3D Analytical Calculation of the Interactions between Permanent Magnets

Hicham Allag, Jean-Paul Yonnet

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Hicham ALLAG and Jean-Paul YONNET*
Laboratoire de Génie Electrique de Grenoble, G2E Lab (UMR 5269 CNRS/INPG-UJF),
Institut Polytechnique de Grenoble, ENSIEG, BP46, 38402 St Martin d'Hères cedex, France

Abstract

Up to now, the analytical calculation has been made only when the magnets own parallel magnetization directions. We have succeeded in two new results of first importance for the analytical calculation: the torque between two magnets, and the force components and torque when the magnetization directions are perpendicular. The last result allows the analytical calculation of the interactions when the magnetizations are in all the directions.

The 3D analytical expressions are difficult to obtain, but the torque and force expressions are very simple to use. As example the analytical expression can be included in optimization software allowing to directly obtaining the shape optimization by a fast way.

Keywords:
Analytical Calculation, Permanent Magnets, 3D, Interaction Energy, Force, Torque

Contact Author:
Jean-Paul YONNET,
G2E Lab, Laboratoire de Génie Electrique de Grenoble,
ENSIEG, BP46, 38402 St Martin d'Hères, France.
Tel: 33 (0)4 76 82 62 97,
Fax: 33 (0)4 76 82 63 00,
Email: **Jean-Paul.Yonnet@g2elab.inpg.fr**

1- Introduction

The analytical expressions are a very powerful and a very fast method to calculate magnetic interactions. It is why the analytical expressions of all the interactions, energy, forces and torques between two cuboïdal magnets are very important results. Many problems can be solved by the addition of element interactions. The simpler shape of elementary volume is the parallelepiped, with its cuboïdal volume. It is why many 3D calculations can be made by the way of 3D interactions between two elementary magnets of cuboïdal shape.

Up to now, only the force components between two magnets with their magnetization direction parallel to one edge of the parallelepipeds have been analytically solved. We have succeeded in two new results of first importance for the analytical calculation:

- the torque between two magnets,
- the force components and torque when the magnetization directions are perpendicular.

Consequently, by combining parallel and perpendicular magnetization directions, the interaction energy and all the components of the forces and the torque can be calculated by fully analytical expressions, for any magnetization direction, and for any relative position between the two magnets. The only two hypotheses are that the magnets own a cuboïdal shape, and they are uniformly magnetized.

2- 3D analytical calculation background

Since the discovery and the development of Samarium Cobalt magnets in the 70 years, the designers can use magnets owning a really rigid magnetization. They are the first magnets which can be used in repulsion without any risk of demagnetization. Their magnetization can be easily modelled by magnetic charges on the poles, or by equivalent currents.

One of the first applications of Samarium Cobalt magnets were the magnetic bearings. Due to the circular symmetry, the calculation can be made in 2D. The first 2D analytical expressions of the forces between magnets were given by Marina Marinescu [1] and Jean-Paul Yonnet [2]. The stiffness of a magnetic bearing can be easily calculated by analytical expressions [3].

The magnetic couplings are another application working by interaction forces between permanent magnets. When their length is long in comparison with the airgap dimensions, the 2D analytical calculation can be used [4]. Otherwise it is necessary to use 3D calculation.

The 3D analytical calculation is obviously more difficult than the 2D. It is not 2 but 4 successive analytical integrations which must be calculated, and the difficulty fastly increases with the number of integrations. Many persons thought that the last analytical integration was not possible, and must be made by a numerical way. We had worked on this problem, and we have succeeded in solving the calculation. At that time all the integrations had been made by brain-work, with a pen and a piece of paper.The first 3D forces analytical expressions were published in 1984 by Gilles Akoun and Jean-Paul Yonnet [5].

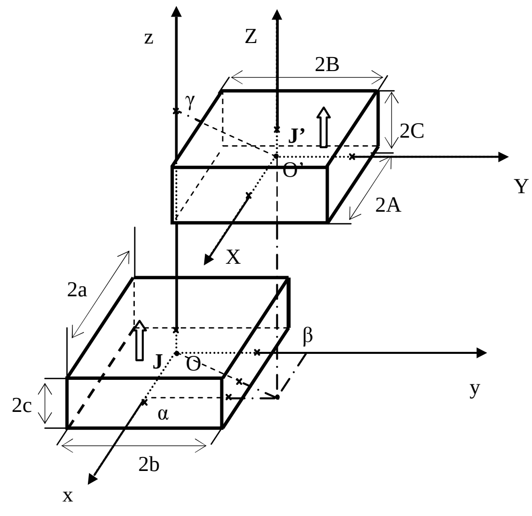

Figure 1: Magnet configuration

The forces were analytically calculated for two cuboïdal magnets with parallel magnetization directions (Figure 1). The calculus was made by the way of interaction energy determination. The forces expressions were obtained by derivation of the energy expression.

A new contribution to the 3D calculation has been presented by Edward P. Furlani [6] in 1993 and Yeong-Der Yao in 95 [7], with the analytical calculation of multipolar magnetic couplings with discoïdal shape. But the last integration is made by a numerical way, or by a series calculation.

In 1998, Fréderic Bancel, a mathematics teacher, reaches to obtain the analytical expressions of forces when the two magnets are in angled position. The force calculus was made by direct integration of magnetic induction. The result has never been published [8]. Several colleagues of his laboratory, around Guy Lemarquand, have diffused the Bancel results by using them to calculate magnetic couplings. They have published a series of articles on the subject [for example 9,10].

An original approach of the 3D analytical calculation has been presented by Fréderic Bancel in 1999, by proposing the notion of magnetic nodes [11]. It comes from the fact that the forces expressions are only function of the distance between the corners of the magnets.

3- New results in 3D analytical calculation

Until now, all the analytical force calculation has been made for cuboïdal magnets which magnetization is parallel to one of the edge of the magnet. It means that the magnetic poles are only on two rectangular faces of the magnet [12]. Recently, we have worked again on the 1984 formulations, and we succeed in the analytical interaction energy calculation when the magnetization directions of the two cuboïdal magnets are perpendicular (Figure 2). From the expression of energy, the forces can be easily calculated by derivation. It is an important step, because its opens the way to the calculation of the interactions when the magnetization directions are in every direction. When the two magnetization directions are not parallel to the magnet edges, it can be decomposed in its three projections on the axes. As it is a fully linear problem, the whole energy can be calculated by addition of the interactions between the magnetization components. From the energy expression, the force can be obtained by derivation. For the first time, the fully analytical force expressions can be written whatever the magnetization directions are.

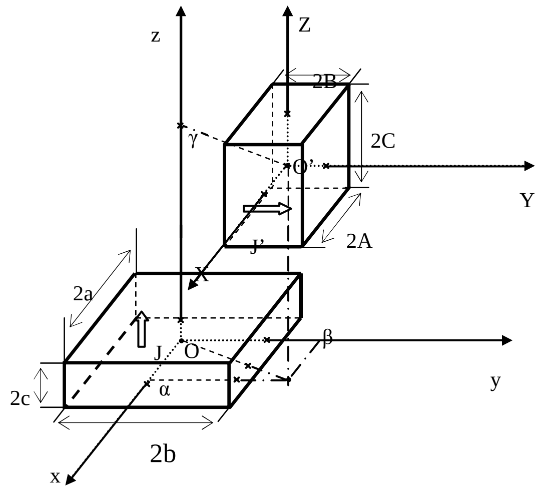

Figure 2: Magnet configuration with perpendicular magnetization directions

Another important step has been made when we succeed in the analytical calculation of the torque component. The use the equivalence between charged surfaces and magnetic nodes allows obtaining an energy repartition on the magnet corners. The torque on the magnets can be analytically obtained by angular derivation of interaction energy. Another method of torque calculation is based on the force components exerted on the parallelogram corners. It leads to the same results. The three torque components between two magnets can be expressed by fully analytical expressions. By combining the magnetization components, the torque can be obtained for all the geometrical and magnetic cases.

In summary, all the interactions (energy, forces and torques) between two cuboïdal magnets can be analytically calculated, whatever the magnetization directions are and whatever the parallelepiped orientations are.

These original results represent a

voluminous work. They have never been published for the moment, except two recent publications in conferences more oriented towards the applications of permanent magnets:

- the full analytical expressions of the torque calculation in the case of magnets with parallel magnetization directions [13],
- the full analytical expressions of the interaction energy and the force components for any magnetization directions [14].

The size allowed for this paper is not sufficient to give full analytical expressions. These original results and their applications will be shown at the time of the oral presentation in the conference.

4- The basic mathematical model

The interactions between two parallelepiped magnets are studied. Their edges are respectively parallel (see Figure 1). The magnetizations J and J' are supposed to be rigid and uniform in each magnet. The dimensions of the first magnet are 2a x 2b x 2c, and its polarization is J. Its centre is O, the origin of the axes Oxyz. For the second magnet, the dimensions are 2A x 2B x 2C, its polarization is J', and the coordinates of its centre O' are (α, β, γ). The side 2a is parallel to the side 2A, and so on.

The magnet dimensions are given on Table 1.

Axis	Ox	Oy	Oz
First Magnet (J)	2a	2b	2c
Second Magnet (J')	2A	2B	2C
Second Magnet Position O'	α	β	γ

Table 1: Magnet dimensions and position

The magnetization directions shown on Figure 1 correspond to the case when the polarizations J and J' have the same direction, parallel to the side 2c. Note that the calculation stay valid when they are in opposite direction; only the expression sign is reversed.

The polarizations J and J' are supposed to be rigid and uniform. They can be replaced by distributions of magnetic charges on the poles. It is the coulombian representation of the magnetization. Their density σ is defined by:

$$\sigma = \vec{J} \cdot \vec{n}$$

On the example of Figure 1, since J is perpendicular to the surfaces 2a x 2b and oriented to the top, these polar faces wear the density σ = +J on the upper face (North Pole), and σ = -J on the lower face (South Pole).

All the analytical calculations have been made by successive integrals, to calculate the interaction energy for the two magnets system. The forces and the torques can be obtained by linear and angular derivation. The analytical calculation of the interaction energy in 3D is made by four successive integrations. The first one gives a logarithm function. In the second one, you have two logarithm and two arc-tangent functions… The last one owns many complex functions based on logarithm and arc-tangent functions. The interaction energy expression has 256 terms. It seems to be complicated, but it can be used very easily. A simple pocket calculator for students is sufficient to obtain the results.

5- Examples of application

The analytical expressions are fully useable for the calculation of many systems working by permanent magnet interactions, like bearings or couplings.

But it is not the only type of application which can be calculated. Very often, permanent magnets are used with iron yokes for flux closing. These iron yokes can be taken into account by image effect. The magnets and the yokes can be replaced by the magnets and the images of the magnets.

Another family of problems are the electromagnetic applications. Some of them can be directly solved by analytical expressions. For example, to calculate the force and torque between coils and magnets, the coils can be seen as Amperian currents around magnets, and the problem can be directly solved by magnet interactions (Figure 3).

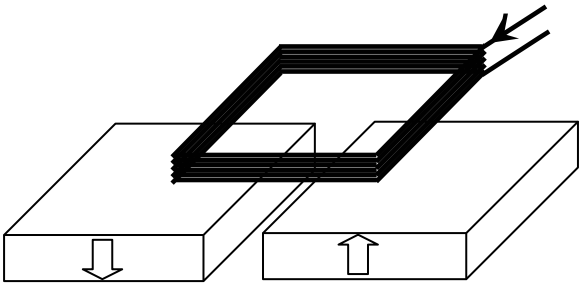

Figure 3: Interaction between a coil and two magnets

The analytical expression can be used in software making direct shape optimization. CADES is an example of such software [14]. In a first step, the problem is described by a linear model, including analytical expressions. In a second step, the problem is reversed, and the optimized shape is obtained. The Figure 4 shows an example of such optimization for a magnetic coupling.

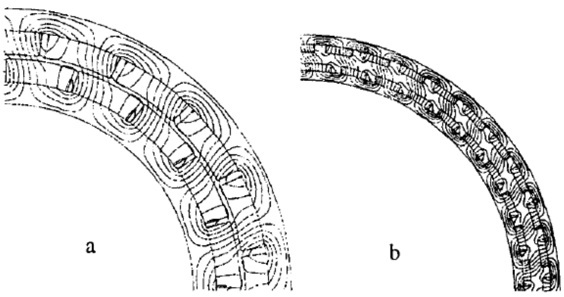

Figure 4: Geometry of the initial (a) and optimised (b) magnetic coupling

10- Conclusion

All the analytical expressions of the interaction energy, the force and torque components between two cuboïdal magnets have been calculated for any magnetization directions. The only hypotheses are that the magnets own a parallelepiped shape, and that are uniformly magnetized. These results allow the analytical calculation of the interactions when the magnetization directions are in any direction.

Many problems can be solved by these results. The simpler shape of elementary volume is the parallelepiped, with its cuboïdal volume. By the superposition of 3D interactions between elementary magnets, many 3D calculations can be made. These results allow calculating not only the direct interaction of permanent magnets, but also many other devices including soft magnetic materials, and currents.

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