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Abstract

We revisit the common view that risk sharing enhances risk taking in the context of heterogenous risk sharing in a small economy. Under low volumes of transfers, we express individual risk level in terms of Bonacich measure. We find that heterogeneity combined to strategic interaction imply that risk sharing enhances risk taking only in average. However, under high transfer volumes, risk sharing may reduce risk taking. We also provide conditions under which agents under or over invest with respect to the risk allocation maximizing the sum of profits.

JEL Classification Numbers: D85, C72, G11

Keywords: Risk taking, Heterogenous risk sharing, Strategic Interactions, Bonacich measure

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1 Introduction

In many economic contexts, a redistribution of incomes in a society of risk averse agents affects risk taking. For instance, in labor markets, unemployment insurance encourages workers to seek higher productivity jobs because they are more willing to endure the possibility of unemployment; similarly, redistributive taxation enhances entrepreneurship¹. Hence, the general message is that social insurance enhances individual risk taking. The conclusion is of sizeable importance, since it has an impact on innovation and growth.

This article revisits this issue in a context of heterogenous risk sharing and strategic interactions. A typical example is that of a small economy, say a developing village, in which agents insure themselves against risk by sharing revenues². First, strategic interactions can emerge from the sharing of correlated incomes. Second, informal insurance may be heterogenous across agents. For instance, if there is no formal institution to enforce a redistribution mechanism, heterogeneity in risk sharing may arise because of self-enforcing mechanisms and trust (for instance social sanctions may be heterogenous), and also the heterogeneity in information flows and moral hazard, in income correlations, in geographic costs, increasing costs to group size, etc.

The main objective of this article is to examine how risk sharing heterogeneity affects individual incentives to take risk. To proceed, we consider a society of risk-averse agents. Each agent has one unit to invest in a specific project. Projects are developed

¹See Acemoglu and Shimer [1999, 2000], Mayshar [1977], Kanbur [1981], Zeira [1988], Boadway *et al.* [1991], Sinn [1996], Garcia-Penalosa and Wen [2008].

²Townsend (1994) emphasizes the importance of informal insurance networks in Indian villages; similarly, Udry (1994) documents that the majority of transfers take place between neighbors and relatives in Northern Nigeria - see also Rosenzweig (1988), Murgai *et al.* (2002), Fafchamps and Lund (2003), Dercon and De Weerd (2006), Fafchamps and Gubert (2007), De Weerd and Fafchamps (2007). Recent theoretical advances about risk sharing in developing economics have studied the impact of the structure of the social insurance networks on the volume of transfers, whether rules are self-enforcing (Ambrus *et al.* (2007), Bloch *et al.* (2007, 2008)) or not (Bramoullé and Kranton (2006, 2007), and Gallegati *et al.* (2008)). With regard to this theoretical literature, our work allows for endogenous income, and heterogenous transfers across agents.

through investment in a portfolio of two correlated technologies A and B . We assume without loss of generality that technology A is more profitable but entails more risk. After investing, incomes are realized, and then agents proceed to transfers.

As a first key ingredient of our model, we consider mean-variance utility functions, and we assume that technologies are correlated. Hence, agents are not only exposed to idiosyncratic factors, but also to systemic factors. In this context, diversification via resources pooling enables to reduce specific risk.

The second key ingredient concerns the rule of transfers, that we assume exogenous and binding³. We represent the structure of transfers by a matrix $\Lambda = [\lambda_{ij}]$, where λ_{ij} represents the *share* of agent j 's income that agent j transfers to agent i . To focus on pure risk sharing and rule out wealth effects⁴, we assume that expected after-transfer revenues are equal. This assumption implies that the matrix is row-stochastic. Moreover, it is column-stochastic by budget constraint (in total the matrix is bi-stochastic).

We ask the following questions. How does the heterogeneity of risk sharing drive individual risk choice? Does more revenue sharing enhance risk taking? Do agents under-invest or over-invest with respect to what is socially optimal? Last, how does cash transfers affect the overall investments in the more risky technology?

We first point out that if the excess return of technology A relative to B is positively (resp. negatively) correlated among agents, risk levels are strategic substitutes (resp.

³The issue of self-enforcement is beyond the scope of this article. Actually, this problem is limited by two factors in developing countries. First, agents may not exchange their whole income with neighbors (of course, a self-enforcing mechanism may have designed low-volume transfers). Second, agents generally know each other for a long time and do not often move during their lives, which enforces trust and cooperation (social sanction are strong). “For developing countries, most researchers [...] argue[...] that information and enforcement problems are likely to be small between the members of a village and this creates a favorable environment for cooperation”, Dercon and De Weerd (2006).

⁴Of course, wealth inequality can generate different risk levels; first risk aversion is in general wealth dependent, and second poor agents may not be able to access all technologies. We study another mechanism producing differentiated risk levels.

complements). We then focus on the case of strategic substitutability, which is more plausible (in particular this is the case if technology B is risk free).

We gradually introduce heterogeneity in income transfers. As a benchmark, we first examine the (full and partial) equal sharing rule, which stipulates that agents share equally an identical proportion of their realized incomes. In the polar case in which projects are not correlated, equal sharing enhances investments in (the higher risk/return) technology A . Due to correlations between transfers, correlated incomes affect the attractiveness of technology A under risk sharing. We therefore state a condition (condition C_0 thereafter) which guarantees that technology A stays more attractive under full equal sharing, and we assume that this condition holds for the rest of the paper. In particular, this condition guarantees technology A stays more attractive under partial equal sharing.

Then we turn to the general case. The transfer matrix can be interpreted as a network, in which the value of connection ij is the share that agent j gives to agent i . First we consider the case in which own shares are homogenous. We find that equilibrium risk levels are homogenous (but not profits) and decreasing in the value of own share. That is, two transfer matrices with same homogenous own share but different off-diagonal elements generate the same equilibrium risk profile. Hence, heterogenous risk levels only emerge from heterogenous own shares.

Second, we examine the case of heterogenous own shares. In general, an optimal risk profile exists and it is unique. We then separate the space of transfer matrices in two regions, region \mathcal{R} in which risk sharing is *weak*, region $\bar{\mathcal{R}}$ in which risk sharing is *large*. Our main theoretical results concern region \mathcal{R} . In this region, risk levels can be interpreted as a Bonacich measure of a transformation of the transfer matrix⁵. Furthermore, with risk sharing, agents take more risk than in isolation. Last, we prove

⁵This measure has been introduced in Bonacich (1987). Ballester *et al.* (2006) renewed the idea in the field of economics. Two recent papers link Bonacich centrality and optimal decisions: see Ghiglino and Goyal (2008) for a model of a pure exchange economy with a positional good; see Corbo *et al.* (2007) and Bramoullé *et al.* (2008) for models of local public goods, Bloch and Qu  rou (2008) for a model of oligopoly with local externalities among consumers. In these latter works, as well as in ours, actions are strategic substitutes.

that increasing revenue sharing enhances risk taking, but only in average; typically certain agents may take less risk.

In region $\bar{\mathcal{R}}$, in which risk sharing is large, simple examples illustrate that more revenue sharing can reduce the average risk level. Moreover, certain risk level may be lower than the level of risk taken by an isolated agent. Therefore, a deep conclusion is that depending on the volume of risk sharing, heterogeneity in risk sharing may affect risk taking in opposite directions.

Then we focus on weak risk sharing. We examine the tension between individually optimal risks and the risk profile maximizing the sum of individual payoffs. We first notice that the game may generate either positive or negative externalities. This arises from a simple tradeoff: when some agent increases investment in technology A , this has an ambiguous effect on the profits of agents who will receive some transfer from her, by raising both expected means and volatility of future transfer.

We first characterize the efficient risk profile as a Bonacich measure defined over a matrix which aggregates all these externalities. Then we turn to the comparison of the average equilibrium risk level and the average efficient risk profile. Given that the sign of externalities is endogenous, agents may either under-invest or over-invest with respect to social welfare. Actually, agents under-invest (resp. over-invest) in situations in which they do not internalize that an increase of their risk level would promote (resp. reduce) significantly the return of technology A for others - because the induced increase in expected means dominates (resp. is dominated by) the raise of variance. We do relate the comparison of the average equilibrium risk level and the average efficient risk profile to conditions on the transfer matrix. We find that if each agent's own share is larger than (resp. is lower than) the sum of squares of shares she transfers to others, then agents under-invest (resp. over-invest) in technology A with respect to the efficient profile.

Finally, an extension examines the issue of cash transfer or wealth redistribution. We determine the impact of a modification of initial wealths on the total amount of investments that the society devotes to technology A . The result has important implications

for economic policy. As a first illustration, we present a set of matrices for which the identity of eligible households does not matter. This case happens to be that of societies with homogenous own shares. In a second application, one agent receives a positive shock on her initial wealth, for instance as being eligible to cash transfer by an aid program. We characterize the agent to be selected in order to generate a maximal increase in the overall investments made in technology A . Typically, agents with maximal equilibrium risk (before the treatment) or with lowest own share may not be those to select.

The article is organized as follows. Section 3 describes the model. Section 4 studies the existence and characterization of equilibrium, and some comparative statics. Section 5 examines efficiency issue, and section 6 is an extension exploring cash transfer issue. The last section concludes. All proofs are in the appendix.

2 The model

We consider a three-stage game in which, first, agents choose their level of risk, second, incomes are realized, and third agents share simultaneously some part of their revenues. To fix ideas, consider a village economy, in which risk averse farmers have one unit of land to use for the plantation of potatoes and strawberries (the latter variety having higher expected means and being more risky). After planting, incomes are realized and farmers share some part of their revenues.

Formally, the society contains a finite set $N = \{1, 2, \dots, n\}$ of agents. Each agent i has one unit to invest in two technologies A and B . The return Y^j of a technology $j \in \{A, B\}$ is random with expected mean μ_j and variance σ_j^2 . Agents are risk-averse and care about expected means and variance of the returns, *i.e.* the utility of agent i investing in technology Y^j is $U_i(Y^j) = E(Y^j) - \frac{\kappa}{2} \text{Var}(Y^j)$. We suppose that both technologies are intrinsically attractive, *i.e.* $\mu_j > \frac{\kappa}{2} \sigma_j^2$ for $j = A, B$. Further, technology A has greater expected return than technology B and is more risky, that is, $\mu_A > \mu_B$ and $\sigma_A^2 > \sigma_B^2$ (one possible interpretation is that technology A is an innovation). Let σ_{AB} represent the covariance between technologies A and B in a same project. Now

we define correlations between projects. We let τ_A (resp. τ_B) represent the correlation between a project that uses technology A (resp. B) and a distinct one that uses technology A (resp. B). Further, τ_{AB} is the covariance between technologies A and B in two different projects. Economically, $\tau_A, \tau_B, \tau_{AB}$ correspond to systemic risks, while $\sigma_{AB}, \sigma_A^2, \sigma_B^2$ incorporate both specific and systemic risk.

A strategy for agent i is a scalar $x_i \in [0, 1]$. It represents the share of investment that agent i devotes to technology A . The study of corner solutions is beyond the scope of this article; we obtain solutions in the interval $[0, 1]$ for a large range of parameters. We interpret x_i as the level of risk chosen by agent i . Note that if agents had a binary choice between A and B , $x_i \in [0, 1]$ could be interpreted as a probability, and we would solve a Bayesian equilibrium. Choosing some level x_i , agent i selects therefore technology $Y_i = x_i Y^A + (1 - x_i) Y^B$. We let $X = (x_1, x_2, \dots, x_n)$ denote a strategy profile of the society.

To insure against risk, agents share part of their realized incomes with others. In our setting, the realization of individual incomes is observable by all others. Furthermore, the sharing rule is exogenous and binding. Transfers are described by a *transfer matrix* $\Lambda = [\lambda_{ij}]$, where $\lambda_{ij} \in [0, 1]$ represents the share of the realized income of agent j that agent i transfers to agent i . Parameter λ_{ii} represents agent i 's *own share*. By budget constraint, $\lambda_{ii} + \sum_{j \neq i} \lambda_{ji} = 1$ for all i . Furthermore, to isolate risk sharing from wealth effects, we suppose that *ex ante* after-sharing revenues are equal. This implies that own share plus the proportions of revenues that agent i receives from others sum up to 1, *i.e.* $\lambda_{ii} + \sum_{j \neq i} \lambda_{ij} = 1$ for all i . Therefore, the matrix Λ is bi-stochastic. This set of rules encompasses the possibility of *ex ante* asymmetric *bilateral* transfers⁶. Among all possibilities covered by our setting, a polar case is the egalitarian sharing of realized incomes⁷.

⁶In line with the existence of asymmetric transfers, the redistributive role of transfers has been emphasized, as in Lucas and Stark (1985) or Azam and Gubert (2006). Recent work on child fostering focuses on the role of extended family ties as key determinant of insurance against persistent shocks, such as how to deal with the loss of a parent or husband, like in Dufló (2003) or Ksoll (2007).

⁷This is for instance the case in Bramoullé and Kranton (2006, 2007). Interestingly, this particular rule happens to be the solution of optimal decisions of costless link formation in our model (see remark

Having defined the matrix of transfers, we relate individual profits to transfers. We consider a profile of choices $X = (x_1, x_2, \dots, x_n)$. Agent i 's profit writes:

$$\pi_i(X) = \sum_j \lambda_{ij} E(Y_j) - \frac{\kappa}{2} \sum_j \sum_k \lambda_{ij} \lambda_{ik} \text{cov}(Y_j, Y_k) \quad (1)$$

where

$$E(Y_j) = \mu_A x_j + \mu_B (1 - x_j)$$

and, letting symbol $\mathbf{1}$ quote for the indicator function,

$$\begin{aligned} \text{cov}(Y_j, Y_k) &= x_j x_k \left(\sigma_A^2 \times \mathbf{1}_{\{j=k\}} + \tau_A \times \mathbf{1}_{\{j \neq k\}} \right) \\ &+ \left(x_j (1 - x_k) + x_k (1 - x_j) \right) \left(\sigma_{AB} \times \mathbf{1}_{\{j=k\}} + \tau_{AB} \times \mathbf{1}_{\{j \neq k\}} \right) \\ &+ (1 - x_j)(1 - x_k) \left(\sigma_B^2 \times \mathbf{1}_{\{j=k\}} + \tau_B \times \mathbf{1}_{\{j \neq k\}} \right) \end{aligned}$$

3 Equilibrium

We search for the existence and characterization of Nash equilibria. Formally, a profile X^* is a (pure) Nash equilibrium if it satisfies that, for all i , for all $x_i \in [0, 1]$, $\pi_i(x_i^*, x_{-i}^*) \geq \pi_i(x_i, x_{-i}^*)$.

The system of first order equations is linear and writes as:

$$\lambda_{ii} x_i^* + F \sum_{j \neq i} \lambda_{ij} \cdot x_j^* = H + K \lambda_{ii} \quad (2)$$

with $F = \frac{\tau_A + \tau_B - 2\tau_{AB}}{\sigma_A^2 + \sigma_B^2 - 2\sigma_{AB}}$, $H = \frac{\mu_A - \mu_B + \tau_B - \tau_{AB}}{\sigma_A^2 + \sigma_B^2 - 2\sigma_{AB}}$, and $K = \frac{\sigma_B^2 - \tau_B + \tau_{AB} - \sigma_{AB}}{\sigma_A^2 + \sigma_B^2 - 2\sigma_{AB}}$. Parameter F , which will be of major interest throughout the paper, can be written $\frac{\text{cov}((A-B)_i, (A-B)_j)}{\text{cov}((A-B)_i, (A-B)_i)}$; that is, F is equal to the ratio of the systemic risk of excess return of A over B to its total risk. If the excess return of technology A relative to B is positively (resp. negatively) correlated among agents, risk levels are strategic substitutes (resp. complements). We assume that $F > 0$, which is the most plausible case (in particular this is the case when technology B is risk free).

4 thereafter), and more, this solution is efficient.

Equation (2) shows that agents take into account correlations between projects. Further, as $F > 0$, individual actions are *strategic substitutes*, which will be a crucial feature of our analysis.

We introduce the $n \times n$ matrix $\Gamma = [\gamma_{ij}]$, with $\gamma_{ii} = 0$ for all i , and $\gamma_{ij} = \frac{\lambda_{ij}}{\lambda_{ii}}$ for all $i, j \neq i$. The element γ_{ij} is equal to the ratio of the share that agent j gives to agent i over agent i 's own share. It is the matrix of interactions, over which the Bonacich measure will be defined⁸.

3.1 Risk sharing with homogenous own shares

We begin with the program of an isolated agent; that is, for all $i, j \neq i$, $\lambda_{ii} = 1$, $\lambda_{ij} = 0$. Individual decisions take into account the following factors. First, technology B is attractive since it is less volatile than technology A . Second, agents are incited to invest some share in technology A for two reasons: for diversification purpose, *i.e.* variance reduction, even if covariance limits diversification, and because technology A has higher expected return than technology B . These factors shape the optimal level of risk of agent i , denoted x^{e*} :

$$x^{e*} = \frac{\frac{\mu_A - \mu_B}{\kappa} + \sigma_B^2 - \sigma_{AB}}{\sigma_A^2 + \sigma_B^2 - 2\sigma_{AB}}$$

We note that $x^{e*} = H + K$, that we assume positive. And under mild conditions on the parameters of the game, we have $x^{e*} \leq 1$. In essence, Markovitz's message about diversification is effective, although limited by the covariance of the lotteries.

Let us turn to socialized agents. We consider here the egalitarian sharing of realized incomes. This corresponds to a transfer matrix Λ^{FES} with $\lambda_{ij}^{FES} = \frac{1}{n}$ for all i, j (where 'FES' quotes for 'Full Equal Sharing'). Adapting the first order equation (2) to matrix

⁸The matrix of interaction is the matrix over which the Bonacich measure is defined. This matrix is a transformation of the initial matrix of bilateral cross effects. In our game, the matrix of cross effects is Λ and the matrix of interaction is Γ . Technically, the transformation of our game echoes that was introduced in Ballester *et al.* (2006), remark 2 pp. 1409.

Λ^{FES} and solving, we obtain

$$x^{FES} = \frac{nH + K}{1 + (n-1)F} \quad (3)$$

We denote $\Delta\mu_c = \kappa \left[\frac{(\sigma_A^2 - \sigma_B^2)\tau_{AB} + (\tau_B - \tau_A)\sigma_{AB} + \sigma_B^2\tau_A - \sigma_A^2\tau_B}{(\sigma_A^2 - \tau_A) + (\sigma_B^2 - \tau_B) - 2(\sigma_{AB} - \tau_{AB})} \right]$. The following condition will be useful:

$$\text{Condition } C_0 \quad \mu_A - \mu_B > \Delta\mu_c$$

In terms of parameters H, K, F , the condition writes $(1-F)H - FK > 0$ or equivalently $H + K < \frac{H}{F}$. We also note that the combination of conditions $H + K > 0$ and C_0 imply that $H > 0$. In the polar case in which projects are not correlated, equal sharing enhances investments in technology A . Due to correlations between transfers, correlated incomes affect the attractiveness of technology A ⁹. Condition C_0 guarantees that the equilibrium risk level under full equal sharing exceeds the equilibrium risk level of some isolated agent; *i.e.* $x^{FES} > x^{e*}$. We assume that **condition C_0 holds until the end of the article.**

We extend the analysis to partial equal sharing. The typical case is that is which a fixed proportion, say τ_0 , of incomes is collected and equally redistributed. Then, agent i receives $\left(1 - \frac{(n-1)}{n}\tau_0\right)y_i + \frac{\tau_0}{n} \sum_{j \neq i} y_j$. Denoting $\lambda_0 = 1 - \frac{(n-1)}{n}\tau_0$, this sharing-rule can be represented by the transfer matrix $\Lambda^{PES}(\lambda_0)$ such that $\lambda_{ii}^{PES} = \lambda_0$, for all i , and $\lambda_{ij}^{PES} = \frac{1-\lambda_0}{n-1}$, for all $i, j \neq i$ (where ‘ PES ’ quotes for ‘Partial Equal Sharing’) - with $\frac{1-\lambda_0}{n-1} = \frac{\tau_0}{n}$. Straightforward computation gives:

$$x^{PES}(\lambda_0) = \frac{H + K\lambda_0}{(1-F)\lambda_0 + F} \quad (4)$$

Noticing that λ_0 is a decreasing function of τ_0 , the equilibrium level of risk is decreasing in the value of λ_0 . In particular, it is larger than the risk taken by an isolated agent. In a word, in the context of egalitarian redistribution, more taxation implies more risk sharing. This leads to an increase of risk taking.

⁹Suppose for instance that τ_A is high and τ_B low. Then, B may become more attractive and the equilibrium levels of risk of socialized agents are lower than in isolation.

We extend the analysis further to transfer matrices with heterogenous off-diagonal elements. We thus define the family of transfer matrices with homogenous own shares $\mathbb{L}^{HOS}(\lambda_0) = \{\Lambda/\lambda_{ii} = \lambda_0, \forall i\}$. A natural issue consists in ordering such matrices in terms of their associated equilibrium risk profile. In fact, for any matrix $\Lambda \in \mathbb{L}^{HOS}(\lambda_0)$, the equilibrium risk level is homogenous and only related to the own share of the matrix Λ :

$$x^{HOS}(\lambda_0) = \frac{H + K\lambda_0}{(1 - F)\lambda_0 + F} \quad (5)$$

(this stems directly from equation (2), exploiting that Λ is row-stochastic) The following proposition summarizes the results of the whole subsection:

Proposition 1 *In societies with homogenous own shares, the equilibrium level of risk is homogenous and decreasing in the value of own share. Furthermore, the equilibrium level of risk exceeds that of isolated agents.*

Proposition 1 confirms that for societies with homogenous own shares, more revenue sharing enhances risk taking. We observe that, in societies with homogenous own shares, risk level is only related to own share. However, profits depend on the whole distribution of transfers. Let $\lambda_{-i} = (\lambda_{i1}, \lambda_{i2}, \dots, \lambda_{ii-1}, \lambda_{ii+1}, \dots, \lambda_{in})$. We say that “the distribution λ_{-i} is more diversified than the profile λ'_{-i} , whenever $\sum_{j \neq i} \lambda_{ij}^2 > \sum_{j \neq i} \lambda'_{ij}{}^2$ ”. Then, it is easily shown that, when own shares are homogenous, more diversification in the distribution λ_{-i} is always beneficial to agent i 's equilibrium profit (proof are available upon request). This means that agents are better off in the society with transfer matrix $\Lambda^{PES}(\lambda_0)$ than in any other society with the same homogenous own share.

3.2 Risk sharing with heterogenous own shares

Until now, we considered risk sharing with homogenous own shares. We found that risk levels are only affected by the value of own share, and we confirmed that more risk sharing enhances risk taking, even in the presence of strategic interactions. We will now focus on heterogenous risk sharing, and we will discuss the view that risk sharing enhances risk taking.

We separate the space of all bi-stochastic matrices in two complementary regions parameterized by F . We let \mathcal{R} be the set of bi-stochastic matrices such that $\lambda_{ii} \in]\frac{F}{1+F}, 1]$ for all i , and $\bar{\mathcal{R}}$ be the set of bi-stochastic matrices such that for some i , $\lambda_{ii} \leq \frac{F}{1+F}$. We note that no own share is below one half in region \mathcal{R} .

We consider a $n \times n$ matrix M with nonnegative elements m_{ij} , any column-vector $a = (a_1, a_2, \dots, a_n)$ with nonnegative real components and we let J denote the column vector of ones. We define $B^a(M; \alpha)$ as the solution of $(I - \alpha M)B^a = a$, $\alpha \in]-1, 1[$ (we write $B(M; \alpha)$ if $a = J$). When both $\alpha > 0$ and the greatest modulus of eigenvalues of M is smaller than $\frac{1}{\alpha}$, $B^a(M; \alpha)$ can be interpreted as a vector of weighted (by a) *Bonacich centrality measure* defined over the network with link m_{ij} representing the intensity of the connection from agent i to agent j . Indeed,

$$B^a(M; \alpha) = \sum_{k=0}^{\infty} \alpha^k M^k a \quad (6)$$

is a well-defined quantity, and $B_i^a(M; \alpha)$ measures the sum of the values of paths from agent i to others through the network, where paths of length k toward agent j are weighted by $a_j \alpha^k$ (where the value of a path represents the product of link intensities). This measure (actually, a slightly modified version) was introduced in Bonacich (1987). Our model containing strategic substitutes, we will consider the case $\alpha \in]-1, 0[$. In such a case, the quantity given in equation (6) is also well-defined, but the contribution of the network to the measure is ambiguous. Odd (resp. even) paths contribute negatively (positively) to the measure.

Remark 1 *Theorem 1, pp. 1408, in Ballester et al. (2006) provides a formulation in terms of Bonacich centrality measure (i.e. a formulation using a positive decay parameter) over another interaction matrix, under some restrictions of our game. We choose not to use that formulation for two reasons. First, the specification is non linear, making also difficult to interpret it. Second, our solutions are valid under less restrictive environment than those required to obtain a positive decay parameter. We will then speak about ‘Bonacich measure’, without reference to the notion of centrality¹⁰*

¹⁰When the matrix of cross effects is binary, a centrality measure can be defined over its comple-

(we keep the reference to Bonacich, as it conveys the notion of network, and further the measure is technically of same nature).

We define the vector D such that $D_i = \frac{1}{\lambda_{ii}}$. Inverting the linear system presented in equation (2), we obtain the following theorem:

Theorem 1 *A unique equilibrium $X^* = (I + F\Gamma)^{-1}(HD + KJ)$ exists ‘almost everywhere’ in the space of admissible values of F . Furthermore, if $\Lambda \in \mathcal{R}$, a unique solution X^* always exists and can be written*

$$x_i^* = \frac{H}{F} - \frac{(1-F)H - FK}{F} B_i(\Gamma; -F) \quad (7)$$

for all $i \in N$, where $B_i(\Gamma; -F)$ is given by equation (6) and belongs to $]0, 1[$.

We note that if own shares exceed one half, Bonacich measures are well defined¹¹. Equilibrium risk levels are lower than unity if $\frac{H}{F} \leq 1$ (i.e. $\mu_A - \mu_B < \kappa(\tau_A - \tau_{AB})$), what we assume for simplicity.

Remark 2 *In region \mathcal{R} , inverting the system presented in equation (2), one expects a characterization of equilibrium risk levels as a weighted Bonacich measure (since the constant $H + K\lambda_{ii}$ contains an idiosyncratic component). Theorem 1 expresses the solution as a simple (non weighted) Bonacich measure. This is specific to the fact that the constant is an affine function of λ_{ii} and that the transfer matrix is row-stochastic (the proof is in lemma 1).*

A lesson of theorem 1 is that under weak risk sharing, interactions are sufficiently low to guarantee that $x_i^* \in]H + K, \frac{H}{F}[$. This means that in any society with transfer matrix in \mathcal{R} , every agent takes more risk at equilibrium than in isolation.

mentary matrix (an off-diagonal element of the complementary matrix is 1 if and only if the associate element is 0 in the initial matrix) - see Corbo *et al.* (2007), Bramoullé *et al.* (2008), Bloch and Quérrou (2008).

¹¹Angelucci and De Giorgi show that “for every 100 pesos transferred by Progresa to the eligible households, the consumption of ineligible households increases by approximately 11 pesos”. Since the proportion of treated household is greater than one half, this suggests that own shares are high proportions of revenues.

We proceed to some comparative analysis of transfer matrices. Two issues emerge when we try to replicate proposition 1 to societies with heterogenous own shares. First, how to define more revenue sharing? We generalize the idea of ‘more revenue sharing’ as follows. Starting from any society, revenue sharing increases when all own shares are diminished and no other share is diminished, in a way that preserves bi-stochasticity. Formally:

Definition [more revenue sharing] Consider two transfer matrices $\Lambda, \Lambda' = \Lambda + \Theta$ such that for all i , $\theta_{ii} = -\sum_{j \neq i} \theta_{ij} = -\sum_{j \neq i} \theta_{ji}$. There is more revenue sharing in matrix Λ' than in Λ if for all i , $\theta_{ii} \leq 0$ and for all $i, j \neq i$, $\theta_{ij} \geq 0$.

Second, for societies with homogenous own shares, individual and average risk taking coincide. In opposite, risk levels are differentiated for societies with heterogenous own shares. A natural question that arises is: does more revenue sharing always increase individual risk taking, or eventually in average? The following theorem compares the risk taking behaviors of two such societies¹² in the region of weak risk sharing:

Theorem 2 Consider two transfer matrices $\Lambda, \Lambda' \in \mathcal{R}$. If there is more revenue sharing in Λ' than in Λ , then $\sum_i x_i'^* \geq \sum_i x_i^*$.

Theorem 2 confirms that more revenue sharing promotes risk taking in average in region with weak risk sharing. Actually, more revenue sharing does not necessarily lead to an increase of all individual risk levels. To illustrate, let $\Lambda' = \Lambda + \Theta$ be such that there is more revenue sharing in Λ' than in Λ and such that $\theta_{1j} = 0$ for all j . Then, combining first order equations applied to agent 1 in both matrices, one obtains easily that there exists some agent, say i_0 , such that $x_{i_0}'^* < x_{i_0}^*$. Moreover, we note that, by continuity, the result holds if there is strictly more revenue sharing in Λ' than in Λ (we just have to select a sufficiently small perturbation of the matrix Θ that yields $\theta_{11} < 0$

¹²Comparing transfer matrices in which diagonal elements vary in opposite directions is difficult. The point can be tackled by perturbing locally a society with homogenous own shares. We find that any small modification, that induces an decrease of diagonal elements in average, leads to more risk taking in average. Results are available upon request.

while preserving both signs of all other terms and bi-stochasticity of the new transfer matrix).

In the region with large risk sharing $\bar{\mathcal{R}}$, more revenue sharing may decrease risk taking in average. Simple examples illustrate the problem in region $\bar{\mathcal{R}}$. Suppose $H = .5, K = 0, F = .9$ and consider the following matrices: $\mathbf{\Lambda} = \begin{pmatrix} .7 & .08 & .22 \\ .12 & .5 & .38 \\ .18 & .42 & .4 \end{pmatrix}$, $\mathbf{\Lambda}' =$

$\begin{pmatrix} .6 & .08 & .32 \\ .12 & .5 & .38 \\ .28 & .42 & .3 \end{pmatrix}$. There is more revenue sharing in $\mathbf{\Lambda}'$ than in $\mathbf{\Lambda}$ in the sense of the

above definition. We find $x_1^* \simeq .499, x_2^* \simeq .496, x_3^* \simeq .578$, implying $\frac{1}{3} \sum_i x_i^* \simeq .5248$; and $x_1'^* \simeq .692, x_2'^* \simeq .783, x_3'^* \simeq .097$, entailing $\frac{1}{3} \sum_i x_i'^* \simeq .5245$. Hence, theorem 2 does not necessarily hold in region $\bar{\mathcal{R}}$. Furthermore, those two cases are such that certain agents can take less risk than in isolation.

More, where we allowing values of $x_i \notin [0, 1]$, what can be interpreted as short-selling, the average level of risk itself may be lower than the risk taken in isolation. To illustrate,

suppose $H = .1, K = 0, F = .9023$ and consider $\mathbf{\Lambda} = \begin{pmatrix} .65 & .05 & .3 \\ .12 & .5 & .38 \\ .23 & .45 & .32 \end{pmatrix}$. Noticing that $x_e^* = .1$, we find $x_1^* \simeq .59, x_2^* \simeq .89, x_3^* \simeq -1.20$, implying $\sum_i x_i^* \simeq 0.094$.

Remark 3 *When one of the two matrices $\mathbf{\Lambda}$ or $\mathbf{\Lambda}'$ is homogenous, the theorem still holds as soon as the other matrix lies in region \mathcal{R} . Considering any transfer matrix $\mathbf{\Lambda} \in \mathcal{R}$, we denote $\underline{\lambda} = \min_i \lambda_{ii}$, $\bar{\lambda} = \max_i \lambda_{ii}$, $\underline{x} = x^*(\underline{\lambda})$ and $\bar{x} = x^*(\bar{\lambda})$. Then, $\bar{x} \leq \frac{1}{n} \sum_i x_i^* \leq \underline{x}$. A direct implication is that there exists a value $\lambda_0 \in [\min_i \lambda_{ii}, \max_i \lambda_{ii}]$ such that $x^*(\lambda_0) = \frac{1}{n} \sum_i x_i^*$. Furthermore, there exists at least one agent i_0 (resp. j_0) with $x_{i_0}^* > \underline{x}$ (resp. $x_{j_0}^* < \bar{x}$).*

To sum up, fixing any transfer matrix in the region of weak risk sharing \mathcal{R} , we provide two broad sets of conditions under which the *average* level of risk taking is enhanced by a (inequality preserving) modification of the structure of risk sharing.

First, this occurs if the modification induces a decrease of all diagonal elements without decreasing any non diagonal element, and such that the resulting matrix stays in region \mathcal{R} ; second, this occurs if the modification induces a decrease of all diagonal elements until some homogenous value not larger than the lowest own share of the former matrix, irrespective of the structure of the modification out of the diagonal, and irrespective of the locus of the resulting matrix (it can be in both regions).

The region of large risk sharing $\bar{\mathcal{R}}$ is more uncertain. Simple examples suggest that the variance of risk levels is higher than in the region of weak risk sharing, and more risk sharing can reduce risk taking; even the average risk level can be lower than the optimal risk of an isolated agent.

4 Over- versus under-investment *w.r.t.* social welfare

We consider the risk profiles that maximize the sum of profits in the society, $W = \sum_i \pi_i$. One particularity of the game is that the *sign* of externalities is endogenous. Indeed, through the transfer of intensity λ_{ji} , an increase in x_i induces higher expected return for agent j , but also higher variance¹³. In consequence, whether agents over- or under-invest in the innovation with regard to social welfare is ambiguous. We will give some partial answers, yet potentially instructive. In particular, the study suggests that the structure of the transfer matrix is crucial to understand whether agents under- or over-invest in technology A .

Let Ψ denote the matrix such that $\psi_{ij} = \sum_k \lambda_{ki} \lambda_{kj}$ for all i, j . We note that $\psi_{ij} = \psi_{ji}$, and since the matrix Λ is bi-stochastic, the matrix Ψ is also bi-stochastic. We also define the matrix $\Phi = [\phi_{ij}]$ such that $\phi_{ij} = 0$ for all i , $\phi_{ij} = \frac{\psi_{ij}}{\psi_{ii}}$ for all $i, j \neq i$. Finally, we define the vector E such that $E_i = \frac{1}{\psi_{ii}}$. We remark that, being bi-stochastic, the matrix Ψ associated with the transfer matrix Λ can be seen as a transfer matrix.

¹³Another originality of the model is that, due to covariances, the sign of the externality that agent i 's risk level generates to agent j is related to the risk levels of *all* agents.

Hence, we obtain:

Proposition 2 *A unique efficient risk profile $\hat{X} = (I + F\Phi)^{-1}(HE + KJ)$ exists ‘almost everywhere’ in the space of admissible values of F . Furthermore, if $\Psi \in \mathcal{R}$, a unique solution \hat{X} always exists and can be written*

$$\hat{x}_i = \frac{H}{F} - \frac{(1-F)H - FK}{F} B_i(\Phi; -F) \quad (8)$$

for all $i \in N$, where $B_i(\Phi; -F)$ is given by equation (6).

When the sign of externalities is constant, it is in general easy to see if agents under invest or over invest with respect to social welfare. Here, since externalities may be either positive or negative, it is difficult to find general conditions under which the equilibrium risk profile either dominates or is dominated by the efficient profile. We will provide two sets of conditions under which we can compare the average value of efficient and equilibrium risk profiles.

Since the efficient risk profile corresponds to the transfer matrix of a modified game, the conditions under which we were able, in the preceding section, to compare equilibrium risk profiles associated to distinct transfer matrices basically hold. Hence, we obtain:

Proposition 3 *Consider a matrix $\Lambda \in \mathcal{R}$. The average equilibrium level of risk is lower (resp. higher) than the average efficient level of risk if those three conditions apply simultaneously: (i) $\Psi \in \mathcal{R}$; (ii) $\lambda_{ii} \geq \psi_{ii}$ (resp. $\lambda_{ii} \leq \psi_{ii}$) for all i ; (iii) $\lambda_{ij} \leq \psi_{ij}$ (resp. $\lambda_{ij} \geq \psi_{ij}$) for all $i, j \neq i$.*

Suppose now that $\Lambda \in \mathbb{L}^{HOS}(\lambda_0)$, and suppose that $\Psi \in \mathcal{R}$. If $\lambda_0 \geq \max_i \psi_{ii}$ (resp. $\lambda_0 \leq \min_i \psi_{ii}$), the average equilibrium level of risk is lower (resp. higher) than the average efficient level of risk.

(proof omitted) Proposition 3 is the mirror of theorem 2 and remark 3 as applied to efficiency issue. The proposition indicates that if matrix Ψ is the transfer matrix of a game in which there is more (resp. less) revenue sharing than in the original game Λ , then agents under-invest (resp. over-invest). Indeed, when agents increase their risk

level, this has two opposite effects on the payoff of neighbors: this increases their risk, but this also increases their expected return. Now, if agents do not exchange much with the society, the expected return effect dominates, *i.e.* in average, agents do not internalize that increasing their risk level would increase the overall expected return of the society. Symmetrically, if agents exchange too much with the society, the risk effect dominates, *i.e.* in average, agents do not internalize that decreasing their risk level would reduce the overall risk of the society.

The condition specifying whether agents under- or over-invest in technology A is only related to the transfer matrix. For instance, in the case of homogenous own shares, the condition is simple, and it compares own shares with a diversification index. Noticeably, under the reasonable condition $\lambda_0 > \frac{1}{2}$, it turns out that $\lambda_0 > \psi_{ii}$ for all i . That is, agents under-invest with respect to societal view¹⁴.

To illustrate, we consider the following transfer matrix. We assume $\lambda_{ii} = \lambda_0$ for all i , $\lambda_{ij} = \frac{1-\lambda_0}{n-1}$ for all $i, j \neq i$. Then, own shares are homogenous, and social links are maximally diversified, so $\psi_{ii} = \psi_0 = \lambda_0^2 + \frac{(1-\lambda_0)^2}{n-1}$ for all i . An immediate observation is that the efficient profile of payoffs is homogenous, and thus Pareto-dominates the equilibrium profile. More, we note that $\lambda_0 > \psi_0$ iff $\lambda_0 > \frac{1}{n}$. Applying the theorem, we deduce the following results. First, if $\lambda_0 > \frac{1}{n}$, and for $\psi_0 > \frac{F}{1+F}$, which is probably the most realistic case, the equilibrium level of risk is lower than the average efficient level of risk. Second, if $\lambda_0 = \frac{1}{n}$, the equilibrium risk profile coincides with the strong efficient one, which is a good news once we have remark 4 in mind.

Remark 4 *Suppose that we let agents fix by themselves the values of the transfers without cost. Simple optimization induces that there is a unique equilibrium¹⁵. At this equilibrium, it can be shown that all agents diversify their social links at maximum, *i.e.* $\lambda_{ij}^* = \frac{1}{n}$ for all i, j . Further, it will be seen thereafter that this sharing-rule uniquely satisfies that equilibrium risk coincides with that maximizing social welfare.*

¹⁴This result provides a possible explanation of the lack of investment in risky innovations in developing villages (Valente [1997]).

¹⁵The proof is available upon request.

5 Extension: cash transfers

Formal institutions can help developing villages by transferring Cash to households. For Progresa program, Angelucci and De Giorgi (2008) document that part of this aid is transferred by informal arrangements through the network and therefore also affects the consumption of the non-treated¹⁶. Beyond consumption, and in direct filiation with the main hypotheses of this article, we address the issue of the impact of Cash transfer or wealth redistribution on the overall investments in technology A .

Suppose that an institution, interested in the promotion of technology A , uses wealth redistribution or cash transfer as a policy tool. For simplicity, we will assume that initial wealths are equal to 1 for all i ¹⁷. A shock in wealths $a = (a_1, a_2, \dots, a_n)$ generates the new vector of wealths $\tilde{\Omega} = (\tilde{\omega}_1, \tilde{\omega}_2, \dots, \tilde{\omega}_n)$ such that $\tilde{\omega}_i = 1 + a_i$ for all i . Let $z_i = 1 \cdot x_i$ (resp. $\tilde{z}_i = \tilde{\omega}_i \cdot x_i$) denote the amount of wealth that agent i invests in technology A and $Z = (z_1, z_2, \dots, z_n)$ (resp. $\tilde{Z} = (\tilde{z}_1, \tilde{z}_2, \dots, \tilde{z}_n)$) the associated profile. We let Z^* (resp. \tilde{Z}^*) denote the equilibrium associated with wealth profile Ω (resp. $\tilde{\Omega}$).

The following proposition relates the variation of the part of wealths invested in technology A to the transpose of matrix Γ :

Proposition 4 *Suppose that $\Lambda \in \mathcal{R}$. The variation in the sum of equilibrium amounts of wealths invested in technology A writes:*

$$\sum_i (\tilde{z}_i^* - z_i^*) = \sum_i a_i \left(\frac{H}{\lambda_{ii}} + K \right) B_i(\Gamma^T; -F) \quad (9)$$

¹⁶“Started in 1997 and still ongoing, Progresas aim is improving poor households education, health, and nutrition through sizeable cash transfers. In our sample of rural villages, more than half of the households are treated. The targeted villages are small and agriculture is the main, and often sole economic activity. The exposure to natural disasters, the absence of formal credit and insurance institutions, and extensive within-village kinship relationships create incentives to engage in informal risk sharing activities. If this is the case, treated households will share part of their higher income with members of their social network through gifts or loans. Therefore, the entire village will benefit from the program.”

¹⁷Extending the analysis to heterogenous initial wealths, results are qualitatively unchanged (results are available upon request).

The institution aiming at promoting technology A should therefore take into account the structure of the matrix Γ , and target certain households rather than others. As a first illustration, we present a case in which the identity of eligible households is not an issue. Suppose that own shares are homogenous, of value λ_0 :

Example 1: suppose that $\lambda_0 \in]\frac{F}{1+F}, 1]$. Then variation in the sum of equilibrium amounts of wealths invested in technology A writes as a function of the shocks on wealths and the transfer matrix (proof in appendix):

$$\sum_i (\tilde{z}_i^* - z_i^*) = x^*(\lambda_0) \sum_i a_i$$

One direct implication is that the amount of cash transfer matters, not the identity of eligible agents. A second implication is that the magnitude of the impact of the wealth shock on investments in technology A is related to $x^*(\lambda_0)$, the equilibrium level of risk of a society with homogenous own shares of value λ_0 ; precisely, $x^*(\lambda_0)$ is the proportion of cash transfer that will be allocated to technology A .

Example 2: this example shows that, in general, risk sharing heterogeneity makes the task of targeting eligible households adequately not obvious. Suppose that Cash transfer is given to a unique agent i_0 ; *i.e.*, $a_{i_0} = 1 > 0$ and $a_j = 0$ for all $j \neq i_0$. The induced variation in the sum of wealths invested in the risky technology writes:

$$\left(\frac{H}{\lambda_{i_0 i_0}} + K \right) B_{i_0}(\Gamma^T; -F) \quad (10)$$

Hence, if one agent is given cash transfer, an institution that would aim at maximizing the investment in the (more) risky technology should give the transfer to the agent who maximizes expression (10). That agent needs not coincide with that investing the maximal amount at X^* .

6 Conclusion

This paper analyzes a model of risk choice in the presence of heterogenous risk sharing and strategic interactions. To focus on pure risk sharing, our model considers agents with homogenous initial wealth, homogenous expected ‘after transfer wealth’, and same risk aversion.

We first point out that solutions generally exist and are unique. In particular, when risk sharing is not too large, optimal risk levels are linearly related to the Bonacich measures of a slight transformation of the transfer matrix.

When risk sharing is not too large, the common view that more risk sharing entails more risk taking is preserved, although only in average. Further, simple statistics over the composition of transfers indicate whether agents over- or under-invest with regard to the efficient allocation. Last, the analysis has important policy implications. For instance, we characterize how to select adequately eligible households for cash transfer or which wealth redistributions is opportune, in order to increase investments in the more profitable/risky technology.

When risk sharing is large, *i.e.* when risk sharing is voluminous, simple examples illustrate that the combination of strategic interactions and heterogeneity can destroy all the results: the average risk level may be reduced by an increase of transfers, and more, it may be even lower than the risk chosen by an isolated agent.

It would be interesting to test the theoretical predictions of this simple model. Moreover, some lines of research should deserve attention: relating the formation of risk sharing rules to wealth and correlated risks, to the nature of risks (health vs incomes shocks), to the interplay between risk sharing and other activities aimed at insuring against volatility (savings, extra income earnings).

APPENDIX

Definition 1 (Strict diagonal-dominance) *A $n \times n$ matrix $M = [m_{ij}]$ is strictly diagonal-dominant if $|m_{ii}| > \sum_{j \neq i} |m_{ij}|$ for all i .*

Definition 2 (row-stochasticity) *A $n \times n$ matrix $M = [m_{ij}]$ is row-stochastic if $m_{ij} \in \mathbb{R}^+$ for all i, j and $\sum_{j=1}^n m_{ij} = 1$ for all i .*

Preliminary result 1 *If a $n \times n$ matrix M is strictly diagonal-dominant, the equation $MZ = J$ admits a unique solution.*

Lemma 1 Consider three parameters $\delta \in]0, 1[$, $\alpha \in \mathbb{R}, \beta \in \mathbb{R}$, such that $\alpha \frac{1-\delta}{\delta} \neq \beta$. Consider also a $n \times n$ row-stochastic matrix $A = [a_{ij}]$ such that $a_{ii} > \frac{\delta}{1+\delta}$ for all i . Define $E = [e_{ij}]$, with $e_{ii} = 0$ for all i and $e_{ij} = \frac{a_{ij}}{a_{ii}}$ for all $i, j \neq i$. The system of equations such that, for all i ,

$$a_{ii}x_i + \delta \sum_{j \neq i} a_{ij}x_j = \alpha + \beta a_{ii} \quad (11)$$

admits a unique solution

$$x_i = \frac{\alpha}{\delta} - \left(\alpha \frac{1-\delta}{\delta} - \beta \right) B_i(E; -\delta) \quad (12)$$

with $B_i(E; -\delta) \in]0, 1[$.

Proof of lemma 1. We consider the following transformation:

$$v_i = \left(\frac{1}{\alpha \frac{1-\delta}{\delta} - \beta} \right) \left(\frac{\alpha}{\delta} - x_i \right) \quad (13)$$

Equation (11) becomes:

$$a_{ii} \left(\frac{\alpha}{F} - \left(\alpha \frac{1-\delta}{\delta} - \beta \right) v_i \right) + \delta \sum_{j \neq i} a_{ij} \left(\frac{\alpha}{F} - \left(\alpha \frac{1-\delta}{\delta} - \beta \right) v_j \right) = \alpha + \beta a_{ii} \quad (14)$$

Dividing all terms by a_{ii} , and taking account of $\sum_{j \neq i} a_{ij} = 1 - a_{ii}$, one obtains:

$$v_i + \delta \sum_{j \neq i} e_{ij} v_j = 1 \quad (15)$$

or in matrix form $(I + \delta E)V = J$. Since $a_{ii} > \frac{\delta}{1+\delta}$ for all i , the matrix $I + \delta E$ is strictly diagonal-dominant. The preliminary result 1 applies with $M = I + \delta E$.

Inverting the system, the solution writes as a Bonacich measure $v_i = B_i(E; -\delta)$ with $B(E; -\delta) = \sum_{k=0}^{\infty} (-\delta)^k E^k J$. Rearranging,

$$B(E; -\delta) = I(I - \delta E)J + (\delta E)^2(I - \delta E)J + (\delta E)^4(I - \delta E)J + \dots \quad (16)$$

Factorizing, one obtains:

$$B(E; -\delta) = \left(\sum_{k=0}^{\infty} \delta^{2k} E^{2k} \right) \cdot (I - \delta E)J \quad (17)$$

Notice that $\delta^{2k}[E^{2k}]_{ij} > 0$ for all k, i, j . Further, if $a_{ii} > \frac{\delta}{1+\delta}$ for all i , the vector $(I - \delta E)J > 0$. Hence, $B(E; -\delta) > 0$. More, note that a solution of $(I + \delta E)Z = J$ also writes $Z = J - \delta EZ$. That is, if $Z > 0$, clearly $Z < J$. \square

Proof of theorem 1. The matrix with diagonal element λ_{ii} and non diagonal elements $F\lambda_{ij}$ represents the system of first order conditions. We note that the system is invertible ‘almost everywhere’ in terms of the parameter F ; indeed, the determinant of the system is a polynomial expression of degree not higher than n in parameter F . Hence, fixing the matrix of transfers, there exists at most n values of parameter $F \in [0, 1]$ for which the system is not invertible. If this matrix is diagonal-dominant (which is the condition on own shares given in the theorem), the system is invertible. This guarantees that the solution is unique, and more, Bonacich measures are well defined.

Since the transfer matrix is row-stochastic, the equation 2 becomes:

$$\lambda_{ii}x_i^* + F \sum_{j \neq i} \lambda_{ij} \cdot x_j^* = H + K\lambda_{ii} \quad (18)$$

Hence, we can apply lemma 1 with $\delta = F$, $\alpha = H$, $\beta = K$, $A = \Lambda$. Note that in this case $\alpha \frac{1-\delta}{\delta} > \beta$ by condition C_0 . \blacksquare

Proof of theorem 2. We use the following lemma:

Lemma 2 (adapted from Farkas’s lemma) *Let Q be an $n \times n$ matrix. If the equation $Q^T x = J$ admits a positive solution, then for all $y \in \mathbb{R}^n$ such that $Qy \geq 0$, we have $\sum_i y_i \geq 0$.*

We let matrix Λ_F denote the matrix with diagonal element λ_{ii} and non-diagonal elements $F\lambda_{ij}$. We will see that the conditions of the lemma apply if we fix $Q = \Lambda_F$ and $y = x'^* - x^*$:

First, we prove that there exists a positive solution to $(\Lambda_F)^T x = J$:

The condition writes:

$$\lambda_{ii}x_i + F \sum_{j \neq i} \lambda_{ji}x_j = 1 \quad (19)$$

Since the matrix Λ is bi-stochastic, the matrix Λ^T is row-stochastic. We can therefore apply lemma 1 with $\delta = F$, $\alpha = 1$, $\beta = 0$, $A = \Lambda^T$, and we conclude that there is a positive solution to the system $(\Lambda_F)^T x = J$.

Second, we see that $\Lambda_F(x'^* - x^*) \geq 0$:

Indeed, we observe that $[\Lambda'_F x]_i^* = H + K(\lambda_{ii} + \theta_{ii})$, and $\Lambda_F x'^* = \Lambda'_F x'^* - \Theta x'^*$. Then, $[\Lambda_F x'^*]_i = H + K(\lambda_{ii} + \theta_{ii}) - [\Theta_F x'^*]_i$. Hence,

$$[\Lambda_F(x'^* - x^*)]_i = K\theta_{ii} - (\theta_{ii}x'_i + F \sum_{j \neq i} \theta_{ij}x'_j) \quad (20)$$

Since the matrix $\Lambda' \in \mathcal{R}$, theorem 1 implies that $H + K < x'_i < \frac{H}{F}$ for all i . Recalling that all $\theta_{ij} > 0$, $x'_i + F \sum_{j \neq i} \frac{\theta_{ij}}{\theta_{ii}} x'_j > H + K + H \sum_{j \neq i} \frac{\theta_{ij}}{\theta_{ii}}$. Since $\sum_{j \neq i} \frac{\theta_{ij}}{\theta_{ii}} = -1$, $x'_i + F \sum_{j \neq i} \frac{\theta_{ij}}{\theta_{ii}} x'_j > K$ and we are done.

Third, we apply lemma 2 and conclude that

$$\sum_i (\underline{x} - x_i^*) \geq 0 \quad (21)$$

which proves the theorem. ■

Proof of proposition 2. Simple computation entails:

$$\begin{aligned} \frac{1}{\kappa} \frac{\partial W}{\partial x_i} &= \frac{\mu_A - \mu_B}{\kappa} + (\sigma_B^2 - \tau_{AB}) \sum_j \lambda_{ji}^2 - (\tau_A + \tau_B - 2\tau_{AB}) \sum_j \lambda_{ji} \sum_{k \neq i} \lambda_{jk} x_k \\ &+ (\tau_B - \tau_{AB}) \sum_j \lambda_{ji} \sum_{k \neq i} \lambda_{jk} - (\sigma_A^2 + \sigma_B^2 - 2\tau_{AB}) x_i \sum_j \lambda_{ji}^2 \end{aligned} \quad (22)$$

That is, $\frac{\partial W}{\partial x_i} = 0$ if and only if

$$\psi_{ii} \hat{x}_i + F \sum_{j \neq i} \psi_{ij} \hat{x}_j = H + K \psi_{ii} \quad (23)$$

Since the matrix Λ is bi-stochastic, $\sum_{j \neq i} \psi_{ij} = 1 - \psi_{ii}$; that is, the matrix Ψ is row-stochastic. We can therefore apply lemma 1 with $\delta = F$, $\alpha = H$, $\beta = K$, $A = \Psi$ (and thus $E = \Phi$). In particular, $\psi_{ii} > \frac{F}{1+F}$ for all i implies that the matrix $I + F\Phi$ is strictly diagonal-dominant. ■

Proof of proposition 4. From the first order condition given by equation (2), we obtain:

$$\begin{cases} \lambda_{ii}z_i^* + F \sum_{j \neq i} \lambda_{ij}z_j^* = H + K\lambda_{ii} \\ \lambda_{ii}\tilde{z}_i^* + F \sum_{j \neq i} \lambda_{ij}\tilde{z}_j^* = (1 + a_i)(H + K\lambda_{ii}) \end{cases} \quad (24)$$

Thus,

$$\tilde{z}_i^* - z_i^* + F \sum_{j \neq i} \gamma_{ij}(\tilde{z}_j^* - z_j^*) = a_i \left(\frac{H}{\lambda_{ii}} + K \right) \quad (25)$$

Defining vector e such that $e_i = \frac{a_i}{\lambda_{ii}}$, we find in matrix form:

$$(I + F\Gamma)(\tilde{Z}^* - Z^*) = He + Ka \quad (26)$$

Denoting $T = (I + F\Gamma)^{-1}$, and summing all terms of the vector, we find:

$$\sum_i (\tilde{z}_i^* - z_i^*) = H \sum_i \sum_j T_{ij} \frac{a_j}{\lambda_{jj}} + K \sum_i \sum_j T_{ij} a_j \quad (27)$$

That is:

$$\sum_i (\tilde{z}_i^* - z_i^*) = H \sum_j \frac{a_j}{\lambda_{jj}} \sum_i T_{ij} + K \sum_j a_j \sum_i T_{ij} \quad (28)$$

Remind that $B_i(\Gamma; -F) = \sum_j T_{ij}$. Now, since $\Lambda \in \mathcal{R}$, we can develop matrix T in its series:

$$\begin{aligned} B_i(\Gamma^T; -F) &= \sum_j ((I + F\Gamma^T)^{-1})_{ij} = \sum_j \sum_{k=0}^{\infty} F^k (\Gamma_{ij}^T)^k = \sum_j \sum_{k=0}^{\infty} F^k ((\Gamma_{ij})^k)^T \\ &= \sum_j \left(\sum_{k=0}^{\infty} F^k (\Gamma_{ij})^k \right)^T = \sum_j ((I + F\Gamma)_{ij}^{-1})^T = \sum_j (I + F\Gamma)_{ji}^{-1} = \sum_i T_{ij} \end{aligned}$$

That is,

$$B_i(\Gamma^T; -F) = \sum_i T_{ij} \quad (29)$$

and we are done. \square

Proof of the result in example 1. Equation (25) writes:

$$\lambda_{ii}(\tilde{z}_i^* - z_i^*) + F \sum_{j \neq i} \lambda_{ij}(\tilde{z}_j^* - z_j^*) = a_i \left(H + K\lambda_{ii} \right) \quad (30)$$

summing over all agents and taking account of the fact that the matrix of transfers is row-stochastic, one obtains:

$$F \sum_i (\tilde{z}_i^* - z_i^*) + (1 - F) \sum_i \lambda_{ii} (\tilde{z}_i^* - z_i^*) = H \sum_i a_i + K \sum_i \lambda_{ii} a_i$$

Taking account that $\lambda_{ii} = \lambda_0$ for all i , the corollary follows. \square

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