Constrained dynamics and parametrized control in biped walking

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Abstract

The intermittent contact with the ground is the main specificity of walking robots, allowing more versatility in their displacements, but resulting in a structural instability of these systems. A walking robot, having a free-floating base, cannot control its global movements directly and must rely on the limited interaction forces in order to move. These constraints on the movements of the robot are an obstacle to the stabilization of a trajectory. We propose then to stabilize not a single trajectory but a parametrized set $q_d(t, p)$ of all the possible trajectories, depending on the lentgth of the steps, the speed of execution... The idea is that a destabilization can be compensated by an adaptation of the walk.

1 Introduction

With service robotics or medical applications, biped walking was studied for a long time, but the control laws proposed so far are not satisfactory in providing robust steady walks. The main specificity of the walking robots is the intermittent contact with the ground: it allows more versatility in their displacements, but it results in a structural instability. In order to move around, a walking robot is dependant on its interaction with the ground, and especially on the contact forces. But these forces are bounded, inducing some constraints on the dynamics and therefore on the stability of the robot.

In order to design a control law for biped walking, some proposed to monitor the contact forces while stabilizing a desired trajectory. The contact forces are monitored directly in [3, 4] or through the position of the centre of pressure (also called Zero Moment Point) in [7]. This way, small perturbations can be compensated without destabilizing the robot, but in case of strong perturbations, when the desired trajectory can't be followed anymore, no recovery option has been proposed so far. Note that some proposed a more global approach to the walking behaviour [5], leading to some interesting stability results, but with no analytic proof.

The walking behaviour is richer than just following one single trajectory. We propose then to stabilize not a single trajectory $q_d(t)$, but a whole parametrized set $q_d(t, p)$ representing a more comprehensive version of the walking behaviour. This way, when one trajectory is not stabilizable anymore, another might be stabilized instead. This approach allows to improve the stability of the robot, and seems to be completely new.

After briefly introducing the dynamics of the robot in section (2.1), we stress upon the effect of the contact with the ground on this dynamics, and especially on the stability of the robot in (2.2) and (2.3). In sections (3.1) to (3.3), we show how a parametrized set of trajectories can be used to improve the stability of the robot, resulting in the design of a new control law for biped walking presented in (3.4).

2 Dynamics and stability of a biped robot

2.1 Dynamics of the robot

The main specificity of the dynamics of a biped robot is the intermittent contact with the ground: we model this interaction as non-penetrating rigid bodies with Amontons-Coulomb friction. For simplicity, we will consider only sticking contact (no slipping occurs during plain walking), therefore we introduce a set of normal constraints $\varphi_n(q) \geq 0$, and the affiliated tangential constraints $\varphi_t(q) = 0$, with q the variables expressing the configuration of the robot. A straightforward derivation of the constrained Lagrangian dynamics leads to:

$$M(q)\,\ddot{q} + N(q, \dot{q}) = T\,u + C(q)^T \lambda \tag{1}$$

with $M(q)$ the inertia matrix, $N(q, \dot{q})$ the gravity and other nonlinear terms, and $T\,u$ the actuation. D'Alembert's principle states that the generalized contact forces can be expressed as $C(q)^T \lambda$ with $C(q)$ the Jacobian of the constraints and λ some Lagrange multipliers.

There is a complementarity condition between the constraints and the contact forces, and since the set of active constraints is varying, this gives rise to hybrid dynamics. For simplicity, we will consider exclusively, at a given state, the constraints (normal and tangential) that remain active (such

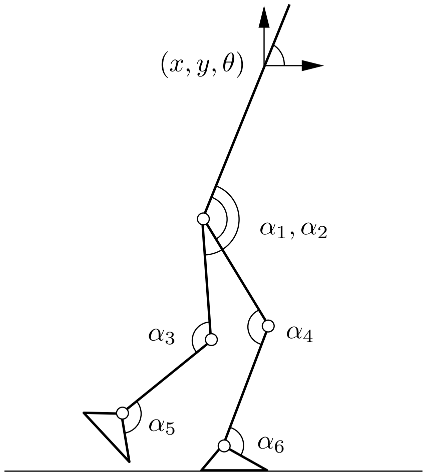

Figure 1: The configuration of a 6-DOF planar biped robot can be expressed through 9 variables, 3 for the global position of the robot (x, y, θ) and 6 for its internal shape $(\alpha_1, \dots, \alpha_6)$, to obtain the state vector $q = (x, y, \theta, \alpha_1, \dots, \alpha_6)^T$.

that $\varphi(q) = \dot{\varphi}(q, \dot{q}) = \ddot{\varphi}(q, \dot{q}, \ddot{q}) = 0$). The unilateral contact and the Amontons-Coulomb friction imply some bounds on the contact forces ($F_n \geq 0$ and $|F_t| \leq \mu F_n$), which we express as a set of inequalities on the Lagrange multipliers:

$$\mathcal{A}(\lambda) \geq 0 \tag{2}$$

When a new contact is reached, an impact may occur, resulting in a jump of the velocities [2], but impacts lead to premature aging of the mechanical structures and should therefore be avoided. For this and for simplicity, we will not consider impacts here. Note that slipping, impacts and hybrid dynamics must definitely be included in any comprehensive modelization of a walking robot, but this is not needed here.

2.2 Feasible movements

The variables q that describe the configuration of the robot consist in two different sets: a first set q_1 describing the global position of the robot in the space, and a second set q_2 describing the joint positions (Fig. 1). Let us split the Lagrangian dynamics (1) in order to dedicate the upper part to the global movements and the lower part to the joint variations:

$$\begin{bmatrix} M_1 \\ M_2 \end{bmatrix} \ddot{q} + \begin{bmatrix} N_1 \\ N_2 \end{bmatrix} = \begin{bmatrix} 0 \\ T_2 \end{bmatrix} u + \begin{bmatrix} C_1 & C_2 \end{bmatrix}^T \lambda$$

The biped robot has no direct actuation of its global movements, what is expressed here by the fact that $T_1 = 0$. It is therefore dependent on the contact forces $C_1^T \lambda$ in order to generate any global movement. But these forces are constrained (2), implying some conditions on the feasibility of the movements of the robot:

$$\begin{cases} M_1 \ddot{q} + N_1 = C_1^T \lambda \\ \mathcal{A}(\lambda) \geq 0 \end{cases} \tag{3}$$

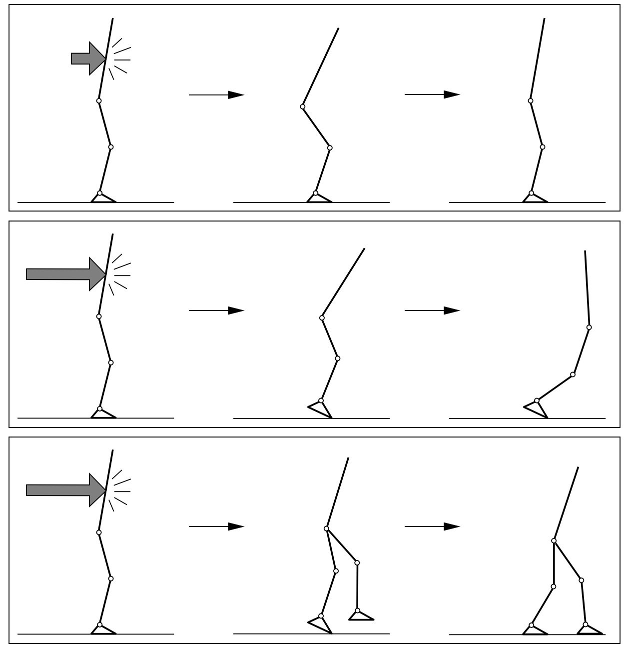

Figure 2: Stabilizing a standing stationary robot, small perturbations (top) can be compensated, but for stronger ones (middle), the only way to avoid to fall is to make a step, returning to the desired state later (bottom).

In order to illustrate the effects of these constraints on the stability of the robot, let's consider the stabilization of a standing stationary position (Fig. 2). For slightly perturbated initial conditions, the robot can be driven back to the equilibrium point (top strip), but over a threshold on the perturbation, the contact with the ground can't generate anymore the forces that would stir the robot to the desired position (middle strip). In this case, the only way for the robot to avoid to fall is to make a step (bottom strip), returning to the desired state later.

2.3 Viability and invariance

We have seen that the stability of a reference state can't be accurately related to the property that the robot can avoid to fall or not. But it's this property we're interested in: it is in fact a viability property [1]. Without going into accurate definitions, we can say that: *a state is viable if it is possible to avoid to fall from it through a suitable selection of the control inputs*. Then, the robot can avoid to fall as long as it remains in the viability kernel (Fig. 3), the set of all the viable states.

For a given control law, a fall is actually avoided only in a sub-set of the viability kernel, an invariant set. Outside this invariant set, the control law can't avoid a fall of the robot, even inside the viability kernel where it would have been possible. The design of the control law must therefore make invariant the largest sub-set of the kernel. Unfortunately, a control law can't be computed directly from the viability properties.

We have seen (Fig. 2) that a perturbed state might quickly

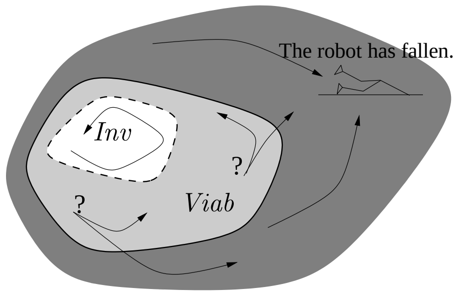

Figure 3: The viability kernel ($Viab$) is the set of all the states from where it is possible to avoid to fall. For a given control law, a fall is avoided only in an invariant set (Inv), a sub-set of the viability kernel.

be outside the invariant set $Inv(u_1)$ of a control law u_1 but that in this case, it might be inside the invariant set $Inv(u_2)$ of another control law u_2. It appears then that u_1 and u_2 can be linked together in order to make invariant $Inv(u_1) \bigcup Inv(u_2)$. We can foresee then the possibility to design a control law u exploiting judiciously a set of control laws u_k in order to obtain $Inv(u) \supset \bigcup_k Inv(u_k)$. Unfortunately, the invariant sets can't be obtained numerically, so a control law can't either be computed directly from the invariance properties.

3 Stabilization of the walking behaviour

3.1 Parametrized trajectories

The robot's walking patterns vary, depending on the length and height of the steps, the speed of execution, and all kinds of reactions to the environment. All these variations can occur during a walk, and should be considered in the definition of the walking behaviour. Moreover, when the desired trajectory is not stabilizable anymore, one of these variations might be stabilized instead.

Let's introduce then a parametrized set of trajectories $q_d(t,p)$, the parameters p expressing the characteristics of each pattern, such as the length, height and speed of the steps. Doing so, the error of stabilization $\tilde{q} = q - q_d(t,p)$ appears depending on the state of the robot q as well as on the selected parameters p. Let's replace then $\ddot{q} = \ddot{\tilde{q}} + \ddot{q}_d(t,p)$ in (3):

$$\begin{cases} M_1 \ddot{\tilde{q}} + M_1 \ddot{q}_d(t,p) + N_1 = C_1^T \lambda \\ \mathcal{A}(\lambda) \geq 0 \end{cases} \tag{4}$$

We can observe here the reciprocal influence between the regulation of the error $\tilde{q}$, the value of the parameters p, and the satisfaction of the constraints on the dynamics of the robot.

We have introduced a new variable p which has an influence on the stabilization of the robot. The idea is to use the possible adaptations of the parameters p in order to compensate the constrained dynamics (4), maintaining this way the regulation of the error $\tilde{q}$. This can be interpreted as a change of walking pattern in order to maintain the stability of the robot. Note that this realizes exactly the behaviour exposed in the bottom strip of (Fig. 2).

One way to achieve this is to consider piecewise constant parameters: when the stabilization of the desired trajectory is no more possible, another stabilizable trajectory is searched for, and an instantaneous modification of p is generated. This modification induces a jump of $\ddot{q}_d(t,p)$, but also of q_d and $\dot{q}_d$, and therefore of $\tilde{q}$, $\dot{\tilde{q}}$ and $\ddot{\tilde{q}}$, giving rise to hybrid dynamics, the stability of which is not obvious.

3.2 Stabilization of a set

If we want the variations of p not to generate any hybrid dynamics, we have to consider that $p(t)$ can be differentiated twice:

$$\dot{q}_d(t,p,\dot{p}) = J(t,p)\,\dot{p} + \frac{\partial q_d}{\partial t}(t,p)$$

$$\ddot{q}_d(t,p,\dot{p},\ddot{p}) = J(t,p)\,\ddot{p} + n(t,p,\dot{p}) \tag{5}$$

with $J(t,p) = \partial q_d(t,p)/\partial p$ and $n(t,p,\dot{p})$ the other terms appearing during the derivation. Replacing $\ddot{q}_d$ in (4), we obtain:

$$\begin{cases} M_1 \ddot{\tilde{q}} + M_1 J\,\ddot{p} + M_1 n + N_1 = C_1^T \lambda \\ \mathcal{A}(\lambda) \geq 0 \end{cases} \tag{6}$$

Here, the variations of p are directly connected to the regulation of the error $\tilde{q}$ and to the constraints on the dynamics that have to be satisfied. The adaptation of p required to compensate the constraints and maintain the regulation of $\tilde{q}$ may be inferred trivially, then. Note that this relation is achieved through the term $J\,\ddot{p}$ which expresses the movements generated by a modification of the walking pattern.

Since $p(t)$ varies, the trajectory actually followed $q_d(t,p(t))$ might be far from the original walking patterns $q_d(t,p)$ (for a fixed p). Gathering all the walking patterns into a set $\mathcal{Q} = \{q_d(t,p) : t \in \mathbb{R},\ p \in \mathcal{P}\}$, it appears that letting p vary amounts to allowing any trajectory inside $\mathcal{Q}$. This way, the set $\mathcal{Q}$, which can be considered as a more comprehensive description of the walking behaviour, is stabilized as a whole.

As long as the constraints in (6) can be compensated by $M_1 J\,\ddot{p}$, the stability of the robot is safe, but this reserve of trajectories is limited: $M_1 J$ might not be of full rank, meaning that the modifications of the walking pattern might not compensate every constraints on the global movements. Moreover, some trajectories might not be realisable due to obstacles in the environment or to some mechanical limitations (maximal step length, for example): p might be constrained to a realisable sub-set of $\mathcal{P}$.

Still, this approach encompasses the traditionnal trajectory following which works with a fixed walking pattern, what should improve the stability of the robot: the invariant set obtained when stabilizing a single trajectory should definitely be included in the invariant set obtained when stabilizing a larger set, of higher dimension... Unfortunately, no inclusion can be proved concerning these invariant sets (cf. section 2.3).

3.3 Monitoring the contacts

Remember that the dynamics of the robot is also constrained by the contact with the ground ($\varphi_n(q) \geq 0$ and $\varphi_t(q) = 0$ in section 2.1). Moreover, the contact forces can only be generated when the contact is active. A control law for the biped robot must therefore carefully monitor and control the active constraints $\varphi(q) = 0$. Replacing $q = \tilde{q} + q_d(t, p)$, these constraints establish another relation between the regulation of $\tilde{q}$ and the variations of p:

$$\varphi(\tilde{q} + q_d(t, p)) = 0$$

From this relation, we can see that when $\tilde{q} = 0$, the selected trajectory must satisfy the active constraints ($\varphi(q_d) = 0$). But when $\tilde{q} \neq 0$, the transient behaviour of $\tilde{q}$ will most probably impose a variation of p which might not be necessary nor even desired. In particular, when $\varphi(q_d) = 0$ and the constraints (6) are satisfied with $\dot{p} = \ddot{p} = 0$, there's no point in changing the reference trajectory, but the transient might induce such a change.

A solution to avoid this unsatisfying transient is to decouple the dynamics with respect to the constraints $\varphi(q) = 0$. For this, let's follow the task function approach [6], and introduce a C^2 diffeomorphism $f(q)$ as an output function to regulate. Differentiating twice the error $e(q, t, p) = f(q) - f(q_d(t, p))$ and using (5), we obtain:

$$\dot{e} = B(q)\,\dot{q} - B(q_d)\,\dot{q}_d(t, p, \dot{p})$$

$$\ddot{e} = B(q)\,\ddot{q} - B(q_d)J(t, p)\,\ddot{p} + b(q, \dot{q}, t, p, \dot{p}) \qquad (7)$$

with $B(q)$ the Jacobian of $f(q)$ and $b(q, \dot{q}, t, p, \dot{p})$ the other terms appearing during the derivation. The dynamics of the error that appears in (7) and (1) can be linearized and decoupled (see section 3.4) in order to obtain the dynamics $\ddot{e} = v$.

The output function $f(q)$ might include any information needed for the stability of the robot, such as the position of the center of mass of the robot. Here, we stress the need to monitor the active constraints. Let's consider then an output function of the form:

$$f(q) = \begin{bmatrix} f_1(q) \\ f_2(q) = \varphi(q) \end{bmatrix}$$

Concerning the regulation of the second part of the output function, we have then:

$$\ddot{e}_2 = \ddot{\varphi}(q, \dot{q}, \ddot{q}) - \ddot{\varphi}(q_d, \dot{q}_d, \ddot{q}_d) = v_2$$

Since $\ddot{\varphi}(q, \dot{q}, \ddot{q}) = 0$ (cf. section 2.1), and differentiating twice $\varphi(q_d)$, we have:

$$v_2 + C(q_d)\ddot{q}_d + s(q_d, \dot{q}_d) = 0$$

with $C(q_d)$ the Jacobian of $\varphi(q_d)$ and $s(q_d, \dot{q}_d)$ the other terms appearing during the derivation. Using (5) then, we obtain:

$$v_2 + CJ\,\ddot{p} + Cn + s = 0 \qquad (8)$$

Where the interaction between the contact with the ground and the regulation of the parameters p appears decoupled from any transient of the rest of the dynamics.

3.4 A control law for biped walking

We have seen that the monitoring and control of the contacts induce a preference for decoupling control laws. Decoupling and linearizing the dynamics of the error $e(q, t, p)$ that appears in (7) and (1) leads to the following scheme:

$$MB(q)^{-1}(v + B(q_d)J\,\ddot{p} - b) + N = T\,u + C^T\lambda \qquad (9)$$

in order to obtain the dynamics $\ddot{e} = v$. Applying this sheme, we must satisfy the constrained dynamics (3) through some appropriate variations of the walking parameters p, having a look also on the constraint (8). The variations of p are hopefully not completely constrained so far, leaving the possibility of some higher level specification for the long term behaviour. A simple specification can be to match as closely as possible to one desired trajectory (one prefered step length and speed, for example). For this, we can try to minimize the distance between a stabilization law $k_p(p - p_d) + k_v\dot{p}$ and the realized acceleration $\ddot{p}_*$.

The control law we propose can be resumed therefore as:

1. Specify a control v_* in order to regulate the error $e(q, t, p)$.

2. Compute the regulation of the parameters $\ddot{p}_*$ according to the high-level specification and satisfying the constraints on the dynamics through the contact forces λ_*. This leads to the following Quadratic Programming with linear constraints:

$$\begin{aligned} &\min_{\ddot{p},\lambda} \; \|\,\ddot{p} - k_p(p - p_d) - k_v\dot{p}\,\|^2 \\ &M_1B(q)^{-1}(v_* + B(q_d)J\,\ddot{p} - b) + N_1 = C_1(q)^T\lambda \\ &v_{2*} + C(q_d)(J\,\ddot{p} + n) + s = 0 \\ &\mathcal{A}(\lambda) \geq 0 \end{aligned}$$

3. Compute then the control actually realized by the actuators (supposing that all the joints are actuated independently, T_2 is invertible):

$$u = T_2^{-1}[M_2B(q)^{-1}(v_* + B(q_d)J\,\ddot{p}_* - b) + N_2 - C_2^T\lambda_*]$$

Note that in order to improve the stability of the robot, this control law is completely dependent on the disponibility of a parametrized set of trajectories $q_d(t, p)$ in which can be found enough variations of the walking pattern for compensating the usual perturbations.

4 Conclusion

We have seen that a walking robot needs to interact with its environment in order to control its global movements. Since this interaction is only performed through limited forces, this leads to a structural instability of the system. First attempts to control this instability led to monitor the contact forces, but the solutions proposed so far don't react properly to strong perturbations of the system. It appears then that the stability of the robot can be improved if we manage to consider a more comprehensive description of the walking behaviour.

This comprehensive description is introduced through a parametrized set of trajectories $q_d(t,p)$. We managed then to relate directly the different constraints acting on the dynamics of the robot to the variations of the walking patterns that will allow to maintain the stability the robot. We can propose then a control law for biped walking with improved stability, based on an approach that seems completely new.

In order to complete the design of this control law, we must achieve now the generation of a set of walking trajectories that will allow to recover from any destabilization. Note that the walking patterns are required for continuous variations of the parameters. Moreover, the more comprehensive will be this set of trajectories, the more perturbations will be compensated. For these reasons, the classical tools for generating one single trajectory are not adapted, and specific developments definitely need to be done.

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