Fuzzy spatial relations for 2D scene

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FUZZY SPATIAL RELATIONS FOR 2D SCENE

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Abstract— *Different models for computing the spatial relations have been developed in the last decade. Separate methods are used for computing topological, directional and distance relations. These models have also been extended in fuzzy domains based on fuzzy or vague objects and less attention has been paid to fuzzy relations. In this paper 8 fuzzy topological relations are computed with the help of 1D Allen relations and directional relations are evaluated by using fuzzy membership functions. A matrix of fuzzy relations is developed which provides complete set of information about a 2D scene. This method deals with two types of fuzziness (i) fuzziness involved in the objects position (ii) fuzziness about directional relations.*

Keywords: Fuzzy objects, Fuzzy relations, Fuzzy topological relations, 1D Allen relations, Matrix of fuzzy relations.

1. Introduction

The space can be studied through spatial relations between different regions included in it. These spatial relations include topological, directional and distance relations. There is an increasing interest in formalisms that describe properties of space in a qualitative way. Usually such a qualitative representation takes the form of topological relations between regions, there are two famous models for finding topological relations between spatial regions. 9-intersection model [1], [2], [3] and Region Connected Calculus ($RCC8$) [4]. Both 9-intersection and RCC models have been extended to the fuzzy domains but each time models are extended for fuzzy objects [5], [6], [7], [8], [9] while in most of the qualitative directional relations models objects are taken as points. The 9-intersection model for directional relations [10] consider the objects by their minimum bounding rectangle (MBR) and then 2D projections are used. To capture internal position of an object internal cardinal directional relations (ICD) model were introduced [11], [12]. All of these models describe the object properties only in one domain and completely ignore the other aspects of the space like 9-intersections for topological relations and $RCC8$ models describe well the topological properties of space and ignore the directional and distance relations meanwhile 9-intersection model for directional relations and internal directional relations models describe the objects regarding their global directional contents and ignore the topological properties, as a result one single model can not be used to perfectly describe the object in embedding space. Fuzzy reasoning about spatial relationship can be viewed fuzzy description of object's relative position. In qualitative domain mostly topological relations are studied between vague or fuzzy objects such as in $RCC8$ and 9-intersection models for topological relations but less attention has been paid to study the fuzzy spatial relations between crisp objects.

Different approaches for directional relations were adopted such as mathematical morphology [13], [14], quantitative approaches [15], called angle histogram and force histograms [16], [17] where evaluation approach for directional relations was fuzzy and deals only *disjoint* objects. Another extension of angle histogram was R-histograms [18] which deals with most of the topological relations like *disjoint, meet, overlap, contains* and *contained*. In all these approaches, the quantitative or fuzzy directional relations are studied and less attention has been paid to fuzzy topological relations while the topological, directional and distance relations are considered essential to understand the image configuration, modeling common sense knowledge and spatial reasoning [19]. The idea of combined topological and directional relations was first initiated by J.Malki et al. [20] by using the 1D Allen [21] relations in spatial domain. This work was revisited by Matsakis and Nikitenko [22] and fuzziness in the 1D Allen relations was introduced and problem of temporal complexity was resolved by using *t-conorms* along with polygonal approximation of objects [23]. This work is related to fuzzy spatial aspects where the topological and directional relations are evaluated according to fuzzy set theoretical viewpoint in 2D scene and it differs from P.Matsakis work [24], [25] where he used 2D fuzzy sets to explore the limited set of 2D topological relations and 1D fuzzy sets are used to express the 1D fuzzy Allen relations. This paper is structured as follows, section 2 discuss in detail the different terms and computations necessary for 1D Allen relations. In section 3, our method for computing the topological relations along with directional aspects and their interpretation is given, results for different situation is given in section 4 and section 5 concludes the paper.

2. Terminology used for computation of fuzzy Allen relations

This section describes different terminology used to decompose the space and computation of different terms used in this paper.

2.1 Oriented lines, segments and longitudinal sections

A and B be two spatial objects and $(v, \theta) \in \mathbb{R}$, where v is any real number and $\theta \in [0, 2\pi]$. $\Delta_\theta(v)$ is an oriented line at orientation angle θ. $A \cap \Delta_\theta(v)$ is the intersection of object A and oriented line $\Delta_\theta(v)$. It is denoted by $A_\theta(v)$, called segment of object A and its length is x. Similarly for object B where $B \cap \Delta_\theta(v) = B_\theta(v)$ is segment and z is its length. y is the difference between the maximum value of $B \cap \Delta_\theta(v)$ and minimum of $A \cap \Delta_\theta(v)$(for details [22]). In case of polygonal object approximations (x, y, z) can be calculated from intersecting points of line and object boundary, oriented lines are considered which passes through at least one vertex of two polygons. If there exist more than one segment then it is called longitudinal section as in case of $A_\theta(v)$ in figure 1. In this paper all the 180 directions are considered with

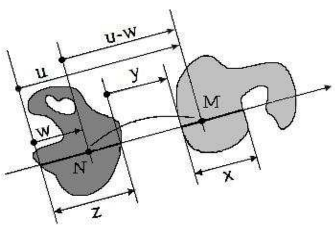

Fig. 1: Oriented line $\Delta_\theta(v)$, segment as in case of object B, longitudinal section as in case of object A [22].

an angle increment of one degree and lines are drawn by 2d Bresenham digital line algorithm. A polygonal object approximation is taken and lines passing through the polygon vertices are taken into account. segments are computed and all pairs of segments can be treated simultaneously. Fuzzy Allen relations are computed for each segment.

2.2 1D Allen relations in space

Allen [21] introduced the well known 13 jointly exhaustive and pairwise disjoint (JEPD) interval relations based on temporal interval algebra. These relations are arranged as $A = \{<, m, o, s, f, d, eq, d_i, f_i, s_i, o_i, m_i, >\}$ with meanings *before, meet, overlap, start, finish, during, equal, during_by, finish_by, start_by, overlap_by, meet_by, and after*. All the Allen relations in space are conceptually illustrated in figure 2. These relations have a rich support for the topological and directional relations. In the neighborhood graph of Allen relations, three paths can be find (because we consider the solid objects and they don't change their size during movement, due to this reason other possible paths are ignored). Depending upon the neighborhood graph of Allen relations, Inverse of a Allen relation can be divided into two categories.

1) Objects Commutativity: There are three continuous paths in this graph. When objects are interchanged, A

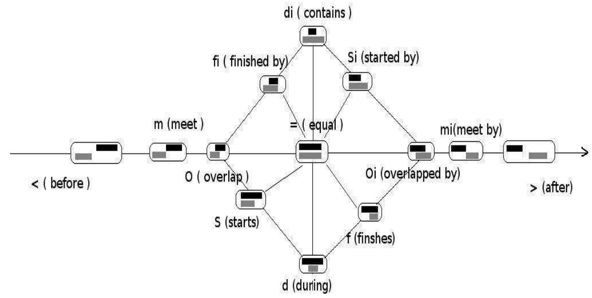

Fig. 2: Black segment represents the reference object and gray segment represents argument object

becomes the reference object and B becomes the argument object. The sequence of relations between objects is changed. This change in path shows that Allen relations $\{<, m, o, S, f, d, =\}$ and their inverses according to the object commutativity are $\{>, m_i, o_i, S_i, f_i, d_i =\}$.

2) Inverse about directions: When the direction is reversed Allen relations and their inverses also change, in this case there are 8 Allen relations and their inverse. These inverses are also called the reorientation of relations. $\{<, m, o, S, d, f_i, d_i, =, S_i, o_i, m_i >\}$ and inverse $\{>, m_i, o_i, f, d, S_i, d_i, =, S, f_i, o, m, <\}$ as a result, we can write $A_1 = \{<, m, o, S, d, f_i, d_i, =\}$ and their inverses as $A_2 = \{>, m_i, o_i, f, d, S_i, d_i, =\}$. This shows that the numeric values $d, =, d_i$ have their own reorientations.

RCC8 relations are possible combination of 8 independent Allen relations in 1D. These relations and their inverse show that the whole 2D space can be explored with the help of 1D Allen relations using oriented lines varying from $(0, \pi)$.

2.3 Fuzzification of Allen relations

In real applications, small errors in crisp values can change the entire result. To cope these problems fuzzification was introduced, it comprises the process of transforming crisp values into grades of membership for linguistic terms of fuzzy sets. Fuzzification process of Allen relations do not depend upon particular choice of fuzzy membership function, trapezoidal membership function is used due to flexibility in shape change. Let $r(I, J)$ is Allen relation between segments I and J where $I \in A$(argument object) and $J \in B$ (reference object), r' is the distance between $r(I, J)$ and its conceptional neighborhood. We consider a fuzzy membership function $\mu : r' \longrightarrow [0, 1]$. The fuzzy Allen relations defined by Matsakis and Nikitenko [22] are

- $f_<(I, J) = \mu_{(-\infty, -\infty, -b-3a/2, -b-a)}(y)$,
- $f_>(I, J) = \mu_{(0, a/2, \infty, \infty)}(y)$
- $f_m(I, J) = \mu_{(-b-3a/2, -b-a, -b-a, -b-a/2)}(y)$,
- $f_{mi}(I, J) = \mu_{(-a/2, 0, 0, a/2)}(y)$
- $f_O(I, J) = \mu_{(-b-a, -b-a/2, -b-a/2, b)}(y)$,

- $f_{Oi}(I,J) = \mu_{(-a,-a/2,-a/2,0)}(y)$
- $f_f(I,J) = min(\mu_5, \mu_3, \mu_1)$
- $f_{fi}(I,J) = min(\mu_6, \mu_4, \mu_2)$
- $f_s(I,J) = min(\mu_6, \mu_4, \mu_1)$
- $f_{si}(I,J) = min(\mu_5, \mu_3, \mu_2)$
- $f_d(I,J) = min(\mu_7, \mu_1)$
- $f_{di}(I,J) = min(\mu_7, \mu_2)$

where $\mu_1 = \mu_{(-\infty,-\infty,z/2,z)}(x)$,$\mu_2 = \mu_{(z,2z,+\infty,+\infty)}(x)$
$\mu_3 = \mu_{(-3a/2,-a,-a,-a/2)}(y)$,
$\mu_4 = \mu_{(-\infty,-\infty,-b,-(b+a)/2)}(y)$,
$\mu_5 = \mu_{(-(b+a)/2,-a,-a,+\infty)}(y)$,
$\mu_6 = \mu_{-b-a/2,-b,-b,-b+a/2}(y)$,
$\mu_7 = \mu_{(-b,-b+a/2,-3a/2,-a)}(y)$, $a = min(x,z)$,
$b = max(x,z)$ and x is the length of longitudinal section of argument object A and z is the length of longitudinal section of reference object B. Most of relations are defined by one membership like $before$, $after$,$meet$, $meet\_by$ and some of them are defined by more than one membership functions like $d(during)$, $d_i(during\_by)$, $f\ (finish)$, $f_i\ (finished\_by)$. In fuzzy set theory, sum of all the relations is one. This gives the definition for relation fuzzy $equal$. Fuzzy Allen relations are not Jointly Exhaustive and Pairwise Disjoint (JEPD) because there exist at least two relations between two spatial objects. All these equations assign a numeric value to a spatial relation.

2.4 Treatment of longitudinal sections

During the decomposition process of an object into segments, there can be multiple segments depending on object shape and boundary which is called longitudinal section. Different segments of a longitudinal section are at a certain distance and these distances might effect end results. In polygonal object approximation, fuzzy *T-norms* and *T-conorms* are used for fuzzy integration of available information. Here for simplicity only *T-conorm* (Fuzzy OR operator)is used.

$$\mu_{(OR)}(u) = max(\mu_{(A)}(u), \mu_{(B)}(u))$$

When fuzzy operator OR is used, only one fuzzy value contributes for the resultant value which represents the *maximum*. In this case each Allen relation has a fuzzy grade objective is to accumulate the best available information. Suppose that longitudinal section of object B has two segments such that $z = z_1 + z_2$ where z_1 is the length of first segment and z_2 is the length of second segment and z is length of longitudinal section. Let $\mu_1(y_1)$ defines the value of fuzzy Allen relations with the first segment and $\mu_2(y_2)$ represents value of fuzzy Allen relations with the second segment where y_1 and y_2 are the distances between object A and two segments of B. Now fuzzy OR operator is used to get consequent information obtained from two sets of fuzzy Allen relations.

2.5 Normalized fuzzy histogram of Allen relations

Histogram of fuzzy Allen relation represents total area of subregions of A and B that are facing each other in given direction θ [22]. Mathematically it can be written as [23]

$$\int_{-\infty}^{+\infty} (\sum_{r \epsilon A} F_r(\theta, A_\theta(v), B_\theta(v))dv = (X+Z)\sum_{k=1}^{n} r(I_k, J_k)$$

where Z is the area of reference object and X is area of augmented object in direction θ, n is total number of segments treated and $r(I_k, J_k)$ is an Allen relation for segments (I_k, J_k). These histograms can easily be normalized by dividing the all the Allen relations by the sum of all the Allen relations for every θ. It is represented by $\lceil F_r^{AB}(\theta) \rfloor$ where $r \in A$. $\lceil F_r^{AB}(\theta) \rfloor = \frac{F_r^{AB}(\theta)}{\sum_{\rho \in A} F_\rho^{AB}(\theta)}$. These fuzzy Allen relations are directional fuzzy numbers and can be used to define the quantitative fuzzy directional relations.

3. Topological and directional relations

This section consists of two subsections where in first subsection, it is described that two different functions can be used to assess the fuzzy qualitative directions and then how the different Allen relations are combined. In the second subsection, the representation method for the fuzzy topological and fuzzy directional relations are described in detail.

3.1 Assessment of fuzzy directional Components

All these equations depicted in section 2.3 assign a numeric value to a spatial relation in a direction θ and it is difficult to analyze the spatial changes in a video scene with the histogram representation. To make the changes visible in a qualitative direction, numeric values are used. The mainstream efforts concentrated on developing computation oriented representation formalisms and their implementations, paying little attention to the process of modeling itself. To cope with the temporal complexity semi interval [26] is used to model the directions. For this purpose 8 compass directions are used to express the directional relations. Directions are represented as $\{E, NE, N, NW, W, SW, S, SE\}$ with meanings *East, North_East, North, North_West, West, South_ West, South* and *South_ East*. To assess these fuzzy directional relations, two trigonometric functions are used. To assess the even directions ($\{E, N, W, S\}$), trigonometric function $cos2\theta$ and to assess the odd direction ($\{NE, NW, SW, SE\}$), trigonometric function $sin2\theta$ is used. As the angle distribution is taken to the half plane so opposite Allen relations are used to define the opposite directions except the direction *East* and *West* where union of both relations are used. Mathematically these relations can be defined as

$$f_E = \begin{cases} A_{r_2} * cos^2(2\theta) & \text{if } \theta \in (0, \frac{\pi}{4}) \\ A_{r_1} * cos^2(2\theta) & \text{if } \theta \in (\frac{3\pi}{4}, \pi) \\ 0 & \text{otherwise} \end{cases}$$

$$f_W = \begin{cases} A_{r_1} * cos^2(2\theta) & \text{if } \theta \in (0, \frac{\pi}{4}) \\ A_{r_2} * cos^2(2\theta) & \text{if } \theta \in (\frac{3\pi}{4}, \pi) \\ 0 & \text{otherwise} \end{cases}$$

$$f_N = \begin{cases} A_{r_2} * cos^2(2\theta) & \text{if } \theta \in (\frac{\pi}{4}, \frac{3\pi}{4}) \\ 0 & \text{otherwise} \end{cases}$$

$$f_S = \begin{cases} A_{r_1} * cos^2(2\theta) & \text{if } \theta \in (\frac{\pi}{4}, \frac{3\pi}{4}) \\ 0 & \text{otherwise} \end{cases}$$

$$f_{NE} = \begin{cases} A_{r_2} * Sin^2(2\theta) & \text{if } \theta \in (0, \frac{\pi}{2}) \\ 0 & \text{otherwise} \end{cases}$$

$$f_{SW} = \begin{cases} A_{r_1} * Sin^2(2\theta) & \text{if } \theta \in (0, \frac{\pi}{2}) \\ 0 & \text{otherwise} \end{cases}$$

$$f_{NW} = \begin{cases} A_{r_2} * Sin^2(2\theta) & \text{if } \theta \in (\frac{\pi}{2}, \pi) \\ 0 & \text{otherwise} \end{cases}$$

$$f_{SE} = \begin{cases} A_{r_1} * Sin^2(2\theta) & \text{if } \theta \in (\frac{\pi}{2}, \pi) \\ 0 & \text{otherwise} \end{cases}$$

Where $A_{r_i} \in A_i, i = 1, 2$ given in section 2.2 and $f \in \{D, EC, PO, TPP, NTPP, TPPI, NTPPI, EQ\}$ with meanings *Disjoint, Externally connected, Partially overlap, Tangent proper part, Non tangent proper part, Tangent proper part inverse, Non Tangent proper part inverse*, and *Equal*. In this way the relations are manipulated as a (8×8) matrix where row hold the topological relations and columns have the qualitative directional aspects of 2D scene information.

3.2 Representation and interpretation of relations

A qualitative direction system will assign a single direction to another object with respect to a reference object and that is the only directional relationship between them. However, in the fuzzy approach, cardinal directions will be have different degrees of truth. This degree of truth value represents the percentage of object which lies in that direction. The relations are manipulated in (8×8) matrix where rows show the topological relations and columns show the directional distribution of each topological relation. Each entity of this matrix represents the percentage surface area of two objects having a topological relation in a specific direction. C(i,j) represents the i^{th} topological relation in j^{th} direction. Rows are arranged in an order of spatial neighboring of RCC8 relations.i.e. $\{D, EC, PO, TPP, NTPP, TPPI, NTPPI, EQ\}$. Different directions may yield different 1D relations in 2D space and different relations may coexist along the same direction. When directional relation for complex 2D objects are computed with the help of 1D Allen relations then one relation may exist in multiple directions. In case of complex 2D objects such as concave objects two opposite relations may coexist along the one direction or the one object may have different relations in different directions. Here a method to understand the representation of fuzzy topological and directional relations and deriving the conclusion about the overall 2D relation is given.

1) If all the rows are zero except the first then objects have fuzzy disjoint (*DC*) topological relation .
2) If non zero values are in first and second row then the overall relation in 2D space is fuzzy meet *EC*.
3) If there is at least one non zero value in third row it means there exist fuzzy topological relation *overlap*. More generally when the two opposite relations exist together i.e *TPP* and *TPPI* or *NTPP* and *NTPPI*, then topological relation in 2D space will be overlap other relations only explain the situation.
4) Relations *PP, PPI, EQ* hold if the corresponding relation holds in all directions, a relation will hold if all elements in a row are non zero and all other rows are zero.
5) If the non zero values also exist in *TPP* along with *NTPP* (*TPPI* long with *NTPPI*) then the relation will be *TPP* (*NTPP*) in the corresponding direction.

This shows that 1D definition of topological relations could not used directly to define 2D topological relations. Whole above explanation shows that *overlap* relation in 2D space is more complex than any other topological relation.

4. Experiments

For the experiment purpose, 180 directions are considered (angle increment is 1 degree) and lines are drawn by 2d Bresenham digital line algorithm. Segments are computed and all pairs of segments can be treated simultaneously. Fuzzy Allen relations are computed for each segment. When the directional relations are evaluated by fuzzy techniques, they can share more than one direction and their directional relations are represented by a degree. Throughout this paper reference object B is represented by dark grey color and light grey object represents the argument object A. In first example, objects are taken at different distances and it is a well known fact that range of directional relations are inversely proportional to the distance between them.i.e. greater the distance between them, smaller will be their range of directional distributions. When the object A is far away from the object B represented in figure 3(a), it represents the topological relation *disjoint* with degree of directional relations *E=0.69, NE=0.16, SE=0.15* as represented in figure

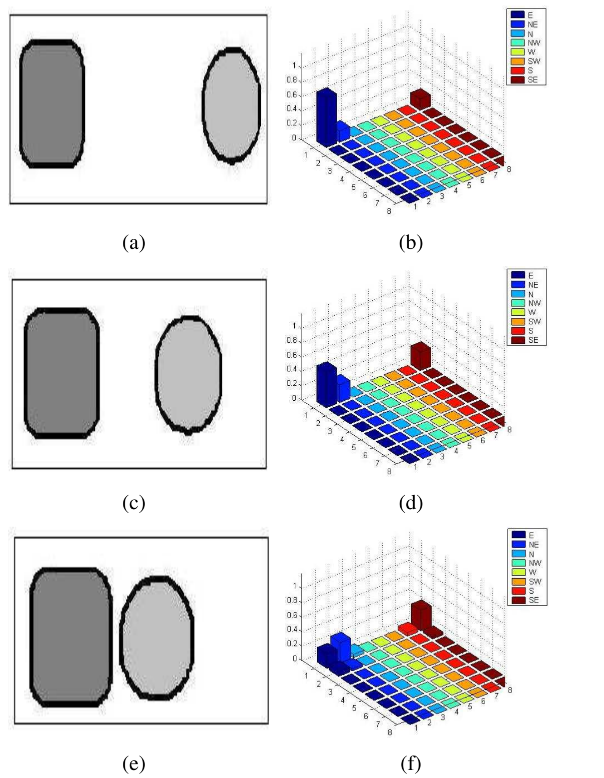

Fig. 3: Object pairs and its spatial relations when distance between them changes

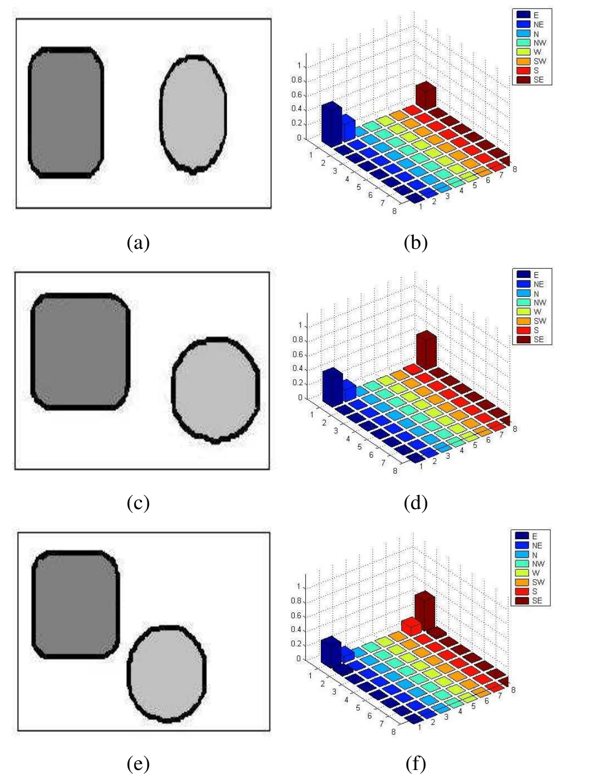

Fig. 4: Object pairs changing the directional and distance information and its spatial relations

3(b). When object A is translated by (50,0) towards object B 3(c), it doesn't change the topological relation but directional relations are changed which are *E=0.5, NE=0.25, SE=0.25* as represented in figure 3(d) and in final figure 3(e) when object is again translated towards object B by (30,0), then topological relations are also changed due to fuzziness in topological relations and now *EC* relation also exist with *Disjoint* relation and results are represented in figure 3(f) where directional relations for *disjoint* topological relations are *E=0.2, NE=0.28, N= 0.05, S= 0.05, SE=0.28* and for *EC* topological relations are *E=0.07,NE=0.03, SE=0. 03*.

In second example (figures 4(a) to 4(f)) objects changing their directional and distance relations are considered. In figures 4(a), 4(c),4(e), at each step object A changes the direction and distance from the reference object B. As a result their topological relation and degree of directional relations is also changing. As the object A moves towards the *SW* i.e. from *E* of reference object B, the degree of their directional relations also changes. The degree of directional relations in figure 4(a) is *E= 0.49, NE= 0.25 SE= 0.26* represented in figure 4(b) and when object moves towards *SW* from its position (object is translated by (-15,-15)) then its new degree of directional relations becomes E= 0.41, NE= 0.17 SE= 0.40, S=0.02. This result is represented in figure 4(d) and finely when object is translated by (-15,-15) again (4(e)), the object becomes closer to the reference object. At this stage two topological relations exist, degree of directional relations for topological relation *D* is *E= 0.30, NE= 0.09 SE= 0.42, S=0.13* and for *EC* relation *E= 0.03, NE= 0, SE= 0.03* represented in figure 4(f).

In third example object A lies inside the object B figure 5(a) and it moves towards the center of object B (figure 5(c)). It topological relations also change as in figure 5(b), there exist a small amount of *TPP* along directions *E,S,SE* and the second (figure 5(d))topological relation is *NTPP* but in next this relation changes and relation *NTPP* exists everywhere. This shows that the 2D topological relation *TPP* changes to *NTPP*.

In the forth example, scenario is taken where objects change topological relations. In figure 6(a), objects are disjoint and fuzzy relations matrix shows that object lies in North of the reference object. Directional components as shown in figure 6(b) specify the object location. Disjoint relation exist in directions *E,NE,N, NW* with different degree. In figures 6(c), 6(e) objects overlap and a topological relation *PO* exists but existence of other relations like *D, EC* represented in figure 6(d), *D,EC, TPP, TPPI, EQ* and *TPP, NTPP, TPPI, EQ* represented in figure 6(f) describes the 2D scene. A small portion of relation *PO* exists in direction *E, NE, N* (figure 6(d)) and in direction *E, NE, SE* figure(6(f)).

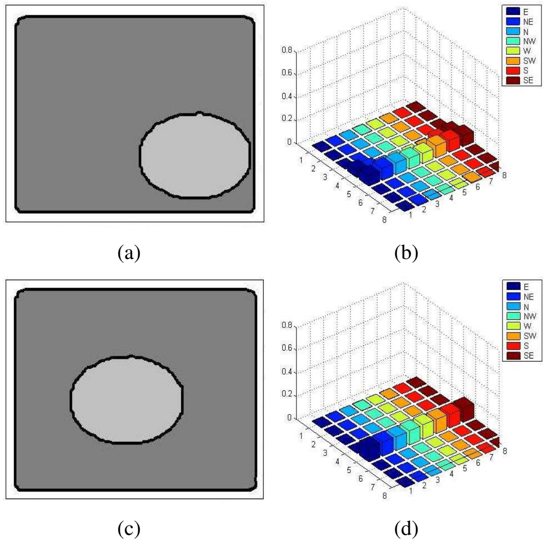

Fig. 5: Object pairs when object A lies inside object B and its spatial relations

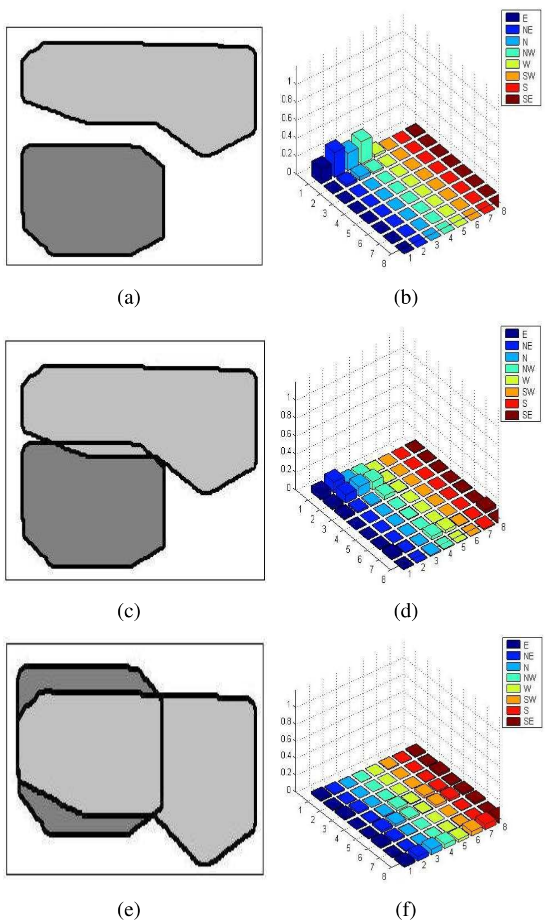

Fig. 6: Object pairs changing the topological relations and its spatial relations

In this last example fuzzy *meet (EC)* relation in topology exist when the exactly meeting or very close to each other and it seems that they are sharing the boundary. Figure (7(a))

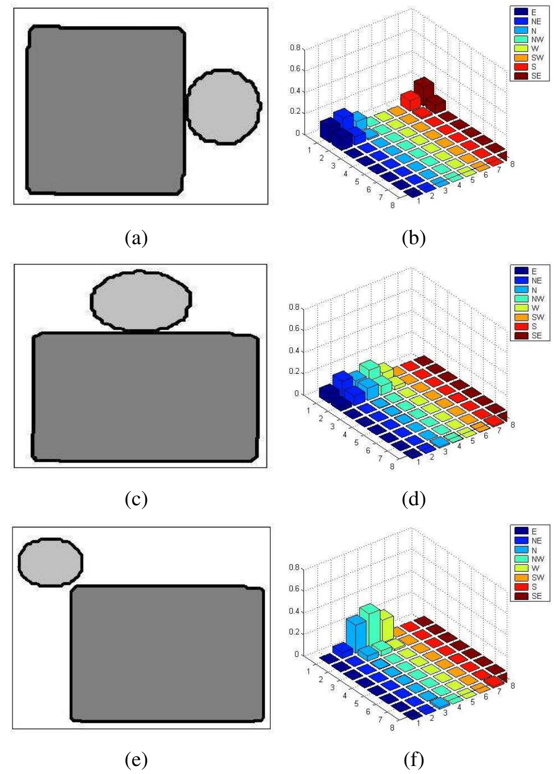

Fig. 7: Object pairs with meet topological relation and their directional relations

shows that argument object A meets the reference object B in direction *East*. Their topological and directional relations show that there exist a topological relation meet in direction East (figure (7(b))). Argument object A in figure 7(c) touches the argument object from *North*, their relation matrix validates the said relation (7(d)). As in figures (7(e)) argument object touches the reference object from *North_West* and *West* directions respectively, their topological relation also change the directional contents (figures 7(f)). All the above results show that the relations are sensitive to the distance between the object pair and their relative positions. These relations changes as the objects change their relative position. It is a numerical description of relative position of object pair. When the objects are disjoint, the relations are simple it becomes more and more complicated when an *overlap* relation exists.

5. Conclusion and future work

In this paper a new method for finding the fuzzy topological and directional relations was studied and all the topological relations are generated by using the fuzzy Allen relations. Directions are evaluated with the help of trigonometric membership functions. This method deals with fuzziness at two levels, fuzziness in the topological relations due to their positions and fuzziness in directional relations. This method differs from P. Matsakis [24] work, where 2D fuzzy sets to generate a limited number of topological relations are used. Here fuzzy Allen relations are combined to generate the 2D topological relations along with the directional assessment of relations and a new representation is used. It is a numerical description of relative position of objects. Spatio-temporal relations are the emerging issue and hopefully these results will be helpful in extending this work to a spatio-temporal aspects and fuzzy spatio-temporal reasoning and natural language processus. These results will be used in future works to develop the motion verbs and spatio-temporal relations between moving objects.

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