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SPATIAL RELATIONS ANALYSIS BY USING FUZZY OPERATORS

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ABSTRACT

Spatial relations play important role in computer vision, scene analysis, geographic information systems (GIS) and content based image retrieval. Analyzing spatial relations by Force histogram was introduced by *Miyajima et al* [10] and largely developed by Matsakis [12] who used a quantitative representation of relative position between 2D objects. Fuzzy Allen relations are used to define the fuzzy topological relations between different objects and to detect object positions in images. Concept for combined extraction of topological and directional relations by using histogram was developed by J.Malki and E.Zahzah [3], and further improved by Matsakis [14]. This algorithm has high computational and temporal complexity due to its limitations of object approximations. In this paper we apply fuzzy aggregation operators for information integration along with polygonal approximation of objects. This approach gives us anew, with low temporal and computational complexity of algorithm for the extraction of topological and directional relations.

keywords: Spatial Relations, Force Histogram, Polygonal Approximation, Temporal Complexity, Fuzzy aggregation operators.

1. INTRODUCTION

Space relations has a remarkable importance in computer vision and image analysis as in Content based image retrieval, similarity based image retrieval [15], to identify forms, manage data bases, support spatial data in artificial intelligence (AI), cognitive science, perceptual psychology, geography particularly geo-information system (GIS), indexation and comparing objects scene and model are major applications of space relations. P.Matsakis et all [13] discussed different applications usable in satellite, robotics and for developing linguistics expressions. Isabelle Bloch [2] discussed different applications in fields of medical imaging, video and vision. Different approaches for finding spatial and topological relations have been developed according to the need for applications and object representations. Qualitative methods for directional and topological relations includes Max J. Egenhofer's method of four intersections [7]. These methods are considered most important in GIS community. Directional relations are defined on relative frame of reference and absolute frame of reference. In relative frame of reference position of a simple object is made with respect to an oriented line or an ordered set forming a vector to some intrinsic properties of reference object. Methods like angle histogram introduced by K.Miyajima and A.Ralescu [10] and statistical method developed by MinDeng.Zalimli [6] depends upon relative frame reference frame. Method introduced by MinDeng.Zalimli is fast but we loose the structural information of objects. Computational complexity for handling the information related to objects increases with increase in dimensions. Matsakis [12] introduced 1D representation of 2d objects by the union of longitudinal sections which is the extension of angle histogram. It processes vector and raster data, net objects as well as fuzzy objects [12, 8, 9]. Combining the different knowledge based information increases the efficiency of the system. Approaches for combining informations has been developed for both qualitative representations such as [1]and quantitative manners like P.Matsakis. The derivation of combined topological and directional relations by using force histogram [14] was first introduced by J.Malki and E.Zahzah [3], then P.Matsakis raised some problems regarding fuzziness of some relations like *meet* and *meet_by* and some others which exist at segmentation level. Extraction of these relations depend on the fuzzy membership functions μ instead of depending on function ϕ . In case of longitudinal section consist of segment or union of segments then they are treated by histograms f and F while in case of histogram of Allen relations μ comprises three histograms. When objects are convex and have the connected boundary (when μ acts as ϕ histogram or f histogram), calculations are very simple but in contrary if objects are concave or objects have the disconnected boundaries then fuzzification

of segments is complex (when μ acts as F histogram). As a result temporal and computational complexity increases. Approximating the object by its boundary, length of longitudinal sections can be computed as distance between the intersecting points of oriented line and object boundary. The degree of fuzzy membership function depends upon three values x, y and z . In tuple (x, y, z) , the pair (x, z) are the lengths of longitudinal sections and y is the difference between maximum value of intersecting points of object B and minimum value of object A , i.e. $y \in R$ or $y \in Z$. By this approach of object approximation, temporal complexity decreases from $n\sqrt{n}$ to $N \log(N)$ where n is number of pixels of objects under consideration and N is the number of vertex of object polygons. Temporal complexity for the said algorithm is not given but in general temporal complexity of force histograms is discussed in [11] for different object types. We assume same temporal complexity for objects because segmentation level problems raised by P.Matsakis forced the object as raster data and in addition to this algorithm for fuzzification of longitudinal sections increases temporal and computational complexity. These problems no more exist if we consider objects by their boundary, then need for P.Matsakis's algorithm remain for objects having disconnected boundaries i.e. objects with holes and for concave objects. Each segment is separated by a certain distance for objects with holes or concave objects. This internal distance has a significant impact on directional and topological relations because values of fuzzy membership functions are distance sensitive (In this case y justifies the grade of a fuzzy membership value). This difficulty is coped off with application of fuzzy disjunctive operators. These fuzzy operators have been developed to summarize information distributed in different sets by grades of fuzzy membership values. This paper is structured as follows. First of all we describe Allen relations in space. In section 2 we describe different fuzzy Allen relations defined by P.Matsakis and changes in mathematical formulation of fuzzy histogram of Allen relations due to object approximation. In section 3 we discuss different fuzzy operators, section 4 describes experimental results. In section 5 temporal complexity is calculated, section 6 describes different affine transformations and section 7 concludes the paper.

Allen relations in space

Allen in [4], introduced the well known 13 mutually exclusive exhaustive interval relations based on temporal interval algebra. These relations are arranged as $A = \{< m, o, s, f, d, eq, d_i, f_i, s_i, o_i, m_i >\}$. where $\{< m, o, s, f, d >\}$ resp. $\{< d_i, f_i, s_i, o_i, m_i >\}$ are the relation *meet*, *overlap*, *start*, *finish*, *during* (resp the inverse relations of the cited ones). The relation *eq* is the equality spatial relation.

All the Allen relations in space are conceptually illustrated in figure extracted from 1. These relations have a rich support for the topological and directional relations.

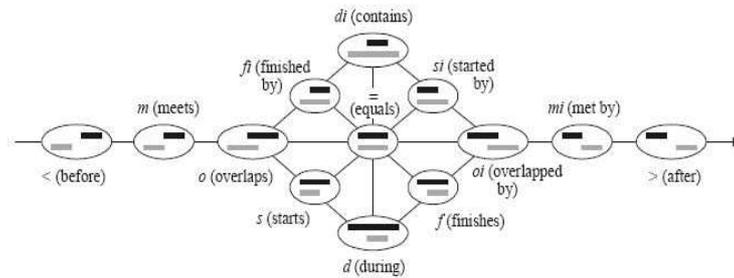


Fig. 1. Black segment represents the reference object and gray segment represents argument object. figure extracted from [12]

2. FUZZY HISTOGRAM OF ALLEN RELATIONS

In real applications, small errors in crisp values can change the entire result when gradual changes of topological relations occur over time. To cope these problems fuzzification was introduced, it comprises the process of transforming crisp values into grades of membership for linguistic terms of fuzzy sets. Fuzzification process of Allen relations does not depend upon particular choice of fuzzy membership function, trapezoidal membership function is used due to flexibility in shape change.

Let $r(i, j)$ is Allen relation and r' is the distance between $r(i, j)$ and its conceptional neighborhood. We consider a fuzzy membership function $\mu : r' \longrightarrow [0, 1]$. The fuzzy Allen relations defined by P.Matsakis [14] are

$$\begin{aligned}
f_b(I, J) &= \mu_{(-\infty, -\infty, -b-3a/2, -b-a)}(y) \\
f_m(I, J) &= \mu_{(-b-3a/2, -b-a, -b-a, -b-a/2)}(y) \\
f_O(I, J) &= \mu_{(-b-a, -b-a/2, -b-a/2, b)}(y) \\
f_f(I, J) &= \min(\mu_{(-(b+a)/2, -a, -a, +\infty)}(y), \mu_{(-3a/2, -a, -a, -a/2)}(y), \mu_{(-\infty, -\infty, z/2, z)}(x)) \\
f_s(I, J) &= \min(\mu_{(-b-a/2, -b, -b, -b+a/2)}(y), \mu_{(-\infty, -\infty, -(b+a)/2)}(y), \mu_{(-\infty, -\infty, z/2, z)}(x)) \\
f_{si}(I, J) &= \min(\mu_{(-(b+a)/2, -a, -a, +\infty)}(y), \mu_{(-3a/2, -a, -a, -a/2)}(y), \mu_{(z, 2z, +\infty, +\infty)}(x)) \\
f_{fi}(I, J) &= \min(\mu_{(-b-a/2, -b, -b, -b+a/2)}(y), \mu_{(-\infty, -\infty, -(b+a)/2)}(y), \mu_{(z, 2z, +\infty, +\infty)}(x)) \\
f_d(I, J) &= \min(\mu_{(-b, -b+a/2, -3a/2, -a)}(y), \mu_{(-\infty, -\infty, z/2, z)}(x)) \\
f_{di}(I, J) &= \min(\mu_{(-b, -b+a/2, -3a/2, -a)}(y), \mu_{(z, 2z, +\infty, +\infty)}(x)) \\
f_{Oi}(I, J) &= \mu_{(-a, -a/2, -a/2, 0)}(y) \\
f_{mi}(I, J) &= \mu_{(-a/2, 0, 0, a/2)}(y) \\
f_a(I, J) &= \mu_{(0, a/2, \infty, \infty)}(y)
\end{aligned}$$

where $a = \min(x, z)$, $b = \max(x, z)$, x is the length of longitudinal section of argument object A , and z is the length of longitudinal section of reference object B . Most of relations are defined by one membership function and some of them by the minimum value of more than one membership functions like $d(during)$, $d_i(during_by)$, $f(finish)$, $f_i(finished_by)$ and by the definition of fuzzy set in fuzzy set theory, sum of all the relations is one. This gives the definition for fuzzy relation equal, then the definition of Histogram of fuzzy Allen relations stated as in [14]:

$$\int_{r \in A} \left(\sum F_r(q, A_q(v), B_q(v)) dv \right) = \frac{x+z}{w} \sum_{k=1}^c \sum_{i=1}^{m_k} \sum_{j=1}^{n_k} [x_i^k z_j^k (a_k - a_{k-1})] r(I_i^k, J_j^k) \quad (1)$$

where $x = \sum_{i=1}^{m_k} x_i^k$, $z = \sum_{j=1}^{n_k} z_j^k$ and $w = \sum_{k=1}^c \sum_{i=1}^{m_k} \sum_{j=1}^{n_k} [x_i^k z_j^k (a_k - a_{k-1})]$

In this case m_k and n_k represents the total number of segments of argument object A and reference object B and c represents number of loops in algorithm described by P.Matsakis [14]. Here terms multiplied by the Allen relation represents the sum of object areas. In our approach, fuzzification of segments is no more required. Fuzzy Allen relations are computed for each segment. Since each fuzzy Allen relation is the grade of a fuzzy membership value, for simplicity fuzzy Allen relation for each segment is a fuzzy set and fuzzy operators or fuzzy aggregation operators are used to combine different values of fuzzy grades. Hence at the next stage fuzzy aggregation operator is used for integration of different fuzzy Allen relations. Fuzzy operators and fuzzy aggregation operators are discussed in next section.

In polygonal approximation of objects, fuzzy Allen relations are calculated for a limited number of segments considering that a complex $2d$ extended object is the union of small and simple $2d$ objects like triangles, parallelograms and trapeziums. We find spatial relations only on the boundary of these simple objects and within this region spatial relations do not change so simply generalize the given relations. Definition of fuzzy histogram of Allen relations given by P.Matsakis [14].

"Histogram of fuzzy Allen relations represents the total area of subregions of A and B that are facing each other in given direction θ "

Area of subregions of object A and B represented by dark gray color in figure 2 represents a histogram of fuzzy Allen relation. By polygonal approximation, area of objects can be calculated by trapezoidal rules and finally definition of histogram of fuzzy Allen relations can be rewrite as:

$$\int_{r \in A} \left(\sum F_r(q, A_q(v), B_q(v)) dv \right) = (x+z) \sum_{k=1}^n r(I_k, J_k) \quad (2)$$

where z is the area of reference object and x is area of augmented object in direction θ which can be calculated by trapezoidal rules, n is total number of segments treated and $r(I_k, J_k)$ is any Allen relation.

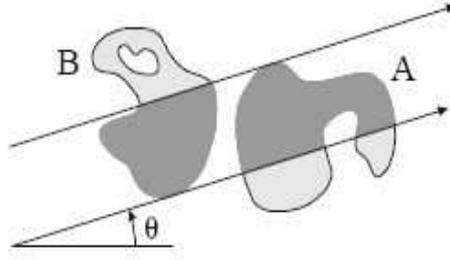


Fig. 2. Gray area represents the histogram of Allen in direction θ

3. FUZZY OPERATORS AND TREATMENT OF LONGITUDINAL SECTIONS

In real word, objects are complex and they may have different boundary types, they are further divided into convex and concave object classes according to shape. During the decomposition process of an object into segments, there can be multiple segments depending on object shape and boundary which are called longitudinal section. We know that different segments of a longitudinal section are at a certain distance and these distances might effect end results. Low temporal complexity and high precision rates are main objectives of different approaches of finding fuzzy directional relations. Different problem raised by P.Matsakis [14] exists at segmentation level, if we define object by its boundary, then all of these problems can be successfully handled by detecting the object boundary and longitudinal sections is the distance between two intersecting points. But need for the fuzzification process developed by P.Matsakis is still there when object has the disconnected boundary or object is concave. In this case there exist number of 1D segments of concave object or object having disconnected boundary. Each segment and its distance from other segment has its own impact on fuzzy Allen relations of whole object. On the other hand if we follow the process of fuzzification of longitudinal sections, we will be away from our objective. To cope with this, we use fuzzy operators. In this case each fuzzy Allen relation is a member of fuzzy set, therefore fuzzy set theory can be used for information integration. In literature of fuzzy set theory there exist variety of operators such as fuzzy T -norms, T -conorms and so on, which can be used for fuzzy integration of available information. Some mostly used operators are [5]

1. $\mu_{(OR)}(u) = \max(\mu_{(A)}(u), \mu_{(B)}(u));$
2. $\mu_{(AND)}(u) = \min(\mu_{(A)}(u), \mu_{(B)}(u));$
3. $\mu_{(SUM)}(u) = 1 - \prod_{i=1}^2 (\mu_{(i)}(u)),$
4. $\mu_{(PROD)}(u) = \prod_{i=1}^2 (\mu_{(i)}(u))$
5. $\mu_{(\gamma)}(u) = [(\mu_{(SUM)}(u))^\gamma * [(\mu_{(PROD)}(u))]^{1-\gamma}$ where $\gamma \in [0, 1]$

When fuzzy operator OR (respectively AND) is used, only one fuzzy value contributes for the resultant value which is *maximum* (respectively *minimum*). For other operators both values contribute. Fuzzy algebraic sum (product)operator makes the resultant set larger than or equal (less than) the participating value. For the fuzzy γ operator resulting value changes from minimum to maximum values depending on the choice of γ . In our case each Allen relation has a fuzzy grade so we use these operators for integration of fuzzy grade values. Our objective is to accumulate the best available information. We consider that there exist number of segments and each segment has a fuzzy Allen relation with segment of other object. Now suppose longitudinal section of object B has two segments such that $z = z_1 + z_2$ where z_1 is the length of first segment and z_2 is the length of second segment and z is length of longitudinal section. Let $\mu_{(A)}(y_1)$ defines the value of fuzzy Allen relations with the first segment and $\mu_{(B)}(y_2)$ represents value of fuzzy Allen relations with the second segment where y_1 and y_2 are the distances between object A and two segments of B . Now Fuzzy OR operator is used to get consequent information obtained from two sets of fuzzy Allen relations. (Fuzzy γ operator can also be used to make possible the contributions of two fuzzy values but in this case finding compensations values of γ is a problem). For more than two segments composition of operator can be used.

4. EXPERIMENTS AND INTERPRETATION

For the experiment purpose 360 directions are considered (angle increment is 1 degree) and lines are drawn by the well known Bresenham digital line algorithm. Instead of considering all the v values, we consider only those lines which passes through

vertex of polygon. Longitudinal sections are computed and all pairs of segments can be treated simultaneously. Fuzzy Allen relations are computed for each segment, if there exist longitudinal section (More number of segments for an object) then fuzzy aggregation operator is applied to obtain the resultant fuzzy Allen relation of each fuzzy Allen relation. Each relation is associated with the gray scale value like *before* with black and white represents *after*. We use the same notations as P.Matsakis, except changing the boundary color of each relation for better visualization of relations. Opposite relations have the same boundary color such as *m(meet)* and *(meet_by)* relations have the yellow boundary color. Object *A* has the light gray color while object *B* is represented by dark gray color. The thirteen histograms that represent directional and topological relations are plotted in the same diagram. For a given angle all the histograms are represented in the layers and each vertical layer represents total area of objects in that direction. Here histograms are not normalized. All relations are symmetric in nature except *d(during)* and *di(during_by)*.

$$f_b^{AB}(\theta) = f_a^{AB}(\theta + \pi), f_{mi}^{AB}(\theta) = f_m^{AB}(\theta + \pi), ; f_{oi}^{AB}(\theta) = f_o^{AB}(\theta + \pi), f_{si}^{AB}(\theta) = f_{fi}^{AB}(\theta + \pi)$$

$$f_f^{AB}(\theta) = f_s^{AB}(\theta + \pi), \text{ and for } d(\text{during}) \text{ and } di(\text{during_by}) \text{ it will be } f_d^{AB}(\theta) = f_{di}^{BA}(\theta).$$

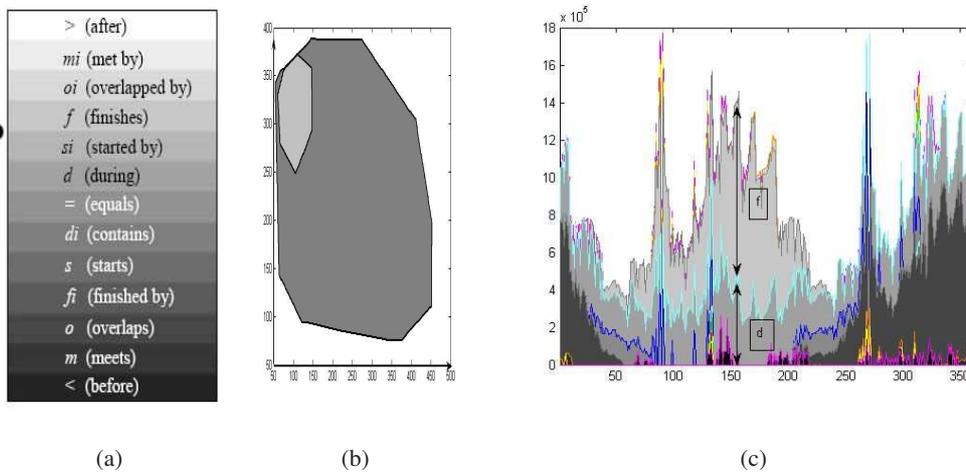


Fig. 3. (a)Explanation of gray level value associated with a relation (b)Object pair representation. (c) Corresponding histogram

In fig.3(a) explains the representation of fuzzy Allen histograms. In fig.3(b) Shows the representation of histograms and explains that each relation is represented by a layer and each layer have a different gray level color associated with a relation, boundary color is not represented here (This figure is taken from [14]). We use the same colors association with a relation and change only boundary color. In fig.3(c) represents object position where *A* is light gray object and object *B* is represented by dark gray color. and fig.3(c) represents histogram associated with objects pair, where y axis represents total area of objects having different relations and directions are represented along x axis. At a certain value *f* represents area under the *finish* relation and *d* represents area having *during* relation and total area is sum of both areas. We consider the different set of examples, in first case we consider both the objects as a convex objects and second case argument object *A* is convex and reference object *B* is concave.

In this experiment Fig.4(a) Pair of objects under consideration are at enough distance. Fig.4(b) represents the corresponding fuzzy histogram of Allen relations, at this stage only *after* and *before* fuzzy relations exist. Fig.4(c) At this stage object *A* moves towards object *B* their internal distance decreases. Fig.4(d) In this histogram of fuzzy Allen relation *meet* and *meet_by* emerges along with relation *after* and *before*. Fig 4(e) Object *A* moves to words center of object *B* and it overlaps *B*. Fig.4(f) represents its histogram at this almost all the relations exist. Fig.4(g) Position of object *A* is at center of object *B*. Fig.4(h) represents its histogram *during* relation exist, There exist *after* and *before* relation near the diagonal direction. Which is due to zigzag of lines in digital space. In this set of examples objects are taken at different distances to show that the relations are sensitive to distance between them and their sensitivity also depends upon relative size.

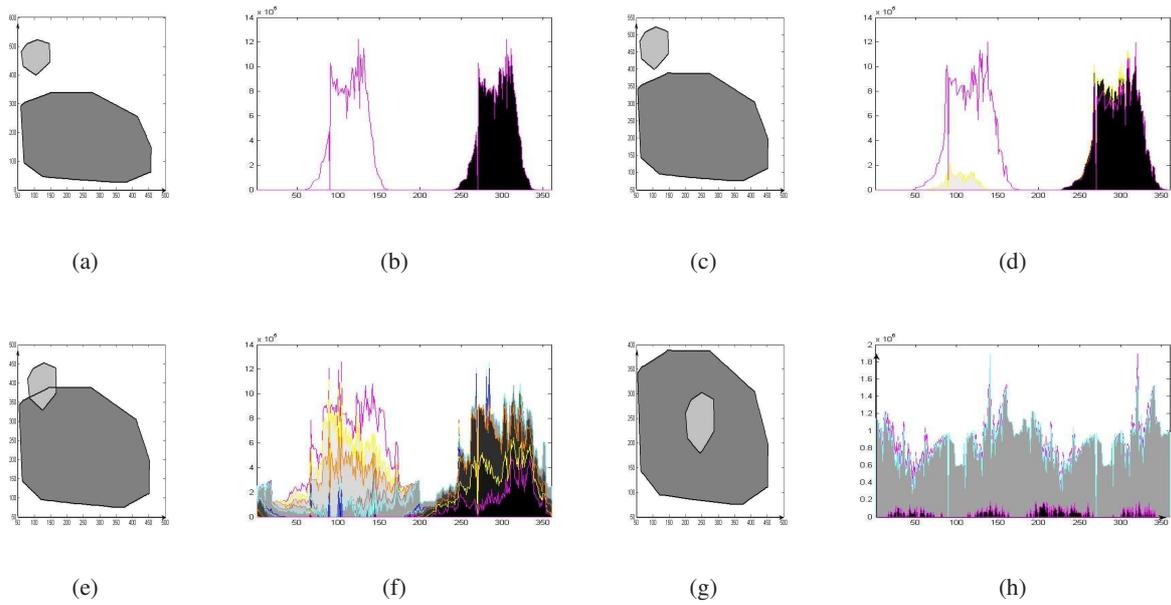


Fig. 4. pairs of convex objects and their corresponding fuzzy histogram of Allen relations

Now we consider second set of examples. In this example we consider the rectangular objects A firstly for away from the U shaped object B . Fuzzy Allen relations are calculated separately for each segment then fuzzy operator is used. Main objective of this example is to show that each segment of longitudinal section has its own impact on Fuzzy Allen relation and each segment may have same, opposite neither opposite nor same Allen relations as in case of fig.5(a) to fig.5(g).

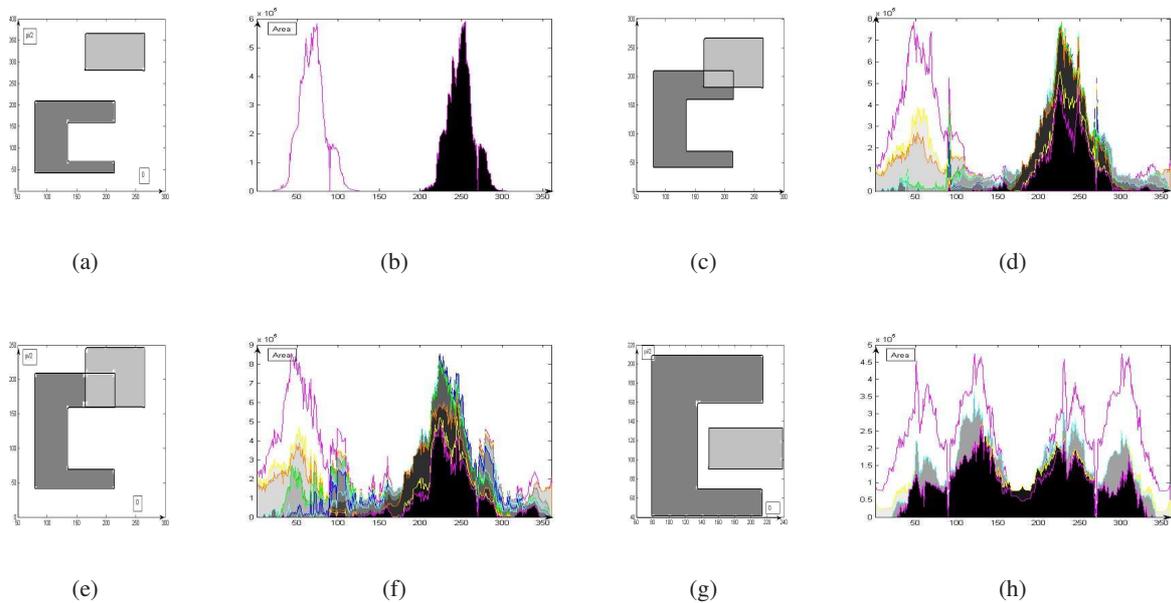


Fig. 5. Convex and concave pair of objects and their corresponding fuzzy histogram of Allen relations

In Fig.5(a) object *A* is at a certain distance to object *B*. Fig.5(b) only *after* and *before* relation exists because both parts of object *B* has the same relation. In fig.5(c) when object *A* is partially overlapped one part of object *B* then both parts of *B* has different relations. First part has relations *after* while second part has relations like *meet_by*, *overlaped_by* along with small value of relation *start_by* (where *meet_by* relation du to zigzag). Information for all existing relations for both segments are summarized for a certain angle. In fig.5(e) when object *A* completely overlapped first part of object *B* as a result relation *finish* and relation *during* grows up. Finely in fig.5(g) when object *A* is between two parts of object *B*, both segments have opposite relations *before* and *after* meanwhile there exist relation *during* which is due to zigzag phenomena of digital space and line algorithm.

5. TEMPORAL COMPLEXITY

Finding the exact temporal complexity is a tough job, major aim of this study is to find time length or amount of memory required by the algorithm. We need a language that expresses the computational time as a function of *N*, grows on the order of *N*. Five symbols for comparing rates are used such as *o*, *O*, *θ*, *Ω* and *~* (Asymptotically equal, irreverently, tiddle). In fact asymptotically equality is an formalism of idea to find the conditions that two functions have same growth rate.i.e. $\lim_{n \rightarrow \infty} (\frac{a_n}{b_n}) = 1$ and $a_n = O(b_n)$ if $|\frac{a_n}{b_n}|$ is bounded

We find temporal complexity of a algorithm by asymptotic analysis. For this purpose we find a function which represents upper bound of our function. In our case time constraint depends upon length of line, contour length and number of polygon vertexes. We summarize time for all 360 directions when line length is fixed to 1000,1200,1400 pixels. Following tables represent different computations where L is length of line and N represents number of polygon vertexes.

Table 1. contour of 1300 pixels

<i>N</i> \ <i>L</i>	1000	1200	1400
24	63	67.14	73
25	66.5	73	78
26	68	74.3	80
27	72	79	84.34
28	73.20	82	86.20

Table 2. contour 3300 of pixels

<i>N</i> \ <i>L</i>	1000	1200	1400
25	97.5	103	113.5
26	102	107	115.5
27	109	115.5	123
28	113	119	126
29	120	125	129.4
30	124	129	135

If we analyze keenly the data in table 1 and table 2, there will be a certain symmetry between the different values of cost function (Time function) and the number of polygons vertexes.

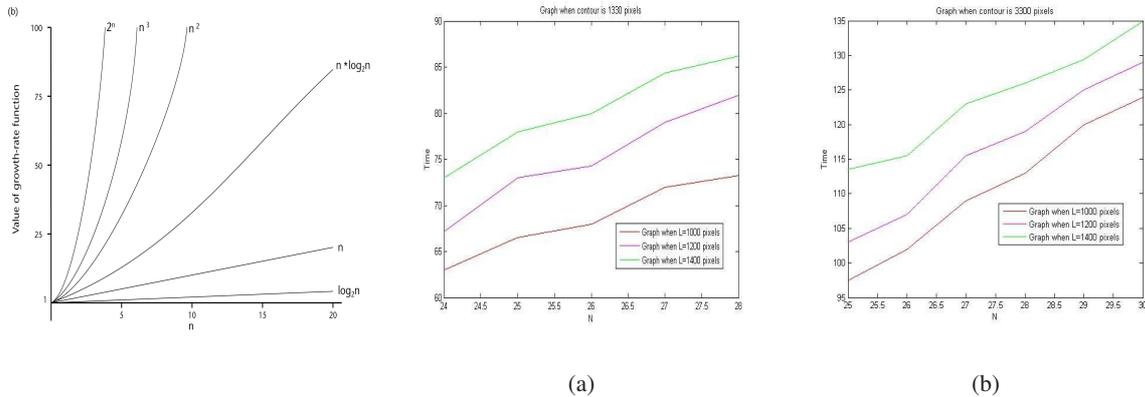


Fig. 6. (a)Graph of some known functions,(b)Graphs of data given in table No.1(c)Graphs of data given in table No.2

Now if we observe the graphical representation of data,(graph fig.6(a)) of data in table 1 and graph fig.6(b) of data in table 2) each time graph for a fixed length of line and given objects sizes (length of contours). It seems that graph is displaced by a constant value of T . It seems that graphically function $f(n) = n \log(n)$ representation the upper bound of our graphs. (Graphes given in fig.5) So histogram of fuzzy Allen relations are of order $O(N \log(N))$

6. CONCLUSION

It is shown that histogram of fuzzy Allen relations associated with pair of objects carry a lot of information. To deal with concave objects or objects having disconnected boundaries, fuzzy operators are used. Use of these operators is simple so polygonal approximation of objects and application of fuzzy aggregation operator simplifies the algorithm given by P.Matsakis [14]. This approach decrease its temporal and computational complexity due to avoiding the fuzzification process developed by P.Matsakis. Certainly this approach of using fuzzy operator will open new fields of applications for fuzzy aggregation operators. Here we calculated all the directions for experimental purpose, in practice we performed only limited number of directions according to the requirement of application. Allen relations are used for describing the relative object position in image understanding.

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