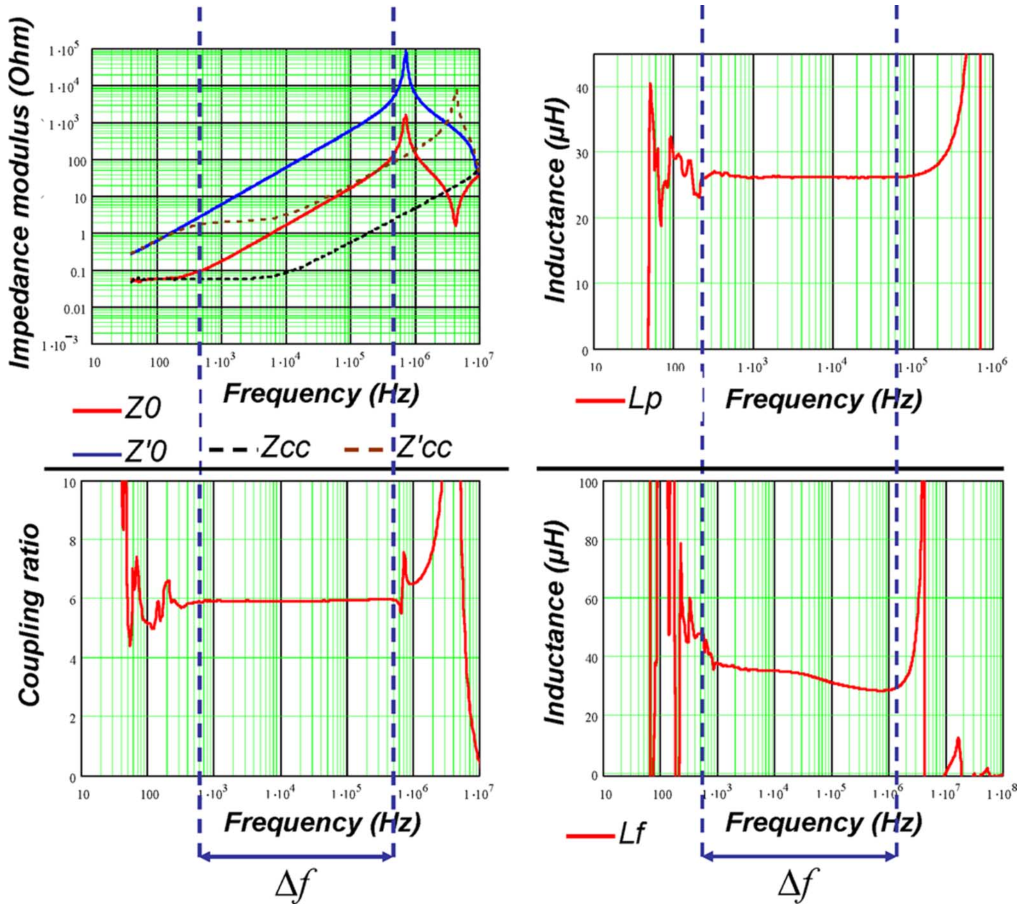


Fig. 8. Identification process for a two-winding 100-kHz 150-W transformer.

having more than two windings, we first characterize every pair of windings. Then, if necessary, we acquire impedances with two or more short circuits in order to precise the elements of the leakage transformer.

Curves obtained on a three-winding, 100-kHz, 150-W ring core transformer are shown in Fig. 8. Because magnetic coupling is masked by series resistances on the low-frequency side and by capacitances on the high-frequency side, its characteristic values are not accurately valued in these regions. Despite this, they are precisely evaluated on a frequency range Δf, which extends over about three decades. In this domain, magnetizing inductance and coupling ratio are quite perfectly constant and leakage inductor decreases with the increase of frequency: that is a well-known effect due to eddy currents. The way of representing such a frequency varying inductance is presented in the following example.

D. Three-Winding Planar Transformer

With an adequate design, planar transformers provide very high power efficiency and efficient power loss draining. The one studied here has three windings (i.e., primary, secondary, and auxiliary). The basic equivalent circuit of such a device is presented in Fig. 2.

Table II shows the 12 considered impedance measurements. We first characterize every pair of windings as independent two-winding transformers. Three measurements are necessary to find the equivalent circuit for the primary/secondary transformer. Then, two more are needed for the primary/auxiliary transformer. Finally, one more is useful to know well the coupling of the leakage transformer drawn between the secondary and the auxiliary. To sum up, six measures are used: three with

TABLE II
MEASUREMENT POSSIBILITIES FOR A THREE-WINDING TRANSFORMER

Measurement	from winding	with	
$Z_{1\_2o\_3o}$	1	$I_2=0, I_3=0$	0 short circuit
$Z_{2\_1o\_3o}$	2	$I_1=0, I_3=0$	
$Z_{3\_1o\_2o}$	3	$I_1=0, I_2=0$	
$Z_{1\_2cc\_3o}$	1	$V_2=0, I_3=0$	1 short circuit
$Z_{1\_2o\_3cc}$	1	$I_2=0, V_3=0$	
$Z_{2\_1cc\_3o}$	2	$V_1=0, I_3=0$	
$Z_{2\_1o\_3cc}$	2	$I_1=0, V_3=0$	
$Z_{3\_1cc\_2o}$	3	$V_1=0, I_2=0$	
$Z_{3\_1o\_2cc}$	3	$I_1=0, V_2=0$	
$Z_{1\_2cc\_3cc}$	1	$V_2=0, V_3=0$	2 shorts circuits
$Z_{2\_1cc\_3cc}$	2	$V_1=0, V_3=0$	
$Z_{3\_1cc\_2cc}$	3	$V_1=0, V_2=0$	

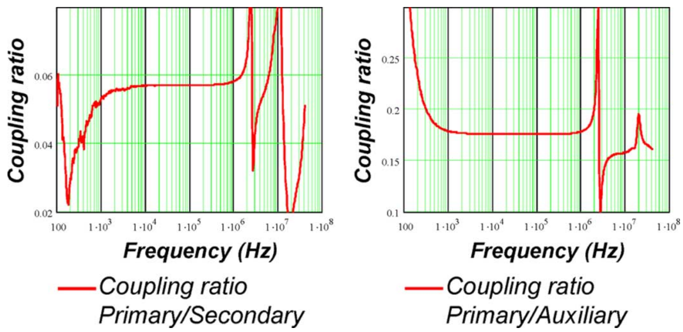


Fig. 9. Primary/secondary and primary/auxiliary coupling ratios.

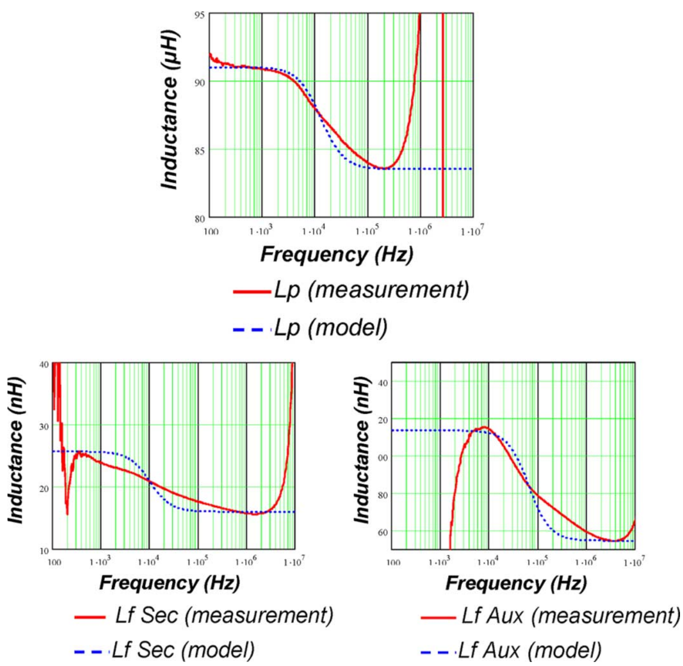


Fig. 10. Magnetizing and leakage inductances.

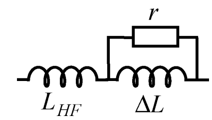


Fig. 11. Inductance variation representation.

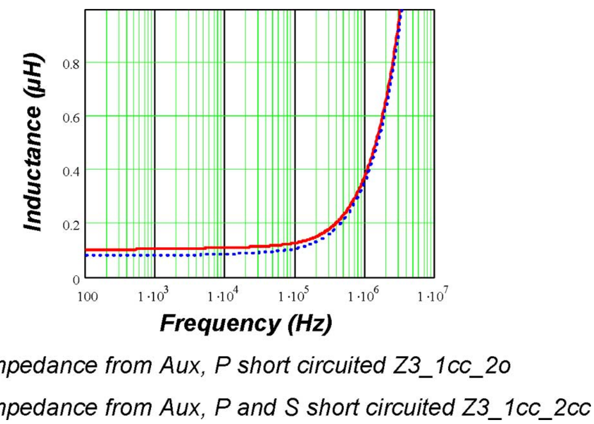


Fig. 12. Influence of leakage coupling.

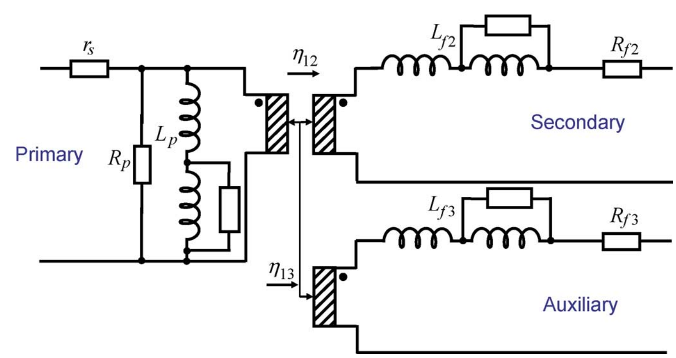


Fig. 13. Three-winding planar transformer equivalent circuit.

open circuits, two with one short circuit, and one with two short circuits.

For this transformer, measurements from secondary are very difficult because of their low impedance values (0.5 mΩ). Inaccuracy of these measurements at low frequencies induce imprecision on all values calculated according to (3)–(5). Coupling ratios (Fig. 9) are quite constant and real between 1 kHz and 1 MHz. Above, capacitors must be taken into account.

Inductances calculated with (3) and (5) are presented with their corresponding models (dotted line) in Fig. 10. Accurate measurement of leakages inductances below 500 Hz for the secondary and 10 kHz for the auxiliary is difficult because, in these regions, resistors dominate inductances. One particularity of this transformer is that all of its inductances decrease with the increase of frequency. Even magnetizing inductance is not constant.

This kind of variation can be represented by the circuit shown in Fig. 11. At low frequency, the inductance is equal to $L_{\mathrm{BF}} = L_{\mathrm{HF}} + \Delta L$. For higher frequencies, equivalent inductance is simply equal to L_{HF}. This rough model often supplies a sufficient representation. If necessary, a small number of parallel cells connected in series can fit the real variation of the complex impedance over a wide frequency range.

To refine the characterization of the coupling between secondary and auxiliary, a measurement with two short circuits is needed. Because of the low impedance value of the secondary, the one seen from the auxiliary is preferred. Fig. 12 shows that the coupling between these two windings is so low that it can be neglected. Consequently, the leakage coupler is removed. In a different case, when leakage coupling is not negligible, using the complete equivalent circuit allow primary resistance to be taken into account during its identification.

The inductive model of the transformer is presented in Fig. 13. Comparisons between measurements and Pspice simulations based on our model (Fig. 14) show a very good agreement. Only two impedances are presented, one with no short circuit and one with one short circuit. Agreement is also good for all other impedances.

The inductive model is now validated. Of course, it must be completed by the capacitive one to allow reliable and accurate simulations. Because equivalent circuits are checked over a wide frequency range, simulations are accurate regardless of the waveforms, as long as the transformer behavior remains linear.

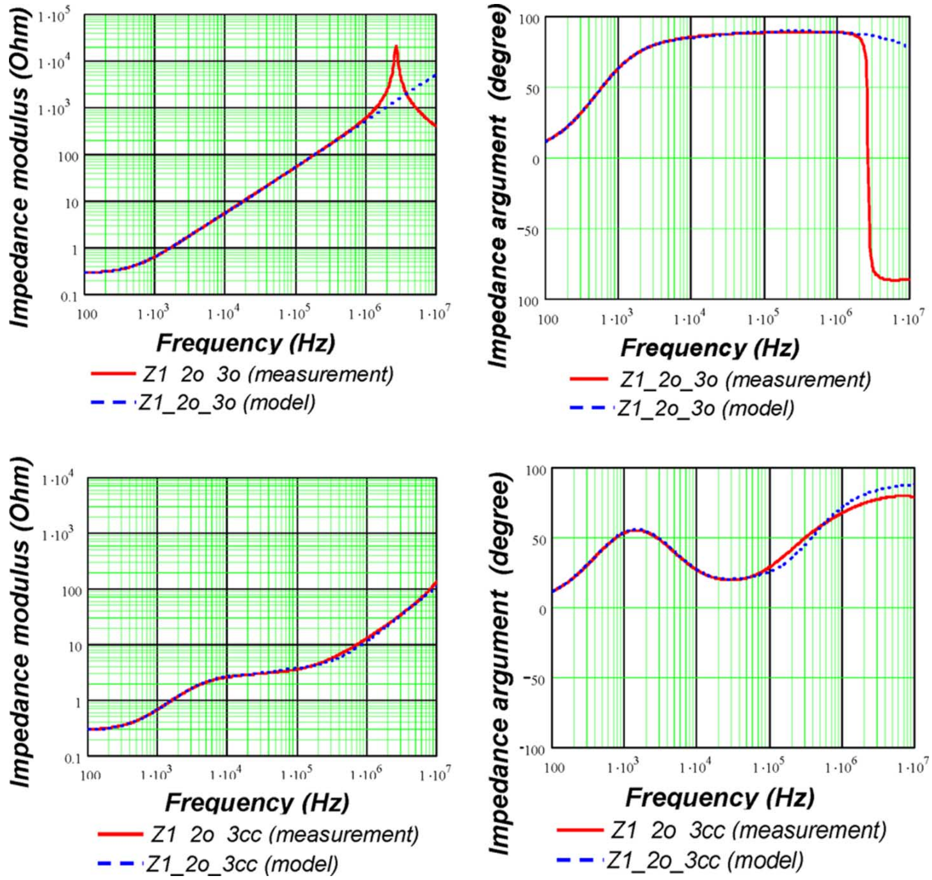


Fig. 14. Comparisons between measurement and model simulation.

V. Conclusion

Experimental characterization of multiwinding transformers is now mature. Theoretical developments have given reliable, general equivalent circuits. This paper presents a general method intended for the experimental identification of these equivalent circuits.

The accuracy target has been clearly enunciated according to the users' needs, and measured impedances have been chosen to reach this target. Indispensable precautions to use the impedance analyzer at best are also described and checked. Results relative to a two-winding ring core transformer and to a three-winding planar one have been given.

References

[1] J. P. Keradec, A. Schellmanns, J. L. Schanen, and K. Berrouche, "Representing electrical behaviour of transformers by lumped element circuits. A global physical approach," in *Proc. IEEE Ind. Appl. Soc.*, Phoenix, AZ, Oct. 3–7, 1999, pp. 2100–2107.

[2] A. Schellmanns, K. Berrouche, and J. P. Keradec, "Multiwinding transformers: a successive refinement method to characterize a general equivalent circuit," in *Proc. IEEE Instrum. Meas. Technol. Conf.*, St. Paul, MN, May 18–20, 1998, pp. 717–722.

[3] B. Cogitore, J. P. Keradec, and J. Barbaroux, "Two winding ferrite core transformer: an experimental method to obtain a wide frequency range equivalent circuit," *IEEE Trans. Instrum. Meas.*, vol. 43, no. 2, pp. 364–371, Apr. 1994.

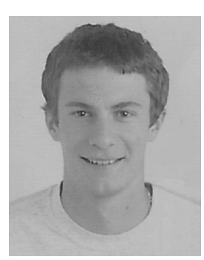
[4] X. Margueron and J. P. Keradec, "Design of equivalent circuits and characterization strategy for n-input coupled inductors," *IEEE Trans. Ind. Appl.*, to be published.

[5] A. Schellmanns, J. P. Keradec, and J. L. Schanen, "Electrical equivalent circuit for frequency dependant impedance: minimum lumped elements for a given precision," in *Proc. IEEE Ind. Appl. Soc.*, Rome, Italy, Oct. 3–7, 2000, vol. 5, pp. 3105–3110.

[6] *"Agilent 4294A Precision Impedance Analyser-Operation Manual,"* Agilent Technologies.

[7] Magnetic Circuits and Transformers ch. XVII, 5, p. 454, by the EE Staff at MIT, McGraw-Hill. New York, 1943.

[8] OM 10 Microhmmetre, Notice d'utilisation, AOIP Instrumentation.



Xavier Margueron was born in Chambéry, France, in 1980. He received the Diplôme d'ingénieur in electrical engineering from the Ecole National Supérieur d'Ingénieur Electricien de Grenoble (ENSIEG), Grenoble, France, and the Diplôme d'études approfonfies (DEA) from the Institut National Polytechnique de Grenoble (INPG), both in 2003. He is currently working toward the Ph.D. degree at the Laboratoire d'Electrotechnique de Grenoble.

Jean Pierre Keradec (M'92) was born in Paris, France, in 1947. He received the doctorat de physique du solide degree from the University of Grenoble, Grenoble, France, in 1973.

He currently teaches analog electronics, signal processing, and measurement systems design at the Institut Universitaire de Technologie of Grenoble and at the Institut National Polytechnique de Grenoble. He joined the Laboratoire d'Electrotechnique de Grenoble in 1985. His research interests mainly include high-frequency power transformer modeling and measurements related to high-frequency static power converters.