Stochastic Analysis of Non-slotted Aloha in Wireless Ad-Hoc Networks

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***Abstract*: In this paper we propose two analytically tractable stochastic models of non-slotted Aloha for Mobile Ad-hoc NETworks (MANETs): one model assumes a static pattern of nodes while the other assumes that the pattern of nodes varies over time. Both models feature transmitters randomly located in the Euclidean plane, according to a Poisson point process with the receivers randomly located at a fixed distance from the emitters. We concentrate on the so-called outage scenario, where a successful transmission requires a Signal-to-Interference-and-Noise Ratio (SINR) larger than a given threshold. With Rayleigh fading and the SINR averaged over the duration of the packet transmission, both models lead to closed form expressions for the probability of successful transmission. We show an excellent matching of these results with simulations. Using our models we compare the performances of non-slotted Aloha to slotted Aloha studied in [6]. We observe that when the path loss is not very strong both models, when appropriately optimized, exhibit similar performance. For stronger path loss non-slotted Aloha performs worse than slotted Aloha, however when the path loss exponent is equal to 4 its density of successfully received packets is still 75% of that in the slotted scheme. This is still much more than the 50% predicted by the well-known analysis where simultaneous transmissions are never successful. Moreover, in any path loss scenario, both schemes exhibit the same energy efficiency.**

***Index Terms*—Medium Access Control; MANET; slotted and non-slotted Aloha; Poisson point process, shot-noise, SINR, stochastic geometry**

I. INTRODUCTION

Aloha is one of the most common examples of a multiple communication protocol; it is presented in many widely used books on data networks such as [7, 14]. A main characteristic of Aloha is its great simplicity: the core concept consists in allowing each source to transmit a packet and back-off for some random time before the next transmission independently of other sources. This, of course, leads to collisions and some packets have to be retransmitted. In order to evaluate the fraction of packets that are transmitted successfully, a simple and widely used model assumes that simultaneous transmissions are never successful. When the aggregate packet transmission process follows a Poisson distribution, the analysis of this pure (or *non-slotted*) Aloha model shows that on average the fraction $1/(2e) \approx 18.4\%$ of successful transmissions can be attained, when the scheme is optimized (tuning the mean back-off time). It also shows that this performance can be multiplied by 2 in *slotted-Aloha*, when all the nodes are synchronized and can send packets only at the beginnings of some universal time slots. This analysis, which is very often taught to well exemplify the protocol's performance, is however based on the simple collision model where two simultaneous transmissions always lead to a collision. This assumption is well adapted to wired networks but is not adequate for wireless networks where *spatial reuse* is generally present.

In this paper we analyze Aloha in a wireless network model featuring transmitters randomly located in the Euclidean plane, according to a Poisson point process, with the receivers randomly located at a fixed distance from the emitters. We assume the SINR coverage context, which is that where each successful transmission requires that the receiver be covered by the transmitter with a minimum SINR. We adopt a path loss model with a power-law mean signal-power decay $l(u) = u^\beta$ on the distance u and we assume some random independent fading model. Whereas the analysis of slotted Aloha in such a MANET model has been done in [5], in this paper we propose an analysis of non-slotted Aloha.

As the *main theoretical contribution* of this paper we build *two analytically tractable stochastic models of non-slotted Aloha*. We believe one of these models to be very close to the reality of a relatively static MANET, while the other assumes that the pattern of nodes is different at each transmission. With Rayleigh fading, and when the interference is averaged in the SINR packet reception constraint, both models lead to closed form expressions for the *probability of successful receptions*. The formula derived in the static MANET requires numerical processing while the formula derived in the other model is more explicit. Moreover, we find an excellent matching of the values obtained for these two analytical models.

We also conduct extensive simulations of non-slotted Aloha, both with averaged and maximal interference in the SINR constraint. We show an excellent matching of both analytical models with the simulations in the mean interference constraint case.

Using our models we also compare the performances of slotted and non-slotted Aloha. The *main findings of this analysis* show that:

- When the path loss exponent β is small (close to its lower theoretical bound $\beta = 2$) slotted and non-slotted Aloha (non-slotted with the mean interference constraint), when appropriately optimized, offer a similar space-time density of successful transmissions.
- For larger values of β, the optimized non-slotted Aloha gives a smaller density of successful transmissions than the optimized slotted Aloha, with the ratio asymptotically going to 0.5 ($\beta \to \infty$) — the value predicted by the widely used simplified model. However, for $\beta = 4$ this ratio is still 75%.
- When optimized, both slotted and non-slotted models

exhibit the same energy efficiency (the mean number of successful transmissions per unit of energy spent) if one ignores the energy spent for maintaining synchronization in the slotted scheme.

The rest of this paper is organized as follows. In the remaining part the current section we recall some previous studies of Aloha. Section II introduces the network model and our two models for non-slotted Aloha. Section III contains the main stochastic analysis results of this paper. In Section IV we show how the performance of the non-slotted Aloha can be optimized and compare it to the optimized slotted Aloha. Our conclusions are presented in Section VI. In the Appendix we present proofs of our mathematical results.

A. Related Work

Aloha and Time Division Multiple Access (TDMA) are the oldest multiple access protocol. Aloha, which is the "mother" of random protocols, was born in the early seventies, the seminal work describing Aloha [1] being published in 1970. Aloha is very simple and also extremely widespread. Another very amazing characteristic of Aloha is that it has an extremely simple analysis which is widely taught (cf. e.g [7, 4.2]). However this analysis is only valid in the rough model where two simultaneous transmissions necessarily lead to a collision. Surprisingly, although Aloha was primarily designed to manage a wireless network, the first models for Aloha were more adapted to wired networks. To the authors' best knowledge the first contribution in which Aloha was explicitly studied in a wireless context is the paper by Nelson and Kleinrock [13]. The propagation model of this paper is very simple and it was only in 1988 that the widely referenced paper [12] was published, in which the first reasonable Aloha model for wireless network with spatial reuse was proposed. However, even in that paper the spatial analysis of the protocol remained simplified. More recently, in [8] a more refined spatial Aloha model was studied with local interactions and the simplified collision model (where simultaneous transmissions always lead to collisions). A little earlier in [5], one evaluated the probability of capture for Aloha in the SINR outage context with Rayleigh fading, showing a direct link between the probability of capture and the Laplace transforms of the thermal noise and of the interference (also called shot-noise). The key factor of this analysis is the explicit formula of the Laplace transform of the interference created by a Poisson pattern of nodes. This analysis of slotted Aloha was completed in [6].

The main contribution of the present paper is the extension of the analysis of [5, 6] to pure (non-slotted) Aloha allowing for a fair comparison of both schemes in the wireless MANET context.

There are many publications on the stability of Aloha and, more generally, random back-off protocols. This problem is not addressed in the present paper.

II. Network and Aloha Models

In this section we present comparable models of slotted and non-slotted Aloha for wireless ad-hoc networks. More precisely we first introduce the geometric model of the network. We then present the access schemes: slotted and non-slotted Aloha and the mathematical models used to represent these schemes. We also model the fading process and the external noise. For non-slotted Aloha, we propose a second model whose analysis is simpler. At the end of this section we show how to relate different model parameters to make a fair comparison of their performance.

A. Location of Nodes — The Spatial Poisson Bipolar Network Model

We consider a *Poisson bipolar network model* in which each point of the Poisson pattern represents a node of a Mobile Ad hoc NETwork (MANET) and is hence a potential transmitter. Each node has an associated receiver located at distance r. This receiver is not part of the Poisson pattern of points.

More precisely, using the formalism of the theory of point processes, we will say that a snapshot of the MANET can be represented by an independently marked Poisson point process (P.p.p) $\widetilde{\Phi} = \{(X_i, y_i)\}$, where the *locations of nodes* $\Phi = \{X_i\}$ form a homogeneous P.p.p. on the plane, with an intensity of λ nodes per unit of space, and where the mark y_i denotes the location of the receiver for node X_i. We assume here that no two transmitters have the same receiver and that, given Φ, the vectors $\{X_i - y_i\}$ are i.i.d with $|X_i - y_i| = r$. [1]

B. Aloha Models — Time Added

We will now consider two time-space scenarios appropriate for slotted and non-slotted Aloha. In both of them the planar locations of MANET nodes and their receivers $\widetilde{\Phi}$ *remain fixed*. It is the medium access (MAC) status of these nodes that will evolve differently over time depending on which of the following two models is used.

1) Slotted Aloha: In this model we assume that the time is discrete, i.e. divided into slots of length B (the analysis will not depend on the length of the time-slot) and labeled by integers $n \in \mathbb{Z}$. The nodes of Φ are *perfectly synchronized* to these (universal) time slots and send packets according to the following *slotted Aloha: each node, at each time slot independently tosses a coin with some bias* p *which will be referred to as the medium access probability (MAP); it sends the packet in this time slot if the outcome is heads and backs off its transmission otherwise*. This evolution of the MAC status of each node X_i can be formalized by introducing its further (multi-dimensional) mark $(e_i(n) : n \in \mathbb{Z})$, where $e_i(n)$ is the medium access indicator of node i at time n; $e_i(n) = 1$ if node i is allowed to transmit in the time slot considered and 0 otherwise. Following the Aloha principle we assume that $e_i(n)$ are hence i.i.d. (in n and i) and independent of everything else, with $\mathbf{P}(e_i(n) = 1) = p$. We treat p as the main parameter to

[1] The fact that all receivers are at the same distance from their transmitters is a simplification. There is no difficulty extending what is described below to the case where these distances are independent and identically distributed random variables, independent of everything else. A further possible extension assumes that transmitters are selected from some, say Poisson, process of potential receivers, common for all MANET nodes, taking the nearest point to the emitter. A more involved model assumes that the transmitters of the MANET choose their receivers in the original set Φ of nodes of the MANET; see [3, Chapter 17] for precise descriptions and the analysis of slotted Aloha.

be tuned for slotted Aloha. We will call the above case *the slotted Aloha model*.

2) Poisson-renewal Model of Non-slotted Aloha: In this *non-slotted Aloha* model all the nodes of Φ independently, without synchronization, send packets of the same duration B and then back off for some random time. This can be integrated in our model by introducing marks $(T_i(n) : n \in \mathbb{Z})$, where $T_i(n)$ denotes the beginning of the nth transmission of node X_i with $T_i(n+1) = T_i(n) + B + E_i(n)$, where $E_i(n)$ is the duration of the nth back-off time of the node X_i. The non-slotted Aloha principle states that $E_i(n)$ are i.i.d. (in i and n) independent of everything else. In what follows we assume that $E_i(n)$ are exponential with mean $1/\epsilon$ and will consider the parameter ϵ as the main parameter to be tuned for non-slotted Aloha (given the packet emission time B). More precisely, the lack of synchronization of the MAC mechanism is reflected in the assumption that the temporal processes $(T_i(n) : n \in \mathbb{Z})$ are time-stationary and independent (for different i). Note also that these processes are of the *renewal* type (i.e., have i.i.d. increments $T_i(n+1) - T_i(n)$). For this reason we will call the above case the *Poisson-renewal model for non-slotted Aloha*. The MAC state of the node X_i at (real) time $t \in \mathbb{R}$ can be described by the on-off process $e_i^{renewal}(t) = \mathbb{1}(T_i(n) \leq t < T_i(n) + B \text{ for some } n \in \mathbb{Z})$.

C. Fading and External Noise

We need to complete our network model by some radio channel conditions. We will consider the following *fading scenario*: channel conditions vary from one transmission to another and between different emitter-receiver pairs, but remain fixed for any given transmission. To include this in our model, we assume a further multidimensional mark $(\mathbf{F}_i(n) : n \in \mathbb{Z})$ of node X_i where $\mathbf{F}_i(n) = (F_i^j(n) : j)$ with $F_i^j(n)$ denoting the *fading* in the channel from node X_i to the receiver y_j of node X_j during the nth transmission. We assume that $F_i^j(n)$ are i.i.d. (in i, j, n) and independent of everything else. Let us denote by F the generic random variable of the fading. We always assume that $0 < \mathbf{E}[F] = 1/\mu < \infty$. In the special case of Rayleigh fading, F is exponential (with parameter μ). (see e.g. [15, pp. 50 and 501]). We can also consider non exponential cases, which allow other types of fading to be analyzed, such as e.g. Rician or Nakagami scenarios or simply the case without fading (when $F \equiv 1/\mu$ is deterministic).

In addition to fading we consider a non-negative random variables $W_i(n)$ i.i.d. (in (i, n)) and independent of $\widetilde{\Phi}$ modeling the power of the external (thermal) noise at the receiver y_i at time n. The Laplace transform of the generic noise variable W will be denoted by $\mathcal{L}_W(s) = \mathbf{E}[e^{-sW}]$.

The slotted Aloha model described above, when considered in a given time slot, coincides with the Poisson Bipolar model with independent fading considered in [6]. It allows an explicit evaluation of the successful transmission probability and other characteristics such as the density of successful transmissions, the mean progress, etc.

An exact analysis of the Poisson-renewal non-slotted Aloha model, albeit feasible, does not lead to similarly closed form expressions. To improve upon this situation, in what follows we propose another model for the non-slotted case. It allows for the results as explicit as these of [6], which are moreover very close to these of Poisson-renewal model.

D. Poisson Rain Model for Non-slotted Aloha

The main difference with respect to the scenario considered above is that the nodes X_i and their receives y_i are not fixed in time. Rather, we consider a time-space Poisson point process $\Psi = \{(X_i, T_i)\}$ with $X_i \in \mathbb{R}^2$ denoting the location of the emitter which sends a packet during time interval $[T_i, T_i + B)$ (indexing by i is arbitrary and in particular does not mean successive emissions over time). We may think of node X_i "born" at time T_n transmitting a packet during time B and "disappearing" immediately after. Thus the MAC state of the node X_i at (real) time $t \in \mathbb{R}$ is simply $e_i(t) = \mathbb{1}(T_i \leq t < T_i + B)$.

We always assume that Ψ is homogeneous (in time and space) P.p.p. with intensity λ_s. This parameter corresponds to the *space-time frequency of channel access*; i.e, the number of transmission initiations per unit of space and time. The point (X_i, T_i) of the time-space P.p.p. Ψ are marked by the receivers y_i in the same manner an described in Section II-A; i.e, given Ψ, $\{X_i - y_i\}$ are i.i.d random vectors with $|X_i - y_i| = r$. Moreover, they are marked by $\mathbf{F}_i = (F_i^j : j)$, with F_i^j denoting the fading in the channel from X_i to the y_j (meaningful only if X_i, X_j coexist for a certain time). We assume that $F_j^j(n)$ are i.i.d. (in i, j) and of everything else, with the same generic random fading F as in Section II-C. We will call the above model the *Poisson rain model for non-slotted Aloha*. It can be naturally motivated by *strong mobility of nodes*.

E. Choice of Parameters for the Fair Comparison of the Models

In order to achieve a fair comparison of the performance of the above models, we have to assume the same offered traffic in all the protocols. Regarding slotted and renewal non-slotted protocol, we observe that the *channel-occupation-time-fraction per node* (i.e., the average fraction of time each node is authorized to transmit) τ is equal to p in the former and $B/(B+1/\epsilon)$ in the latter one. Thus a fair comparison between these two models requires

$$\tau = p = \frac{B}{B + 1/\epsilon} \tag{2.1}$$

and we will consider p, τ as the main parameters to optimize the performance of the respective Aloha models. To compare the Poisson rain model the other two models we assume the same *space-time density of channel occupation* (i.e., the expected total channel-occupation-time by all the nodes, evaluated per unit of space and time):

$$\lambda_s B = \lambda \tau \, . \tag{2.2}$$

III. Successful Transmission

A. Path-loss Model

Assume that all emitters, when authorized by Aloha, emit packets with unit signal power and that the receiver y_i of node X_i receives a power from the node located at X_j (provided this node is transmitting) equal to $F_j^i / l(|X_j - y_i|)$, where $|\cdot|$

denotes the Euclidean distance on the plane and $l(\cdot)$ is the path loss function. An important special case consists in taking

$$l(u) = (Au)^{\beta} \quad \text{for } A > 0 \text{ and } \beta > 2. \tag{3.1}$$

Other possible choices of path-loss function avoiding the pole at $u = 0$ consist in taking e.g. $\max(1, l(u))$, $l(u+1)$, or $l(\max(u, u_0))$.

B. SINR Condition

1) Slotted Aloha : It is natural to assume that transmitter X_i *covers* its receiver y_i in time slot n if

$$\text{SINR}_i(n) = \frac{F_i^i(n)/l(|X_i - y_i|)}{W_i(n) + I_i(n)} \geq T\,, \tag{3.2}$$

where T is some SINR threshold and where $I_i(n)$ is the *interference* at receiver y_i at time n; i.e., *the sum of the signal powers received by y_i from all the nodes in $\Phi^1(n) = \{X_j \in \Phi : e_j(n) = 1\}$ except X_i*, namely,

$$I_i(n) = \sum_{X_j \in \Phi^1(n),\, j \neq i} F_j^i(n)/l(|X_j - y_i|)\,. \tag{3.3}$$

When condition (3.2) is satisfied we say that X_i can be *successfully received* by y_i or, equivalently, that y_i *is not in outage* with respect to X_i in time slot n.

2) Non-slotted Aloha: When transmissions are not synchronized (as is the case for non-slotted Aloha) the *interference* (defined, as previously, as the sum of the signal powers received by a given receiver from all the nodes transmitting in the network except its own emitter) *may vary during a given packet transmission*. Indeed, other transmissions may start or terminate during this given transmission. In our Poisson-renewal model of Section II-B2 this interference process $I_i(n,t)$ during the nth transmission to node y_i can be expressed using (3.3) with $\Phi^1(n)$ replaced by $\Phi^1_{ren}(t) = \{X_j \in \Phi : e_j^{ren}(t) = 1\}$. Similarly, in the Poisson rain model of Section II-D, the interference process, denoted by $I_i(t)$, during the (unique) transmission of node X_i admits the above representation (3.3) with $\Phi^1(n)$ replaced by $\Psi^1(t) = \{X_j \in \Psi : e_j(t) = 1\}$, and $F_j^i(n)$ replaced by F_j^i.

Below, we propose two different ways of taking into account this varying interference in the SINR condition (3.2):

- To take the *maximal interference value* during the given transmission $I_i^{\max}(n) = \max_{t \in [T_i(n), T_i(n)+B]} I_i(n,t)$ or $I_i^{\max} = \max_{t \in [T_i, T_i+B]} I_i(t)$ for the Poisson-renewal or the Poisson rain model, respectively; this choice corresponds to the situation where bits of information sent within one given packet are not repeated/interleaved so that the *SINR condition needs to be guaranteed at any time of the packet transmission* (for all symbols) for the reception to be successful.
- To take the *averaged interference value* over the whole packet duration $I_i^{mean}(n) = 1/B \int_{T_i(n)}^{T_i(n)+B]} I_i(n,t)\,\mathrm{d}t$ or $I_i^{mean} = \int_{T_i}^{T_i+B} I_i(t)\,\mathrm{d}t$ for the Poisson-renewal or the rain model, respectively; this condition corresponds to a situation where some coding with repetition and interleaving of bits on the whole packet duration is used.

More precisely, we will say that in *non-slotted Aloha with maximal interference constraint* X_i can be successfully received by y_i (in time slot n in the case of the Poisson-renewal model), if condition (3.2) holds with $I_i(n)$ replaced by $I_i^{\max}(n)$ or $I_i^{\max}$ in the Poisson-renewal or the Poisson rain model, respectively.

Similarly, we will say that in *non-slotted Aloha with average interference constraint* X_i can be successfully received by y_i (in time slot n in the case of the Poisson-renewal model), if condition (3.2) holds with $I_i(n)$ replaced by $I_i^{mean}(n)$ or I_i^{mean} in the Poisson-renewal or Poisson rain model, respectively.

In what follows we will be able to express in closed form expressions the coverage probability for both the Poisson-renewal and the Poisson rain model when the average interference constraint is considered. The maximal interference constraint case is studied by simulations in Section V-B.

C. Coverage Probability

In slotted Aloha and the Poisson-renewal model of non-slotted Aloha let $\mathbf{E}^0$ denote the expectation with respect to the Palm probability $\mathbf{P}^0$ (cf. [2, Sec. 10.2.2]) of the P.p.p. Φ. Under this distribution, the nodes and their receivers are located at $\widetilde{\Phi} \cup \{(X_0 = 0, y_0)\}$, where $\widetilde{\Phi}$ is a copy of the original (stationary) marked P.p.p., and where y_0 is independent of $\widetilde{\Phi}$, distributed like the other receivers. Moreover under $\mathbf{P}^0$ all other marks of points in $\widetilde{\Phi}$ and X_0 (MAC status, fading, packet emission renewal processes in the renewal model) are i.i.d. and have their original distributions. Under $\mathbf{P}^0$, the node X_0 at the origin is called the *typical node*. (For more details on Palm theory cf. e.g. [2, Sections 1.4, 2.1 and 10.2].) Further conditioning on the time scale, in the case of the slotted Aloha model denote by $\mathbf{P}^{0,e_0=1}\{\,\cdot\,\} = \mathbf{P}^0\{\,\cdot \mid e_0 = 1\}$ the conditional probability of $\mathbf{P}^0$ given the node X_0 emits at time $n = 0$.

In the Poisson-renewal model we denote by $\mathbf{P}^{0,T_0(0)=0} = \mathbf{P}^0\{\,\cdot \mid T_0(0) = 0\}$ the probability $\mathbf{P}^0$ given the node $X_0 = 0$ starts transmitting at time 0. Formally this means that the renewal process of transmission times $T_0(n)$ of node $X_0 = 0$ is so called zero-delayed (and we denote by 0 the transmission that starts at time 0). By the independence (lack of synchronization) other node transmission times are not affected by this conditioning.

Finally let us denote by p_{slot} the *probability* $\mathbf{P}^{0,e_0(0)=1}$ *of the successful transmission of the node X_0 at the time 0 given it is selected by the Aloha*; i.e., the that the condition (3.2) holds for $i = 0$ at time $n = 0$. Similarly, denote by p_{ren}^{mean} the *probability* $\mathbf{P}^{0,T_0(0)=0}$ *of the successful transmission, with the average interference constraint, of the node X_0 started at time 0 given it is selected by the Aloha.*

In the case of the Poisson rain model we consider the Palm distribution $\mathbf{P}^{0,0}$ of the space-time P.p.p. Ψ given a point $X_0 = 0, T_0 = 0$ and denote by p_{rain}^{mean} the *probability* $\mathbf{P}^{0,0}$ *that the transmission from X_0 started at time $T_0 = 0$ is successful with the average interference constraint.*

Similar notation $p_*^{\max}$ with $* = ren, rain$ will be used for the probability of successful transmission for non-slotted Aloha with the maximal interference constraint.

1) Slotted Aloha: For the sake of completeness we recall first a result for slotted Aloha (cf [6]).

Proposition 3.1: *Assume the slotted Aloha model of Section II-B1 with Rayleigh fading (F exponential with mean $1/\mu$). Then*

$$p_{slot} = \mathcal{L}_W(\mu T l(r)) \times \exp\Big\{ - 2\pi\lambda p \int_0^\infty \frac{u}{1 + l(u)/(Tl(r))}\, \mathrm{d}u \Big\}. \quad (3.4)$$

In particular if $W \equiv 0$ and that the path-loss model (3.1) is used then

$$p_{slot} = \exp\Big\{ -\lambda p r^2 T^{2/\beta} K(\beta) \Big\}, \quad (3.5)$$

where

$$K(\beta) = \frac{2\pi\Gamma(2/\beta)\Gamma(1-2/\beta)}{\beta} = \frac{2\pi^2}{\beta\sin(2\pi/\beta)}. \quad (3.6)$$

We remark that the successful transmission probability p_{slot} can also be evaluated in the case of a general fading distribution F; cf [6, Prop. 2.2].

2) Non-slotted Aloha, Poisson-renewal Model: Here we present our result for non-slotted Aloha in the Poisson-renewal model. Its proof, as well as all other proofs is given in the Appendix.

Proposition 3.2: *Assume the Poisson-renewal non-slotted Aloha model of Section II-B2 with Rayleigh fading (F exponential with mean $1/\mu$). Then*

$$p^{mean}_{ren} = \mathcal{L}_W(\mu T l(r)) \times \exp\Big\{ -2\pi\lambda \int_0^\infty u \Big(1 - \frac{1}{1+\epsilon B} \times \Big(e^{-\epsilon B} + \int_0^B \frac{\epsilon e^{-\epsilon s}}{1 + \frac{(B-s)Tl(r)}{Bl(u)}}\, \mathrm{d}s + \int_0^B \frac{\epsilon}{1 + \frac{(B-t)Tl(r)}{Bl(u)}} \int_0^t \frac{\epsilon e^{-\epsilon s}}{1 + \frac{(t-s)Tl(r)}{Bl(u)}}\, \mathrm{d}s\mathrm{d}t \Big)\Big)\, \mathrm{d}u \Big\}.$$

As we can see, the above expression for the successful transmission in the Poisson-renewal model, albeit numerically tractable, is not very explicit. In the following section we show that the Poisson rain model leads to much more tractable results.

D. Non-slotted Aloha — Poisson Rain Model

Here we present our main result for this model.

Proposition 3.3: *Assume the Poisson Rain model for non-slotted Aloha of Section II-D with Rayleigh fading (F exponential with mean $1/\mu$). Then*

$$p^{mean}_{rain} = \mathcal{L}_W(\mu T l(r)) \times \exp\Big\{ - 4\pi\lambda_s B \int_0^\infty u \Big(1 - \frac{l(u)}{l(r)} \log\Big(1 + \frac{l(r)T}{l(u)}\Big)\, \mathrm{d}u \Big\}. \quad (3.7)$$

In particular if $W \equiv 0$ and that the path-loss model (3.1) is used then

$$p^{mean}_{rain} = \exp(-\lambda_s B r^2 T^{2/\beta} K'(\beta)), \quad (3.8)$$

where

$$K'(\beta) = \frac{4\pi}{\beta} \int_0^\infty u^{2/\beta - 1}(1 - u\log(1 + u^{-1}))\, \mathrm{d}u. \quad (3.9)$$

The successful transmission probability p^{mean}_{rain} can also be effectively evaluated in the case of a general fading distribution F; see Appendix.

Remark: We consider the Poisson-rain model to be a simplified model for non-slotted Aloha. In particular it has only one parameter λ_s (time-space density of transmission initiations) that does not allow us to distinguish between the spatial density λ of nodes and the channel-occupation-time-fraction per node. However, in Section V-A we validate this model by comparing the probability of successful transmission p^{mean}_{rain} to p^{mean}_{ren} under equality (2.2) with $\tau = B/(B+1/\epsilon)$. We will see a very good matching. Given this observation we can use (3.7) with $\lambda_s B = \lambda\tau$ to express the basic performance metric of the non-slotted Aloha (probability of successful transmission) in terms of all the parameters of the (real) non-slotted system. In the case of $W = 0$ and the path-loss function (3.1) this relation has the following simple form

$$p_{ns} = \exp\Big\{ -\frac{\lambda}{1 + 1/(\epsilon B)} r^2 T^{2/\beta} K'(\beta) \Big\}. \quad (3.10)$$

E. Slotted Versus Non-slotted Aloha — First Comparison

Note that the expression in (3.8) has exactly the same form as that for the slotted Aloha in (3.4), provided (2.2) holds (i.e., when the both schemes exhibit the same space-time density of channel occupation, with the only difference being in the path-loss dependent constant $K'(\beta)$. This observation allows for an explicit comparison of several performance metrics of the slotted and non-slotted Aloha. The simplest one consists in comparing the blocking probabilities p_{slot} to p_{ns} given the same tuning of both systems.

Result 3.4: *Assume the same density of nodes λ, transmission distance r, and the same channel-occupation-time-fraction per node τ (2.2). In the case of Rayleigh fading the non-slotted Aloha offers*

$$\frac{p_{ns}}{p_{slot}} \times 100\% = e^{-(K'(\beta) - K(\beta))\lambda r^2 T^{2/\beta}\tau} \times 100\%$$

of the good-put (frequency of the successful transmissions) per node of the slotted Aloha.

In Figure 1, the ratio $\frac{p_{ns}}{p_{slot}} \times 100\%$ is shown for different values of the path-loss exponent β and SINR threshold T. For other parameters we take $p = \frac{\epsilon}{1+\epsilon} = 0.05$, $\lambda = 0.001$, $r = \sqrt{1000}$. For $T = 10$ and β close to 2.5 slotted and non-slotted Aloha have similar performances. For higher values of β non-slotted Aloha offers a good-put ranging from 70% to 80% of this of slotted Aloha. Note that the above comparison concerns performance of the non-optimized (in p and τ) schemes. We compare both models under their respective optimal tuning in what follows.

IV. Optimal Tuning of Non-slotted Aloha

In what follows we are interested the following MANET performance metrics introduced in [6] (p_c denotes p_{slot}, or p_{ns} in the slotted or non-slotted Aloha case, respectively):

- *(space-time) density of successful transmissions $d_{suc} = \lambda\tau p_c$,*
- *mean progress $prog = r\, p_c$.*

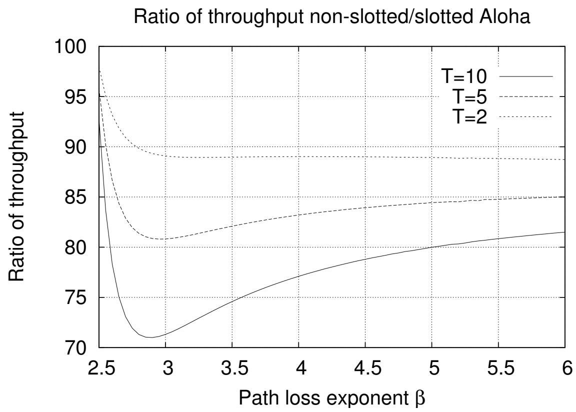

Fig. 1. The ratio (in %) of the good-put offered by the non-slotted Aloha with respect to the slotted one, as a function of the path loss exponent β, for various choices of the SINR threshold T; other parameters are $p = \frac{\epsilon}{1+\epsilon} = 0.05$, $\lambda = 0.001$, $r = \sqrt{1000}$.

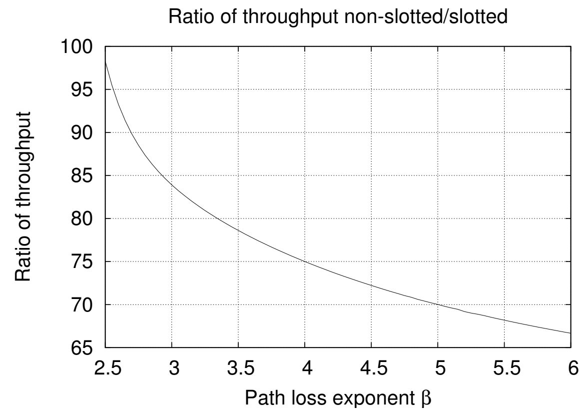

Fig. 2. The ratio (in %) of the good-put offered by the non-slotted Aloha with respect to the slotted one, when both are optimized so as to to maximize the density of successful transmissions as a function of the path loss exponent β. This ratio does not depend on any other parameters.

A. Optimal MAC for the Density of Successful Transmissions

Assume λ, r to be fixed. A good tuning of the non-slotted Aloha renewal parameter ϵ (or equivalently of τ, given B) should find a compromise between the average number of concurrent transmissions per unit area and the probability that a given authorized transmission will be successful. To find such a compromise, one can e.g. maximize the time-space frequency of successful transmissions d_{suc}. The following result follows immediately from (3.10).

Result 4.1: *Assume no noise $W = 0$, Rayleigh fading and path-loss (3.1). Given r, the maximum value of the density of successful transmissions $d_{suc} = 1/(eK'(\beta)r^2T^{2/\beta})$ in the non-slotted Aloha is attained for the space-time density of channel access $\lambda\tau = 1/(K'(\beta)r^2T^{2/\beta})$. Moreover, given the spatial density of nodes λ the optimal mean channel-access-time-fraction τ per node $\tau_{\max}$ for d_{suc} is equal to*

$$\tau_{\max} = \frac{1}{\lambda K'(\beta)r^2T^{2/\beta}}\,.$$

if $\lambda > 1/(K'(\beta)r^2T^{2/\beta})$ and ∞ (interpreted as no back-off; i.e., immediate retransmission) otherwise.

Remark: Recall from [6] that similar optimal value of $d_{suc} = 1/(eK(\beta)r^2T^{2/\beta})$ for slotted Aloha is attained for $\lambda p_{\max} = 1/(K(\beta)r^2T^{2/\beta})$. Since $K'(\beta) > K(\beta)$ we have $p_{\max} > \tau_{\max}$, which means that optimally tuned non-slotted Aloha occupies less channel than optimally tuned slotted Aloha. However, both schemes exhibit the same energy efficiency. Indeed if one assumes that each transmission requires a unit energy, the number of successful transmissions per unit of energy spent is $1/e$. Remark that the latter comparison does not take into account the energy spent to maintain synchronization in the slotted scheme.

The following result compares the optimal density of transmission in slotted and non-slotted Aloha.

Result 4.2: *Under the assumptions of Result 4.1, for a given density of nodes λ non-slotted Aloha with the optimal tuning τ_{max} offers*

$$\frac{\tau_{\max}p_{ns}(\tau_{\max})}{p_{\max}p_{slot}(p_{\max})} \times 100\% = \frac{K(\beta)}{K'(\beta)} \times 100\%$$

of the good-put of the optimally tuned slotted Aloha.

In Figure 2 we present this good-put ratio for the optimized systems in function of β (note that it does not depend on other parameters like λ, r, T). We observe that for small values of path-loss exponent β (close to 2 the performances of slotted and non-slotted Aloha are similar but for large values of β non-slotted Aloha performs significantly worse than the slotted one. In fact, more extensive numerical computations (not presented here) allow us to conjecture that $\lim_{\beta\to 2+} K(\beta)/K'(\beta) = 1$ and $\lim_{\beta\to\infty} K(\beta)/K'(\beta) = 0.5$. The second part of this conjecture means that the good-put ratio for the optimized systems only asymptotically, when $\beta \to \infty$, goes to 0.5 — the value predicted by the widely used simplified model with the simplified collision model (see [7, Section 4.2]). However, e.g. for $\beta = 4$ this ratio is still 75% and even for $\beta = 6$ the ratio still remains significantly larger than 50%.

When trying to explain the above asymptotic value of 50%, one may argue that in the presence of a very strong path-loss the only significant interferers are those, closer to the given receiver than its own emitter, and that their impact is the same as if they were all located in an immediate vicinity to the receiver. This makes the channel between a given emitter and its receiver compatible with the classical, "geometryless" model.

B. Optimal Transmission Distance Given MAC

We assume now that the density of nodes λ as well as some tuning of MAC (τ) is given. We are interested in finding the transmission distance r that maximizes the mean progress *prog* in the network. The following result follows immediately from (3.10).

Result 4.3: *Assume no noise $W = 0$, Rayleigh fading and path-loss (3.1). Given λ and τ, the maximum mean progress prog in the non-slotted Aloha is attained for the transmission*

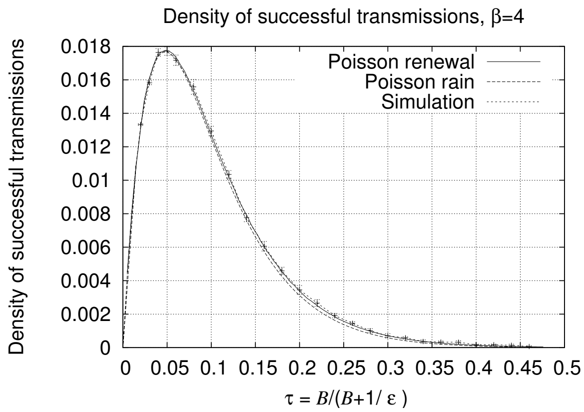

Fig. 3. Density of successful transmissions versus $\tau = \frac{B}{B+1/\epsilon}$. Comparison of the Poisson-renewal and the Poisson rain model to simulation results.

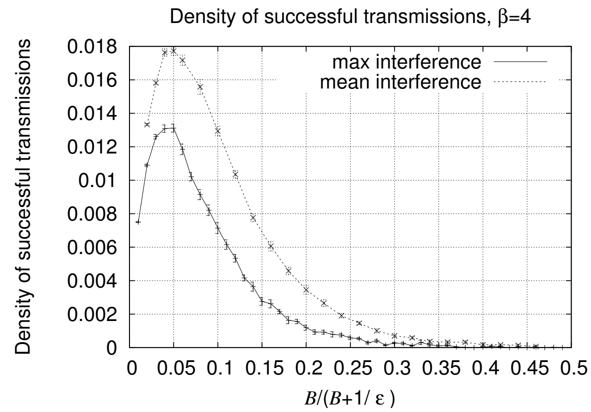

Fig. 4. Density of successful transmissions versus $\tau = \frac{B}{B+1/\epsilon}$ for mean and maximal interference constraint; simulation results.

distance

$$r_{\max} = \frac{1}{\sqrt{2K'(\beta)T^{2/\beta}\lambda\tau}} \tag{4.1}$$

Recall from [6] that the similar optimal tuning of r in slotted Aloha is equal to (4.1) with $K'(\beta)$ replaced by $K(\beta)$. It is thus larger than this for non-slotted Aloha. Here is the comparison of the mean progress in both systems.

Result 4.4: *Under the assumptions of Result 4.3 the non-slotted Aloha with the optimal transmission distance offers a mean progress*

$$\frac{prog_{ns}(r_{\max})}{prog_{slot}(r_{\max})} \times 100\% = \frac{\sqrt{K(\beta)}}{\sqrt{K'(\beta)}} \times 100\%$$

of the mean progress in the slotted Aloha with the optimal transmission distance and the same tuning of MAC $\tau = p$.

The curve corresponding to this result is very similar to the one presented in Figure 2. In particular for $\beta = 4$, the ratio of the mean progress is close to 87%.

V. Further Numerical Results

A. Validation of the Poisson Rain Model

In this section we validate our Poisson rain model by comparing the successful transmission probability p_{rain}^{mean} to the same characteristic p_{ren}^{mean} evaluated numerically with the formula of Proposition 3.2 for our Poisson-renewal model of non-slotted Aloha. We use the same numerical assumptions as previously: $\lambda = 0.001$, $r = \sqrt{1000}$, $T = 10$ and $\beta = 4$. We also compare the results of these two models with simulations carried out in a square of 1000 m $\times$ 1000 m with the same numerical assumptions; this is shown in Figure 3. We observe an excellent matching of the two models with simulations, the differences being almost imperceptible. [2] We perform the same comparison between the two models and the simulations for $\beta = 5$ and $\beta = 3$. For $\beta = 5$ the matching of the two models and the simulation is perfect. For $\beta = 3$ the two models provide the same results whereas the simulations give a larger density of throughput. This can be explained by the fact the analytical models regard infinite-plane models, while in simulations the network area is finite, and the border effects have stronger impact for small values of β.

[2]The error bars in all simulation results correspond to a confidence interval of 95%.

B. Mean Versus Maximum Interference Constraint in SINR

In this section we show the impact of the assumption on maximum interference constraint in the SINR on the probability of a successful transmission.

In Figure 4 we compare p_{ren}^{max} to p_{ren}^{mean} for $\lambda = 0.001$, $r = \sqrt{1000}$, $T = 10$ and $\beta = 4$. The loss in performance when the SINR is computed with the maximum interference can be large and may be up to 45%. But when the throughput is optimized in ϵ, the loss in performance is only 26%. We observe that the throughput is optimized in both cases for the same value of $B\epsilon \simeq 0.045$, this value is also optimal for the Poison rain model. The density of successful transmissions for non-slotted Aloha when the SINR is not averaged is 55% that of slotted Aloha. In this case, the comparison is close to those of the 'standard' model of slotted/non-slotted Aloha on a wired network.

In Figure 5 we compare the density of successful transmissions for slotted Aloha and non-slotted Aloha when the maximum or average SINR is considered. For slotted Aloha we use the analytical model and optimize the density of throughput in p. For non-slotted Aloha we use simulation results and the Poison rain model to optimize the schemes in $\frac{B}{1+1/\epsilon} = \tau$. We observe that for $\beta \leq 4$ non-slotted Aloha with the averaged SINR provides 50% more throughput than with the maximum SINR. For $\beta \geq 5$ non-slotted Aloha with the averaged SINR provides only around 35% more throughput than with the maximum SINR. When we compare slotted Aloha with non-slotted Aloha with maximum SINR, we find that slotted Aloha offers 66% more throughput for $\beta = 3$ and 100% for $\beta = 6$.

VI. Conclusion

We have developed two stochastic models to analyze non-slotted Aloha in SINR based scenarios. If we consider Rayleigh fading, a power-law signal-power decay and the interference to be averaged over the duration of the transmission slot, our two models lead to closed formulas for the probability of capture and the density of throughput.

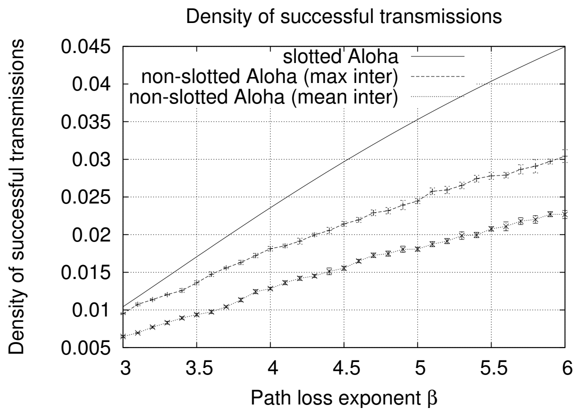

Fig. 5. Density of successful transmissions versus path loss exponent β. Slotted Aloha and non-slotted Aloha (mean and maximal interference constraint) are tuned to maximize the density of successful transmissions.

The formula of the Poisson rain model can be very simply used to provide straightforward results whereas the Poison-renewal model requires more computational effort. The two models also provide very close results, which are confirmed by simulations. The analysis can also be extended for a general fading F using the Plancherel-Parseval theorem.

Our two models for non-slotted Aloha allow extensive performance comparisons with slotted Aloha. We compare non-slotted and slotted Aloha both for a given value of transmission attempts as well as when the two models are optimized. Slotted Aloha does indeed offer better performances than non-slotted Aloha but for realistic path-loss assumptions the ratio is far smaller than the "well-known" factor 2 obtained for Aloha in the wired model,

Using the simulation results we have studied non-slotted Aloha when the maximum value rather than the average value of the interference is considered in the SINR constraint. This change introduces a significant loss in the density of throughput but this loss is greatly reduced when the density of successful transmissions is optimized.

APPENDIX

In this section we prove Propositions 3.3 and 3.2, and show some extensions of these results.

A. General Approach

We begin with the simple observation that the successful transmission probability p_c can be expressed in all cases considered in this paper (slotted Aloha, Poisson-renewal and Poisson rain, both with maximum or average interference constraint) in therms of the following *independent* random variables

$$p_c = \mathbf{P}\{\, F \geq Tl(r)(I+W)\,\}$$

where F is the generic fading random variable, W is the external noise and I is the appropriate interference (maximum or averaged during the reception of the given packet in the non-slotted case; cf. Section III-B2). Thus, in the case of Rayleigh fading, we have.

Fact A.1: *Assume exponential F with mean $1/\mu$ (Rayleigh fading). Then*

$$p_c = \mathbf{E}[e^{-\mu Tl(r)(I+W)}] = \mathcal{L}_W(\mu Tl(r))\mathcal{L}_I(\mu Tl(r))\,,$$

where $\mathcal{L}_I(\xi)$ is the Laplace transform of the interference in the respective model.

The above formula was used for the first time in [5] in the case of slotted Aloha. The following result proved in [6] (for slotted Aloha) allows the analysis to be extended to the case of a general fading F. It follows from the Plancherel-Parseval theorem (see e.g. [9, Th. C3.3, p.157]).

Fact A.2: *Assume that*

- *F has a finite first moment and admits a square integrable density;*
- *Either I or W admit a density which is square integrable* [3]

Then the probability of a successful transmission is equal to

$$p_c = \int_{-\infty}^{\infty} \mathcal{L}_I(2i\pi l(r)Ts)\,\mathcal{L}_W(2i\pi l(r)Ts)\,\frac{\mathcal{L}_F(-2i\pi s)-1}{2i\pi s}\mathrm{d}s\,,$$

where $\mathcal{F}(\xi) = \mathbf{E}[e^{-\xi F}]$ is the Laplace transform of F.

The results of Propositions 3.3 and 3.2 follow now from Fact A.1 and the particular form of the Laplace transform of the corresponding *averaged interference* $I = I^{mean}$ of the typical user transmission in the Poisson-renewal and Poisson rain model. We are unable to give an analytical expression for the successful transmission in either of the non-slotted Aloha models under the *maximal interference constraint* due to the fact that we are not aware of any explicit representation of the Laplace transform of the maximal interference in the considered models.

In what follows we develop expressions for the Laplace transforms of I^{mean}. We use the following result giving an explicit formula for the Laplace transform of the *generic shot-noise* $J = \sum_{Y_i \in \Pi} f(G_i, Y_i)$ generated by some homogeneous Poisson p.p. with intensity α, the *response function* $f(\cdot,\cdot)$ and i.i.d. (possibly multi-dimensional) marks G_i distributed as a generic r.v. G. It can be derived from the formula for the Laplace functional of the Poisson p.p. (see e.g. [10]).

Fact A.3: *Consider the shot-noise random variable defined above. Then*

$$\mathcal{L}_J(s) = \mathbf{E}[e^{-sJ}] = \exp\Big\{-\alpha\int(1-\mathbf{E}[\exp\{-sf(G,y)\}])\,\mathrm{d}y\Big\}\,, \tag{A.1}$$

where the integral is evaluated over the whole state space of on which P.p.p. Π lives and the expectation $\mathbf{E}$ in the exponent is taken with respect to the distribution of the generic mark G.

B. Interference in the Poisson Rain Model

We begin with the simpler — Poisson-rain case. Recall that, in this case, we have

[3] The square integrability of the density of a given random variable (in particular of I) is equivalent to the square integrability of its Fourier transform (in particular to the integrability of $|\mathcal{L}_I(is)|^2$ in the case of I); see [11, p.510].

$$I^{mean} = 1/B \int_0^B \sum_{X_j \in \Psi^1(t)\, j\neq 0} F_j^0 / l(|X_j - y_i|)\, \mathrm{d}t$$

considered under the Palm probability $\mathbf{P}^{0,0}$ of the space-time P.p.p. Ψ given a point $X_0 = 0, T_0 = 0$. By the stationarity of $\Psi \setminus \{X_0\}$ under $\mathbf{P}^{0,0}$ we can replace y_i by 0 in the above formula. Moreover using the representation $\Psi^1(t) = \{X_j : 1 = e_j(t) = \mathbb{I}(T_j \leq t < T_j + B)\}$ for nodes that emit at time t, changing the order of integration and summation we obtain the following representation for (the distribution of) I^{mean}

$$I^{mean} =_{distr.} \sum_{(X_j,T_j)\in\Psi} F_j h(T_j)/l(|X_j|)\,, \tag{A.2}$$

where Ψ is the stationary space-time P.p.p. of the Poisson rain model, F_j are i.i.d. copies of the fading variable F and

$$h(s) = \int_0^B \frac{\mathbb{I}(s \leq t < s + B)}{B}\, \mathrm{d}t = \frac{(B - |s|)^+}{B}\,, \tag{A.3}$$

where $a^+ = \max(0, a)$. Note that the random variable on the right-hand-side of (A.2) is an example of the shot-noise random variable J, with respect to the P.p.p. Ψ on $\mathbb{R}^2 \times \mathbb{R}$ with the response function $f(F,(x,t)) = F\, h(t)/l(|x|)$. Using Fact A.3 we can obtain an explicit formula for the Laplace transform of I^{mean} in this case. For the sake of simplicity, below we only give it below for the case of Rayleigh fading.

Fact A.4: *The Laplace transform of the averaged interference during the packet reception of the typical packet in the Poisson rain model with the Rayleigh fading (exponential F with mean $1/\mu$) is equal to*

$$\mathcal{L}_{I^{mean}}(\xi) = \exp\Big\{-4\pi\lambda_s B \int_0^\infty u\Big(1 - \frac{\mu l(u)}{\xi}\log(1 + \frac{\xi}{\mu l(u)})\Big)\,\mathrm{d}u\Big\}\,.$$

C. Interference in the Poisson-Renewal Model

As for the above demonstration, we consider the transmission at $X_0 = 0$ starting at time $T_0 = 0$ under the $\mathbf{P}^{0,T_0(0)=0}$. Each node X_j can send at most two packets interfering with the given transmission. To identify them let us denote by n_j^* the unique integer such that $T_j(n_j^*) \leq 0 < T_j(n_j^*+1)$. To simplify the notation denote $R_j = T_j(n_j^*)$ and $S_j = T_j(n_j^* + 1)$. These are the arrival times of the two (potentially) interfering packets transmitted by X_j. We have for the distribution of the mean interference I^{mean} at node $X_0 = 0$:

$$I^{mean} =_{distr.} \sum_{X_j\in\Phi} F_j h(R_j)/l(|X_j|) + F_j' h(S_j)/l(|X_j|)\,, \tag{A.4}$$

where $(F_j, F_j' : j)$ are independent copies of the fading variable F, independent of (R_j, S_j). Using Fact A.3 we obtain the Laplace transform of the above shot-noise variable:

$$\mathcal{L}_{I^{mean}}(\xi) = \exp\Big\{-\lambda \int_{\mathbb{R}^2} \Big(1 - \mathbf{E}^{0,T_0(0)=0}\Big[\frac{\mu}{\mu + \xi\frac{h(R)}{l(|x|)}} \times \frac{\mu}{\mu + \xi\frac{h(S)}{l(|x|)}}\Big]\Big)\mathrm{d}x\Big\},$$

where the expectation is with respect to (R, S) — a generic copy for (R_j, S_j) (the expectation with respect to the exponentially distributed fading variables F, F' has already been taken into account in the formula).

According to renewal theory (see e.g. [4, eq. 1.4.3]), the joint distribution of (R, S) is given by $\mathbf{P}\{-R + S \leq B + a\} = \int_0^a \frac{(B+s)\epsilon}{B+1/\epsilon} e^{-\epsilon s}\mathrm{d}s$ for $a \geq 0$. From this formula we derive first the marginal law of R that is with probability $\frac{\epsilon B}{1+\epsilon B}$ uniformly distributed on $[-B, 0]$ and with probability $\frac{1}{1+\epsilon B}$ equal to $-(B + e_\epsilon)$ where e_ϵ is an exponentially distributed random variable of rate ϵ. Next, the conditional distribution of S given R can be derived: if $R \geq -B$ then $S = B + R + e_\epsilon$ and $R = e_\epsilon$ otherwise. Using these distributions, the required expectation can easily computed:

$$\mathcal{L}_{I^{mean}}(\mu T l(r)) = \exp\Big\{-\lambda \int_{x\in\mathbb{R}^2} \Big(1 - \frac{\epsilon B}{1+\epsilon B} \times \frac{1}{B}\int_0^B \frac{1}{1 + \frac{(B-t)Tl(r)}{Bl(|x|)}} \int_0^\infty \frac{\epsilon e^{-\epsilon s}}{1 + \frac{(t-s)^+ Tl(r)}{Bl(|x|)}}\mathrm{d}s\mathrm{d}t - \frac{1}{1+\epsilon B}\int_0^\infty \frac{\epsilon e^{-\epsilon s}}{1 + \frac{(B-s)^+ Tl(r)}{Bl(|x|)}}\mathrm{d}s\Big)\mathrm{d}x\Big\}\,.$$

After some simplifications and a change in polar variables this gives the result announced in Proposition 3.2.

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