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Michel Bellet, Serge Moto Mpong. Determination of the constitutive equation parameters of a thixotropic Al alloy and FEM modelling of the thixoforming process. 7th International Conference on Numerical Methods in Industrial Forming Processes (NUMIFORM 2001), Jun 2001, Toyohashi, Japan. p. 1087-1092 - ISBN 90-2651-822-6. hal-00536356

**HAL Id: hal-00536356**

**<https://minesparis-psl.hal.science/hal-00536356>**

Submitted on 11 Mar 2011

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# Determination of the constitutive equation parameters of a thixotropic Al alloy and FEM modelling of the thixoforming process

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*in Simulation of Material Processing: Theory, Methods and Applications, Proc. NUMIFORM'2001, 7<sup>th</sup> Int. Conf. on Numerical Methods in Industrial Forming Processes, Toyohashi, Japan, 18-20 June 2001, K. Mori (ed.), Balkema, Lisse, pp. 1087-1092 (2001)*

**ABSTRACT:** This paper is focused on the thixoforming of A356 Al alloy. In the present study, an original rheological test has been developed. The identification of the constitutive equation parameters has been carried out by the finite element modelling. On another hand, the adaptation of a three-dimensional FEM code to the simulation of the process is presented, together with and its first application to an industrial test part.

## 1 INTRODUCTION

Thixoforming is a process in which metallic alloys are formed in semi-solid state, taking advantage of a specific globular structure. The industrial development of this process, which is intermediate between forging and casting, requires the use of predictive numerical models. This suppose the identification of appropriate constitutive equations and their implementation in numerical simulation codes. The present study contributes to these both points.

The first part of the paper is dedicated to the development of a rheological test that could be used to characterize the flow rule of semi-solid aluminium alloys in conditions that are representative of thixoforming process. In this case, the interpretation of the rheological test is carried out by means of a finite element simulation. After a brief review of the different types of constitutive equations for semi-solid materials (section 2), the experimental device is described in section 3 and the identification strategy and results are discussed in section 4.

The second part of the paper deals with the numerical simulation of the thixoforming process. A three-dimensional finite element method has been used and is described in section 5. Finally, section 6 gives an example of application of the numerical model on a test part.

## 2 CONSTITUTIVE EQUATIONS FOR SEMI-SOLID MATERIAL

According to the literature, there are two main general classes of models, which will be briefly described hereunder, with comments on their respective advantages and drawbacks.

### 2.1 Models deriving from the theory of suspensions (Kumar et al. 94)

These models are essentially used for metallic slurries with low or medium solid fraction.

At very low solid fraction ( $0 < f_s < 0.2$ ), the flow of the metal can be described by a simple Newtonian law.

At medium solid fraction ( $0.2 < f_s < 0.6$ ), the slurry cannot be considered as Newtonian any more and its behaviour depends on the past of the material deformation :

- In the transient state, and when put under stirring conditions, the semisolid slurry behaves as a thixotropic fluid. The thixotropy is the characteristic of a material for which the viscosity decreases with time (destruction) and gradually recovers its initial viscosity at rest (restructuration) ;
- In the stationary state, the semi-solid material behaves as a pseudoplastic (i.e. non Newtonian viscoplastic) material (see 2.3) ;

The Kumar-Brown model has provided results that have shown good agreement with experiments made in isothermal condition, considering that the most important parameter is the thixotropic effect. It can be summarised by the following equations :

$$\left\{ \begin{array}{l} \sigma = -p\mathbf{I} + \eta \dot{\epsilon} \\ \eta = \eta_{hyd} + \eta_{pla} \\ \eta_{hyd} = A(\lambda)(c/c_{max})^{1/3} \left(1 - (c/c_{max})^{1/3}\right)^{-1} \eta_l \\ \eta_{pla} = (m+1)C_0 \exp(mQ/RT) f_s \lambda \eta_l^{n+1} \dot{\epsilon}^{m-1} \end{array} \right. \quad (1)$$

where  $\sigma$  is the stress tensor,  $p$  the hydrostatic pressure,  $\eta$  the dynamic viscosity, and  $\dot{\epsilon}$  the strain rate tensor.  $\eta_{hyd}$  is the viscosity contribution due to the hydrodynamic interactions,  $\eta_{pla}$  is the viscosity contribution due to the disruption of the particle bonds,  $\lambda$  is an internal variable which represents the degree of agglomeration of the solid particles in the semi-solid material ( $\lambda = 0$  : no agglomeration ;  $\lambda = 1$  : full agglomeration),  $A(\lambda)$  is a function which reflects the effect of the structure on the hydrodynamic part of the viscosity,  $c$  is an effective volume fraction of solid,  $c_{max}$  is the maximum value of  $c$ ,  $\eta_l$  is the liquid viscosity.  $m$  is the power law exponent,  $C_0$  is a constant,  $\bar{\epsilon}$  is the equivalent strain rate,  $T$  is the temperature,  $R$  is the universal gas constant,  $Q$  is the activation energy.

The model is then complemented by an equation that describes the evolution of the internal variable  $\lambda$ , which depends on the competition of agglomeration and disagglomeration kinetics:

$$\frac{d\lambda}{dt} = H(f_s)(1 - \lambda) - R(f_s)\lambda\bar{\epsilon}^m \quad (2)$$

The following types of  $c$ ,  $A$ ,  $H$  and  $R$  functions can be used:

$$\begin{cases} c(\lambda) = c_1 + c_2\lambda \\ A(\lambda) = a_1 + a_2\lambda \\ H(f_s) = h_1 f_s^{h_2} \\ R(f_s) = r_1 f_s^{r_2} \end{cases} \quad (3)$$

The weakness of this kind of model is the important number of parameters which makes their practical identification extremely difficult.

## 2.2 Two-phase models (Lalli 85, Toyoshima 91, Nguyen et al. 94, Martin et al. 97, Bay et al. 98)

These models derive from the theory of soils saturated with liquid. At higher solid fraction than 0.6, the solid phase is continuous and constitutes a viscoplastic skeleton. The deformation of this skeleton determines the flow of the liquid phase, which is governed by the Darcy law (equation of a liquid flow through a permeable medium). The pressure of the interstitial liquid determines in turn the deformation of the solid skeleton. This kind of approach has been found quite relevant for slow deformation of semi-solid specimen, especially when liquid segregation can be observed during the deformation.

Regarding the application of such models to thixotropic injection, it is less convincing for several reasons:

- the solid fractions are at the lower bound of the domain of application of this kind of model ;
- the flow is quite fast in thixotropic injection ;

- because of this flow velocity, liquid segregation is not observed generally during the process.

## 2.3 Viscoplastic models

These models are common to model hot deformation of metallic alloys and are the more frequently used for thixotropic application. In the literature, they are generally of power law type. The most frequent is the Norton-Hoff law :

$$s = \sigma + pI = 2\eta_{app}\dot{\epsilon} \quad (4)$$

where the apparent viscosity  $\eta_{app}$  is a function of the viscoplastic consistency  $K$ , times an exponential function of the equivalent strain rate  $\bar{\epsilon}$  :

$$\eta_{app} = K(T)(\sqrt{3}\bar{\epsilon})^{m(T)-1} \quad (5)$$

The coefficient  $m$  is the strain rate sensitivity of the flow stress and depends - like the consistency  $K$  - on the temperature  $T$ . The uniaxial expression of the constitutive model is then given by:

$$\bar{\sigma} = \sqrt{\frac{3}{2}} s : s = K(\sqrt{3})^{m+1} \bar{\epsilon}^m \quad (6)$$

- For hot solid state, the value of the strain rate sensitivity  $m$  is typically 0.2.
- For liquid state,  $m=1$ , which corresponds to the pure Newtonian behaviour. The apparent viscosity  $\eta_{app}$  is then equal to  $K(T)$ .
- In semi-solid state, the strain rate sensitivity will vary from 1 (zero solid fraction) to a value around 0.2 (solid fraction 1).

This kind of model presents the great advantage of being simple. It should be noted however that often, it is used in an over-simplified version, assuming constant coefficients  $K$  and  $m$ .

In conclusion of this brief review, a viscoplastic model with temperature dependent coefficients is probably the best compromise between a realistic approach to the semi-solid material flow and the practical capability of the determination of the parameters.

## 3 EXPERIMENTAL WORK: RHEOLOGICAL TEST

The objective was to carry out rheological experiments, reproducing the thixotropy forming process conditions, in order to determine a constitutive equation and the values of its parameters for the studied aluminium alloy (A356). A dedicated original experiment (thixotropic injection of a disk) has been developed at Cemef, and the constitutive parameters have been determined thanks to finite element simulations of experimental injection tests.

The rheological test consists basically of an axisymmetrical injection of preheated material between two parallel discs (fig. 1). This geometry has been chosen because it has been thought representative of the actual thixotropic injection process and, at the same time, it is simple enough to permit instrumentation with sensors and numerical modelling by finite element technique.

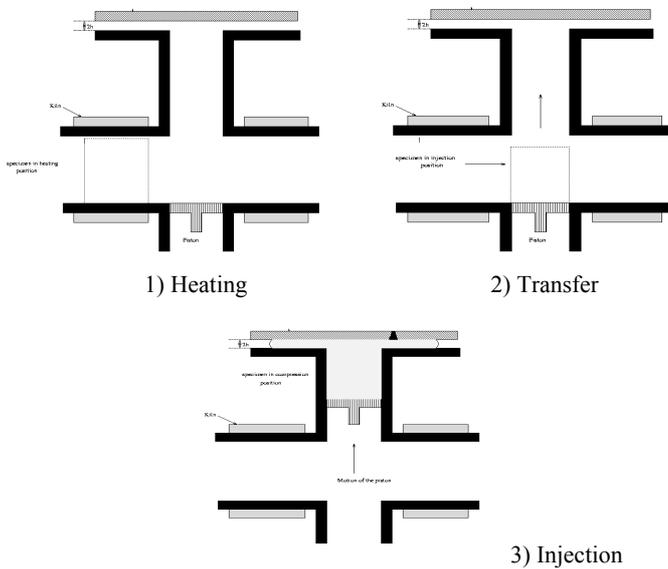


Figure 1. Principle of the rheological test.

### 3.1 Description of the set-up

The set-up has been developed on an hydraulic tension-compression INSTRON machine (fig. 2). The piston is connected to the ram of the machine, permitting a control of its velocity and the measurement of the piston force by a load cell.

As illustrated by fig. 1, a billet section of the studied material is first heated during 30 mn, to reach a prescribed temperature between 570 and 580°C, corresponding to a solid fraction of about 0.6. The specimen is then transferred laterally on to the piston head. Then the piston moves upwards at a maximum velocity of 160 mm/s, while the piston force is recorded. The deformation of the material begins as it touches the surface of the upper disk. The material is then forced to fill the space between the two horizontal disks.

The mould is made of a refractory steel. This material is usually used for injection moulding of light alloys. It has a good resistance to oxydation at hot temperature. The mould is pre-heated at 220°C and lubricated.

The kiln is made of the same refractory steel. It has been recovered by a surface treatment in order to limit friction and attack by liquid aluminium. The heating power is 2.4 kW.

The piston is made of copper-beryllium alloy.

The semi-solid specimen is a cylindrical A356 aluminium alloy of diameter 25 mm and height 45 mm.

The duration of the pre-heating of the semi-solid specimen is about 20 mn. Three initial temperatures have been used: 575, 580 and 585°C.

Experiments have been carried out at two different piston velocities (160 mm/s and 80 mm/s) and two mould thickness (2 mm and 4 mm).

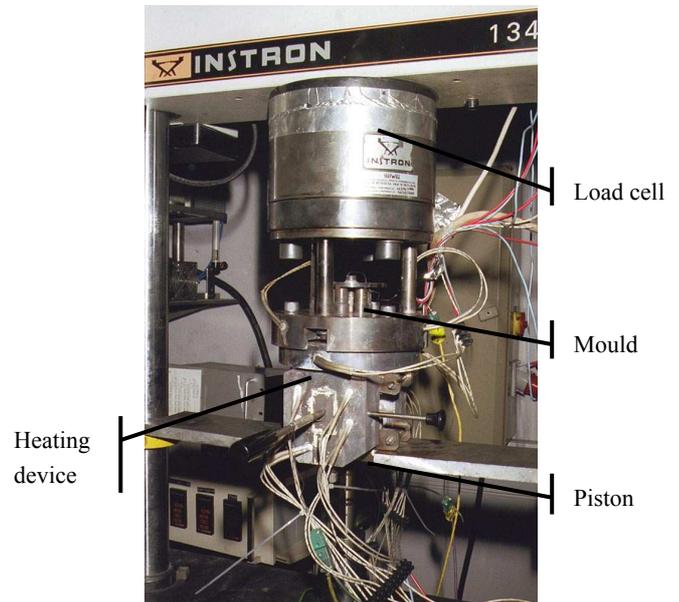


Figure 2. Experimental rheological set-up.

### 3.2 Results of the experiments

In order to achieve good results, it was necessary to redesign a part of the original set-up, namely the heating system. Initially, it was axial and induced thermal dilatation effects, causing injection problems: non axisymmetry of flow, artificial friction, deterioration of the surface quality of the injection channel... A new design, with lateral separate heating, permitted to overcome these problems. Lubrication problems have also been solved thanks to our partner Stampal. The aspect of injected specimens is shown in fig. 3.

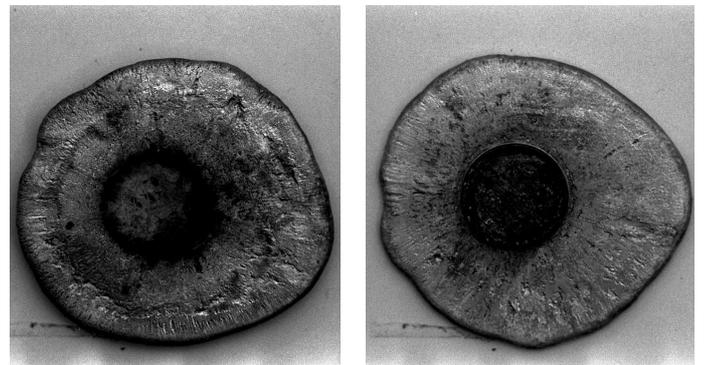


Figure 3. Two examples of injected specimens.

The injection parameters were the following:

- Initial specimen temperature: 575, 580, 585°C
- Mould temperature: 220°C
- Piston velocity: 80 and 160 mm/s
- Mould thickness: 2 and 4 mm

An example of recorded injection force is plotted on fig. 4. The evolution of the force during the tests consists basically of two steps. A first step during which the force level is rather low. The semi-solid slurry flows into the mould without any major resistance. A second step during which the force increases dramatically. This corresponds to the influence of the heat transfer. The material being cooled down by the mould, some regions of the flow have a higher viscosity, giving rise to a much more important deformation power, *i.e.* an increased injection strength. The lower the initial temperature, the higher the injection strength. As expected, this influence is reinforced when the flow velocity is lower and also when the mould thickness is lower.

It was also noted that the injection forces are higher with a 2 mm mould thickness for similar temperature and velocity conditions. The average equivalent strain rate is indeed higher in this case, causing an increase in the deformation power and so the injection force. Such an effect is consistent with the use of a viscoplastic power law. In addition, it was noted that the distinction between the two main stages of the evolution of the injection force (low constant force / rapidly increasing force) is favoured by a decrease of the flow velocity and a decrease of the mould thickness. This is again an illustration of the main role of heat transfer, which will be confirmed by the finite element simulation of the tests performed with Forge2<sup>®</sup>.

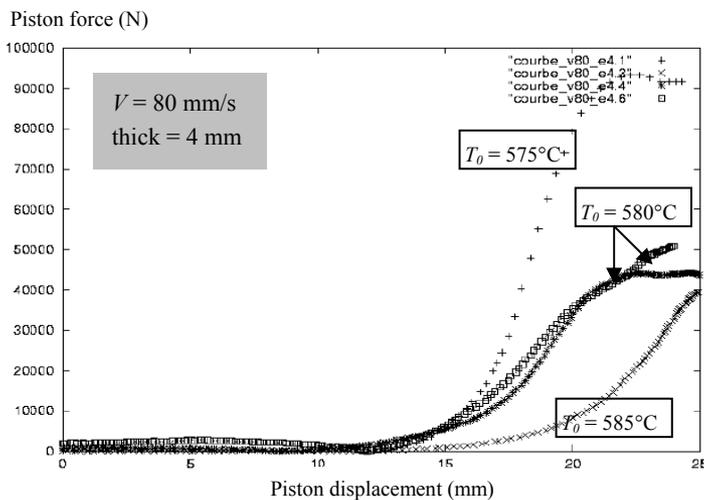


Figure 4. Injection force (N) vs piston displacement (mm). Piston velocity: 80 mm/s, mould thickness: 4 mm.

#### 4 IDENTIFICATION OF PARAMETERS

Once the constitutive model has been chosen (the temperature dependent viscoplastic model), and the experimental data base has been constituted, the identification task consists in determining the temperature dependency of coefficients  $K$  and  $m$ .

#### 4.1 Typical evolution of the viscoplasticity parameters in the solidification interval

From the literature, the qualitative temperature dependence of coefficients  $K$  and  $m$  is known. In the solidification interval, when  $T$  decreases,  $K$  increases by several orders of magnitude, from the value in the liquid state (liquid viscosity) up to the value in the solid state. Coefficient  $m$  decreases from 1.0 in liquid state (Newtonian behaviour) down to a typical value of 0.2 in solid state. In the semi-solid state, there is a critical temperature  $T_C$  - the *coherency temperature* - below which the semi-solid medium can support stresses, due to the setting up of a continuous solid skeleton. The semi-solid metal which previously behaved as a fluid with solid particles in suspension, changes its behaviour into the one of a solid saturated with liquid. Therefore below this coherency temperature, the apparent viscosity increases a lot and depends more and more on the strain rate (as  $m$  decreases).

When the semi-solid material flows in the rheological set-up described above, its temperature and strain rate vary. The associated variation of apparent viscosity within the solidification interval will give rise to changes in the value of the piston force. By measuring the evolution of the strength during tests performed at different initial temperature and different flow velocities, it should be possible to deduce the temperature dependence of  $K$  and  $m$ . As the material flow is non isothermal and quite complex, a numerical simulation of the test is necessary. We have used the software Forge2<sup>®</sup> to proceed to these simulations (example of computation in fig. 5). These simulations were used first to fix the heat transfer coefficient between specimen and mould  $h$  ( $1000 \text{ W}\cdot\text{m}^{-2}\cdot\text{K}^{-1}$ ).

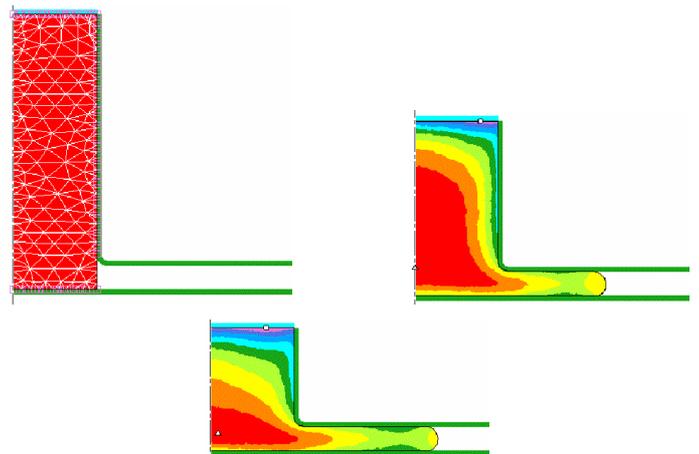


Figure 5. Simulation of the rheological test with Forge2<sup>®</sup> (including temperature distribution).

#### 4.2 Results of identification

The identification of the parameters has been carried out on a trial-and-error basis. From a set of parameters taken from the literature on different aluminium alloys, the values have been modified to provide a

good fit between the injection forces computed by Forge2<sup>®</sup> for the different tests and the measured forces (fig. 6). There is a reasonable agreement for all injection conditions. Due to the complexity of this rheological identification (complex flow, temperature coupling), this is quite satisfying.

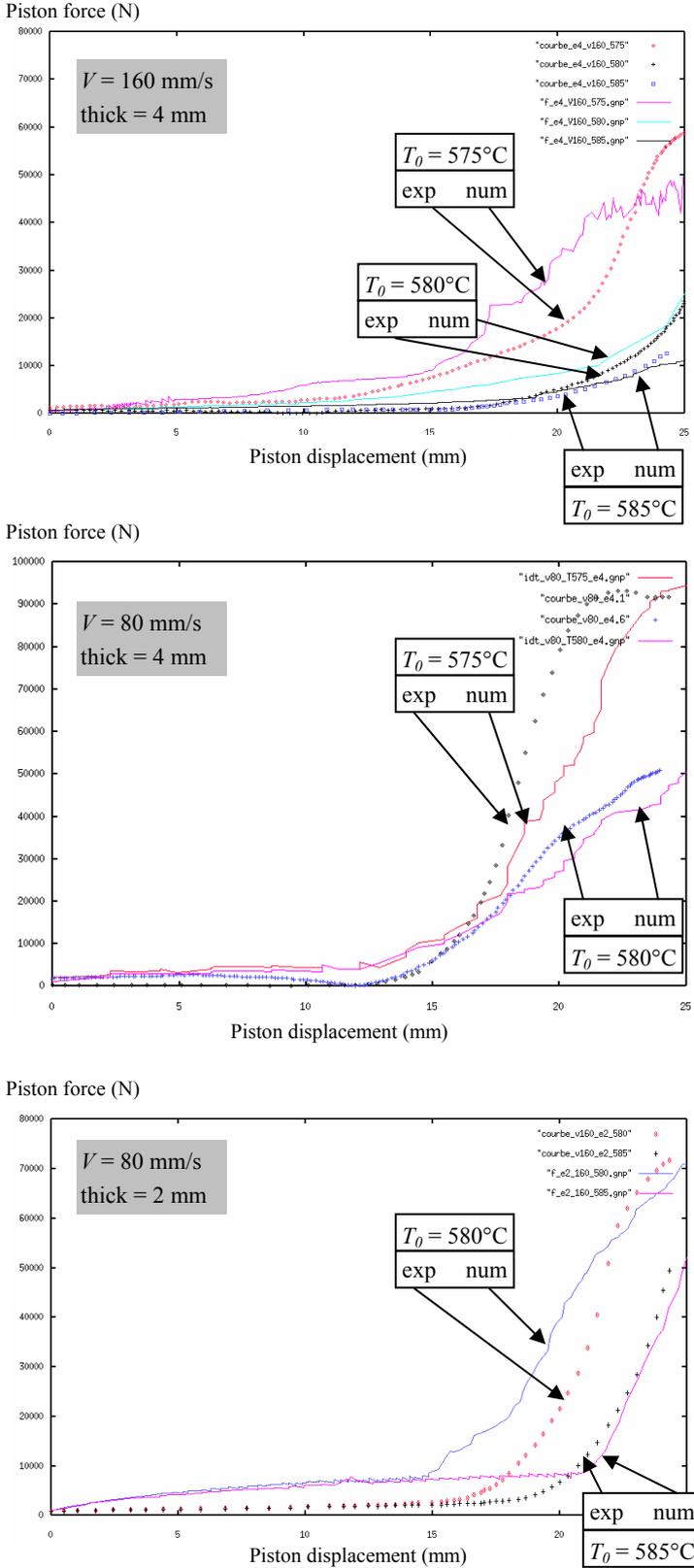


Figure 6. Comparison of measured injection forces and predicted ones.

All the major characteristics of the force curves are reproduced correctly: strength level in the initial

stage and in the final one, time of transition between the two regimes, rate of increase of the force. This means that the identification, although somewhat rough, is reliable within the frame of the experimental data base. The identified coefficients  $K(T)$  and  $m(T)$  are given in table 1. Figure 7 gives the dependency of the apparent viscosity with respect to temperature and strain rate.

$T$ (°C)	540	560	567	570	585	600	620
$K$ (Pa.s <sup>m</sup> )	$2 \cdot 10^7$	$2 \cdot 10^7$	$5 \cdot 10^6$	$5 \cdot 10^5$	20	0.1	0.1
$m$	0.2	0.2	0.3	0.4	0.9	1.0	1.0

Table 1: Set of identified parameters  $K(T)$  and  $m(T)$  for A356 alloy (values under 560°C (solid state) and over 600°C are out of the scope of this study).

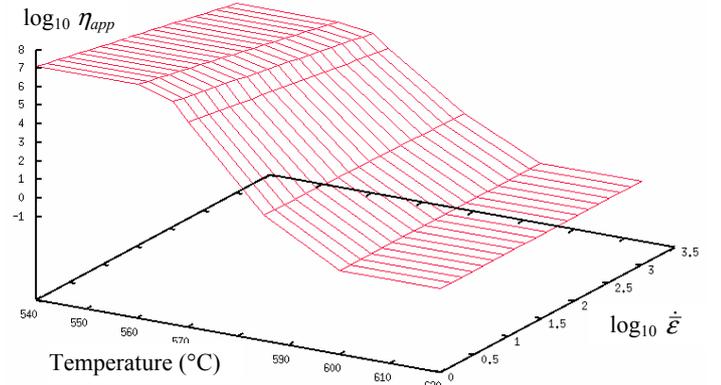


Figure 7. Variation of the apparent viscosity with respect to temperature and strain rate.

## 5 FEM MODELLING OF THE THIXOFORMING PROCESS

The Rem3D<sup>®</sup> software has been initially developed to model the injection of thermoplastic polymers (Pichelin & Coupez 98). The generalized Stokes equations are solved by means of a velocity-pressure formulation:

$$\begin{cases} \nabla \mathbf{v}^* \int_{\Omega} s : \dot{\boldsymbol{\varepsilon}}^* dV - \int_{\Omega} p \nabla \cdot \mathbf{v}^* dV - \int_{\partial\Omega} \mathbf{T} \cdot \mathbf{v}^* dS - \int_{\Omega} \rho \mathbf{g} \cdot \mathbf{v}^* dV = 0 \\ \nabla p^* \int_{\Omega} p^* \nabla \cdot \mathbf{v} dV = 0 \end{cases} \quad (7)$$

where  $s$  is given by (1-2),  $\mathbf{T}$  is the external stress vector,  $\rho$  the specific mass,  $\mathbf{g}$  the gravity vector and  $\mathbf{v}$  the velocity field. The resolution is obtained after a spatial discretization using P1+/P1 tetrahedral elements. The formulation is Eulerian: the cavity to be filled in is meshed and the evolution of the free surface at each time increment is found by the resolution of the following front tracking equation:

$$\frac{df}{dt} = \frac{\partial f}{\partial t} + \nabla f \cdot \mathbf{v} = 0 \quad (8)$$

where  $f$  is the characteristic function of the material ( $f = 1$  in zones filled in,  $f = 0$  elsewhere). This advection equation is solved by a Taylor-Galerkin formulation applied to a P0 field for  $f$ .

In order to model thixoforming, the inertia terms have been added to the previous formulation by addition to the left hand side of (7a) of:

$$\int_{\Omega} \rho \gamma \cdot \mathbf{v}^* dV = \int_{\Omega} \rho \left( \frac{\partial \mathbf{v}}{\partial t} + (\nabla \mathbf{v}) \mathbf{v} \right) \cdot \mathbf{v}^* dV \quad (9)$$

In a first approach, these terms have been implemented with an explicit scheme, using the following time discretization:

$$\frac{d\mathbf{v}}{dt} = \frac{\partial \mathbf{v}}{\partial t} + (\nabla \mathbf{v}) \mathbf{v} = \frac{\mathbf{v}^t - \mathbf{v}^{t-\Delta t}}{\Delta t} + \nabla \mathbf{v}^{t-\Delta t} \cdot \mathbf{v}^{t-\Delta t} \quad (10)$$

The version of Rem3D<sup>®</sup> that has been used does not account for the heat transfer within the mould. The extracted heat flux through the material/mould

contact surface can only be expressed with a constant mould temperature. This provides a considerable overestimation of the extracted heat and of the cooling down of the material. Therefore, it has not been yet possible to test the temperature dependent constitutive equation identified in section 4. Only constant viscoplastic coefficients have been used. However, those preliminary simulations have shown a reasonable agreement with interrupted formings for an industrial test part (fig. 8). It can be seen that the computed flow is different from the real one at the beginning of the filling. In the real flow, the alloy tends to go straight on into the mould, whereas in the simulation, it tends to spread laterally. This is probably due to the fact that only a sticking contact condition can be used in Rem3D<sup>®</sup> at the present time. However, later on, the two flows are similar. The mesh of the cavity comprises 21,000 nodes and 100,000 elements and the computation time is about 20 h on one node of a Sun4000 computer.

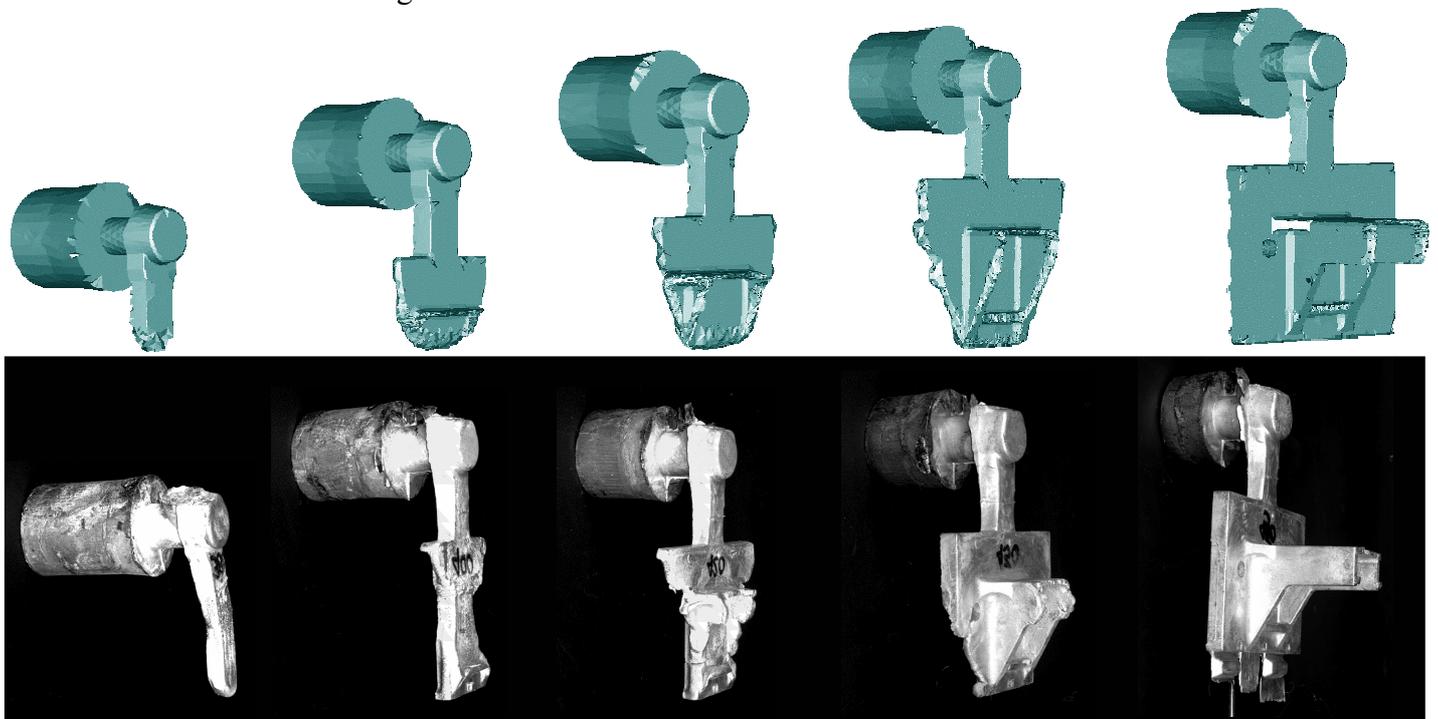


Figure 8. Simulation of an industrial test part with Rem3D<sup>®</sup>. Comparison between the finite element computation (first line) and the experimental incomplete formings (second line). NB: the process times corresponding to each couple of figures is not exactly the same for computations and experiments.

## ACKNOWLEDGEMENT

The present study has been conducted in the frame of the European project Adform (BE96-3106) with CR Fiat, Regienov Renault, Aerospatiale Matra, Stampal, Derbi and the University of Barcelona as partners.

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