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A model for scheduling drug deliveries in a french homecare[1]

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Abstract

In this paper, we deal with a drug delivery problem in a French homecare. The carriers are assigned to specific areas and must visit each patient once a day while minimizing the total travelled distance. In this context, we propose to explore four strategies of delivery: (i) starting deliveries when a specified number of deliveries is achieved, (ii) starting deliveries if a specified distance is reached regarding to the planned deliveries, (iii) starting deliveries on a fixed number of deliveries per carrier and, (vi) starting deliveries on fixed hours. We introduce a mixed linear program which takes into account various constraints and minimize the total travelled distance. The results obtained for each strategy are compared in order to identify which one is the most efficient to solve the drug delivery problem at the homecare.

Key words: Integer programming, vehicle routing problem, staff scheduling, drug delivery, homecare logistic.

1 Introduction

"Soins et santé" is a homecare based in Lyon (France). It was created since 1972 and is, from June 2007, the only French homecare which has its own pharmacy. Thus, this structure must face several delivery problems. It is difficult to schedule regular deliveries with only four carriers taking into account urgent prescriptions which arrive at any time and must be treated rapidly whereas all carriers are gone. Moreover, the pharmacy does not have any decision support tool to establish the planning of deliveries along the week. In fact, deliveries schedule consists in daily assigning an area to each carrier where he must visit each patient respecting their time windows and achieving all the deliveries into his working hours. In this context, we propose a decision support tool based on mixed integer programming which takes into account various constraints and minimize the total travelled distance. The following sections are structured as follow. First, we describe the delivery problem of the homecare. Second, we present some previous works which deal with delivery problems, vehicle routing problem, and transportation scheduling problems. In a third part, we propose a mathematical formulation for scheduling deliveries. Next, we proceed to an analysis of the results obtained for the data of the homecare

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pharmacy and then, we compare the results of the four strategies. Finally, we conclude with some prospects for future researches.

2 Description of the problem

The homecare pharmacy uses four carriers which are organised as follow: 2 carriers work from 8 a.m to 16 p.m, 1 carrier works from 9 a.m to 17 p.m and 1 carrier works from 10 a.m to 18 p.m. This latter realizes only the urgent deliveries. Each day, the pharmacist prepares the list of deliveries for each area. Each carrier verifies if the delivery list and the number of packets are coherent. Next, he schedules his deliveries and starts his tour regarding to his working hours including a break if it is required. Each carrier manages his time as he wants, however he must complete all deliveries regarding to his working hours without overtime if possible. The deliveries are organized by areas and each carrier is assigned to one of them according to his delivery list. The patients are assigned to areas from their admission to the homecare and each area has a defined day of delivery. Moreover, each patient receives one delivery per week and must sign the roadmap of the carrier when he receives the drugs packet. In addition of these scheduled deliveries, the pharmacist receives urgent prescriptions which arrive at any time and must be treated rapidly. So, the pharmacist must decide to assign these deliveries to the carriers if one of them can do it. Otherwise, the pharmacist calls one of the two external delivery services which assist the homecare.

Our purpose is to provide a decision support tool to the pharmacist to help him to schedule urgent deliveries and to insert them into the delivery planning. Besides, the problem of drug deliveries can be considered as a vehicle routing problem with time window (VRPTW) in which vehicles are carriers and homecare is the depot. The VRPTW problem involves a fleet of vehicles to serve a number of customers, at different geographic locations, with various demands and within specific customer time windows. The objective of the problem is to find routes for the vehicle to satisfy all the customers at a minimal cost (in terms of travelled distance) without violating the customer time windows constraints. Thus, in our drug delivery problem, it consists in finding feasible tours for carriers to deliver all the patients at a minimal cost without violating their availability time window.

3 Previous works

We have studied researches which focus on delivery problem, vehicle routing problem, and transportation scheduling problems. In a first part, we investigated in researches which focused on the Vehicle Routing Problem (VRP). In fact, VRP are omnipresent in industries, ranging from distribution problem to fleet management. They account for a significant portion of the operational cost of many companies. In this context, the operations research fields has produced in the last decades numerous algorithms and optimization methods which are both effective and efficient. These methods, based on mathematical programming, column generation, metaheuristics or tabu search are used to solve most of logistic problems.

Rousseau and Gendreau [1] presented a method using constraint programming as a neighborhood-searching algorithm for three operators in order to solve Vehicle Routing Problems. The authors used a neighborhood which can be constructed by moving a single customer from one route to another. Similarly, another method called Local Neighborhood Search (LNS) allows exploring a large neighborhood of the current solution by selecting a number of related customer visits to remove from the set of planned routes, and re-inserting these visits using a constraint-based tree search [2]. A specific local search approach is also used to schedule the tours of mixed vehicles over a working day from producing depots to demanding customers and vice versa [3].

Some researches based on tabu search studied the VRPTW problem. Indeed, Schulze and Fahle [4] introduce a new parallel tabu search heuristic for the vehicle routing problem with time window constraints (VRPTW). The reason for applying a parallel algorithm is to accelerate the tabu search process by performing several search threads in parallel. The neighborhood solution is generated by performing a sequence of simple customer shifts. Lau and al [5] proposed a tabu search approach characterized by a holding list and a mechanism to force dense packing within a route. They allow also relaxed time windows by introducing the notion of penalty lateness.

Bent and Van Hentenryck [6] propose a two-stage hybrid algorithm for the VRPTW problem. The algorithm based on simulated annealing minimizes first the number of vehicles. It then minimizes travel cost by using a large neighborhood search that relocates a large number of customers. Another research based on heuristics which are third generation artificial intelligent algorithms, namely simulated annealing, tabu search and genetic algorithm are used to reach optimal solutions [7].

Goel [8] presents a column generation approach which is particularly suited for the General Vehicle Routing Problem (GVRP). The GVRP is represented as a Set Partitioning Problem (SPP) and the objective is to maximise the profit which is determined by the accumulated revenue of all served transportation requests. Besides, the SPP has a vast number of vehicles and it can not be solved directly. Instead, the author proposes to solve a restricted version of the problem using a column generation approach where columns represent a limited number of vehicles. In [9], the manpower allocation problem with time windows, job-teaming (cooperation between teams) and a limited number of teams, is solved by column generation in a branch-and-price approach. The problem is divided into a generalized set-covering master problem and an elementary shortest path pricing problem using Dantzig-Wolfe decomposition. Westphal and Krumke [10] investigate a vehicle routing problem which can be modelled as an integer linear program. The problem consists in assisting people whose cars break down on their way and is considered as a vehicle dispatching problem with soft time windows and real-time requirements (decisions have to be made within a short time 10-15s). Thus, computing an optimal dispatch must be done with a short computation time. In order to speed up the resolution, the authors apply a Branch-and-Bound method (master problem) to explore only parts of the search tree knowing that they skip all tours having greater costs than an acceptance threshold cost. Thus, the pruning (procedure of skipping a sub tree) scheme (auxiliary problem) proposed speeds up the Branch-and Bound enumeration in the column generation process.

The GA-TSP (Genetic Algorithm-Travel Salesman Problem) is also used to solve VRPs. The author improved the genetic algorithm operators and developed a GUI (Graphic User Interface) type computer program according to the proposed method [11]. The VRPTW was also studied in the case of considering the delivery of perishable food products [12]. The objective of this approach is to maximize the expected total profit of the supplier depending on the value and the transaction quantity of perishable products carried to trailers. VRPTW is also used to assist distribution centers solving their special vehicle dispatching and routing problem [13]. The authors decompose the problem to a clustering problem (main problem) and a set of Traveling Salesman Problem (TSP) with time windows constraints. To solve the TSP, a genetic algorithm is developed with a simple heuristic algorithm.

Bin and al [14] propose an improved ant colony optimization which possesses a new strategy to update the increased pheromone and a mutation operation to solve VRP. Homberger and Gehring [15] propose an approach based on meta heuristics to solve the VRPTW problem. Their objective function combines the minimization of the number of vehicles and of the total travelled distance minimization.

Some researches dealt with the extended version of the vehicle routing problem. An extension of the split delivery vehicle scheduling problem with time windows and grid network distances was considered in order to model more realistic problems [16]. A deterministic tabu search algorithm was studied to solve the mix VRP problem in which the authors define the type and the number of vehicle as well as the order in which to serve the customers for each vehicle [17].

Vehicle routing problem with fuzzy time windows where time windows are not strictly respected was studied with the purpose of minimizing the travel distance and maximizing the service level of the supplier to customers [18]. Another researches studied the VRP with Soft Time Windows (VRPSTW) where they introduced a penalty when the time-windows are not respected [19].

Other researches focused on delivery scheduling using heuristics and ant colony algorithm. Irnich [20] developed new models and algorithms to improve the planning of letter mail delivery. He proposed a model of a very general incapacitated arc-routing problem capturing most of the practically relevant side constraints encountered by Deutsche post world net. Irnich suggested heuristic algorithms based on a transformation to an Asymmetric Travelling Salesman Problem (ATSP). Wang [21] proposed a multiple ant colony algorithm to solve a Vehicle Scheduling Problem with Route and Fuelling Time Constraints (VSPRFTC). The author considers a more elaborated VSPRFTC with two objective functions: the minimization of the number of tours (or vehicles) and the minimization of the makespan.

Some researches studied the nurse and staff scheduling problems. In fact, the homecare staff scheduling problem using combined vehicle routing and scheduling with temporal precedence and synchronization constraints, was studied in [22]. The authors present a mathematical model in order to make a computational study by comparing a direct use of a commercial solver (CPLEX) against a heuristic. Jaumard and al [23] present a generalized linear model for the complex nurse scheduling problem which considers workload, rotations and day-off…etc minimizing salary costs and maximizing both employee preferences and team balance. The auxiliary problem provides a feasible schedule for a given nurse and the master problem finds a configuration of individual schedules to satisfy the demand coverage constraints. Eveborn and al [24] focused also in staff planning in homecare and developed a decision support system “Laps Care” to aid the planners. The system consists of a number of components including information data bases, maps, optimization routines, and report possibilities. The authors formulate the problem using a set partitioning model and, for a solution method, they make use of a repeated matching algorithm. In [25], column generation approach combining integer programming and heuristics is used to solve the nurse scheduling problem. The integer program is used to provide the set of alternatives schedules for nurses and a double swapping heuristic is used to generate the columns.

Many problems faced by Operational Research in industry can be applied to problems in health care. In fact, the two fields treat the same problems and thus, the industrial problems can be adapted to homecare logistic by adjusting the different parameters to the given situation. Thus, in our work, we made a literature review about works that treated the vehicle routing problem which can be adapted to our drug delivery problem. Moreover, we studied also the researches which focused on nurse and staff scheduling. However, we did not found works which focused on the drug delivery problem in homecare, as for, we propose a mathematical formulation to solve the problem studied.

4 Mathematical formulation

In this section, we introduce the notation which will be used in the mathematical formulation of the problem. We aim to provide a decision support tool based on mathematical modelling in which we take into account urgent and planned deliveries.

The delivery problem can be described as a VRPTW problem. Given a set of customers (patients), a set of vehicle (carriers) and a depot (homecare), find a set of routes (tours), starting and ending at the depot such that each customer (patient) is visited by exactly one vehicle (carrier). Each customer (patient) has a specific demand which must be realized into his time window of availability. In our model, we do not take into account capacity constraints. In fact, the homecare does not have problems of vehicle capacity in their deliveries. In our delivery problem, the number of carrier is fixed. Thus, we are in the case of m-VRPTW where the number of vehicle is fixed. We describe the four strategies of deliveries which are proposed to the pharmacist:

- (i) starting tours when a specified number of deliveries is reached,
- (ii) starting deliveries if a specified distance is reached by the calculated tour,
- (iii) starting the deliveries of each carrier when a fixed number of assigned deliveries is reached,
- (iv) starting deliveries on fixed hours.

These four strategies correspond to the situation. The pharmacist can manage their deliveries as he want according to the number of deliveries, to the availability of the carriers, to the urgency of the delivery…etc.

4.1 Hypothesis

We consider that:

- Each patient receive at most one delivery per day,
- Each delivery has a time window for being accomplished,
- Each day of the week is assigned to an area of deliveries,
- Each delivery has a duration,
- Urgent deliveries are added to the list of planned deliveries.

4.2 Notation

Let p be the number of deliveries. We introduce a dummy delivery denoted by 0 which represents the homecare. Let also n be the number of carriers and let Hd_l the beginning working hour of the carrier *l*. Each delivery *j* has a duration D_j and a time window $[e_j, l_j]$ which specifies when each delivery must be accomplished. This time window represents the earliest and the latest dates (in minutes) in which each delivery *j* must be done. Let $d_{j,k}$ denote the distance (in minutes) between two consecutives deliveries *j* and *k*. Our model is described with the following notations.

Data

p : number of deliveries indexed by $j, k, h = 1,..., p$,

n : number of carriers indexed by $l = 1,..., n$,

D_j : duration of the delivery *j* in minutes,

e_j : earliest date to accomplish delivery *j*,

l_j : latest date to accomplish delivery *j*,

$d_{j,k}$: distance in minutes between two deliveries *j* and *k*,

$dur\_tour\_\max$: maximum length of a tour corresponding to working hours of each carrier,

HV : a high value;

$dist\_\max$: maximum distance between two successive deliveries in minutes,

Hd_l : work starting time of each carrier *l*,

We based our approach on a model used for tour planning of nurses in a homecare [26]. We adapted this model for the drug delivery problem by adding constraints and by adjusting it for each delivery strategy.

4.3 *Model formulation*

To model the problem, we define for each delivery *j* a binary variable $X_{l,j,k}$. We define also a positive variable $sl_{l,j}$ which correspond to the arrival date of the carrier *l* to the delivery point *j*.

Binary variables

$$X_{l,j,k} = \begin{cases} 1 & \text{if the carrier } l \text{ realizes the delivery } j \text{ immediately before the delivery } k \\ 0 & \text{Otherwise} \end{cases}$$

Integer variables $sl_{l,j}$: arrival date of the carrier *l* to the delivery location *j*.

The problem can then be formulated as the following integer linear programming model:

$$\min \left[\sum_{l=1}^{n} \sum_{j=0}^{p} \sum_{k=0/k\neq j}^{p} d_{jk} \cdot X_{ljk} \right] \tag{1}$$

s.t.

$$\sum_{l=1}^{n} \sum_{j\,/\,j\neq k=0}^{p} X_{ljk} = 1 \quad \mathit{for} \quad k \in [1,\ p] \tag{2}$$

$$\sum_{l=1}^{n} \sum_{k\,/\,j\neq k=0}^{p} X_{ljk} = 1 \quad \mathit{for} \quad j \in [1,\ p] \tag{3}$$

$$\sum_{l=1}^{n} \sum_{j\,/\,j\neq h=0}^{p+n} X_{ljh} \cdot l = \sum_{l=1}^{n} \sum_{k\,/\,k\neq h=0}^{p+n} X_{lhk} \cdot l \quad \mathit{for} \quad h \in [1,\ p+n] \tag{4}$$

$$sl_{lj} + D_j + d_{jk} \leq Dur\_tour\_\max \quad \mathit{for}\ l \in [1,n] \quad \mathit{and}\ \ j \in [1,p]\ \mathit{and}\ \ k = 0 \tag{5}$$

$$sl_{lk} \geq sl_{lj} + D_j + d_{jk} + (X_{ljk} - 1) \cdot HV \quad \mathit{for}\ j \in [0,p]\ \mathit{and} k \in [1,p]\ \mathit{and}\ j \neq k\ \mathit{and} l \in [1,n] \tag{6}$$

$$e_j \leq sl_{lj} \quad \mathit{for}\ \ j \in [1,p]\ \mathit{and}\ \ l \in [1,n] \tag{7}$$

$$l_j \geq sl_{lj} \quad \mathit{for}\ \ j \in [1,p]\ \mathit{and}\ \ l \in [1,n] \tag{8}$$

$$\sum_{k=0}^{p} \sum_{j\,/\,j\neq k=p+1}^{p+n} X_{ljk} = 1 \quad \mathit{for} \quad l \in [1,n] \tag{9}$$

$$\sum_{j=0}^{P} \sum_{k\,/\,k\neq j=p+1}^{p+n} X_{ljk} = 1 \quad \mathit{for} \quad l \in [1,n] \tag{10}$$

$$\sum_{k=0/k\neq j}^{p+n} X_{ljk} = 1\ \ \mathit{for}\ l \in [1,n]\ \mathit{and}\ \ j = 0 \tag{11}$$

$$\sum_{j=0}^{p+n} X_{ljk} = 1 \quad \mathit{for}\ l \in [1,n]\ \mathit{and}\ k = 0 \tag{12}$$

$$X_{ljk} \cdot d_{jk} \leq dist\_\max \quad \mathit{for}\ k \neq j\ \mathit{and}\ k \in [0,p]\ \mathit{and}\ \ j \in [0,p]\ \mathit{and}\ l \in [1,n] \tag{13}$$

$$sl_{l1} \geq Hd_l \ \ \mathit{for}\ \ l \in [1,n] \tag{14}$$

$$sl_{lj} \geq 0 \ \ \mathit{for}\ l \in [1,n]\ \mathit{and}\ \ j \in [1,p] \tag{15}$$

$$X_{ljk} \in \{0,1\} \ \ \mathit{for}\ l \in [1,n]\ \mathit{and}\ \ j \in [0,p]\ \mathit{and}\ k \in [0,p] \tag{16}$$

The objective function (1) minimizes the total travelled distance. The constraints (2) and (3) ensure that all the deliveries are processed only once. Constraints (4) ensure that each tour is coherent i.e. realized by the same carrier. Constraints (5) impose a maximum length for the carrier tour which corresponds to allowed working hours. Constraints (6) calculate the arrival date of the carrier to the delivery location *k* after visiting the delivery location *j*. The arrival date takes into account the distance between two locations visited consecutively and the duration of the delivery. Constraints (7) and (8) impose that each delivery is completed into a time window which correspond to the availability of patients (customers) at home. Constraints (9) and (10) ensure that each carrier has a break of one hour during his working hours. Constraints (11) and (12) guarantee that each carrier starts and return to the homecare after finishing his tour. Constraints (13) make sure that a maximal distance *dist_max* between two delivery locations visited consecutively is respected.

The model described above is used to describe and solve the four strategies for drugs deliveries. For each strategy, we allow us to change this model by adding or removing constraints.

<u>First strategy</u>: starting tours when a specified number of deliveries is achieved

In this case, we define a number of deliveries *N* from which we start the resolution of the mixed linear program. For each delivery, the pharmacist introduces the postcode and the corresponding time window. Once, the number of deliveries *N* is reached, the solver calculates the planning of deliveries. So, for this strategy, the model does not change and we do not introduce any other constraints.

<u>Second strategy</u>: starting deliveries if a specified distance is reached by the calculated tour

We modify the constraints (5) by adding a new variable $length_l$ which is equal to the length of the tour made by each carrier *l*. So, when we have several deliveries to realize, we start the resolution and we check if the tours obtained have a sufficient length to begin the deliveries. When one of the tour length is equal or greater to the fixed distance, we start the deliveries. Otherwise, we wait for other deliveries.

$$sl_{lj} + D_j + d_{jk} \leq length_l \quad for \quad l \in [1, n] \; and \quad j \in [1, p] \; and \quad k = 0 \tag{5'}$$

<u>Third strategy</u>: starting the deliveries of each carrier when a fixed number of assigned deliveries is reached

Each tour must at least be constituted of a specified number *M* of deliveries. For this strategy, we add the constraints (17) to ensure that each carrier start only if *M* deliveries are reached. Besides, we alter the constraints (2) and (3) in order to assess the model feasibility. In this strategy, we suppose to have *m* carriers which allow us to complete the total number of deliveries to achieve and which can take into account *M* deliveries per tour. In fact, we retain m carriers knowing that the number m is equal to (n - 1) in order to assume that we have a dummy carrier which takes all the remaining deliveries which can not constitute a regular tour of M deliveries. The minimum number of deliveries is equal to M*m.

$$\sum_{l=1}^{m} \sum_{j=0 \,/\, j \neq k}^{L} X_{ljk} = 1 \quad for \quad k \in [1, p] \tag{2'}$$

$$\sum_{l=1}^{m} \sum_{k=0 \,/\, j \neq k}^{L} X_{ljk} = 1 \quad for \quad j \in [1, p] \tag{3'}$$

$$\sum_{j=0}^{P} \sum_{k=1 / k \neq j}^{P} X_{ljk} = M \quad for \;\; l \in [1, m] \tag{17}$$

<u>Forth strategy</u>: starting deliveries on fixed hours

We define three hours to start deliveries: 10a.m, 14p.m and 18p.m. We organize for each carrier three tours per day. We impose the three departing hours by the time window of each delivery.

5 Resolution

To solve the model presented above, we use the solver *LINGO* which is an academic solver from LINDO SYSTEMS INC. We conducted all the tests on an Intel® Pentium® M with 1500MHz CPU, using LINGO_8. First, we present the results obtained with the different strategies described based on an example of 3 carriers and 10 deliveries. Second, we experiment the mathematical model for different sizes in order to compare the trend of results obtained.

5.1 Example of resolution

In order to illustrate the results of our mathematical model, we solve the mixed linear program for 10 deliveries with 3 carriers for the three first strategies. In this part, we aim to show the applicability of the model developed. We studied the data of February 2008 of the homecare pharmacy and we choose to experiment the model for 10 deliveries which were delivered by the homecare using postcodes in order to define the data $d_{j,k}$ which represent the real distance separating the delivery locations *j* and *k* in minutes [27]. The following tables (table 1 to table 4) illustrate the results obtained.

Table 2 illustrates the results obtained when the model is solved using the first strategy (starting tours when a specified number of deliveries is reached). The solver calculates a planning for the 3 carriers in 24 seconds with an optimum objective value of 492mn. Each carrier has a break of one hour and achieves the deliveries regarding his working hour. We observe that the third carrier has a longer tour than the two other carriers and complete 5 deliveries when the first accomplish only 2 deliveries and the second 3 deliveries. All the deliveries are completed.

Table 1
Results obtained for each strategy

Strategy	***Computation time***	***Objective value***[2]	***Optimum***
1	24s	492	Yes
2	4mn11s	492	Yes
3	1s	232	Yes
4	18s	412	Yes

Table 2
Planning obtained for the first strategy

	Carriers		***Tour***							
Strategy 1	*Carrier$_1$*	Tour	0	4	B[3]	10	0			
		Arrival date (mn)	0	90	180	260	287			
	Carrier $_2$	Tour	0	9	1	B	7	0		
		Arrival date (mn)	0	125	160	215	335	357		
	Carrier$_3$	Tour	0	5	B	2	3	6	8	0
		Arrival date (mn)	0	60	180	280	330	385	420	465

Table 3
Planning obtained for the second strategy

	Carriers		***Tour (case 10 + 2 deliveries)***							
Strategy 2	*Carrier$_1$*	Tour	0	4	B	12	11	10	0	
		Arrival date (mn)	0	90	180	260	265	270	297	
	Carrier $_2$	Tour	0	9	1	B	7	0		
		Arrival date (mn)	0	125	160	240	400	422		
	Carrier$_3$	Tour	0	5	B	2	3	6	8	0
		Arrival date (mn)	0	60	180	280	330	385	420	465

Table 3 illustrates the results obtained when the model is solved using the second strategy (starting deliveries if a specified distance is reached by the calculated tour). We start the resolution of the mathematical model with 10 deliveries. However, we obtain tours that do not satisfy the length required to start deliveries (420mn). After that, we add two deliveries and solve the model with 12 deliveries and we obtain a planning after a computation time of 4mn11s. Only one tour has reached the length required (third tour). For the two other tours (first and second), the length is lower than the length required to start the deliveries. Thus, only one carrier can start his tour.

In the third strategy (starting the deliveries of each carrier when a fixed number of assigned deliveries is reached) (Table 4), each tour must be constituted of 4 deliveries. We obtain a planning for two carriers after one second of computation time with an objective value of 232mn. The third carrier has no deliveries to accomplish because the number of deliveries remaining is lower than 4 deliveries. He can not start his tour with only 2 deliveries.

Table 4
Planning obtained for the third strategy

	Carriers		***Tour***						
Strategy 3	*Carrier$_1$*	Tour	0	5	B	2	3	7	0
		Arrival date (mn)	0	60	180	268	330	361	383
	Carrier $_2$	Tour	0	9	1	B	6	8	0
		Arrival date (mn)	0	125	160	200	360	420	465

With only 10 deliveries, we considered that it is not interesting to present the results for the forth strategy (starting deliveries on fixed hours).

5.2 Results of the different strategies

In this section, we present the results obtained (sum of distances between deliveries) for different sizes of the problem in the table 5.

[2] The objective value is part of the sum of the total length of all the tours. The objective function does not take into account the duration of the deliveries.
[3] Break

In the case of planning 15 deliveries, we notice that the computation time vary from 9s (forth strategy) to 51mn (third strategy) and for all the strategies, we obtained the optimal solution.
In the case of scheduling 20 deliveries, the solution of the first strategy is optimal with an objective value of 363mn. For the second strategy (starting deliveries if a specified tour length is reached**)**, we choose to interrupt the resolution after the first solution found by the solver. Thus, we obtain after 4h39mn an objective value of 377mn. We obtain three planning for which the length of tour is greater then the length specified, so, the carriers can start their deliveries. The third strategy (starting the deliveries of each carrier when a fixed number of deliveries assigned is reached) consists in planning each tour for each carrier only if he has 6 deliveries to do. The objective value is 377mn obtained after a computation time of 2h46mn. For the forth strategy (starting deliveries on fixed hours), we interrupted the resolution after the first solution found and we find an objective value of 366mn. We obtain a tour for each carrier which corresponds to his starting time of deliveries. So, only the solutions of the first and the third strategies proved to be optimal.

Table 5
Results of resolution

Number of deliveries	strategy	computation time	objective value	optimum
15 deliveries	strategy 1	9mn43s	339	yes
	strategy 2	9mn43s	339	yes
	strategy 3	51mn28s	378	yes
	strategy 4	9s	312	yes
20 deliveries	strategy 1	1h32mn34s	363	yes
	strategy 2	4h39mn17s	377	unknown
	strategy 3	2h46mn17s	377	yes
	strategy 4	1h05mn09s	366	unknown
30 deliveries	strategy 1	3h03mn	X	unknown
	strategy 2	2h53mn	X	unknown
	strategy 3	2h30mn	X	Unknown
	strategy 4	2h30mn	X	unknown

In the case of planning 30 deliveries, we do not obtain solution. The solver does not solve the mixed integer program for this size of problem. We observe that the model developed has some limitations. However, we can divide the problem into two sub-problems of 15 deliveries regarding to areas and thus, we will obtain a planning of deliveries.
To conclude, we observe that in the case of 15 and 20 deliveries, we obtain results for all the strategies. In the case of planning 15 deliveries, all the results are optimal and are obtained after a short computation time. However, in the case of 30 deliveries, we do not have any results.
The different strategies are compared in table 7. The first strategy (starting deliveries when a specified number of deliveries is reached) consists in planning a specified number of deliveries taking into account various constraints and we do not set any conditions to the assignment of carriers to deliveries.
The second strategy consists in starting deliveries when the length of tour required is reached. On the one hand, we optimize the works of the carriers and we maximize the load of his tour respecting his working hours. On the other hand, the pharmacist can have at least one carrier available when all the others are gone and can face the urgent deliveries which must be accomplished rapidly.
The third strategy is based on assigning a fixed number of deliveries to each carrier. If the number of deliveries is not important, the carrier will return early at the homecare, and will be able to begin another tour. Hence, the pharmacist could manage efficiently their planned and urgent deliveries. Otherwise, if the number of deliveries assigned to the carriers is important then the length of tours is great. Thus, the carriers will not return to the homecare early and the pharmacist will need to turn to external delivery services. Facing alternatively to these two cases, it could be efficient to assign a great number of deliveries to a carrier and to keep the others in the homecare. They will be employed for possible urgent deliveries or for deliveries which must be completed later at the end of the day.
The forth strategy consists in starting deliveries on fixed hours. This method allows the pharmacist to adjust the deliveries during the day. All the carriers will be available three hours per day to start their tours and return to the homecare, so the delivery lists can be adjusted taking into account the urgent deliveries. Table 6 syntheses the advantages and the drawbacks of each strategy.

6 Conclusion

In this paper, we focused on a drug delivery problem in a French homecare. The drug delivery problem is due to planned deliveries and urgent prescriptions which can arrive at any time when all the carriers are gone. Hence, the pharmacist does not have available carriers to accomplish those urgent deliveries and so, he must use one of the two external deliveries services which are quite expensive. If the deliveries are better planned, these costs can be minimized. In this context, we proposed a mathematical model based on mixed linear programming. We notice that the mathematical model does not give results for more than 20 deliveries. However, our principal purpose was to experiment and to show the applicability of this method for the homecare drug delivery problem. We have noticed that in the literature review, many researches have already studied the VRPTW and these researches based essentially on column generation, tabu search, simulating annealing…, and all these applications were used in industry. Thus, we tried to employ the VRPTW in homecare using mathematical modelling and solvers. We added the consideration of the availability of carrier by considering their work starting hours and their geographical repartition. We used this mathematical model to experiment four strategies for deliveries that can be managed according to the delivery orders arrival, to the prescriptions change, to geographic areas, to carrier load…etc. After that, we proposed four strategies in order to provide a decision support tool to the pharmacist. The four strategies consists respectively in scheduling a specified number of deliveries (first strategy), starting a tour if only a defined distance is reached by the calculated tour (second strategy), starting a tour if it is composed of M deliveries (third strategy) and fixing three hours for starting deliveries per day (forth strategy). We use a mathematical model based on mixed integer programming for each strategy and compare the results obtained in order to identify which strategy suits better to the drug delivery problem in the homecare. We conclude that each strategy has his advantages and drawbacks. However, these four strategies can better suits to given situations and thus, the pharmacist can plan the deliveries using the right strategy. Indeed, this work constitutes a contribution to help the pharmacist by facilitating the delivery planning.

Table 6
Advantages and drawbacks of the four strategies

Strategy	**Advantages**		**Drawbacks**
1- starting tours when a specified number of deliveries is achieved	Start deliveries at any time Choose the number of deliveries to plan		No carrier available for possible urgent deliveries and the pharmacist can need to call the external delivery services Unequal workload between carriers
2- starting deliveries if a specified distance is reached by the tour calculated by the solver	The pharmacist could manage efficiently their deliveries	Carriers can accomplish urgent deliveries Optimize the work of the carriers	If the distance specified is important, the carrier will not return to the homecare and will not be able to deliver the possible urgent deliveries.
3- a number of deliveries is fixed to begin a tour		We can divide the deliveries equally between the carriers The urgent deliveries can be accomplished at any time if a carrier is available	If the number of deliveries is important, the carrier will not return to the homecare early. Thus, the pharmacist will not have available carriers for urgent deliveries
4- starting deliveries on fixed hours	We define three hours of deliveries according to the number of deliveries to be accomplished		All the carriers start their tours and the pharmacist can need to call the external delivery services Fixed hours to start deliveries

To improve the proposed approach, we suggest adding cut constraints which will reduce the computational time minimizing studied solutions number. Besides, we can take into account the overtime when it can not be avoided and minimize not only the distance travelled, but also the amount of overtime hours done by the carriers. It could also be interesting to add constraints which will balance the workload between carriers.

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