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# Suppression of decoherence effects in the quantum kicked rotor

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We describe a method allowing transient suppression of decoherence effects on the atom-optics realization of the kicked rotor. The system is prepared in an initial state with a momentum distribution concentrated in an interval much sharper than the Brillouin zone; the measure of the momentum distribution is restricted to this interval of quasimomenta: As most of the atoms undergoing decoherence processes fall outside this detection range and thus are not detected, the measured signal is effectively decoherence-free.

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The kicked rotor has been studied for many years in the helm of both classical and quantum Hamiltonian dynamics, where it has the status of a paradigm. Its formal simplicity, and the richness of its dynamics make it one of the most studied dynamical systems. Experiments started with the observation by Raizen and co-workers [1] of quantum chaos in an atom-optics realization of the kicked rotor. An impressive number of works are devoted to different aspects of the kicked rotor dynamics, including dynamical localization [1–6], quantum resonances [7–11], sub-Fourier resonances [12], chaos-assisted tunneling [13–15], ratchets [16–18], and, recently, the observation of the Anderson metal-insulator transition [6, 19, 20]. The atomic kicked rotor is a very clean and flexible system, but it is limited by the unavoidable presence of decoherence, mainly due to spontaneous emission [2, 3]. In the present work, we propose and analyze a method for suppressing to a large amount, if only transiently, decoherence effects in the kicked rotor's dynamics, which is directly applicable to existing experimental setups.

Consider a two-level atom placed in a standing wave generated by two counterpropagating beams of intensity  $I$  and wavenumber  $k_L$ . On the one hand, the atom may interact non-dissipatively with the radiation by absorbing a photon in one of the beams and emitting a photon in the *other* beam by *stimulated* emission. This is a conservative elementary process in which a “quantum” of momentum  $2\hbar k_L$  is exchanged between the atom and the field. It generates a potential acting on the center of mass of the atom, which turns out to be proportional to  $\Omega(x)^2/\Delta_L \sim (I/\Delta_L) \cos(2k_L x)$ , where  $\Omega$  is the (resonant) Rabi frequency,  $\Delta_L$  is the laser-atom detuning, ( $\Delta_L \gg \Gamma$  where  $\Gamma$  is the natural width of the excited state) [21, 22]. This “optical potential” only couples momentum states separated by multiples of  $2\hbar k_L$ , therefore, quasimomentum is conserved. On the other hand, there can be spontaneous emission (SE) process which are dissipative and thus are fatal to quantum effects. The total momentum exchange (along the direction of the standing wave) in a process involving SE is  $\hbar k_L(1 + \cos\theta)$  where  $\theta$  is a random angle in the range  $[0, 2\pi)$ . After such a process the atom momentum distribution is not anymore

concentrated around multiples of  $2\hbar k_L$ .

The atom-optics realization of the kicked rotor consists in placing laser-cooled atoms in a periodically-pulsed standing wave. The pulse (*kick*) duration  $\tau$  is very short compared to the atom dynamics, so that the pulses can be assimilated to a Dirac delta function. The Hamiltonian for the center of mass motion of atoms of mass  $M$  and momentum  $p$  along the laser beam propagation direction  $x$  is thus

$$H = \frac{P^2}{2} + K \cos X \sum_{m=0}^{N-1} \delta(t - m), \quad (1)$$

where we introduced normalized units [1] where time is measured in units of the pulse period  $T$ ,  $X = 2k_L x$ ,  $P = 2k_L T p/M$ , and  $K = 4\hbar k_L^2 \Omega^2 T \tau / \Delta_L M$ . With such definitions,  $[X, P] = i\hbar$  where  $\hbar = 4\hbar k_L^2 T / M$  plays the role of a normalized Planck constant. Note that  $P/\hbar = p/2\hbar k_L$ .

For  $K \gtrsim 5$  the *classical* dynamics obtained from Hamiltonian (1) is an ergodic chaotic diffusion [23] which leads to a Gaussian momentum distribution whose broadening corresponds to an average kinetic energy increasing linearly with time,  $E_{cl} = D_{cl} t$ , with  $D_{cl} \sim K^2/4$ . In the quantum case, after a characteristic *localization time*  $t_L$ , quantum interference effects lead to a *saturation* of the kinetic energy  $E$  to the constant value  $E_L = P_L^2/2$  [24, 25], and the average momentum distribution takes an exponential shape  $\sim \exp(-|P|/P_L)$ , hence the name of “dynamical localization” (DL) given to this phenomenon. The instantaneous diffusion coefficient  $D_0(t) = dE/dt$  starts with an initial value  $D_0(0) = D_q$  [26] and tends to zero asymptotically. Following [2], we will simply model this behavior by an exponential with a characteristic time  $t_s$  [27]

$$D_0(t) = D_q e^{-t/t_s}. \quad (2)$$

DL, which is a quantum interference effect, is highly sensitive to decoherence and will serve as a decoherence probe in the present work. Decoherence effects in the kicked rotor have been studied both theoretically [28] and

experimentally [2, 3]. Here, decoherence is essentially due to SE, whose probability per kick is

$$\Pi \equiv \frac{\Gamma\tau}{2} \frac{\Omega^2/2}{\Delta_L^2 + \Omega^2/2 + \Gamma^2/4} \approx \frac{\Gamma\tau\Omega^2}{4\Delta_L^2}, \quad (3)$$

where the approximate value on the right corresponds to the limit  $|\Delta_L| \gg \Gamma, \Omega$ . Typical values of  $\Pi$  in experiments are a fraction of  $10^{-2}$ ; DL can thus be observed only for a hundred kicks or so [20].

Suppose one prepares a sample of cold atoms in a well defined momentum state, say  $P = 0$ . Interaction with the optical potential populates only states of momentum  $P_n = n\bar{k}$ , ( $n$  integer) and the corresponding wavefunction is of the form  $|\psi\rangle = \sum_n c_n e^{i\varphi_n} |n\bar{k}\rangle$ . Stimulated processes keep *well-defined* phases  $\varphi_n$ , and allow the interference effects responsible for DL. Spontaneous emission introduces *random* phases and puts the atom into a mixture of momentum states; in other words, it “resets” quantum interference, restoring the initial diffusion coefficient  $D_q$ . The form of the diffusion coefficient  $D(t)$  with SE is thus a weighted mean of the evolution without SE, given by  $D_0(t)$ , and of the effect of SE events. As has been shown in [2], SE restores diffusion after a time  $t \sim t_s(1 + \tau_s)^{-1}$  (where  $\tau_s \equiv \Pi t_s$ ) with an asymptotic diffusion coefficient given by

$$D_\infty \equiv \frac{D_q \tau_s}{1 + \tau_s}. \quad (4)$$

Fig. 1a shows the typical behavior of the kinetic energy for increasing levels of SE obtained by numerical simulation of the quantum dynamics corresponding to Hamiltonian (1). Clearly, DL is destroyed and diffusion restored as the SE rate increases. The initial momentum distribution is a square (for simplicity) of width  $\Delta_R = 0.04\bar{k}$  centered at  $P = 0$ . For each kick, a Monte Carlo procedure is used to decide whether a photon is spontaneously emitted; if so, the entire momentum distribution is translated of a quantity  $\bar{k} \cos \theta$ , with  $\theta$  picked randomly in the interval  $[0, 2\pi)$ , which produces the mixing of quasimomenta. In order to simplify notations, we will measure momentum in units of  $\bar{k}$ , so that, from now on,  $P_n \equiv n$  and  $\Delta \equiv \Delta_R/\bar{k}$ .

Momentum distributions  $f(P) = |\psi(P)|^2$ , are shown in Fig. 1b. For  $\Pi = 0$ , the distribution is fitted by an exponential (red triangles); decoherence effects are clearly seen for  $\Pi \neq 0$ ; for instance, the  $\Pi = 0.02$  is fitted by a Gaussian (black squares).

Let us now present our method for a transient suppression of decoherence effects. If no SE event happens, the momentum distribution  $f(P)$ , at any time will be nonzero only for momenta in the range  $\Delta_n = [n - \Delta/2, n + \Delta/2]$ . As  $\Delta \ll 1$ , atoms undergoing a SE process will mostly populate other momentum classes. If, at the end of the kick series, one measures the momentum distribution by *probing only the  $\Delta_n$  momentum*

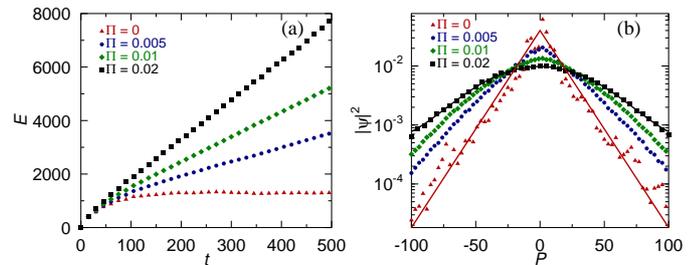


Figure 1: (Color online) Effect of spontaneous emission: (a) Kinetic energy  $E(t)$  for different values of the spontaneous emission rate  $\Pi$ ; (b) momentum distributions (log scale) at  $t = 500$ , the  $\Pi = 0$  curve (red triangles) is well fitted by an exponential, whereas the  $\Pi = 0.02$  curve (black squares) is fitted by a Gaussian. Parameters are  $K = 10$ ,  $\bar{k} = 2.9$  ( $t_L \approx 12$ ) and the initial momentum distribution is a square of width  $\Delta = 0.04$ .

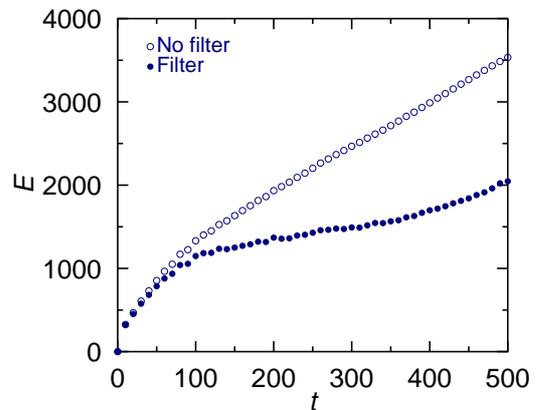


Figure 2: (Color online) Average kinetic energy with filtering (full circles) and without filtering (empty circles) for  $\Pi = 0.005$  and at  $N = 300$  kicks (1.5 times the typical SE time,  $\Pi^{-1} = 200$  kicks). The filtering reduces the average energy at of 60 %. Referring to the saturation value of the kinetic energy in absence of SE, the SE effect has been reduced by a factor of about 5.5.

*classes*, most of the atoms having performed SE will be excluded from the resulting signal, *which will be effectively decoherence-free*. Experimentally, preparation and measurement of  $f(P)$  with a precision  $\Delta \ll 1$  can be made e.g. using Raman stimulated spectroscopy [29–31]. However, the probability that an atom having performed a SE falls back into a detection range  $\Delta_n$  increases with the number of fluorescence cycles, so this filtering is a transient effect.

Fig. 2 shows the kinetic energy of filtered atoms,  $\bar{E} = \sum_n \mathcal{P}_n^2/2$  (full circles), with  $\mathcal{P}_n \equiv \int_{n-\Delta/2}^{n+\Delta/2} f(P) P dP$ . The comparison with the result in the case where there is no filtering (empty circles) shows the efficiency of the filtering.

Let us make the reasonable assumption that an atom emitting spontaneously a photon has a probability  $\Delta$  of

falling in one of the ranges  $\Delta_n$  and  $(1 - \Delta)$  of falling outside any of these ranges. We then consider three populations of atoms:  $F_0$ , the population of atoms which have *never* performed (at time  $t$ ) any SE (the momentum of these atoms is thus in the ranges  $\Delta_n$ );  $F_\Delta$ , the population of atoms in one of the ranges  $\Delta_n$  but having performed at least one SE, and  $F_{1-\Delta}$ , the population of atoms outside any detection range  $\Delta_n$ . The rate equations for these populations are straightforwardly obtained (noting that  $F_0 + F_\Delta + F_{1-\Delta} = 1$ ) as:

$$\frac{dF_0}{dt} = -\Pi F_0 \quad (5)$$

$$\frac{dF_\Delta}{dt} = -\Pi F_\Delta + \Pi \Delta, \quad (6)$$

The integration of these equations gives:

$$F_0(t) = e^{-\Pi t} \quad (7)$$

$$F_\Delta(t) = \Delta (1 - e^{-\Pi t}). \quad (8)$$

Note that the population of *detected* (or *filtered*) atoms  $F_0 + F_\Delta = \Delta + (1 - \Delta)e^{-\Pi t}$  is not constant in time and tends, as expected, to  $\Delta$  as  $t \rightarrow \infty$ .

The kinetic energy of *filtered* atoms is

$$\bar{E}(t) = \frac{E_0 + E_\Delta}{F_0 + F_\Delta}, \quad (9)$$

where  $E_0$  and  $E_\Delta$  are the *total* kinetic energy calculated over, resp., the populations  $F_0$  and  $F_\Delta$ . The evolution of  $E_0$  can be obtained from the equation  $dE_0/dt = D_0(t)F_0 - \Pi E_0$ : the first term is the contribution due to the kicks [see Eq. (2)] and the second term is the depletion by SE. Thus

$$E_0(t) = D_q t_s \left(1 - e^{-t/t_s}\right) e^{-\Pi t}, \quad (10)$$

which has a maximum at time

$$t_1 = t_s \ln(1 + \tau_s^{-1}). \quad (11)$$

The evolution of  $E_\Delta$  is governed by two contributions: *i*) the contribution of the atoms in the population  $F_\Delta$  which performed at least one SE event and whose kinetic energy is due to all past fluorescence cycles [28], and *ii*) the contribution of the atoms in the population  $F_0$  that perform a fluorescence cycle at time  $t$  and fall into  $F_\Delta$ , that is

$$\frac{dE_\Delta}{dt} = \int dt' \frac{dF_\Delta}{dt'} D(t') + \Delta \Pi E_0(t), \quad (12)$$

which gives, after some algebra:

$$E_\Delta = \frac{\Delta D_q t_s}{1 + \tau_s} \left( \Pi t - (1 + \tau_s) e^{-\Pi t} + \frac{\tau_s(1 + 2\tau_s)}{1 + \tau_s} e^{-(1+\tau_s)t/t_s} + \frac{1 + \tau_s - \tau_s^2}{1 + \tau_s} \right). \quad (13)$$

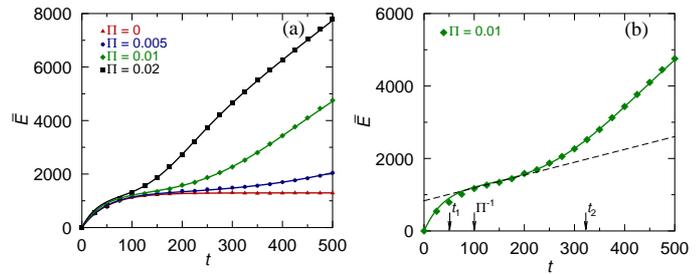


Figure 3: (Color online) (a) Plot of  $\bar{E}(t)$  obtained from the numerical simulation (same curves as in Fig. 1) and best fits obtained from Eq. (9) with the fit parameters  $D_q$  and  $t_s$  (full lines). (b) The  $\Pi = 0.01$  curve of plot (a), indicating the characteristic times  $t_1 \approx 51$  and  $t_2 \approx 320$ , and the reduced diffusion (dashed line) with coefficient  $D_r$  (cf. text).

$\Pi$	0	0.005	0.01	0.02
$D_q$	25.5	26.3	30.7	36.6
$t_s$	50.7	48.8	41.3	32.5

Table I: Fit parameters  $D_q$  and  $t_s$  corresponding to the curves displayed on Fig. 3.

From Eqs. (9), (10) and (13) one obtains an explicit expression for  $\bar{E}(t)$  that we do not give here as it somewhat cumbersome; let us just consider the limit  $\Delta, \tau_s \ll 1$  in which it can be written:

$$\bar{E}(t) = D_q \tau_s \Pi \Delta e^{\Pi t} (1 + \Pi t). \quad (14)$$

Fig. 3a shows the numerical results for  $\bar{E}(t)$  for different values of  $\Pi$ . The full lines are fits of the numerical data obtained from Eqs. (9), (10) and (13), using  $D_q$  and  $t_s$  as adjustable parameters (see notes [26, 27]), whose values are given in Table I.

The main features of the curves in Fig. 3 can be understood by simple arguments. The curve in Fig. 3b shows two characteristic times: *i*) time  $t_1$  [Eq. (11)] at which the SE starts depleting the population  $F_0$ , *ii*) time  $t_2$  when the contribution of the “incoherent atoms”  $F_\Delta$  exceeds the “coherent” contribution due to  $F_0$ ;  $t_2$  is thus given by the condition  $F_\Delta(t_2) = F_0(t_2)$  which leads to  $t_2 \approx -\ln \Delta \Pi^{-1}$ . In the absence of filtering,  $t_2 \sim \Pi^{-1}$ , the filtering thus increases this time by a factor  $|\ln \Delta|$ .

The energy  $\bar{E}(t)$  exhibits a transient reduced diffusion regime that corresponds to the quasi-plateau which is clearly seen in Fig. 3b; the corresponding diffusion coefficient  $D_r$  (indicated by the dashed line) can be estimated by expanding Eq. (14) around  $t = \Pi^{-1}$ , which gives  $D_r \approx 2e D_q \tau_s \Delta \approx 2e(1 + \tau_s) \Delta D_\infty$  with  $D_\infty$  given by Eq. (4). Thus the condition for the efficiency of filtering is  $\Delta < [2e(1 + \tau_s)]^{-1}$  ( $\sim 0.1$  for current parameters).

Finally, let us consider the effect of filtering process on momentum distribution. Fig. 4 displays momentum distributions after  $N = 500$  kicks, corresponding to the curves in Fig. 3a. One sees that in the case  $\Pi = 0.005$

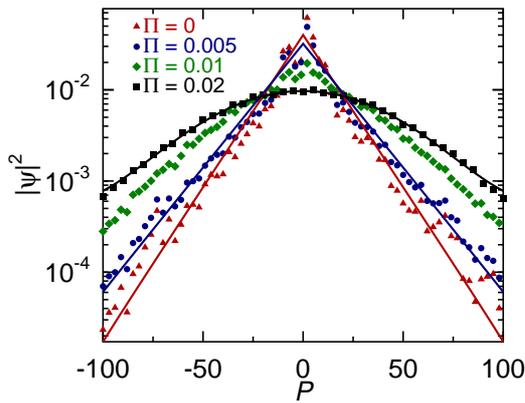


Figure 4: (Color online) Momentum distributions at  $N = 500$ . The  $\Pi = 0$  (red triangles) and  $\Pi = 0.005$  (blue circles) curves are fitted by exponentials, while the  $\Pi = 0.02$  (black squares) curve is well fitted by a Gaussian. The curve corresponding to  $\Pi = 0.01$  (green diamonds) has an intermediate shape.

( $\Pi N = 2.5$ ) the distribution can still be fitted by an exponential, showing that DL has survived up to 500 kicks (cf. the corresponding kinetic energy curve in Fig. 1). The  $\Pi = 0.02$  ( $\Pi N = 10$ ) curve is fitted by a Gaussian and is thus affected by decoherence. The case  $\Pi = 0.01$  ( $\Pi N = 5$ ) has an intermediate shape between exponential and Gaussian.

In conclusion, we have proposed an original method allowing the suppression of decoherence effects in the dynamics of the quantum kicked rotor for a finite but quite long time compared to the typical duration of current experiments. The method can be applied to state of art experiments and shall allow more precise studies of systems in which decoherence is the main limitation [19, 20]. Although spontaneous emission was the only source of decoherence considered here, it is worth noting that the the method is in principle applicable to any source of decoherence which does not conserve quasimomentum (e.g. collisions).

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