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# An urban multi-scale modeling using fuzzy evaluation of accessibility and morphological constraints

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## **Abstract**

Increasing mobility has contributed since a couple of years to favor urban sprawl. The negative impacts of urban sprawl are well known, in particular the increase of traffic flows and vulnerability of natural resources. Addressing those questions we propose a spatial decision support system based on a multi-scale approach. On the one hand, fractal geometry is used to determine where new urban developments are possible. Here, the central assumption is the fractal nature of urban growth allowing access to various types of amenities: central amenities (retailing, cultural offer...) and peripheral amenities (open landscape). On the other hand, the accessibility to retail centers offering different amenities is evaluated. Synthetic indicators are proposed taking into account the offer of each retail center as well as the distance to residential areas. The formalization is based on fuzzy sets theory. The fractal modeling and the indicators of accessibility are both integrated in a decision support system by means of a set of rules. The outputs of the system are cartographic representations at several scales where the most interesting locations for new urban developments are identified.

## **Key-words**

Multi-scale modeling, planning support system, urban developments, fractal, accessibility

## **1. Introduction**

Managing urban sprawl is a major concern for urban planning. Indeed its negative effects on the environment are of great significance: air pollution, noise, destruction of natural resources... On a socio-economic point of view, the negative effects are mainly the increase of costs for housing and travel, and their consequences (social segregation and social inequity). Considering the case of European countries, the management of urban sprawl can be expressed through four main stakes. The first is to limit development of diffuse built-up patterns that are too sparse and too far away from centers to allow the creation of efficient public transport systems and to ensure a good accessibility to various amenities (shops, working places, leisure places...). The second stake is to limit fragmentation of urban patterns in order to preserve (even develop) biodiversity, to avoid isolated buildings in areas characterized by a high quality of their natural landscape, and to maintain agriculture in suburban areas. The third stake is to increase (at least preserve) the diversity of urban forms to avoid landscapes standardization and to favor social diversity. The fourth stake is to preserve penetration of green alleys into the built-up areas, in order to ensure a good ventilation of dense central areas as well as a good accessibility to proximity places for leisure and recreation.

Hence managing urban sprawl supposes to take into account numerous conflicting phenomena that occur at several interacting scales. Considering such complexity, spatial decision support systems (SDSS) or planning support systems (PPS) can help urban planners and designers to achieve their task. A DSS can be defined as a computer program that assists individuals or groups of individuals

in their decision process, supports rather than replaces judgements of individuals, and improves the effectiveness rather than the efficiency of a decision process (Uran & Janssen, 2003). A SDSS is used to support decision processes where the spatial aspect of a problem plays a decisive role (Uran & Janssen, 2003). SDSS are close to PSS. But PSS specifically support the whole of or some part of a unique professional planning task whereas SDSS can be regarded as systems designed specifically to support a decision research process for complex spatial problems (Geertman & Stillwell, 2004).

In this paper, we focus on one question related to the field of urban planning and design that is: where are the most relevant locations for new urban developments? More precisely, we aim at helping planners to select locations for residential developments with respect to four objectives:

- good accessibility to various retail and service amenities;
- good accessibility to various open spaces (small squares, parks, forest...);
- limit fragmentation of non built-up spaces;
- reduce space consumption.

To deal with these objectives, several approaches are interesting. Location-allocation models could give relevant answers to the first objective. Multi-criteria evaluation methods are well appropriate to find relevant locations considering accessibility constraints (Arentze & Timmermans, 2000). Spatial interaction/choice model (SIM) can be used to specify scenarios in terms of planned or anticipated developments (e.g. opening a new facility, population forecasts) and the system gives feedback in terms of impacts on criterion variables (e.g. travel demands) (Arentze & Timmermans, 2000). Constrained cellular automata can be used for urban scenario generation, when only the final state is analysed but not the trajectory of the system in course of time (White et al., 1997) . Sometimes researchers have combined several of these approaches in PSS or SDSS. For instance, Arentze et al. (2006) have developed a method that combines location-allocation models and land-use models for land-use plan generation. Saarlös et al. (2005) have also proposed a multi-agent model for alternative plan generation. To our knowledge, however, none of the existing PSS or SDSS answers all of the four stakes exposed above.

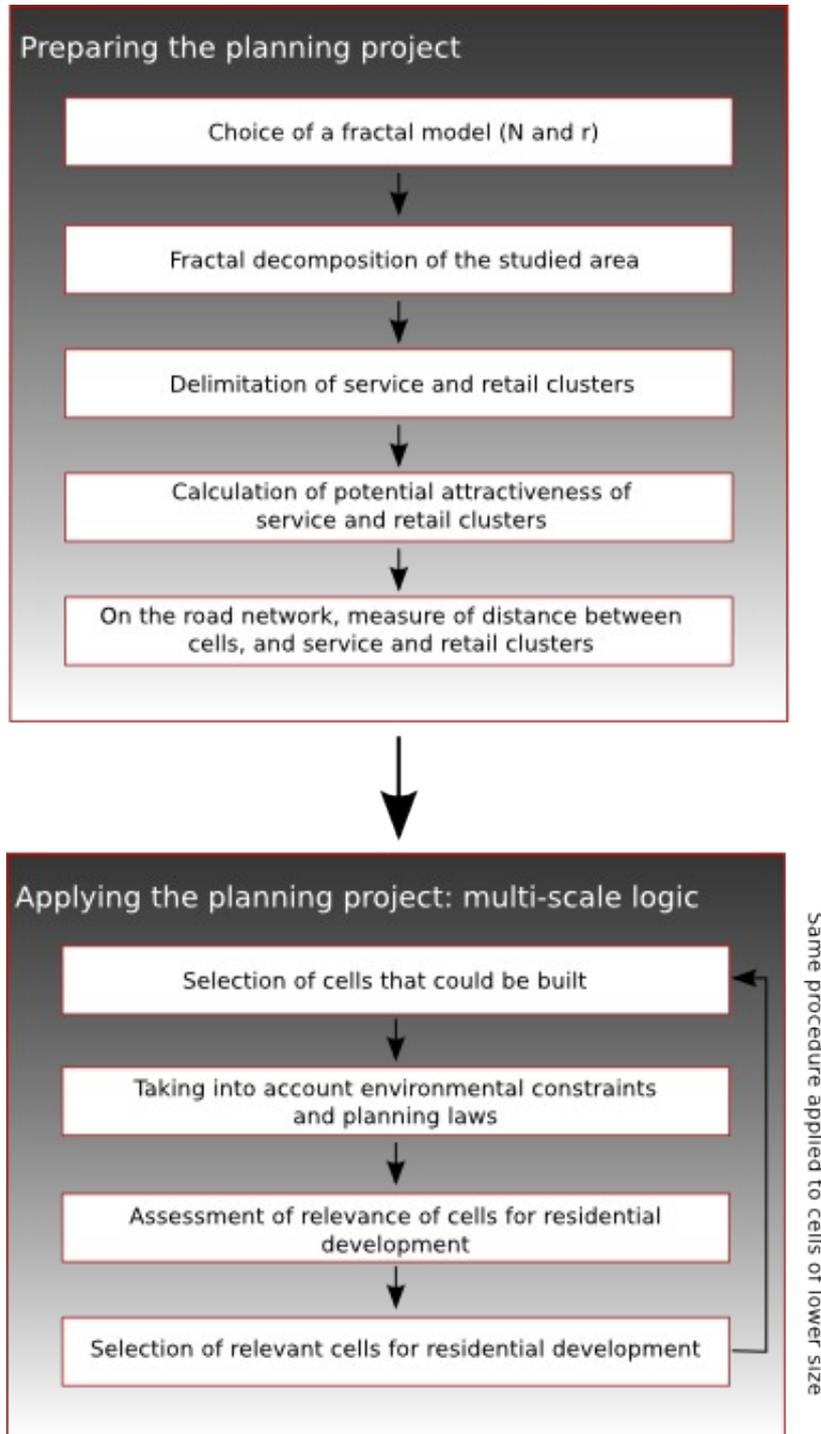
We chose to develop a multi-scale SDSS for urban planning and design that includes fractal urbanization rules and accessibility constraints. The modeling concept has been developed in the framework of the research program PREDIT 3 of the French Ministry of Sustainable Development (Frankhauser et al., 2007; Frankhauser et al., 2008). This concept is based on a multi-scale approach referring to fractal geometry. The name of the SDSS is MUP-City: multi-scale urban planning for a sustainable city. Indeed, better address the multi-scale characteristics of land use systems is a real challenge in urban modeling (Verburg et al., 2004). A multi-scale approach allows to take into account the multi-scalar structure of the phenomena studied (European Spatial Planning Observation Network, 2006). In MUP-City, fractal urbanization rules and the multi-scale modeling introduce morphological constraints whereas accessibility constraints are used for modeling behaviors of people. Thus the system takes into account the two aspects of the urban dynamics: form and processes.

## **2. Objectives of the spatial decision support system MUP-City**

*The first objective is to minimize the number and the length of trips by car while ensuring a good accessibility to various amenities (urban and rural).* Considering that little modifications of the urban structure can lead to strong modifications of the urban functioning (Batty, 2001), our aim is to identify relevant locations for new urban developments that lead to a decrease of the global number

of trips (and distances traveled, especially by car), which are required to join the different spatial components of the urban fabric. Thus, we want to act on the urban form to influence the urban processes: reduce car reliance and promote cycling, walking and transit use. Urban amenities are retail and service centers of different orders; rural amenities are open spaces of different sizes and functions (small squares, parks, periurban forests...) (Cavailhès et al., 2004).

Figure 1. Process of an analysis with MUP-City



*The second objective is to reduce space consumption while satisfying the housing demand (qualitatively and quantitatively).* The idea is that space could be “better consumed” which means that urban growth should be rather canalized than forbidden. Even if the model of compact city is often put forward, increasing density seems not to be a sovereign remedy for negative effects of urban sprawl: besides the increase of cost of land and estate, people are rarely satisfied by an increase of density (Fouchier, 1999). Moreover, several studies, in particular (Garcia & Riera, 2003), have showed that people prefer small individual open spaces located near their housing than bigger but more distant open spaces. Hence, the objective is to propose alternative urban models that allow to reduce space consumption without imposing to increase density.

*The third objective is to avoid fragmentation of built-up areas and open spaces.* Answering this objective will in turn protect the ecological environments, maintain agricultural activities in the urban peripheries, preserve the landscapes quality, and allow to develop profitable and efficient public transport systems. Obviously, urbanization rules that avoid fragmentation of built-up patterns avoid in turn fragmentation of non built-up patterns.

### 3. Methodology

#### 3.1 Preparing the planning project (figure 1)

##### *Step 1: Choice of a fractal model for urbanization*

The choice of introducing fractal urbanization rules in MUP-City follows from two hypotheses. The main hypothesis is that fractal organization of an urban pattern allows a good accessibility to various urban amenities, offered by the central city and the surrounding secondary urban centers, and rural amenities (several types of open spaces) (Cavailhès et al., 2004), while minimizing spatial fragmentation (Frankhauser, 2000). The second hypothesis is that some urban fractal models minimize space consumption without necessarily increase density: in a fractal form, built-up mass can be locally concentrated (Thomas et al., 2007).

We consider a relation between  $N$  (number of elements) and  $\varepsilon$  (scale of analysis) corresponding to the fractal law (Mandelbrot, 1982):

$$\log(N) = D \cdot \log(\varepsilon)$$

Fractal dimension describes some properties of built-up patterns: non-uniformity through the scales, fragmentation, morphological connectivity.... (De Keersmaecker et al., 2003; Thomas et al., 2007). Hence, a fractal dimension of built-up surface close to 2 characterizes a uniform pattern: buildings are organized following a one-scale logic; there is no local concentration of built-up mass. A fractal dimension of built-up surface comprised between 2 and 1 corresponds to a mix of connected elements forming large clusters, connected elements forming small clusters, and isolated elements. A fractal dimension of surface lower than 1 describes a pattern made up of unconnected elements (a high number of built-up clusters separated one from another) (Thomas et al., 2008).

Because fractal dimension summarizes the statistical self-similarity of a built-up pattern, it can be considered as an urban model. We chose to introduce it in the system by means of two variables:

- $N$  that is the number of cells in each mesh;
- $r$  that is the reduction factor between a higher size of mesh and a lower one.

In other words, the choice of a fractal model corresponds to the choice of values for  $N$  and  $r$ . For instance, we consider the case of a built-up pattern characterized by a fractal dimension of 1.46, which corresponds to  $N = 5$  for  $r = 1/3$ . The spatial structure of this pattern is multi-scale. Without

loosing the multi-scale properties of the pattern, the fractal dimension could be increase to 1.6, which corresponds to  $N = 6$  for a reduction factor  $r = 1/3$ . In any case, however, the fractal dimension should not go over 1.8 in order to avoid uniformity of the built-up pattern through the scales (Frankhauser, 2004).

### *Step 2: Multi-scale decomposition of the study area*

The study area is covered by a regular grid, the size of which is reduced from one analysis level to another through the application of a constant reduction factor (Figure 2a). Each mesh of the grid contains a fixed number of cells. At the first stage of the decomposition, the size of meshes is  $l_1$  (bigger size of mesh). In each mesh, built-up cells are identified and counted. At the second stage of the decomposition, each mesh of size  $l_1$  is decomposed into meshes of size  $l_2$ , which correspond in fact to the cells of the first decomposition stage. As previously, built-up cells in each mesh are identified and counted. The same procedure is applied to meshes of lower size until the size of meshes comes near the size of buildings.

The number of analysis levels varies according to the size of the study area and to the reduction factor from a higher analysis level to a lower one. To give an example, for an area of size equal to 4860 m. and a reduction factor equal to 1/3, the system considers five analysis levels. The corresponding sizes of cells are:

- first analysis level: 1620 m.
- second analysis level: 540 m.
- third analysis level: 180 m.
- fourth analysis level: 60 m.
- fifth analysis level: 20 m.

When a cell of size  $l_2$  is not built, cells of size  $l_3$  belonging to the corresponding mesh of size  $l_2$  can not be built. Hence, the multi-scale decomposition of the built-up pattern allows the addition of a spatial component to the fractal model defined in step 1. On the one hand, the multi-scale decomposition determines dramatically results that will be obtained when applying a planning project. On the other hand, it allows to take into account crucial aspects of the multi-scale organization of a city, in particular the multi-scale nest of open spaces.

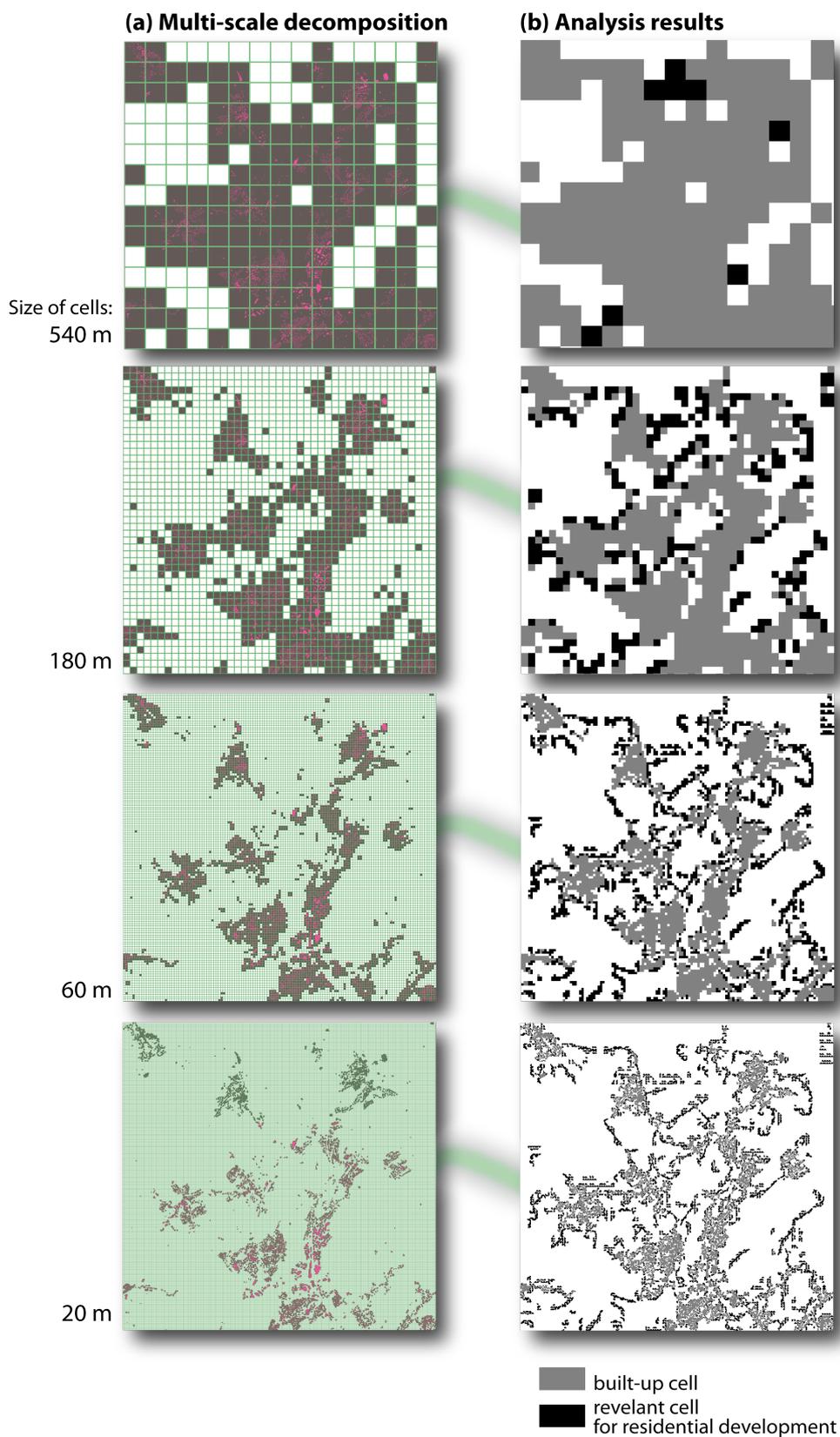
### *Step 3: Delimitation of service and retail clusters*

Two types of service and retail clusters and thirteen types of service and retail outlets are taken into account. They have been distinguished according to their frequency of recourse.

- First order clusters are characterized by a daily or almost daily frequenting. They can comprise five types of outlets, which are mainly convenience stores: butcher/caterer/delicatessen, baker, school, tobacconist/newsagent, supermarket-hypermarket.
- Second order clusters are characterized by a weekly frequency of recourse. They can comprise seven types of outlets: garage, supermarket-hypermarket., surgery, grocery, pharmacy, post office, cafe/bar.

A service and retail cluster can be simultaneously of first and second order. It comprises either only one outlet or more than one belonging to the same morphological set. Obviously, the choice of those types of retail outlets as well as the definition of their frequency of recourse corresponds to the French periurban context. It may be necessary to modify this for the application of the system to another context.

Figure 2. Examples of outputs given by MUP-City



*Step 4: Calculation of potential attractiveness of service and retail clusters*

The potential attractiveness of a retail cluster results from a fuzzy valuation, which depends on the number of outlets in the cluster and their diversity.

*Step 5: On the road network, measure of distance between cells and service and retail clusters*

The distance between each free cell and each retail cluster is measured for every size of cell. Distance is simply the shortest path on the network.

**3.2 Applying the planning project following a multi-scale logic** (figure 1)

The application begins at the higher analysis level that corresponds to the bigger size of mesh ( $l_i$ ).

*Step 1: Selection of meshes that could be built*

At this step, the system identifies meshes in which cells could be built with respect to the chosen fractal model. Meshes are selected if they already contain buildings and if they contain less built-up cells than allowed by the fractal model.

*Step 2 : Taking into account environmental constraints and planning laws*

Cells that could be built after step one are kept in this state only if:

- their suitability for residential use is good enough (for example, slope must not be too steep; soil must not be too wet...);
- planning rules and laws allow developing the cells for residential use.

*Step 3: Assessment of relevance of selected cells for residential development*

The system comprises four accessibility rules for assessing relevance of cells. Each rule consists in the combination of assessment criteria. Valuation of each criterion is described by a fuzzy variable; combination of criteria in a rule is done by means of fuzzy aggregation operators (Yager, 1978; Zimmermann, 1987; Zimmermann & Zysno, 1983). The recourse to fuzzy set theory is especially interesting because it provides a way to handle imprecise information in a well-defined and expressive mathematical framework (Zadeh, 1965; Zadeh, 1980).

Table 1. Assessment rules

	<b>Rules for assessing relevance of cell for future residential developments</b>	<b>Corresponding assessment criteria</b>
1.	Assessed cell must be close to a built-up cell. Building the assessed cell must not hamper the access to open spaces for neighboring built-up cells.	Fuzzy valuation of the number of non built-up cells around each built-up cell directly contiguous to the assessed cell (3.3 Moore neighborhood).
2.	Assessed cell must be crossed by or near to a transportation axis.	Fuzzy valuation of the distance to an existing transportation axis.
3.	Distance between assessed cell and the nearer service and retail cluster of first order must not exceed 600 m.	Fuzzy valuation of the distance, which takes into account the potential attractiveness of all surrounding service and retail clusters of first order.
4.	Distance between assessed cell and the nearer service and retail cluster of second order must not exceed the range of theoretical market area of all the service and retail outlets of second order.	Fuzzy valuation of the distance, which takes into account the potential attractiveness of surrounding service and retail clusters of second order

When all assessment criteria are completely satisfying, the relevance of a cell for residential use is equal to one.

*Step 4: Selection of the most relevant cells for residential development*

For the moment, the system selects cells characterized by the highest assessment value. However, for a planning purpose, it would be more interesting that the user of the system selects himself or herself the most relevant cells with respect to his or her knowledge of the local situation and his or her planning objectives.

*Step 5: The four previous steps are applied to smaller meshes ( $l_2, l_3, l_4...$ )*

#### **4. Data, materials**

The system requires three types of GIS data (ArcGIS shapefiles):

- detailed road network (lines);
- buildings (polygons);
- location of retail and service outlets described by few simple attributes, in particular the type of outlets (points).

It may seem that the system has high spatial data requirements. However, even if spatial data considered are precise, it is easy to obtain them at least in the French context. Indeed, most of the French urban planning agencies have very precise databases from the French National Geographic Institute, which describe buildings and networks (BD Topo and BD Adresse). Data describing retail and service activities are downloadable on line free of charge (e.g. data file SIRENE).

#### **5. Examples of results**

One series of results is presented in this paper (Figure 2b). The parameters of the analysis have been chosen as follows.

Definition of the planning project:

- Step 1: The fractal model chosen is  $N = 5$  and  $r = 1/3$ .
- Step 2: The smaller size for a cell is 20 m.
- Step 3: Service and retail outlets belong to the same cluster if they are separated by less than 200 m.
- Step 4: Calculation of the potential attractiveness of clusters (*i.e.* fuzzy variables for evaluating criteria and choice of aggregation operators) has been defined considering the French periurban context.

Application of the planning project:

- Step 2: No environmental constraint or planning law has been taken into account.
- Step 3: Parameters of the accessibility rules have been estimated considering French periurban areas. For instance, the system considers that people do not walk more than 600m. to go to the convenience store. The parameters should be revised if applying the system to different contexts.
- Step 3: Results given by the four assessment rules have been aggregated using the arithmetic mean.
- Step 4: Cells characterized by the higher assessment value are considered to be the most relevant for residential use.

Results presented on figure 2b are the locations of the most relevant cells for urban residential development. As one might expect, the number of relevant cells becomes higher when the size of the mesh becomes smaller. For the smaller size of cell (20 m.), built-up patterns generated by the model are quite fingering; in the same way open spaces are nested and connected. When cells are bigger, however, open spaces are often disconnected. These observations are arguments for a multi-scale approach in urban planning.

## **6. Discussion: does the proposed SDSS answer the objectives exposed in section 2?**

At the moment, the properties of the spatial patterns generated by MUP-City have not yet been assessed. Hence, we can only answer to the question considering the structure of the model and not its results.

*First objective is to minimize the number and the length of trips by car while ensuring a good accessibility to various urban and rural amenities.* Answers given by MUP-City are as following:

- Assessment rule 1 defines accessibility constraints to open spaces (rural amenities).
- Assessment rules 3 and 4 define accessibility constraints to retail and service centers of two orders (urban amenities).
- Due to its multi-scale nature, the model tends to increase the length of the urban border and, consequently, the accessibility to open spaces. At least, it preserves the current number of contacts between built-up and non built-up cells.
- The multi-scale modeling allows to consider different sizes of open spaces, that can be considered as quite good indicators of their functions.

*Second objective is to reduce space consumption while satisfying the housing demand (qualitatively and quantitatively).* Answers given by MUP-City are as following:

- Assessment rule 2 aims at reducing space consumption resulting from roads construction.
- Assessment rule 1 combined with the fractal model increases the heterogeneousness of urban forms through the scales. It avoids the development of large areas characterized by uniform housings (that could lead to the homogenization of social patterns of population, and then to segregation processes).

*Third objective is to avoid fragmentation of built-up areas and open spaces.* Answers given by MUP-City are as following:

- Assessment rule 1 allows to avoid fragmentation of built-up areas. In turn, fragmentation of open spaces (natural or agricultural) is also avoided.
- The fractal logic of urban growth allows to preserve (even develop) penetration of green alleys into built-up areas.

## **7. Conclusion**

MUP-City identifies and assesses potential places for future residential developments considering two objectives: 1) fractal aspect of the urban growth and 2) proximity to services and retailing, to open spaces and to existing roads. However, it does not tell where urban developments have to be.

Moreover it deals only with some aspects of a planning project. In particular, it does not consider the housing market, the accessibility to work places, the relevance of places with respect to risks... MUP-City is also not an impact model: it does not assess the impact of the selected new urban developments in terms of induced trips for commuting or purchasing, energy consumption, traffic congestion, housing prices... Hence, it should be applied jointly with analysis of environmental impacts of land use plans (Geneletti et al., 2007).

First results obtained with MUP-City are promising. Above all, the architecture of the system allows to answer the objectives for which it has been conceived. The next step of research is to go further in the assessment of results that can be obtained. We will more particularly consider two points. The first point will be to perform a series of tests in order to find criteria for optimising the positioning of the spatial decomposition grid. The second point will be the *ex-ante* and *ex-post* assessment of analysis results. This will consist in the calculation of spatial indexes (e.g. fragmentation index, fractal measures...) and accessibility measures before and after the application of a planning project with MUP-City. Thus, it would be possible to assess the performance of the system considering the improvement of accessibilities and the minimization of spatial fragmentation.

## References

- Arentze, T., Borgers, A., Timmermans, H. (2006). A heuristic method for land-use plan generation in planning support systems. In: *Progress in design & decision support systems in architecture and urban planning*. Eindhoven, NL.
- Arentze, T., Timmermans, H. (2000). A spatial decision support system for retail plan generation and impact assessment. *Transportation Research Part C: Emerging Technologies*, 8(1-6), 361-380.
- Batty, M. (2001). Polynucleated urban landscapes. *Urban Studies*, 38(4), 635–655.
- Cavallès, J., Frankhauser, P., Peeters, D., Thomas, I. (2004). Where Alonso meets Sierpinski: an urban economic model of fractal metropolitan area. *Environment and Planning A*, 36, 1471-1498.
- De Keersmaecker, M., Frankhauser, P., Thomas, I. (2003). Using fractal dimensions for characterizing intra-urban diversity. the example of Brussels. *Geographical Analysis*, 35, 310-328.
- European Spatial Planning Observation Network (2006). *The modifiable areal unit problem*. ESPON Scientific Support Project 3.4.3 - Lead Partner: University Paris 7, UMS 2414 RIATE, CNRS, DATAR (France). [http://www.espon.eu/mmp/online/website/content/projects/261/431/index\\_EN.html](http://www.espon.eu/mmp/online/website/content/projects/261/431/index_EN.html) (access: February 2008).
- Fouchier, V. (1999). Maîtriser l'étalement urbain : une première évaluation des politiques menées dans quatre pays (Angleterre, Norvège, Pays-Bas, Hong-Kong). *2001 Plus (Synthèse et Recherches)*, n°49, DRAST - Ministère français de l'équipement, des transports et du logement.
- Frankhauser, P. (2000). La fragmentation des espaces urbains et périurbains : une approche fractale. In: P.H. Derycke (Ed.), *Structure des villes, entreprises et marchés urbains* (pp. 25-50). Paris: Harmattan.
- Frankhauser, P. (2004). Comparing the morphology of urban patterns in Europe: a fractal approach. In: A. Borsdorf & P. Zembri (Eds.), *European cities - insights on outskirts* (pp. 79-105). Brussels.
- Frankhauser, P., Tannier, C. & Vuidel, G. Houot, H. (2007). Approche fractale de l'urbanisation - méthodes d'analyse d'accessibilité et simulations multi-échelles. In *11th world conference on transportation research*. Berkeley, USA.
- Frankhauser, P., Tannier, C. & Vuidel, G. Houot, H. (2008). A multi-scale approach of accessibility for managing urban sprawl. In *Mobil.tum 2008 – International conference on mobility and transport*. Munich, Germany.
- Garcia, D., Riera, P. (2003). Expansion versus density in Barcelona: a valuation exercise. *Urban Studies*, 40(10), 1925-1936.
- Geertman, S., Stillwell, J. (2004). Planning support systems: an inventory of current practice. *Computers, Environment and Urban Systems*, 28, 291–310.
- Geneletti, D., Bagli, S., Napolitano, P., Pistocchi, A. (2007). Spatial decision support for strategic environmental assessment of land use plans. a case study in southern Italy. *Environmental Impact Assessment Review*, 27, 408 – 423.
- Mandelbrot, B. (1982). *The fractal geometry of nature*. San Francisco: Freeman.

- Saarlos, D., Arentze, T., Borgers, A., Timmermans, H. (2005). A multiagent model for alternative plan generation. *Environment and Planning B: Planning and Design*, 32, 505-522.
- Thomas, I., Frankhauser, P., De Keersmaecker, M. (2007). Fractal dimension versus density of the built-up surfaces in the periphery of Brussels. *Papers in Regional Science*, 86(2), 287-307.
- Thomas, I., Tannier, C., Frankhauser, P. (2008). Is there a link between fractal dimension and residential environment at a regional level?. *Cybergeo: European Journal of Geography*, 413. <http://www.cybergeo.eu/>
- Uran, O., Janssen, R. (2003). Why are spatial decision support systems not used? Some experiences from the Netherlands. *Computers, Environment and Urban Systems*, 27, 511–526.
- Verburg, P., Schot, P., Dijst, M., Veldkamp, A. (2004). Land use change modelling: current practice and research priorities. *GeoJournal*, 61, 309–324.
- White, R., Engelen, G., Uljee, I. (1997). The use of constrained cellular automata for high-resolution modelling of urban land use dynamics. *Environment and Planning B*, 24, 235-246.
- Yager, R. R. (1978). Fuzzy decision making including unequal objectives. *Fuzzy sets and systems*, 1, 87-95.
- Zadeh, L. A. (1965). Fuzzy sets. *Information and Control*, 8(3), 338-353.
- Zadeh, L. A. (1980). *Citation Classics Commentaries*. <http://www.garfield.library.upenn.edu/classics.html> (access: May 2008).
- Zimmermann, H. (1987). *Fuzzy sets, decision making and expert systems*. Boston [u.a.]: Kluwer Academic Publishers.
- Zimmermann, H., Zysno P. (1983). Decisions and evaluations by hierarchical aggregation of information. *Fuzzy sets and systems*, 10, 243-260.