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Qualitative Analysis of Fundamental Theoretical Tools Employed in Modelling Decision Making Problems

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by

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Abstract

The objective of this paper is to analyze fundamental theoretical tools employed in modelling decision making problems which involve multivariate risks. By exploiting the richness of the literature, we highlight the qualities as well as the "weaknesses" specific to a wide class of decision models. These are not free from critiques, especially when real world problems are considered. A little effort has been spent on improving this type of models, often conceptually convenient but not always adequate. This study emphasizes the need for a consistent framework to cover a broad area of research on the strategic behavior of potentially risk-averter policy-makers in a controlled dynamic stochastic environment. Work remains to be done in this exciting area.

JEL classification: C10, C51, C52, C53, P51.

Keywords: Active (passive) learning, Rational decision-maker, Closed-loop (open-loop) strategy, Stochastic (deterministic) control, Numerical (analytical) solutions, Reversible decisions, Increasing uncertainty, Risk-aversion.

Résumé

L'objectif de cet article est d'analyser des outils théoriques fondamentaux utilisés dans la modélisation des problèmes de prise de décision qui intégrent des risques multivariés. En exploitant la richesse de la littérature, nous mettons en évidence les qualités ainsi que les "faiblesses" spécifiques à une large classe de modèles de décision. Ce ne sont pas exempts des critiques, en particulier si des problèmes complexes sont considérés. Peu d'effort a été investi sur l'amélioration de ce type de modèles, souvent conceptuellement commodes mais pas toujours adéquats. Cette étude fait ressortir la nécessité d'un cadre cohérent pour couvrir un large domaine de recherche sur le comportement stratégique des décideurs potentiellement

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risk-averse dans un environnement dynamique stochastique contrôlé. Du travail reste à faire dans ce domaine passionnant.

Classification JEL: C10, C51, C52, C53, P51.

Mots-clés: Apprentissage actif (passif), décideur rationnel, Stratégie en boucle fermée (ouverte), Contrôle stochastic (déterministe), Solutions numériques (analytiques), Décisions réversibles, Incertitude croissante, Aversion pour le risque.

1. Introduction

We often ascertain in the decision making literature models which are not free from critiques, especially when real world problems are considered. An implication of an inappropriate modelling is the divergence between theoretical results and empirical findings. More general models could provide greater robustness being less restrictive but they have greater modelling difficulties. The advantage of allowing a much broader structure consists in more appropriate decisions taken by policy-makers. However, difficulties emerge when computational costs are taken into account. In this article we analyze and compare qualitative theoretical tools employed in the context of decision theory, providing a consistent review of the existing literature. In doing so, we focus on their advantages and limitations, the purpose being to emphasize the need for a consistent framework which better describe decision problems under risk and uncertainty. Section 2 presents the learning advantages in the context of a dynamic stochastic system and compares active learning with passive learning. Sections 3 discusses the concept of rationality and highlights its "weaknesses". Section 4 analyzes two types of strategies often employed in problems of decision and control, namely the closed-loop and respectively open-loop strategies. Section 5 examines the role of numerical solutions for dynamic programing models and compares these with analytical solutions generally obtained in more restrictive conditions. Section 6 treats several types of decisions employed in the context of stochastic optimization problems. Section 7 deals with several types of aversions encountered in the decision making process and points out important characteristics of environmental policy-making problems, given that in this area of research the development of specific formal tools is very awaited. Section 8 concludes and discusses possible future work.

2. Active Learning versus Passive Learning

Learning is one of the three aspects (beside the parametric uncertainty and stochasticity) of the economic agent's uncertainty problem and has many dimensions. This can take place at various levels of a policy problem, ranging from economic agents who are learning in order to adapt to changes in the environment to policy-makers who are trying to formulate the best policy (i.e., a rule for choosing an optimal action at each point in time) in an uncertain and changing world. An optimal behavior may arise from a process of learning. We recall here

the importance of learning in a NASH equilibrium (SEE, FUDENBERG & LEVINE, 1993). The decision-maker may by learning reduce the uncertainty with which he is faced. The relative efficiency of the learning generally depends on the method chosen. Learning has some normative and positive implications for policy-making after historical episodes of structural change. It is the case of an optimal monetary policy under uncertainty concerning the relationship between policy instruments and targets.

Potential non-linearities may be introduced by learning (SEE, AMONG OTHERS, BRAY & SAVIN, 1986, AND MARCET & SARGENT, 1989). In function of the success of model approximation, the learning may be more or less efficient. As learning constitutes a form of economic estimation, it is desirable to develop learning theory in a context that allows for dynamic structure. The optimization policies are essentially dynamic in nature. Unlike in the single-period decision problem, in multi-period control problems we have the opportunity to benefit from learning. Learning possibility can occur only in dynamic models, and appears more likely with longer planning horizons. Economic models involving learning often have the potential for converting independent shocks into correlated movements in observables. Thus, models with learning induce persistent effects of transitory shocks. This is an important feature of the models with stochastic endogenous fluctuations.

Accurate representations of the reality generally involve active learning, allowing economic agents to experiment. This is the case when decisions are made as much to acquire information. Thus, the agents have some influence over the rate at which information arrives, so that their behavior may generate information. The timing of information is a crucial aspect. The agents can acquire additional information by receiving a noisy signal about the true state of the world. We can affirm that the active learning makes the agents more experienced over time. Generally, the uncertainty depreciates the decision-maker's activity and produces a temporary stability followed by a longer or shorter period of adaptation in instability which implies for the agent an additional effort allocated in the active learning. One of his objectives is to diminish the costs of learning.

It is well-known that investment in research and development yields information. This is an obvious way in which information is actively acquired and an example of active learning. An other example is the case of a consumer who expects to make future purchases and knows that a purchase today will yield information which may lead to a better choice tomorrow. Consideration of active investment in information will be important in principle, in models in which single agents have market power. For instance, a monopolist (social planner) expecting to be in business for many periods can vary output in early periods in order to accumulate information about the unknown parameters. A form of active learning is, for example, the learning from experience (SEE, BALVERS & COSIMANO, 1990).

It exists benefits from active learning in a stochastic optimization model (SEE, EASLEY & KIEFER, 1988, OR KIEFER & NYARKO, 1989, AMONG OTHERS). The value of learning will be incorporated into the optimization problem. The algorithm will anticipate future learning when choosing the control for each period, and thus will perturb the system early in time in

order to reduce the variance of the parameters estimate later in time. However, if the model is very noisy, then the potential for learning is limited. Note also that nonregularities in the form of nonconvexities may arise in active learning problems (SEE, MIZRACH, 1991). For a short-run period or in the presence of nonconvexities, active learning bias. It is also important to underline that generally, active learning can be ruled out a priori for macro-economic control problems.

Other times, the learning is passive (and without experimentation), the acquisition of information non depending on the agents' decisions (they do not actively invest in information). In other words, it involves the exogenous arrival of information. This may occur all at once, as in MANNE & RICHELS (1992), or gradually as a function of time, as in KOLSTAD (1996). In this context, the estimation problem will be separated from the stochastic optimization problem. This generally induces an important bias in the control and target variables and so, a myopic decision rule.

With passive learning, the strategies might be expected to be superior to a strategy in certainty equivalence but inferior to a strategy which allows for active learning, that is, a strategy which considers the information value of an action at each date (SEE, AMMAN & KENDRICK, 1994).

In systems with substantial variance noise or with enough time periods and parameters, active learning performs much better than passive learning and certainty equivalence. The passive learning algorithms take account of the uncertainty of the parameters, but does not consider the possibility of future learning. The certainty equivalence method ignores the parameters uncertainty when choosing the controls, but does learn in a passive way by reestimating the parameters at each period.

3. Rational Decision-Maker

The notion of rational decision making in an uncertain environment is associated with the expected utility function maximization behavior. Rationality lies in the correspondence of the decision-maker's choice with some goal or objective. For example, this does not refuse to act in accordance with the efficient outcome-oriented behavior (SEE, DREZE, 1990). Even though the expected utility representation has been credited for its normative appeal and convenient mathematical form, there is fairly strong empiric evidence in disfavor of this theory, whether it is considered objective or subjective. In general, there are two approaches for a preference relation to have an expected utility representation, depending on whether one treats the distribution function as objective or subjective. VON NEWMAN AND MORGENSTERN (1944) introduced the former approach by assuming that the agents know the distribution functions. An alternative approach proposed by SAVAGE (1954) considers the distribution function as unknown and subjective to the economic agents.

Very often, the decision-maker's behavior violates the rationality axioms. Experimental studies as well as observed behavior for several classes of choice problems have revealed systematic violations of expected utility theory. A substantial body of empiric evidence

shows that individuals overweight extreme events and act in conflict with the expected utility theory. The criticism against the expected utility theory originates from Allais (1953) and his famous paradox. He observes, for example, that people who in many cases are risk-averter appear to be willing to purchase lottery tickets as well. The explanation of this behavior may be due to a substitution of decision weights for probabilities. Other similar experiments have been constructed which provide inconsistent results with the expected utility hypotheses. We note, for instance, the violation of the axiom of transitivity of preferences (see, Kahneman & Tversky, 1979, or Loomes et al., 1991) as well as the violation of the independence axiom (see, among others, Cohen & Jaffray, 1982, or Machina, 1983). Numerous alternative theories to the expected utility have been proposed, as for example the non-expected utility criteria (see, Quiggin, 1982, Yaari, 1987, Segal, 1987).

Generally, the preferences are incomplete. It is very rare in econometrics to be able to fully specify the utility function. No decision-maker has sufficient a priori knowledge to fully specify his utility function. In theory, it is generally supposed that the objective function is correctly specified by the decision-maker (econometrician). It is very unlikely that the decision-maker exactly maximizes his utility at each stage of the control (see, Varian, 1990). He rather has a nearly optimization behavior, where the control variable is adjusted continuously over time in order to maximize his objective function. The rationality can also be viewed as consistency (internally consistent choice or action) or, as well as the pursuit of agents' self-interest, an egoistical behavior (see, Walsh, 1996). The rationality of the decision-makers is characterized by the anticipation (forward-looking behavior) that the system will be affected by other factors than their control instruments. These factors are completely or partially observed and may be exogenous or endogenous variables as well as continuous unobserved random shocks (because of the misobservation or measurement errors). It is useful to note that sometimes, the decision-maker is unable to make a rational decision, in which case it will be used an ad hoc decision rule, called a rule of thumb. Some (empiric) rules of thumb for choice under uncertainty can be, for example, rationalized by the rank-dependent utility theory. We can also imagine the case of a bounded rationality (in the sense that the decision-maker can predict the behavior of the system with only a very limited steps ahead). This may limit the agent's ability to understand the system response to some of his actions. In the case of a game, this may limit the agent's ability to understand a rival's pattern of play (see, Shapiro, 1980).

4. Closed-Loop Strategy versus Open-Loop Strategy

It exists two different alternative (adaptive) strategies according to the decision-maker's objective and the way he observes the evolution of the system in decision making situations. One can speak about a selection among rules in function of the information processing:

1) the case of an open-loop strategy, when the decision-maker has access only to the initial state of the process. He sets his control vectors as functions of time. This strategy is specifically to a long run behavior but it has the disadvantage that the decision-maker cannot benefit from learning (everything is changing in the long run). More the period of control is longer, more

the effect of cumulated errors on the agent's optimal policy is significant. Disadvantages of the open-loop controls are that they require much information about the future development of the system and that they are not robust. Very often we constate that in an open-loop strategy the agent is supposed to have perfect forecast of the state space in the future (a very strong assumption).

2) the case of a closed-loop strategy, when the decision-maker observes the state of the process at each period of the control. Thus, he attempts to reduce the uncertainties associated with his actions by acquiring new information. The agent constantly monitors the output of the process under control. He utilizes the information in real time. His decision rule is revised as more experience is accumulated. Due to the cumulated disturbances in the system, the process of monitoring will be imperfect. Accumulations of stock external effects alter the environment faced by the economic agent and force him to react optimally. In a closed-loop strategy, the policy does not require some large periods of engagement from the part of the decision-maker. In other words, the control rules will be sensitive to the choice of the working horizon. This may be the case when the relevant information acquisition cost is high, most likely due to the permanent random shocks in the system or because of the slow inertia of the economic environment (generally, the system changes very slowly in relation to the speed at which the economic actors learn). However, the agent actively seeks to learn about the environment, being incited to obtain more information. This is the context of an active learning.

The decision-maker optimally chooses the control instruments on the basis of a non-decreasing endogenous information set (which includes unexpected shocks and new information about the system). Because exogenous shocks in the future are not predictable, the closed-loop strategy cannot incorporate them into the decision implemented by the agent. He will set his control vectors as functions of the history of the state vector from the beginning of the control to the moment of decision. This type of strategy is specifically to a short run behavior and has the advantage to "continuously" improve the decision-maker' optimal policy. The optimality of a strategy is defined relative to the information the decision-maker has at the time the strategy is used. It exists so a relation between the instruments efficiency and the optimal policy chosen by the decision-maker. The closed-loop control processes can be viewed as a method of control alternative to economic planning. This control strategy must include feedback (memoryless) information that describes the actual state of the system. Therefore, this responds not only to the effects of the random inputs, but also to the measurement errors as well. Thus, it will not be necessary to be able to identify and measure the sources of disturbance (a potential effect of such information is that it can prevent unexpected shocks). The controller adjusts to keep small the difference between actual and assumed system characteristics. Dynamic feedback (from the future to the present) entails measurements, and these may be uncertain or indirect. With uncertain or indirect measurements, it is necessary to estimate the state that is most likely to have caused the measurements. The control principle and the estimation principle can be designed concurrently (i.e., one depends upon the other) to solve the stochastic optimal control problem. It can be said that a closed-loop strategy is

robust, in the sense that it anticipates the possibility of a disturbance. Moreover, the decision-maker has the possibility to learn this error and to monitor the performance of his optimal policy at each stage of the control. Thus, his behavior is competitive, even if the monitoring is generally imperfect. The agent can decide on the intensity of monitoring (by gradually enforcing the active learning) as long as perfect monitoring is either not possible or non-optimal. The process of monitoring can be deterministic (at each period of control) or stochastic.

The open-loop solution is generally incapable of rendering the qualitative properties of a closed-loop solution. Different from the pre-committed open-loop strategy, the closed-loop strategy allows to the agent to condition his decision on the history of the system. The closed-loop strategy can be regarded as an extension and a refinement of the open-loop concept. The open-loop and closed-loop strategies are equivalent only under the perfect forecast assumption (unrealistic in most circumstances). Generally, the closed-loop solution deviates from the open-loop solution. It is also useful to note that in a static context it is not generally possible to derive the closed-loop feedback rule.

The decision-maker has to estimate the effect of the feasible control policies on the system behavior and to select the most effective policy. He can use a knowledge base of past and present information to effect a control strategy, but future information is unavailable. Any policy would be preferred, the uncertainty will alter this one. It is very important in practice the quality of the control and so to design an appropriate control for the economic problem. The best control must include a strategical qualitative learning.

In a wide range of problems, the control and estimation strategies of the stochastic process are derived separately / independently, then concatenated to form the optimal solution. This is the case of a myopic behavior. The resulting myopic decision rule will hence disregard the potential advantages from experimentation. The ability to separate these strategies depends upon the manner in which the uncertainties enter the problem and the statistics of the uncertainties themselves (SEE, SINN, 1983). The only advantage of a myopic (sub-optimal) decision rule is that this can easily be derived analytically.

In function of his objective, the decision-maker can choose some long-run or short-run economic targets. This corresponds with some long-period, respective short-period expectations and thus with the long-run respective short-run equilibrium of the system subject to the control. Notice that generally, the long-run equilibrium is more sensitive with respect to the short-run one, being highly sensitive to the decision-maker initial decision. A natural question which comes in mind is: how must one fix the working horizon so that it can be regarded as "large"? In general, its length does not only depend on the number of periods but also on the unity of measure chosen.

We point out that the performance of the closed-loop (or, feedback) control is superior to any open-loop control in the stochastic dynamic context (SEE, CRUZ, 1975, XEPAPADEAS, 1992, AND WIEDMER ET AL., 1996). It is this decisive advantage which makes the "feedback" so valuable. It is at such level that economic theory can be exploited at best. For empiric work (optimal policy experiments) and associated hypotheses testing of the optimal control problem,

the closed-loop or feedback solution will be preferable. If the decision-maker is interested in determining the effect of parameter changes on the optimal control policy, then the closed-loop strategy is generally the best way to do so.

5. Numerical Solutions versus Analytical Solutions

It is very instructive to point out the distinction between a deterministic optimal control problem and a stochastic optimal control problem. In the first case, the state of the system at any time t can be deduced from the initial state and the control used up to time t. Thus, observing the current state of the system at each time t does not really give more information that knowing the initial data. For stochastic control systems, there are many path which the system states may follow given the control and initial data. In this case, the best system performance depends on the information available to the controller at each time t. This distinction is important. An a priori analysis of the deterministic problem is often crucial (SEE, SARGENT, 1987).

In practice, there are very few cases where one can obtain analytical or "closed-form" solutions for dynamic programing models, using the BELLMAN's (1961) optimization principle (backwards through time). The optimal policy will ensure the minimal deviation between the state of the system and the fixed targets (a priori levels of aspiration). In order to obtain analytically tractable results, restrictions have to be imposed which are less attractive from an economic point of view. When the analytical formula cannot be obtained easily, the analysis of the problem requires the use of numerical methods. If, in addition, the dimension of the non-linear dynamic model is high, then the analytical treatment becomes even more difficult. It appears that dynamic programming amounts to a solution algorithm that allows to obtain numerical solutions to specific problems rather than analytical characterizations of a wide class of problems.

Numerical procedures suffer from the curse of dimensionality which often occurs in solving the BELLMAN equation for the value function in dynamic programming. In general, the complexity of the problem grows exponentially with the dimension of the state space. If higher-dimensional problems should be treated, there is an urgent need for approximating methods that escape the curse of dimensionality. In this sense, RUST (1997) has shown that randomization can break the curse of dimensionality in certain classes of linear problems but it is generally not able to break the curse of dimensionality in non-linear problems. On the other hand, high non-linear models present considerable difficulties in terms of initialization and convergence. Such models are often randomly initialized. In this case, further analysis is needed in order to find more advantageous algoritms in terms of regions and rates of convergence. In order for numerical experiments to be effected rigorously, it will be important to dispose of error bounds or accuracy estimates of the computed solutions (SEE, MANUEL & AGUIAR, 1998). The approximation errors must evolve as predicted by the theoretical analysis. These numerical errors may propagate in unexpected ways over the iteration process.We recall here the possibility of rounding errors when numerical methods are used. Unfortunately, progress

in finding closed-form solutions of dynamic stochastic models is very slow; the only model which can be solved in any generality is the linear-quadratic approximation model (i.e., linear model and quadratic criterion) which gives linear decision rules under certain conditions, very convenient for theoretical analyses. This is attractive on analytical and computational grounds. In non-linear models, very special assumptions have to be made in order to obtain analytical solutions. The nonlinearity makes the determination of an analytical solution difficult, if not impossible. In recent years, a number of numerical methods have been proposed in the economic literature. We mention here, among others, Rust (1996), Amman (1995), Marcet (1994), and Taylor & Uhling (1990).

One cause for all these disadvantages is the set of constraints imposed by the computer (computational constraints) and data. Because it is not possible to obtain analytical solutions for the optimal policy, an alternative to do it is to appeal to simulation techniques-based algorithms. However, a weakness of these techniques is that the properties of the model may depend on a particular set of parameters and functions chosen for the simulation, and one may get a distorted picture of the properties. On the other hand, differences in the level of optimization can have a direct effect on the speed of convergence.

Due to the recent increasing (exponential growth) computational speed in the computer power as well as the statistical and econometric methods (e.g., the bootstrap, method of simulated moments, nonparametrics), new feasible adaptive stochastic control rules (learning algorithms) have enlarged the class of the models that can be approached by simulation data (using the DGP). It permits us to gain experience from large models.

Nowadays, the power of the computer make it possible to solve the complicated recursive equations obtained from the non-linear specification (see, Amman, 1989, 1990). Moreover, it is possible to implement empiric tests and study the behavior of the model under different environments. Advances in computer-based inferential techniques (such as the bootstrap and computational Bayes methods) facilitate and stimulate the research in the context of the models with high computational costs. Following Pitchford (1977), we point out that several-state-variable problems in specific theoretical or empiric contexts can be more interesting than one-variable problems (i.e., one-dimensional), which are popular due to their tractability, because they allow for a richer set of interactions. Moreover, the solution of several-state-variable problems may lead to progress towards a solution for a wider class of problems. However, it must said that the application of numerical methods becomes increasingly difficult for models with higher dimensionality (see, Rothenberg, 1973). This is essentially due to the fact that a function with more variables presents more information than a function of one variable and of comparable "smoothness", so that the cost of computation algorithms may become extremely tedious. The computational burden increases more than proportionally as the dimension of the model increases. (see, for example, Hendry, 1984).

Even if a method requires a much larger number of iterations than another, it is perfectly possible that for this one, each iteration is much simpler and less computer time consuming

with respect to the other. Therefore, when differentiation is performed numerically it is not really clear which of the methods has a better numerical behavior and so should be generally preferred.

6. Types of Decisions

It is well-known that economics agents behave on average risk-neutral for small and repeated decisions, but risk-averse in one-shot and all important decisions (SEE, AIGINGER, 1987). We distinguishes two different types of decisions in the context of a stochastic optimization problem:

i) Uncertain decision, where at least one parameter or functional form must be estimated. Even in the simplest case of uncertainty (with a single unknown parameter), difficulties arise in solving the uncertain decision. Because of the curse of dimensionality, the computational complexity of the uncertainty case even for a simple non-linear model may well be exponential in the time horizon.

ii) Risk decision, where the parameters and the functional forms required to determine the optimal decision are known. The difficulties in making risk and uncertain decisions can be defined in terms of computational complexity (SEE, NORMAN & SHIMER, 1994).

The decision-maker bases his decisions on some body of knowledge. When that base of knowledge evolves over time, regulatory decisions evolve over time. The knowledge upon which the decisions are based increases gradually with the passage of the time (there is an asymmetry between the past and the future) and due to the wisdom derived from experience. The decisions made in the past will be reflected in changes in the state of the system itself, and they will influence the perception of the future actions to be analyzed. Because the source of randomness may differ from one application to another, the decision making response may vary.

In the face of anticipated increases in information and its possible exogenous changes over time, it might be better to take a reversible rather than an irreversible decision, all the more as the decision-maker's initial knowledge level is low. It is important here the rate at which the information is acquired. In this way, the decision-maker will act optimally, taking the future into account. We speak here about the practical importance of strategic considerations in the timing of decisions. In reality, a decision problem does not start with a definition of the problem and even if it does, the problem is often redefined during the decision process itself. What was in a decision-maker's ex-ante best interest is not necessarily in his ex-post best one. The prospect of acquiring better information in the future should induce less irreversible decisions. This is particularly the case for dynamic behavior situations where uncertainty is only gradually solved through time and the decision-maker may be afforded at different points in time (or continually) the opportunity to revise his plan of action (i.e., his strategy).

A decision-maker who neglects the prospect of receiving more complete information at later stages of a sequential decision problem will in certain cases too easily take an irreversible decision, or he ignores the existence of a positive option value in favour of reversible decisions. This option value represents the decision-maker's flexibility to adapt subsequent decisions to the obtained information. We note here the link between irreversibility and flexibility, namely

that a more irreversible decision is in a sense a less flexible decision. We remember that an irreversible decision is one which, if taken, results in not being able to exercise (for a long time or forever) some option that was available earlier. In other words, under uncertainty, the optimal sequence of decisions depends on not only the expected losses, but also the flexibility, in terms of availability of future options associated with each decision. Notice that the implications of irreversibility for sequential decision making under uncertainty were first analysed in economic literature by Arrow & Ficher (1974) and by Henry (1974).

7. Types of Aversions

There are three types of aversions which can be encountered in the decision making process of an economic agent:

1) Uncertainty aversion, when the decision-maker prefers to bet on an urn of known composition rather than on an urn of unknown composition. In other words, the decision-maker is less willing to accept bets about an event when he does not know the true probability of that event (see, James Dow et al., 1992). In general, the objective probabilities are never known.

2) Aversion to increasing uncertainty, if any convex combination of two acts is preferred to the least favorable of these acts. Consequently, aversion to increasing uncertainty implies uncertainty aversion, but the opposite does not hold (see, Aldo Montesano et al., 1996).

3) Risk-aversion, when the decision-maker prefers the expected value of the lottery over the lottery itself (see, James Dow et al., 1992). Notice that the concept of uncertainty aversion is more general than the risk-aversion concept.

It is well-known the link between aversion and statistic uncertainty. In general, the aversion is associated with increasing uncertainty while the uncertainty is naturally associated with incomplete information about future behavior of the system. The decision-maker does not know which state in the future will in fact hold. In this sense, the knowledge must be viewed as future oriented expertise. We have the following sequence of logical implications (which defines the structure of uncertainty): shocks $\Rightarrow$ information loss $\Rightarrow$ forecast errors $\Rightarrow$ forecast dispersion (as a measure of uncertainty). It is useful to note here that there may be periods in which forecast errors are low but uncertainties great. For a risk-averter decision-maker there are two important effects of the increased uncertainty on optimal decisions:

1) The direct effect of increased uncertainty (in the sense of a finer information structure to be available in the future) holding fixed the agent's choice. This is the context when an increase in uncertainty generates a reduction in the flexibility of the decision (see, Marschak & Nelson, 1962).

2) The indirect effect due to the change in the optimal choice as a result of increased uncertainty. It is the case when an increase in uncertainty implies an increase in decision flexibility (see, Jones & Ostroy, 1984, or Freixas & Laffont, 1984). This time, the uncertainty is viewed as determining decisions. It is well-known that both effects diminuate the decision-maker's utility.

In many models of choice / decision under uncertainty, increased risk-aversion induces the decision-maker to take a lower level of risky activity.

The most common attitude in the decision making problems in economics is the one taken in risky situations, namely in the risk-aversion context. A consistent behavior generally implies aversion characterized by risky actions from the part of the decision-maker (SEE, DREZE, 1990). The assumption of risk-aversion must be associated (given the complexity of the environment) with the assumption of rationality imposed in problems of optimal decisions.

In the literature on risk, very often it is adopted the risk-neutrality assumption for the decision-maker. This is a "weakness" of the economic modelling. In reality, the decision-maker is confronted with multiple risks (generally, different phenomena being characterized by different risks). His decision is not made independently; it is made together with other decisions that place the economic agent in risky situations. Decisions made to avoid (even partially) one source of risk may be affected by the presence of others. One can interpret the risk as the degree of confidence in the future (the variance of the decision-maker's forecast provides a first-order measure of the "riskiness" of the future). Generally, the agent's confidence will decrease with uncertainty. Such behavior characterizes most decision-makers, at least for large gains or large losses. Note that the behavior towards losses is not of the same type as the behavior towards possible gains.

The uncertainty always attends the risk. We recall here some efficient methods in order to reduce uncertainty, as the control of the future, the increased power of prediction, or by diffusion (SEE, KNIGHT, 1971). It is useful to make the difference between (measurable / calculable / quantitative) quantifiable "risks" (when the objective probabilities are supposed to be known) and (inherently ummeasurable) unknown "uncertainties" (when the objective probabilities are not given in advance). In other words, it must distinguish between decision under risk and decision under uncertainty. We can so define the risk as a measurable uncertainty. However, not all risks can be quantified. It is the case of uncertainty proper. Notice that even with known probabilities, the decision-makers face great difficulty in applying the economic theory.

Risk-aversion is equivalent to the concavity of the utility function (or, a decreasing marginal utility). Consequently, we will have a particular type of dependence or form of utility function. However, the concavity of the utility function is just a way of expressing risk-averse preferences. An individual is risk-averter if for any arbitrary risk he prefers the sure amount equal to the expected value of the risk to the risk itself. In other words, a more risk-averter agent must be willing to pay more for insurance. We can say that the risk-aversion is the rule in an economy where there exist opportunities for gambling at fair odds. An agent who expects in the future a high deviation from the target because of possible large shocks on the system (social or financial events, natural catastrophes, and so on) can be considered to be risk-averter, while an other who expects not very large shocks can be considered risk-lover. We can imagine two distinct situations: when the risk increases additively, respective multiplicatively.

In order to model the degree of risk-aversion, we need to generalize the traditional

approach of Arrow-Pratt (1964, 1971a, 1971b) which takes into account only risks totally exogenous which are not under the control of the decision-maker. This is inconsistent in a dynamic stochastic context (see, D. Protopopescu, 2003a, 2003b). It is useful to remember here some important implications of the risk, as for example in insurance theory (see, Mossin, 1968, and Schlesinger, 1981), portofolio theory (see, Arrow, 1971), in problems of production under price uncertainty (see, Baron, 1970), in taxation theory, in asset markets theory or life-cycle savings theory as well as environmental problems. Risk theory must naturally be present in all economic models, but we often constate its lack in many situations of interest.

We finish our analysis by underlying three important characteristics of environmental problems, given that in this area of research the development of specific formal tools is very awaited:

1) There is almost always uncertainty over the future costs and benefits of adopting a particular policy.

2) There are always important irreversibilities associated with environmental policy. These irreversibilities can arise not only with respect to environmental damage itself, but also with respect to the costs of adopting policies to reduce the damage.

3) Policy adoption is rarely a now or never proposition; in most cases it is feasible to delay action and wait for new information. We speak here about a potential evolution of the information structure (as well as some institutional constraints). Generally, we have a two-level hierarchy of the environment dynamic: the first level is the dynamic of the environment given the information structure, and the second level is the dynamic of the information structure itself.

These uncertainties, irreversibilities and the possibility of delay can significantly affect the optimal timing of policy / decision adoption (it may be the case of investments, for example). We keep to point out two types of uncertainties that are relevant:

a) Economic uncertainty, that is, uncertainty over the future costs and benefits of policy adoption. It is the case of environmental damages and their reduction.

b) Ecological uncertainty (uncertainty over the evolution of relevant ecosystems), that is, uncertainty over the future evolution of key environmental variables. It is useful to note that if an environmental policy reform is restricted to a subset of policy instruments, then its efficiency can be greatly undermined.

One can also distinguish two types of environmental risks in the behavior of economic agents:

i) Risks induced by agents' actions which are to as certain extent under their control. Situations where the risks are at least partially controllable are considered endogenous risks. Risks which are partly induced by economic actions are called endogenous uncertainty.

ii) Risks induced by nature's moves which are beyond the control. Situations where the risks are not controllable are considered totally exogenous risks. Risks which are generated from acts of nature are called exogenous uncertainty.

8. Concluding Remarks

This paper reviews the theoretical and empirical literature on decision making and analyzes important tools employed in this exciting area. We briefly survey several insights about the complexity of a decision process, especially when real world problems are considered. Generally, simplified models do not permit to capture the effects of environmental uncertainty. The determination of good models is an important element in many practical econometric research. The objective was to explore the advantages as well as the limitations of several specific tools when modelling decision making problems. This briefly study is intended to suggest to the reader how complexity-based research might be developed and how it might impact policy-making analyses in the future. One area that could be fruitfully explored further is the effect of the dynamic complexity of a stochastic system on the behavior to risk of a rational decision-maker in the context of a closed-loop strategy. Alternative treatments may be implemented by considering a more realistic risk-aversion approach which allows for a better caracterization of the economic agents' optimal decisions (SEE, D. PROTOPOPESCU, 2003A, 2003B). An other possibility would be to investigate the implications for the system evolution of the agents' actions when they integrate an endogenous risk-aversion. These suggestions cover a broad area of research.

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