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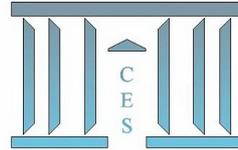
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Optimal Immigration Policy When the Public Good Is Rival

Stefano BOSI, Eleni ILIOPULOS, Hubert JAYET

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Optimal Immigration Policy When the Public Good Is Rival*

Stefano Bosi[†], Eleni Iliopoulos[‡] and Hubert Jayet[§]

November 24, 2010

Abstract

In this model, we characterize optimal immigration and fiscal policies in presence of a rival public good and heterogeneous discounting. Surprisingly, even if the government is benevolent towards natives only, it is optimal to keep borders open. Indeed, in the long run, patient natives hold the whole stock of capital, while impatient immigrants work. Moreover, since capital intensity is stationary, capital per native, consumption and the public good increase with the number of (immigrant) workers. This positive effect offsets the disutility deriving from the congestion of the public good. However, when we account for the costs associated to cultural heterogeneity, we find that it is optimal to regulate immigration inflows. We also interpret the long-run sensitivity of the optimal policy mix with respect to the fundamentals.

Keywords: heterogenous discounting, public good, immigration policy, cycles.

JEL Classification: D91, E32, H41, J61.

1 Introduction

"When a man migrates from the one country to another, he abandons his share of public property [...] in the former country and acquires a share of public property in the latter [...] imposing a cost upon the original residents of the country to which he goes [...]; however the "public property" effect is contrasted with other sources of costs and benefits of migration" (Usher, 1977).

The economic effects of migration flows into developed economies have been widely analyzed by a rich literature.¹ Indeed, migration inflows are either beneficial or harmful depending upon manifold contingent features of both host countries and immigration flows. As Nannestad (2006) suggests, one of the main issues concerns the composition of migration flows. Clearly, in presence of adverse-selection effects, migration flows are not welfare improving.² Secondly, the economic impact of immigrants depends on the extent and the features of their absorption into the labor market.

Moreover, one of the most controversial issues of debate in Western countries concerns the economic drain of immigrants on welfare-states social programs. While there is a general agreement on the fact that, if immigrants are not absorbed in host-countries labor force, they are a burden on natives, little agreement has been found on the effects of perfectly absorbed and substitutable inflows of workers. In his article, Wildasin (1994) provides a characterization of the income distribution frontier in host countries both in presence and in absence of migration inflows; his static analysis shows that, if immigrants are beneficiaries of income transfers, migration can lead to Pareto-inferior outcomes in all destination regions. Michael and Hatzipanayotou (2001)

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¹See Borjas (1994) for a general discussion on welfare implications deriving from immigration. See Nannestad (2006) for a focus on welfare implications of immigration when destination countries are welfare states.

²According to the "welfare magnets hypothesis" (Borjas, 1999b), generous welfare states could act as magnets for "bad" immigrants.

provide a more detailed focus on welfare policies; their analysis shows that the effect of immigrants crucially depends on public good provision and on how welfare policies are financed. When consumption tax revenues finance the provision of a public good, marginal migration increases natives' welfare in destination countries; when the consumption tax is equally redistributed, marginal migration decreases the welfare of natives. More ambiguous outcomes arise from tariffs revenue. However, their static analysis provides a one-shot picture and does not allow for long term considerations. Using a similar framework in presence of internationally mobile capital, Michael (2003) shows that income taxes and transfers can improve the welfare of natives even if immigrants provide smaller tax revenues to the state; the hypothesis of international capital mobility is thus crucial. The key role of capital is also stressed by Usher (1977); while his back-to-envelope calculations suggest that immigrants cannot but be a drain on welfare states, he stresses the importance of a long-term perspective.

In light of the above considerations, we focus on long-term optimal immigration policies in welfare states. We consider a dynamic general equilibrium framework with capital accumulation. Indeed, another crucial factor for evaluating the impact of migration flows into destination countries is the time perspective. Most of the existing literature evaluating welfare effects of immigration tends to see the phenomenon as a static one; therefore, the focus is on a static comparison between pre and post-immigration welfare in the country. These works generally find a significant increase in the post-immigration domestic output. More precisely, according to Borjas (1999b), the "immigration surplus"³ is conditional upon the fact that at least some natives see their wage decrease with respect to the pre-immigration situation. The extent of these effects has been questioned by using a dynamic framework. In a neoclassical growth model of overlapping dynasties, Ben-Gad (2004) shows that both the benefits and the decrease of wages are much smaller; his elegant analysis demonstrates that static models that do not incorporate endogenous capital accumulation, elastic labor supply and transition paths, do overestimate the impact of immigration on both the distribution of income and the welfare of natives.

In our exercise, we assume the perspective of a policy maker who aims at maximizing the long run welfare of the native population. Her/his optimization program takes into account the arrival of foreign workforce: its impact on natives' incomes and on natives' public-good consumption. As in Michael (2003), we suppose that the government provides his citizens with a rival but non-excludible public good;⁴ marginal immigration has thus a negative effect on natives' public-good utility. This effect is however offset by the contribution of immigrants (who are also allowed for a degree of free riding) to government revenues. The determination of the optimal immigration policy is linked to the determination of the optimal fiscal policy.

Optimal fiscal and immigration policies result from the incorporation of natives' utility maximization programs into the government welfare function; immigrants' optimal decisions on saving and consumption are also considered. Notice in particular that we account also for natives' attitudes towards migration. Indeed, the recent political debate in Europe shows that even if immigration flows have been for most countries "[...] a boon for their economy, [...] hostility to immigration is becoming a mainstream [...]" (*The Economist*, 2008). In support of this evidence, O'Rourke and Sinnott (2006) and Gang, Rivera-Batiz and Yun (2002) stress that natives' preferences towards immigration are significantly affected by nationalistic sentiments. On the other hand, Bisin and Verdier (2000) and Bisin, Topa and Verdier (2004) remark the significance of immigrants' assimilation resilience. These considerations suggest that the impact of foreign workforce on host countries goes beyond issues related to income distribution; immigration inflows represent a cultural challenge for host countries' natives and, in this light, a disutility.⁵ In order to account for the disutility related to the cultural challenge, we introduce cultural heterogeneity costs in natives' utility function.

Our analysis aims at contributing to the actual debate on optimal immigration policies in the Western world.⁶ It allows us to characterize optimal immigration and fiscal policies. It shows that even if policy-makers optimize the welfare of natives only and account for both the effects of congestion (of public goods) and of hostility towards immigrants, it is optimal to let (at least some) immigrants enter the country (by regulating flows). The analysis also shows that, in absence of costs of cultural heterogeneity, it is optimal for natives to let borders open. Indeed, because of the heterogeneous discounting, patient natives hold the entire stock of

³It is defined as the difference between the post-immigration increase in output and the share of output earned by immigrants.

⁴Think for instance of public health and education: too many recipients of these services can negatively affect the quality of the service for them (congestion).

⁵Alternatively, according to Nannestad (2004), integration can be seen as a public good.

⁶Throughout the analysis, our assumptions will be based on a relatively conservative stand. Nonetheless, immigration results to be optimal.

capital in the long run, while the impatient immigrants work. Since capital intensity is stationary, capital per native eventually increases with the number of (immigrant) workers; thus, in turn, natives' consumption of private and public good increases with their capital stock. This positive effect offsets the disutility deriving from the congestion of the public good.

In order to account for the possibility of costs of cultural heterogeneity, we focus on homogeneous functional forms; costs of cultural heterogeneity lead us to characterize an interior solution for frontier openness. We show that an increase in the degree of intolerance has a negative effect on the optimal number of immigrants, but, surprisingly, no effects on the optimal fiscal policy. The impact of an increase in free riding and on the immigration side of the optimal policy mix is not univocal and depends on the degree of substitutability between the public and the private good in natives' consumption: given the stronger contribution of immigrants to the provision of the private good, the higher substitutability, the stronger the incentive to let immigrants come in (even if the provision of the public good diminishes).

The article is organized as follows. In the next section, we introduce the model, while in the third we define the intertemporal equilibrium. Section 4 is devoted to the optimal fiscal and immigration policies in general. An example with explicit functional forms is studied in Section 5. Concluding remarks are provided in Section 6.

2 The model

In this article, we focus our attention on a rich welfare-state country. The country is populated by n_1 identical infinite-lived natives and n_2 infinite-lived immigrants. For the purpose of contributing to the current debate on migration policies, we will assume a relatively conservative stand; incorporating in our analysis elements that are currently under discussion will indeed contextualize and remark the importance of our results.

We assume that when immigrants enter the country they do not dispose of (human or physical) capital; however, once arrived in the destination country, they can choose to accumulate capital. Conversely, natives are endowed with human and physical capital, but supply no (unskilled) labor.⁷ We also suppose that natives are more patient than immigrants, *i.e.*, they have a higher discount rate. This assumption is consistent with data; indeed, as stressed by Lawrance (1991), rich individuals show higher discount rates than the poor ones.⁸ Finally, immigrants supply inelastically one unit of labor.

Both capital and labor are employed in the production of a privately provided consumption good; an imperfect public good, which is non-excludible but rival, is also provided and consumed.

We assume that the policy maker of the domestic country is benevolent toward the native population only; s/he maximizes a domestic welfare function that only accounts for natives' utility. S/he chooses the fiscal pressure (and thus, the amount of public good) and the number of immigrants to allow to enter the country.

In the following, all variables referring to natives are labelled with 1; all variables referring to immigrants are labeled with 2. In addition, k_i , c_i and g_i (referred to the i th agent) denote respectively the amount of capital, the consumption good and the public good. Both goods are produced with the same technology and, therefore, have the same price (normalized to one for simplicity). Finally, β_i and u_i will denote the discount factor and the utility function, respectively.

2.1 Firms

Suppose that in every period a competitive (representative) firm maximizes the profit $F(K, L) - rK - wL$ taking factor prices (the interest rate r and the wage rate w) and technology as given. Let $f(k) \equiv F(k, 1)$,

⁷In practice, we assume that the native population of Western developed countries is relatively more endowed of physical and human capital. Here we could alternatively suppose that natives do also supply inelastically one unit of labor. As long as the native's capital endowment is sufficiently large, this wouldn't affect our qualitative results.

⁸According to Fisher: "A small income, other things being equal, tends to produce a high rate of impatience, partly from the thought that provision for the present is necessary both for the present itself and for the future as well, and partly from lack of foresight and self-control" (Fisher 1930, p. 73). Time preference can also be related to religious and cultural factors.

Notice that in a two-country Ramsey model, discount factors heterogeneity among countries implies that in the long run the most patient country attracts the whole stock of world capital. Thus, consumption in the other country tends to zero. The country where inhabitants are the most patient ones is thus, *ex post*, the richest. This result would be mitigated by presence of credit constraints (see, among others, Barro and Sala-i-Martin (2003)).

where $k \equiv K/L$ is capital intensity. The constant return to scale production function $F(K, L)$ satisfies additional neoclassical features.

Assumption 1 (*technology*) F is homogeneous of degree 1 with $F \in C^2$, $F_K > 0$, $F_L > 0$, $F_{KK}F_{LL} > F_{KL}^2$ for every (K, L) such that $K, L > 0$.

Assumption 2 (*Inada*) Moreover, $f(0) = 0$, $\lim_{k \rightarrow 0} f'(k) = +\infty$ and $\lim_{k \rightarrow +\infty} f'(k) = 0$.

Under Assumption 1, profit maximization requires

$$r = f'(k) \equiv r(k) \quad (1)$$

$$w = f(k) - kf'(k) \equiv w(k) \quad (2)$$

Second-order restrictions for profit maximization are also satisfied under Assumption 1.

The following elasticities summarize the technological features and will play a role in the sequel:

$$\varepsilon_r(k) \equiv \frac{kr'(k)}{r(k)} = -\frac{1-s(k)}{\sigma(k)} < 0 \quad (3)$$

$$\varepsilon_w(k) \equiv \frac{kw'(k)}{w(k)} = \frac{s(k)}{\sigma(k)} > 0 \quad (4)$$

where $s(k) \equiv f'(k)k/f(k) \in (0, 1)$ is the capital share in total income and $\sigma(k) \equiv [s(k) - 1]f'(k)/[kf''(k)] > 0$ is the elasticity of capital-labor substitution.⁹

2.2 Consumers

We focus now the attention on the inhabitants of the host country. The country is populated both by natives and immigrants; both types of agents choose their optimal consumption levels, given their own budget constraint. Notice in particular that agents' utility depends on consumption levels of both the consumption good – that is produced by firms – and on the public good – provided by the government.

Consider first the perspective of native citizens. Their utility function positively depends on the consumption good (c_{1t}) and on public good levels (g_{1t}). The arrival of newcomers entails a cultural challenge. In order to proxy for the disutility associated to it, we have introduced costs of cultural heterogeneity into their utility function. We assume that this disutility is a positive function of the number of foreigners (n_2) in total population ($n_1 + n_2$). This assumption is supported by historical considerations. Indeed, when in 1913 this ratio reached its peak (15%) in US, hostility to immigration pushed the population to demand (and eventually obtain) tough limits on migration that slowed the inflow for decades (*The Economist*, 2008).¹⁰ More precisely, in order to capture the increasing hostility associated to a larger amount of newcomers in total population, we introduce convex subjective costs of cultural heterogeneity, v . Natives solve the program:

$$\max_{(k_{1t+1}, c_{1t})_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta_1^t [u_1(c_{1t}, g_{1t}) - \mu^t v(x)]$$

subject to their budget constraint.

We denote by

$$x \equiv \frac{n_2}{n_1 + n_2} \in (0, 1) \quad (5)$$

the share of immigrants in total population; it represents a measure of cultural diversity. We will show in the following that the density of immigrants (5) plays a role in public decision making.

We denote by $\mu < 1/\beta_1$ the discount factor associated to the costs of cultural heterogeneity; it captures the evolution of hostility over time. Diversity costs are a positive function of the number of newcomers but

⁹When s is constant, the technology is Cobb-Douglas. When σ is constant, the technology is CES (Constant Elasticity of Substitution).

¹⁰The evidence of Canada and US southern states shows also that for high levels of the ratio foreigners/total population, the national language of a country can also be challenged: the higher the ratio, the tougher the cultural challenge.

decrease during time as soon as immigrants gradually mix with the native population. Notice however, that this does not necessarily imply that immigrants are assimilated over time (*i.e.* converge) to natives. Indeed, while recent empirical evidence on the state of New York shows that hostility is only addressed towards recent immigration inflows, the melting-pot assimilation pattern has proved wrong (Bisin and Verdier, 2000). More generally, the case of an increasing negative bias towards immigrants is captured by $\mu > 1$, while $\mu < 1/\beta_1$ ensures the convergence of the series.

Natives are subject to a sequence of budget constraints:

$$c_{1t} + k_{1t+1} - (1 - \delta)k_{1t} \leq (1 - \tau)r_t k_{1t}$$

where τ is the constant tax rate and δ is the depreciation rate, with $k_{10} > 0$. We assume therefore that natives have positive initial endowments of capital. Indeed, capital should be here interpreted as physical, financial but also as human capital. Therefore, starting from the first period, natives do supply a positive quantity of (physical and human) capital.

We now focus the attention on the latecomer population. We suppose that all immigrants have access to both the consumption good and the public good;¹¹ however, in order to capture phenomena such as clandestine immigration and black labor-market networks, we allow immigrants to contribute only partially to government revenues. Clearly, if latecomers can enjoy the provision of public goods without paying for them, this creates a potential free-riding problem for welfare states.¹²

We denote by $\pi \in [0, 1]$ the degree of participation to government's public expenditure; equivalently, the free-riding degree is represented by $1 - \pi$. Clearly, one of these possibilities may arise: (1) $\pi = 1$ (citizens and immigrants pay the same tax rate), (2) $\pi < 1$ (free riding). The utility of immigrants positively depends on the consumption good, c_{2t} , and on the public good, g_{2t} , *i.e.*, they solve the program:

$$\max_{(k_{2t+1}, c_{2t})_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta_2^t u_2(c_{2t}, g_{2t})$$

under a sequence of budget constraints:

$$c_{2t} + k_{2t+1} - (1 - \delta)k_{2t} \leq (1 - \pi\tau)(r_t k_{2t} + w_t)$$

with $k_{20} = 0$, implying that when immigrants enter the country they have no capital. However, once entered the country, it is possible for them to enjoy greater incomes and start accumulating (human and physical) capital.

We introduce standard assumptions on the structure of preferences.

Assumption 3 (*preferences*) $u_i \in C^2$, $u_{ic} > 0$, $u_{ig} > 0$, $u_{icc} < 0$, $u_{icc}u_{igg} > u_{icg}^2$ and $\lim_{c_i \rightarrow 0} u_{ic}(c_i, g_i) = \lim_{g_i \rightarrow 0} u_{ig}(c_i, g_i) = +\infty$ for every (c_i, g_i) with $c_i, g_i > 0$. In addition, $v \in C^2$, $v > 0$, $v' > 0$, $v'' \geq 0$ for every $x > 0$.

In what follows, we introduce debt limits. More in particular, we assume for simplicity that both immigrants and natives cannot have access to credit. It is possible to show that in the long-run equilibrium the debt limit is binding only for immigrants (see the discussion in the sequel).¹³ This equilibrium result is consistent with reality. Indeed, given that access to credit market is strongly related to the value of collaterals,¹⁴ immigrants have a limited access to the credit market.

Assumption 4 (*borrowing constraints*) $k_{it} \geq 0$ for $i = 1, 2$ and $t = 0, 1, \dots$

¹¹Myers and Papageorgiou (2000) show that, if illegal immigrants can be excluded from the redistribution of public services, it is optimal for the host country to impose a zero quota for legal immigration and spending nothing to enforce it, letting therefore illegal immigrants enter the country.

¹²Schultz and Sjöström (2001) propose the accumulation of public debt as a possible tool to mitigate the free-rider problem. Indeed, if the public good is financed by local debt, immigrants will eventually need to pay it by sharing the debt burden. Their two-periods analysis shows that in equilibrium there will be either too much debt or too little public good; this policy recommendation is far from being optimal.

¹³In equilibrium, this assumption (together with discounting heterogeneity) is equivalent to imposing debts limits on immigrants only.

¹⁴For some discussion and empirical evidence, see, among others, IMF (2008).

In additions agents are heterogenous: immigrants discount future more.

Assumption 5 (*heterogeneous discounting*) $\beta_1, \beta_2 \in (0, 1)$, $\mu \in (0, 1/\beta_1)$ and

$$\beta_2 < \left[1 - \delta + \frac{1 - \pi\tau}{1 - \tau} \left(\frac{1}{\beta_1} - 1 + \delta \right) \right]^{-1} \quad (6)$$

Assumptions 4 and 5 are important and deserve some comments. Unlike the case with one representative agent, discounting heterogeneity gives rise to borrowing transactions. Borrowing allows impatient agents to tilt consumption toward the present and, when labor supply is elastic, to shift labor effort towards the future. Thus, in absence of debt limits, their consumption vanishes asymptotically in the long run.¹⁵ Conversely, in presence of borrowing constraints, debt accumulation is limited; there exists thus a stationary state where the consumption level of impatient agents is positive. Under borrowing constraints, persistent cycles also arise. Stationary and non-stationary equilibria are characterized in the seminal contribution by Becker (1980) and Becker and Foias (1987, 1994), and in Sorger (1994).

Focus now on condition (6).

(1) It generalizes the basic restriction in Becker (1980): indeed, the original assumption $\beta_2 < \beta_1$ is recovered, by setting $\pi = 1$ (no free riding).

(2) It imposes a restriction on free riding, given β_1, β_2 . Indeed, we can rewrite inequality (6) in terms of π , *i.e.*:

$$\pi > \frac{1}{\tau} \left[1 - (1 - \tau) \frac{1/\beta_2 - 1 + \delta}{1/\beta_1 - 1 + \delta} \right] \equiv \pi^*$$

(3) It implies discounting heterogeneity. Indeed $\pi^* < 1$ implies $\beta_2 < \beta_1$.

Assumption 6 (*participation degree*) $\pi > \max \{ \pi^*, 1 - \sigma / (1 - s) \}$.

Assumption 6 reduces to $\pi > \pi^*$ if inputs are sufficiently substitutable ($\sigma > 1 - s$).

Notice finally that the above assumptions have important implications for immigrants' capital endowments as shown in the following proposition.

Proposition 1 *Under Assumptions 1-5, if k_{10} is sufficiently close to the steady state value (expression (19) below), then $k_{2t} = 0$, for every $t \geq 0$.*

Under inequality (6), the patient agents (the natives) hold the entire stock of capital in the long run, while the impatient agents (the immigrants) need to work. As shown by Becker (1980), the heterogeneous discounting (*i.e.* the different degree of impatience) determines the emergence of a "dominant consumer", (that is, a class of capitalists). Notice that, even if immigrants enter the country and finish their life with no capital holdings, they could *a priori* choose to hold it during at least one period of their life.

Proof. The basic proof is in Becker (1980) and Becker and Foias (1987, 1994). For a formal proof in a model with elastic labor supply, see also Bosi and Seegmuller (2010). In order to ensure the existence of a dominant consumer under free-riding ($\pi < 1$), we generalize Becker's restriction $\beta_2 < \beta$. Comparing the Euler equation of the patient agent, *i.e.*:

$$\frac{u_{1c}(c_{1t}, g_{1t})}{u_{1c}(c_{1t+1}, g_{1t+1})} = \beta_1 [1 - \delta + (1 - \tau) r_{t+1}] \quad (7)$$

with that of the impatient agent:

$$\frac{u_{2c}(c_{2t}, g_{2t})}{u_{2c}(c_{2t+1}, g_{2t+1})} = \beta_2 [1 - \delta + (1 - \pi\tau) r_{t+1}]$$

¹⁵Few authors have focused on Ramsey models with heterogeneous discounting and without market imperfections. See Le Van and Vailakis (2003) for a model with inelastic labor supply and Le Van, Nguyen and Vailakis (2007) for a model with elastic labor supply.

we require at the stationary equilibrium:

$$\beta_2 [1 - \delta + (1 - \pi\tau) r] < \beta_1 [1 - \delta + (1 - \tau) r] = 1$$

Solving for r the equation on the right-hand side and substituting r in the inequality on the left-hand side, we get the inequality (6) in Assumption 5. ■

In the following, we will focus only on the equilibria around the steady state. In this case, Proposition 1 simplifies the immigrants' optimization program. It states that immigrants' relative impatience prompts them not to hold capital. Moreover, in every period, immigrants consume only and completely their labor income, *i.e.*:

$$c_{2t} = (1 - \pi\tau) w_t \quad (8)$$

We thus now shift the attention towards the optimization problem of the native population. Under Assumptions 3-5, natives' optimization program entails an Euler equation:

$$\frac{u_{1c}(c_{1t}, g_{1t})}{u_{1c}(c_{1t+1}, g_{1t+1})} = \beta_1 [1 - \delta + (1 - \tau) r_{t+1}]$$

a binding budget constraint:

$$c_{1t} + k_{1t+1} = [1 - \delta + (1 - \tau) r_t] k_{1t} \quad (9)$$

and a transversality condition: $\lim_{t \rightarrow +\infty} \beta_1^t u_{1c}(c_{1t}, g_{1t}) k_{1t+1} = 0$, which will be satisfied in a neighborhood of the steady state (since $\beta_1 < 1$).

Notice finally that the representative citizen's capital depends on the capital intensity as follows:

$$k_{1t} = k_t \frac{n_2}{n_1} = \frac{x k_t}{1 - x} \quad (10)$$

It is worthy to stress the difference between the capital intensity k_t and the capital k_{1t} owned by the natives. The former determines the price of factors, while the latter affects the demand for goods through the natives' budget constraint: $c_{1t} + k_{1t+1} = [1 - \delta + (1 - \tau) r_t] k_{1t}$. The mechanisms at work on the producer's and the consumer's sides are different. When $n_1 \neq n_2$ the difference between k_t and k_{1t} in equation (10) matters.

2.3 Government

Assume now the perspective of a benevolent government who chooses the optimal (fiscal and immigration) policy by taking into account the welfare of citizens only. Given that immigrants do not vote, we have realistically supposed that the policy maker is not influenced by their preferences.

Since the public good is rival but non-excludible, the individual amounts consumed by citizens and immigrants are the same, *i.e.*:

$$g_{1t} = g_{2t} = \frac{G_t}{(n_1 + n_2)^\rho} \quad (11)$$

where G is the aggregate public good, financed through the tax receipt:

$$G_t = \tau (n_1 r_t k_{1t} + \pi n_2 w_t) \quad (12)$$

and $\rho \in [0, 1]$ is the degree of rivalry of the public good: according to the well-known definition of Samuelson, the public good is pure when $\rho = 0$ while it is fully rival when $\rho = 1$.

By substituting (12) in (11) and using (10), we obtain:

$$g_{1t} = g_{2t} = \tau x (r_t k_t + \pi w_t) (n_1 + n_2)^{1-\rho} \quad (13)$$

The government maximizes a welfare function that corresponds with natives' utility function. Indeed, s/he takes into account their best replies by incorporating the equilibrium prices $r_t(\tau, n_2)$ and $w_t(\tau, n_2)$, and the equilibrium quantities $k_t = k_t(\tau, n_2)$ and $c_{1t} = c_{1t}(\tau, n_2)$ into her/his maximization program, *i.e.*:

$$\begin{aligned}
& \omega(\tau, n_2) \\
\equiv & \sum_{t=0}^{\infty} \beta_1^t [u_1(c_{1t}, g_{1t}) - \mu^t v(x)] \\
= & \sum_{t=0}^{\infty} \beta_1^t u_1\left(c_{1t}(\tau, n_2), \tau x[r_t(\tau, n_2)k_t(\tau, n_2) + \pi w_t(\tau, n_2)](n_1 + n_2)^{1-\rho}\right) - \sum_{t=0}^{\infty} (\mu\beta_1)^t v(x)
\end{aligned} \tag{14}$$

with respect to the fiscal policy τ and the immigration policy n_2 . The share of immigrants x depends on the choice variable n_2 according to definition (5).

3 Equilibrium

The mechanisms that lead the system to equilibrium are based on the following timing:

- (1) The government commits itself to a policy-mix, *i.e.* to a fiscal pressure τ and to allowing n_2 immigrants to enter.
- (2) Citizens and immigrants maximize their respective utility functions.
- (3) Markets clear.

3.1 Equilibrium in the labor market

We have assumed for simplicity that natives provide both human and physical capital; at the same time, immigrants provide (unskilled) labor. As above mentioned, as long as natives provide more human capital than labor, qualitative results wouldn't change if we allowed also natives to provide labor. In equilibrium, total labor coincides thus with the number of immigrants, *i.e.*, $L_t = n_2$.

Notice finally that we have implicitly assumed that all n_2 individuals are willing to migrate from their source country. We suppose in practice that immigrants' wage in their source country is lower than the one they receive in the destination country (for simplicity, we assume that their reservation wage is equal to zero) and labor supply is inelastic; therefore, they always find profitable to migrate.

3.2 Equilibrium in the goods market

Aggregate demand is the sum of aggregate consumption, investment and public spending; in each period, it is equal to aggregate production:

$$n_1 c_{1t} + n_2 c_{2t} + n_1 [k_{1t+1} - (1 - \delta) k_{1t}] + G_t = n_2 f(k_t) \tag{15}$$

By substituting (8), (10), (12) in (15), we obtain $r_t k_t + w_t = f(k_t)$, due to the homogeneity of the production function (Assumption 1). In other words, firm's technology, (1)-(2), and the lack of pure profit ensure the equilibrium in the goods market.

3.3 Intertemporal equilibrium

By replacing (1), (2), (10), (13) in the Euler equation (7) and in natives' budget constraint (9), we obtain the following two-dimensional dynamic system in the pair (k_t, c_{1t}) . k_t is a predetermined variable, while c_{1t} is a jump variable.

$$\frac{u_{1c}\left(c_{1t}, \tau x[r(k_t)k_t + \pi w(k_t)](n_1 + n_2)^{1-\rho}\right)}{u_{1c}\left(c_{1t+1}, \tau x[r(k_{t+1})k_{t+1} + \pi w(k_{t+1})](n_1 + n_2)^{1-\rho}\right)} = \beta_1 [1 - \delta + (1 - \tau)r(k_{t+1})] \tag{16}$$

$$[1 - \delta + (1 - \tau)r(k_t)]n_2 k_t = n_1 c_{1t} + n_2 k_{t+1} \tag{17}$$

3.4 Steady state

It is now possible to compute the steady state. The stationary interest rate is determined by the Euler equation (16):

$$r = \frac{1/\beta_1 - 1 + \delta}{1 - \tau} \quad (18)$$

which in turn determines capital intensity:

$$k(\tau) \equiv r^{-1} \left(\frac{1/\beta_1 - 1 + \delta}{1 - \tau} \right) \quad (19)$$

We notice that under Assumptions 1 and 2, the non-zero steady state is unique. (19), jointly with (10), gives the individual capital level:

$$k_1(\tau, n_2) \equiv \frac{n_2}{n_1} k(\tau) \quad (20)$$

Natives' budget constraint (17) gives their consumption level:

$$c_1(\tau, n_2) \equiv \frac{1 - \beta_1}{\beta_1} \frac{n_2}{n_1} k(\tau) \quad (21)$$

while (13) the individual provision of public good:

$$g_1(\tau, n_2) = g_2(\tau, n_2) \equiv \tau [r(k(\tau))k(\tau) + \pi w(k(\tau))] \frac{n_2}{(n_1 + n_2)^\rho} \quad (22)$$

The private/public good ratio,

$$\frac{c_1}{g_1} = \frac{1 - \tau}{\tau} \frac{(n_1 + n_2)^\rho}{n_1} \frac{1 - \beta_1}{1 - \beta_1(1 - \delta)} \frac{s}{s + (1 - s)\pi} \quad (23)$$

will play an important role in the following.

In order to obtain parametric solutions for k_1 , c_1 , g_1 , simply replace (19) in (20), (21) and (22), respectively. Notice that neither aggregate capital nor the interest rate depend on the number of immigrants; to the contrary, the individual consumption levels of both the private and the public good are increasing functions of n_2 .

3.5 Comparative statics

In order to shed light on the long-run effects of fiscal and immigration policies we now focus on their impact on the steady state; we thus compute the following elasticities with respect to the policy parameters, (τ, n_2) . Let us denote with $\varepsilon_{ab} \equiv (\partial a / \partial b) b / a$ the partial elasticity of variable a with respect to variable b . The following proposition characterizes the sign of these elasticities.

Proposition 2 *The elasticities of the steady state values with respect to the policy parameters have the following signs:*

$$\begin{bmatrix} \varepsilon_{k\tau} & \varepsilon_{kn_2} \\ \varepsilon_{k_1\tau} & \varepsilon_{k_1n_2} \\ \varepsilon_{c_1\tau} & \varepsilon_{c_1n_2} \\ \varepsilon_{g_1\tau} & \varepsilon_{g_1n_2} \end{bmatrix} = \begin{bmatrix} -\frac{\tau}{1-\tau} \frac{\sigma}{1-s} (< 0) & 0 \\ -\frac{\tau}{1-\tau} \frac{\sigma}{1-s} (< 0) & 1 \\ -\frac{\tau}{1-\tau} \frac{\sigma}{1-s} (< 0) & 1 \\ 1 - \frac{\tau}{1-\tau} \frac{s}{1-s} \frac{s+(1-s)\pi+\sigma-1}{s+(1-s)\pi} (\leq 0) & 1 - \rho x (\in (0, 1]) \end{bmatrix} \quad (24)$$

Thus, $\varepsilon_{g_1\tau} > 0$ if and only if $\tau < \bar{\tau}$, where, under Assumption 6,

$$\bar{\tau} \equiv (1 - s) \frac{s + (1 - s)\pi}{s\sigma + (1 - s)\pi} \in (0, 1) \quad (25)$$

Proof. See the Appendix.

We observe that, according to (3), the tax rate has a positive effect on the interest rate and a negative effect on both the capital intensity and consumption, independently of the immigration policy. Moreover, (as expected from an inspection of the steady state) the number of immigrants, n_2 , has no effect on capital intensity but has a positive impact both on private consumption and the provision of the public good.

4 Natives-oriented policy

We now focus on government's optimization program. As stated above, s/he only accounts for the welfare of native citizens (see expression (14)); in practice, while deciding over fiscal and immigration policies, s/he neglects immigrants' welfare. Indeed, for the purpose of optimizing natives' long-run welfare, s/he maximizes the welfare function (14) evaluated at the steady state, with respect to the policy (τ, n_2) , *i.e.*:

$$\omega(\tau, n_2) = \frac{u_1(c_1, g_1)}{1 - \beta_1} - \frac{v(x)}{1 - \mu\beta_1} \quad (26)$$

In order to justify such a benevolent attitude towards natives, we can assume that the government is elected by native citizens only or that natives are more involved in politics than immigrants.¹⁶

We notice that c_1 and g_1 are functions of (τ, n_2) (see equations (21) and (22)). According to (26), the welfare evaluated at the steady state is in turn a function of (τ, n_2) :

$$\omega(\tau, n_2) = \frac{u_1(c_1(\tau, n_2), g_1(\tau, n_2))}{1 - \beta_1} - \frac{v(x)}{1 - \mu\beta_1}$$

where x depends on n_2 according to (5).

We compute the gradient of the welfare function:

$$\frac{\partial \omega}{\partial \tau} = \frac{1}{1 - \beta_1} \left(\frac{\partial u_1}{\partial c_1} \frac{c_1}{\tau} \varepsilon_{c_1 \tau} + \frac{\partial u_1}{\partial g_1} \frac{g_1}{\tau} \varepsilon_{g_1 \tau} \right) \quad (27)$$

$$\frac{\partial \omega}{\partial n_2} = \frac{1}{1 - \beta_1} \left(\frac{\partial u_1}{\partial c_1} \frac{c_1}{n_2} \varepsilon_{c_1 n_2} + \frac{\partial u_1}{\partial g_1} \frac{g_1}{n_2} \varepsilon_{g_1 n_2} \right) - \frac{n_1}{(n_1 + n_2)^2} \frac{v'}{1 - \mu\beta_1} \quad (28)$$

where $\varepsilon_{c_1 \tau}$, $\varepsilon_{c_1 n_2}$, $\varepsilon_{g_1 \tau}$, $\varepsilon_{g_1 n_2}$ are given by (24).

We have assumed for simplicity that the government does not announce an optimal non-stationary policy $(\tau_t, n_{2t})_{t=0}^{\infty*}$, but commits itself to implement a constant policy over time.¹⁷

In the following, we first characterize the optimal fiscal policy for a given immigration policy; then, we compute the optimal immigration policy for a given fiscal policy. Finally, we will consider the optimal policy mix in the simple case of Cobb-Douglas technology and preferences.

4.1 Optimal fiscal policy

Equations (27) and (28) allow us to focus on optimal fiscal and immigration policies, respectively. We first concentrate our attention on the optimal fiscal policy. As shown in the next proposition, equation (27) determines the optimal level of taxation, for a given immigration policy.

Proposition 3 *Given the immigration policy n_2 , the optimal fiscal pressure is given by*

$$\tau^* = \bar{\tau} \left(1 + \bar{\tau} \frac{\sigma}{1 - s} \frac{\varepsilon_{u_1 c_1}}{\varepsilon_{u_1 g_1}} \right)^{-1} \quad (29)$$

where $\bar{\tau}$ is given by (25), and $\varepsilon_{u_1 c_1}$, $\varepsilon_{u_1 g_1}$ are the elasticity of utility with respect to the private and the public good, respectively. Under Assumption 6, $\tau^* \in (0, 1)$.

¹⁶It is interesting to observe that in France the share of immigrants in the parliament is lower than the one in the population; it is close to zero for communities resorting from Africa.

¹⁷Clearly, the solution $(\tau, n_2)^*$ is time-consistent at the steady state, *i.e.*, if the economic system is at the steady state from the beginning. Indeed, the objective (26) is invariant over time (since (c_1, g_1) are invariant) so that the optimal solution does not change over time.

However, when initial conditions are outside the steady state, one may question whether the optimal sequence computed at $t + 1$: $(\tau'_s, n'_{2s})_{s=t+1}^{\infty*}$, is in general the consistent continuation of the sequence announced at time t : $(\tau_s, n_{2s})_{s=t}^{\infty*}$, (that is $(\tau'_s, n'_{2s}) = (\tau_s, n_{2s})$ for every $s > t$), and also whether a constant policy announced at time t : $(\tau, n_2)_{s=t+1}^{\infty*}$, is consistent with the optimal sequence at time $t + 1$: $(\tau', n'_2)_{s=t+1}^{\infty*}$, (that is $(\tau', n'_2) = (\tau, n_2)$).

Proof. By substituting (24) in (27), we obtain: $\partial\omega/\partial\tau \geq 0$ iff $\tau \leq \tau^*$. Then, the maximum of ω is global at τ^* . Moreover, Assumption 6 entails $\bar{\tau} \in (0, 1)$ and, therefore, $\tau^* \in (0, 1)$ since $\varepsilon_{u_1 c_1}/\varepsilon_{u_1 g_1} > 0$. ■

Notice that $\bar{\tau}$ is a critical value: indeed, a taxation degree beyond $\bar{\tau}$ implies a negative effect (a reversal) on the individual provision of public good ($\varepsilon_{g_1 \tau} < 0$: see Proposition 2).

We consider an explicit functional form so as to provide more explicit policy recommendations. Focus on the case of a homogeneous utility function of degree $1 - 1/\eta$:

$$u_1(c_1, g_1) \equiv \frac{\varphi(c_1, g_1)^{1-1/\eta}}{1-1/\eta} \text{ if } \eta \neq 1; u_1(c_1, g_1) \equiv \ln \varphi(c_1, g_1) \text{ if } \eta = 1 \quad (30)$$

where φ is a positive function homogeneous of degree one, while u_1 is homogeneous of degree $1 - 1/\eta$. Notice that η can be interpreted as the elasticity of intertemporal substitution of a composite good φ .

The homogeneity of the utility (30) implies that the private consumption share in the composite good φ depends only on the ratio private/public good: $\alpha = \alpha(c_1/g_1)$, where r and c_1/g_1 are explicitly given by (18) and (23). Simple computations yield:

$$\varepsilon_{u_1 c_1} = \frac{\eta - 1}{\eta} \alpha \left(\frac{c_1}{g_1} \right) \quad (31)$$

$$\varepsilon_{u_1 g_1} = \frac{\eta - 1}{\eta} \left[1 - \alpha \left(\frac{c_1}{g_1} \right) \right] \quad (32)$$

We obtain thus:

$$\tau^* = \bar{\tau} \left[1 + \bar{\tau} \frac{\sigma}{1-s} \frac{\alpha(c_1/g_1)}{1-\alpha(c_1/g_1)} \right]^{-1} \quad (33)$$

where $\alpha = c_1 \varphi_c / \varphi$ is the private consumption share in the composite good, which is a function of the private/public good ratio (23).

4.2 Optimal immigration policy

We now shift our attention to the optimal immigration policy under a given fiscal policy. We will first consider the case of costless cultural heterogeneity; we will then shift our attention to the case of heterogeneity cultural costs. Notice that, in the first scenario, we obtain very general results; indeed, they do not depend on specific functional forms for the fundamentals.

4.2.1 Costless cultural heterogeneity

In this section we aim to disentangle the impact of public-good congestion and immigrants' free riding on natives' welfare. In particular, we study whether the congestion of public good coupled with free riding in immigrants' fiscal participation represents a sufficient economic argument to limit the openness of frontiers (and to introduce thus immigration quotas).

Our results show that, in absence of costs of cultural heterogeneity, the optimal immigration policy consists in opening borders. Surprisingly, this result holds for any degree ρ of impureness of the public good. Indeed, natives' gains coming from immigrants' labor force more than offset the disutility deriving from the congestion of the public good. This finding is far from trivial and is formalized in the following proposition; it can indeed be considered one of the main results of our work.

Proposition 4 *With no costs of cultural heterogeneity ($v = 0$), $\partial\omega/\partial n_2 > 0$.*

Proof. According to (24) and (28), we have

$$\frac{\partial\omega}{\partial n_2} = \frac{1}{1-\beta_1} \left[\frac{\partial u_1}{\partial c_1} \frac{c_1}{n_2} + \frac{\partial u_1}{\partial g_1} \frac{g_1}{n_2} (1-\rho x) \right] > 0 \quad (34)$$

because, under Assumptions 3, marginal utilities are strictly positive. ■

Notice that, for a given τ , neither the stationary state k nor r depend on n_2 (see (18)). However, individual wealth $k_1 = kn_2/n_1$, current and future consumption $c_1 = [(1-\tau)r - \delta]kn_2/n_1$ and the provision of public good $g_1 = \tau x(rk + \pi w)(n_1 + n_2)^{1-\rho}$ are all positive functions of n_2 . Thus, the presence of immigrants has a positive impact on welfare.

4.2.2 Costly cultural heterogeneity

In presence of costs of cultural heterogeneity, (34) needs to be generalized so as to account for hostility towards immigrants. Equation (26) is now the welfare function we need to maximize with respect to n_2 . By replacing (24) in (28), we obtain:

$$n_2 \frac{\partial \omega}{\partial n_2} = \frac{u_1}{1 - \beta_1} [\varepsilon_{u_1 c_1} + (1 - \rho x) \varepsilon_{u_1 g_1}] - \frac{v}{1 - \mu \beta_1} (1 - x) \varepsilon_v \quad (35)$$

where $\varepsilon_v(x) \equiv xv'(x)/v(x)$ is the elasticity of the costs of cultural heterogeneity with respect to share of foreigners.

Consider now the case of homogeneous utility function. Substituting (31) and (32) in (35) gives

$$n_2 \frac{\partial \omega}{\partial n_2} = \frac{\varphi^{1-1/\eta}}{1 - \beta_1} [1 - \rho(1 - \alpha)x] - \frac{v}{1 - \mu \beta_1} (1 - x) \varepsilon_v \quad (36)$$

where $\varphi = \varphi(c_1, g_1)$ and $\alpha = \alpha(c_1/g_1)$. Welfare maximization requires $\partial \omega / \partial n_2 = 0$.

4.2.3 Results interpretation

Costless cultural heterogeneity Broadly speaking, one could explain the need for immigrants on the ground of capital-labor complementarity. Having said that, the crucial mechanism driving the dynamics of the system in our model is the following: in the long run, patient natives hold the entire stock of capital, while impatient immigrants only work. Capital intensity is stationary and thus, capital per native increases with the number of (immigrant) workers. In turn, natives' consumption of private and public good increases with their capital stock. This positive effect more than offsets the disutility deriving from the congestion of the public good.

Notice thus that the Beckerian feature of heterogenous discounting is the dominant mechanism at work. It is at the roots of the resulting social segmentation; it also ensures significant gains of welcoming immigrants, even in presence of a strongly rival public good. Therefore, if citizens are not hostile to immigrants (no costs of cultural heterogeneity) but the government considers natives' welfare only (neglecting the welfare of immigrants) and the public good is rival (congestion costs), the optimal policy consists in letting frontiers completely open. Things change if people are hostile to foreigners: in this case, the degree of openness depends on the degree of intolerance.

Finally, we also observe that $\partial \omega / \partial n_2$ decreases with ρ . This allows us to provide an additional (somehow obvious) result: the purer the public good (i.e., the smaller the congestion costs), the stronger the positive impact of migrants on natives' welfare.

Costly cultural heterogeneity Welfare maximization requires an arbitrage between physical wealth and cultural hostility. Indeed, according to (36), $\partial \omega / \partial n_2 = 0$ if and only if

$$\frac{\varphi^{1-1/\eta}}{1 - \beta_1} \left(1 + [1 - \rho(1 - \alpha)] \frac{x}{1 - x} \right) = \frac{v \varepsilon_v}{1 - \mu \beta_1}$$

Therefore, the optimal immigration policy is the result of an arbitrage between the benefits of a larger number of newcomers, resulting in a higher level of capital per native (left-hand side), and the costs associated to hostility (right-hand side).

4.3 Local dynamics

We now provide some economic intuitions about the equilibrium dynamic properties and the occurrence of endogenous fluctuations. We focus in particular on the destabilizing power of black-labor market through informal arguments.¹⁸

As shown by Becker and Foias (1994), heterogenous discounting promotes the occurrence of endogenous cycles under weak elasticity of capital-labor substitution σ . More precisely, Becker and Foias prove that there

¹⁸These considerations are supported by computations that are available upon request.

exists a critical value $\sigma^* > 0$ such that (1) if $\sigma > \sigma^*$, the steady state is a saddle-point, (2) if $\sigma < \sigma^*$, the steady state is a source surrounded by a two-period cycle or a 2^n -period cycle with $n > 1$. Generically, the system undergoes a flip bifurcation at $\sigma = \sigma^*$.

Our model relies on the assumption of heterogeneous discounting and inherits the dynamic properties of Becker and Foias (1994). Linearizing the system (16)-(17) and computing the characteristic polynomial gives the critical value σ^* of flip bifurcation. One can show that if the consumption good and the public good are substitutes or weakly complements, then $\sigma^* > 0$: as in Becker and Foias (1994) the system generically undergoes a flip bifurcation at $\sigma = \sigma^*$. The transitional equilibrium is unique in the case of saddle point ($\sigma > \sigma^*$). Indeed, k_t is a predetermined variable, while c_{1t} is a jump variable in the two-dimensional system (16)-(17). The equilibrium is also unique in the case of source ($\sigma < \sigma^*$) when the cycle around is supercritical. Thus, economies where inputs are sufficiently complements can experience endogenous cycles.

One may wonder whether any fundamental parameter affects the size of the instability region $(0, \sigma^*)$.

We reinterpret free riding $1 - \pi$ as the relative extent of the black labor in the total labor market. When the composite good φ is Cobb-Douglas, it is possible to prove that the extent of the black-labor market promotes the occurrence of cycles and destabilizes the economy ($1 - \pi$ widens the instability region $(0, \sigma^*)$), if and only if the private and public good are complements.¹⁹

The following can explain the economic intuition. When $\eta > 1$, public and private goods are complements and the intertemporal substitution dominates the income effect. If k_t increases, capital income also increases at time t , entailing both an increase in tax revenues and public good provision. Because of the complementarity between c_1 and g_1 , we observe also an increase in private consumption. This eventually reduces investment and thus k_{t+1} (due to the budget constraint). The negative impact of k_t on k_{t+1} is amplified by the black-labor market (*i.e.*, by the free riding). When free riding matters, the provision of public good mostly weights on natives' capital income: the higher the contribution of capital income to the provision of the public good, the stronger the effect.

5 The Cobb-Douglas case

We have computed the optimal fiscal policy, given the number of immigrants, and the optimal immigration policy, given the tax rate. We now compute the optimal policy mix and maximize thus the welfare function (26) jointly with respect to τ and n_2 . In order to characterize the optimal policy mix, in what follows we introduce Cobb-Douglas technology and preferences.

Assumption 7 (technology) *The production function is Cobb-Douglas:*

$$F(K, L) \equiv AK^s L^{1-s} \quad (37)$$

In the Cobb-Douglas case, the capital share in total income $s \in (0, 1)$ is constant and the elasticity of capital-labor substitution is $\sigma = 1$. The average and marginal productivities become respectively $f(k) = Ak^s$ and $r = sAk^{s-1}$. Solving (19) gives the capital intensity at the steady state:

$$k = \left[\frac{sA(1-\tau)}{1/\beta_1 - 1 + \delta} \right]^{\frac{1}{1-s}} \quad (38)$$

Focus now on consumers' preferences.

Assumption 8 (preferences) *The composite good is Cobb-Douglas:*

$$\varphi(c_1, g_1) \equiv c_1^\alpha g_1^{1-\alpha} \quad (39)$$

¹⁹Complementarity means that the consumption of public good increases the marginal utility of private good and *viceversa*. This happens if and only if $\eta > 1$.

Notice that φ is now a weighted geometric average of both the private and public good; the interpretation of φ as composite good is thus appropriate.

The Cobb-Douglas technology implies $\sigma = 1$. At the same time, Cobb-Douglas preferences imply that α , the private consumption share in the composite good φ , does no longer depend on the ratio private/public good, $\alpha(c_1/g_1) = \alpha$.

5.1 Optimal policy mix

In order to compute the optimal policy mix (τ^*, n_2^*) we proceed by solving the welfare maximization problem. In the following, we will first study the role of the fiscal policy and then the one of the immigration policy.

5.1.1 Fiscal policy in the optimal mix

The following proposition represents the fiscal side of the optimal policy mix.

Proposition 5 *Under Assumptions 7 and 8, the tax rate of the optimal policy mix is given by*

$$\tau^* = (1 - \alpha)(1 - s) \quad (40)$$

Proof. Since $\sigma = 1$ and $\alpha(c_1/g_1) = \alpha$, (25) simplifies to $\bar{\tau} \equiv (1 - s)$ and (33) to (40). Eventually, we notice that τ^* is independent from the number of immigrants n_2 . Thus, τ^* corresponds also with the optimal-mix taxation level. ■

The above proposition implies that the stronger the preference for the public good $(1 - \alpha)$, the greater the tax rate and the provision of public good. Moreover, the larger the capital share in total income, the lower the taxation on capital income (natives' income). Surprisingly, the latter does not depend on other parameters than the public good share in the composite good and the capital share.

5.1.2 Immigration policy in the optimal mix

Costless cultural heterogeneity In absence of heterogeneity costs, the presence of immigrants has a positive impact on welfare (see Proposition (4)). This conclusion is general and does not depend on the adoption of Cobb-Douglas fundamentals. In addition, it holds for any level of taxation and, so, for what concerns the optimal policy mix.

Costly cultural heterogeneity When cultural heterogeneity entails a disutility, Cobb-Douglas technology and preferences allow us to find explicit solutions.

Under the production function (37), the capital share in total income s is constant. Under the utility function (39), the consumption share α in the composite good is also constant and does no longer depend on the ratio c_1/g_1 .

We now specify natives' disutility deriving from cultural heterogeneity. For simplicity, we consider a linear function. In fact, introducing convexity would not change the qualitative properties of the optimal policy mix.

Assumption 9 *(cultural cost) The cost of cultural heterogeneity is linear:*

$$v(x) \equiv \gamma x \quad (41)$$

where γ can be interpreted as a degree of intolerance.

Assumption 9 entails $\varepsilon_v = 1$. Replacing (21), (39), (41) in (36), we find

$$\frac{\partial \omega}{\partial n_2} = \frac{\gamma h(x, y)}{(1 - \mu\beta_1)n_2 y}$$

where

$$h(x, y) \equiv b[1 - (1 - \alpha)\rho x] - x(1 - x)y \quad (42)$$

with

$$b \equiv \frac{1}{\gamma} \frac{1 - \mu\beta_1}{1 - \beta_1} > 0$$

and

$$y \equiv \left[\frac{\beta_1}{1 - \beta_1} \frac{1 - x}{xk} \left(\frac{c_1}{g_1} \right)^{1 - \alpha} \right]^{1 - \frac{1}{\eta}} \quad (43)$$

where c_1/g_1 and k are now given respectively by (23) and (38).

(1) Case $\eta = 1$ (logarithmic utility).

When $\eta = 1$, the private and the public good are neither substitutes nor complements in the individual basket of consumption.

The following proposition determines the critical degree of intolerance beyond which the government decides to introduce immigration quotas.

Proposition 6 *Let Assumptions 4-9 be satisfied. We introduce the following critical values:*

$$x^* \equiv \frac{1 + (1 - \alpha)\rho b}{2} - \sqrt{\left[\frac{1 + (1 - \alpha)\rho b}{2} \right]^2 - b} \quad (44)$$

$$\gamma^* \equiv \frac{1 - \mu\beta_1}{1 - \beta_1} \frac{[(1 - \alpha)\rho]^2}{2 - (1 - \alpha)\rho - 2\sqrt{1 - (1 - \alpha)\rho}} > 0 \quad (45)$$

- (a) If $\gamma < \gamma^*$ (low natives' cultural aversion), it is optimal to open the borders to newcomers.
- (b) If $\gamma > \gamma^*$ (high cultural aversion), the optimal density of immigrants is given by $x^* \in (0, 1)$.

Proof. See the Appendix.

Proposition 4 belongs in this sense to case (a) in Proposition 6. Indeed, $\gamma = 0 < \gamma^*$.

Focus on (44) and (45). We notice that:

- (i) $\partial x^*/\partial \rho < 0$ and $\partial \gamma^*/\partial \rho < 0$: the more rival the public good, the less tolerant the natives.
- (ii) $\partial x^*/\partial (1 - \alpha) < 0$ and $\partial \gamma^*/\partial (1 - \alpha) < 0$: the stronger the preference for a rival public good ($\rho > 0$), the less tolerant the natives.
- (iii) $\partial \gamma^*/\partial \mu < 0$: the higher the weight on cultural disutility, the less tolerant the natives.
- (iv) Because $\eta = 1$, neither the tax rate (τ) nor the free riding ($1 - \pi$) nor the capital share (s) affect the optimal density of immigrants x^* or the critical degree of tolerance γ^* .

We know that, in the Cobb-Douglas case, the optimal tax rate τ^* does no longer depend on n_2 (see (40)). In addition, when $\eta = 1$, the optimal immigration density x^* does no longer depend on the tax rate τ (see (44)). Since $n_2 = n_1 x / (1 - x)$, we obtain the following result:

Proposition 7 *Under the Assumptions 4-9 and $\eta = 1$, the optimal policy mix is*

$$\begin{aligned} \tau^* &= (1 - \alpha)(1 - s) \\ n_2^* &= n_1 \frac{x^*}{1 - x^*} \end{aligned}$$

where the optimal share of immigrants x^* is given by (44).

(2) General case $\eta \neq 1$.

For simplicity, we consider a fully rival public good: $\rho = 1$.

By replacing (40) in (23) and (38), and substituting (23) and (38) in (43), we get

$$y(x) \equiv \left[\frac{\beta_1}{1 - \beta_1} \frac{1 - x}{x} \left(\frac{1/\beta_1 - 1 + \delta}{sA[1 - (1 - \alpha)(1 - s)]} \right)^{\frac{1}{1 - s}} \left(\frac{c_1}{g_1} \right)^{1 - \alpha} \right]^{1 - \frac{1}{\eta}}$$

with

$$\frac{c_1}{g_1} = \frac{1}{1 - x} \frac{1 - (1 - \alpha)(1 - s)}{(1 - \alpha)(1 - s)} \frac{1 - \beta_1}{1 - \beta_1(1 - \delta)} \frac{s}{s + (1 - s)\pi}$$

where we have taken into account the optimal fiscal policy (40).

The optimal immigration density x^* is the solution of the following equation:

$$h(x, y) \equiv b[1 - (1 - \alpha)\rho x] - x(1 - x)y(x) = 0 \quad (46)$$

Differentiating (46) with respect to x and a given parameter p allows us to compute the partial derivative $\partial x^*/\partial p$ and the partial elasticity:

$$\varepsilon_{x^*p} \equiv \frac{p}{x^*} \frac{\partial x^*}{\partial p}$$

We have studied above the role of ρ (rivality in the consumption of public good) and $1 - \alpha$ (preference for the public good) in the case $\eta = 1$. Let us now focus on the impact of γ , μ and π in the more general case $\eta \neq 1$. By totally differentiating (46) with respect to γ , μ , π and x , we find the following elasticities:

$$\begin{aligned} \varepsilon_{x^*\gamma} &= -bz(x^*) \frac{1 - (1 - \alpha)\rho x^*}{x^*} \\ \varepsilon_{x^*\mu} &= -bz(x^*) \frac{1 - (1 - \alpha)\rho x^*}{x^*} \frac{\mu\beta_1}{1 - \mu\beta_1} \\ \varepsilon_{x^*\pi} &= (1 - \alpha)(1 - x^*)y(x^*)z(x^*) \frac{\eta - 1}{\eta} \frac{(1 - s)\pi}{s + (1 - s)\pi} \end{aligned}$$

where

$$z(x^*) \equiv \left[(1 - \alpha)\rho b + y(x) \frac{1 - x(1 - \alpha + \eta + \alpha\eta)}{\eta} \right]^{-1}$$

In order to be able to interpret these results, we introduce a suitable assumption.

Assumption 10 $0 < x^* < (1 - \alpha + \eta + \alpha\eta)^{-1} < 1$.

Assumption 10 allows us to study the role of free riding when the optimal share of immigrants x^* is not too significant. When for instance η is close to one, Assumption 10 becomes $0 < x^* < 1/2$.

Proposition 8 *Under Assumption 10, (1) $\varepsilon_{x^*\gamma} < 0$ and $\varepsilon_{x^*\mu} < 0$. In addition, (2) $\varepsilon_{x^*\pi} > 0$ if and only if $\eta > 1$.*

Proof. Assumption 10 implies $z(x^*) > 0$. Results follow immediately. ■

Let us provide some economic intuition.

(2.1) $\varepsilon_{x^*\gamma} < 0$ and $\varepsilon_{x^*\mu} < 0$. As expected, an increase in the degree of intolerance towards immigrants (either γ or μ) has a negative impact on the optimal immigration policy: the stronger the intolerance, the lower the number of welcomed immigrants.

(2.2) $\varepsilon_{x^*\pi} > 0$ if and only if $\eta > 1$. An increase in fiscal participation has a positive impact on the optimal number of immigrants if and only if $\eta > 1$.²⁰ Indeed, an increase in free riding ($1 - \pi$) has always a negative effect on the provision of the public good. When goods are complements ($\eta > 1$), this entails in turn a decrease in the marginal utility of private-good consumption; therefore, a decrease in the number of immigrant workers is required for the production of both goods. To the contrary, if goods are substitutes ($\eta < 1$), a decrease in the provision of public good rises the marginal utility of the private good and incentives its production. The increased production of private good more than offsets the reduction of public good; thus, the optimal number of (welcomed) immigrant workers increases.

6 Conclusions

In this article, we have characterized optimal fiscal and immigration policies in a dynamic framework of general equilibrium. We have accounted for the possibility of free riding in immigrants' contribution to government revenues and for natives' intolerance towards immigrants. We have assumed the perspective of a government who provides a rival public good and maximizes the welfare of natives only.

²⁰As discussed above, when $\eta > 1$ the private and the public good are complements in the individual basket of consumption

Our work proves that, in absence of costs of cultural heterogeneity, it is optimal to keep frontiers open; on the other hand, in presence of costs of cultural heterogeneity, the optimal immigration policy is an interior solution. Indeed, in the log run, heterogenous discounting results in social segmentation between native capitalists and immigrant workers; moreover, a larger number of newcomers implies in turn higher levels of capital and consumption per native, even in presence of a rival public good.

We have characterized analytically the optimal policy mix under Cobb-Douglas fundamentals. Surprisingly, the degree of intolerance and the degree of rivalry of the public good do not affect the optimal taxation. Indeed, while the degree of congestion dampens the utility deriving from consumption, the provision of the public good depends on agents' incomes.

We have shown that an increase in the degree of intolerance towards immigrants has (as expected) a negative impact on the optimal number of welcomed immigrants. Moreover, an increase in the degree of free riding on optimal quotas is not univocal and depends on the substitutability between the private good and the public good in natives' consumption basket. Indeed, an increase in free riding entails less public good provision; if goods are complements, a decrease in the public good implies a decrease in the marginal utility of private consumption. Therefore, firms produce less private good and less immigrants workers are needed. To the contrary, if goods are substitute, a decrease in the provision of the public good raises the marginal utility of private good. This prompts to produce more private good, and thus, to hire (*i.e.*, welcome) a larger number of immigrants.

Before concluding, we need to spend a word of warning. Our work is meant to give a theoretical contribution to the economic literature on immigration policies. To this purpose, we have limited our analysis to a stylized picture of the reality. The policy implications of our work should be thus interpreted with caution. Nevertheless, our analysis contributes to shed light on the current debate on immigration policies.

7 Appendix

Proof of Proposition 2 Deriving the inverse function on the right-hand side of (19) and using (3), we obtain

$$\varepsilon_{k\tau} = \frac{\tau k'(\tau)}{k(\tau)} = \frac{\tau}{1-\tau} \frac{r(k)}{kr'(k)} = -\frac{\tau}{1-\tau} \frac{\sigma(k)}{1-s(k)} \quad (47)$$

Moreover $k(\tau)$ does not depend on n_2 , thereby $\varepsilon_{kn_2} = 0$.

Focus now on (20). k_1 is proportional to $k(\tau)$, then $\varepsilon_{k_1\tau} = \varepsilon_{k\tau}$. In addition, k_1 depends linearly on n_2 , thus $\varepsilon_{k_1n_2} = 1$.

Consider (21). c_1 is proportional to $k(\tau)$, then $\varepsilon_{c_1\tau} = \varepsilon_{k\tau}$. Moreover c_1 depends linearly on n_2 , thus $\varepsilon_{c_1n_2} = 1$.

Finally, we derive (22) with respect to τ and n_2 . We find first

$$\begin{aligned} \varepsilon_{g_1\tau} &\equiv \frac{\tau}{g_1} \frac{\partial g_1}{\partial \tau} = 1 + \tau \frac{r'(k) k'(\tau) k + r(k) k'(\tau) + \pi w'(k) k'(\tau)}{r(k) k + \pi w(k)} \\ &= 1 + \frac{\tau k'(\tau)}{k(\tau)} \frac{1 + \frac{kr'(k)}{r(k)} + \pi \frac{kw'(k)}{w(k)} \frac{w(k)}{kr(k)}}{1 + \pi \frac{w(k)}{kr(k)}} = 1 + \varepsilon_{k\tau} \frac{1 + \varepsilon_r(k) + \pi \varepsilon_w(k) \frac{w(k)}{kr(k)}}{1 + \pi \frac{w(k)}{kr(k)}} \end{aligned}$$

Replacing (3), (4), (5) and (47), and noticing that

$$\frac{w(k)}{kr(k)} = \frac{1-s(k)}{s(k)}$$

we obtain the elasticity $\varepsilon_{g_1\tau}$ in (24). Finally, we get

$$\varepsilon_{g_1n_2} \equiv \frac{n_2}{g_1} \frac{\partial g_1}{\partial n_2} = 1 - \rho \frac{n_2}{n_1 + n_2}$$

that is the elasticity $\varepsilon_{g_1n_2}$ in (24).

We observe that the signum of the elasticity of the individual provision of public good with respect to the tax rate is positive ($\varepsilon_{g_1\tau} > 0$) if and only if $\tau < \bar{\tau}$. The right-hand side in (25) is always positive and, under Assumption 6, less than one. ■

Proof of Proposition 6 If $\eta = 1$, then $y = 1$ and (42) writes

$$h(x, y) \equiv b[1 - (1 - \alpha)\rho x] - x(1 - x)$$

The roots of $h(x, y)$ are x^* and

$$x_2 \equiv \frac{1 + (1 - \alpha)\rho b}{2} + \sqrt{\left[\frac{1 + (1 - \alpha)\rho b}{2}\right]^2 - b}$$

We notice that $0 < x^* < x_2$ if the discriminant is positive.

Let us introduce two critical values for γ :

$$\begin{aligned} \gamma_1 &\equiv \frac{1 - \mu\beta_1}{1 - \beta_1} \frac{[(1 - \alpha)\rho]^2}{2 - (1 - \alpha)\rho + 2\sqrt{1 - (1 - \alpha)\rho}} \\ \gamma_2 &\equiv \frac{1 - \mu\beta_1}{1 - \beta_1} (1 - \alpha)\rho \end{aligned}$$

We observe that $0 < \gamma_1 < \gamma_2 < \gamma^*$ because $(1 - \alpha)\rho \in (0, 1)$. In addition:

- (i) the discriminant of x^* and x_2 is positive if and only if $\gamma < \gamma_1$ or $\gamma > \gamma^*$,
- (ii) $x^* < 1$ if and only if $\gamma > \gamma_2$.

We have three cases.

(a1) $0 < \gamma < \gamma_1$. The discriminant is positive. Then, $h(x, y) > 0$ if and only if $x < x^*$ or $x > x_2$. However, $\gamma < \gamma_2$ implies $x^* > 1$. Since $x < 1$, we obtain $h(x, y) > 0$, that is $\partial\omega/\partial n_2 > 0$: opening borders is optimal.

(a2) $\gamma_1 < \gamma < \gamma^*$. In this case, the discriminant is negative. Then, $h(x, y) > 0$, that is $\partial\omega/\partial n_2 > 0$: opening borders is still optimal.

(b) $\gamma^* < \gamma$. The discriminant is positive and $0 < x^* < x_2$. Thus, $h(x, y) < 0$ if and only if $x^* < x < x_2$. In other terms, $\partial\omega/\partial n_2 > 0$ if $x < x^*$; $\partial\omega/\partial n_2 < 0$ if $x^* < x < x_2$; $\partial\omega/\partial n_2 > 0$ if $x_2 < x$. Then x^* locally maximizes the welfare ω . Moreover, $x^* < 1$ because $\gamma > \gamma_2$. ■

8 References

References

- [1] Barro, R.J. and X. Sala-i-Martin (2004), *Economic Growth*, MIT Press, Cambridge.
- [2] Becker, R. A. (1980), On the Long-Run Steady State in a Simple Dynamic Model of Equilibrium with Heterogeneous Households, *Quarterly Journal of Economics* **95**, 375-382.
- [3] Becker, R. A. and C. Foias (1987), A Characterization of Ramsey Equilibrium, *Journal of Economic Theory* **41**, 173-184.
- [4] Becker, R. A. and C. Foias (1994), The Local Bifurcation of Ramsey Equilibrium, *Economic Theory* **4**, 719-744.
- [5] Ben-Gad, M. (2004), The Economic Effects of Immigration – A Dynamic analysis, *Journal of Economic Dynamics and Control* **28**, 1825-1845.
- [6] Bisin, A. and T. Verdier (2000), Beyond the Melting Pot: Cultural Transmission, Marriage and the Evolution of Ethnic and Religious Traits, *Quarterly Journal of Economics* **3**, 955-988.
- [7] Bisin, A., G. Topa and T. Verdier (2004), Religious Intermarriage and Socialization in the US, *Journal of Political Economy* **112**, 615-665.
- [8] Borjas, G. J. (1994), The Economics of Immigration, *Journal of Economic Literature* **32**, 1667-1717.
- [9] Borjas, G.J. (1999a), *Heaven's Door*, Princeton University Press, Princeton.

- [10] Borjas, G.J. (1999b), Immigration and Welfare Magnets, *Journal of Labor Economics* **17**, 607-637.
- [11] Bosi, S. and T. Seegmuller (2010), On the Ramsey Equilibrium with Heterogeneous Consumers and Endogenous Labor Supply, *Journal of Mathematical Economics* **46**, 475-492.
- [12] *Economist, The* (2008), Special report on migration, January 5th-11th.
- [13] Fisher, I. (1930), *The Theory of Interest*, Macmillan, London.
- [14] Gang, I.N., F.L. Rivera-Batiz and M.-S. Yun (2002), Economic Strain, Ethnic Concentration and Attitudes Towards Foreigners in the European Union, *Working Paper 2002-14*, Department of Economics, Rutgers University.
- [15] International Monetary Fund (2008), The Housing Sector and the Business Cycle, *World Economic Outlook* ch. 3.
- [16] Lawrance, E.C. (1991), Poverty and the Rate of Time Preference: Evidence from Panel Data, *Journal of Political Economy* **99**, 54-77.
- [17] Le Van, C., M.H. Nguyen and Y. Vailakis (2007), Equilibrium Dynamics in an Aggregative Model of Capital Accumulation with Heterogeneous Agents and Elastic Labor, *Journal of Mathematical Economics* **43**, 287-317.
- [18] Le Van, C. and Y. Vailakis (2003), Existence of a Competitive Equilibrium in a One Sector Growth Model with Heterogeneous Agents and Irreversible Investment, *Economic Theory* **22**, 743-771.
- [19] Michael, S.M. (2003), International Migration, Income Taxes and Transfers: a Welfare Analysis, *Journal of Development Economics* **72**, 401-411.
- [20] Michael, S.M. and P. Hatzipanayotou (2001), Welfare Effects of Migration in Societies with Indirect Taxes, Income Transfers and Public Good Provision, *Journal of Development Economics* **64**, 1-24.
- [21] Myers, G.M. and Y.Y. Papageorgiou (2000), Immigration Controls and the Welfare State, *Journal of Public Economics* **75**, 183-207.
- [22] Nannestad, P. (2004), A Game, Real Actors Won't Play? Integration of Ethnic Minorities in Denmark as a Collective Action Dilemma, *International Migration Review* **38**, 287-308.
- [23] Nannestad, P. (2006), Immigration and Welfare States: A Survey of 15 Years of Research, *European Journal of Political Economy*, doi: 10.1016/j.ejpoleco.2006.08.007.
- [24] O'Rourke, K. and R. Sinnott (2006), The Determinants of Individual Attitudes Towards Immigration, *European Journal of Political Economy* **22**, 838-861.
- [25] Schultz, C. and T. Sjöström (2001), Local Public Goods, Debt and Migration, *Journal of Public Economics* **80**, 313-337.
- [26] Sorger, G. (1994), On the Structure of Ramsey Equilibrium: Cycles, Indeterminacy, and Sunspots, *Economic Theory* **4**, 745-764.
- [27] Usher, D. (1977), Public Property and the Effects of Migration upon Other Residents of the Migrants' Countries of Origin and Destination, *The Journal of Political Economy* **85**, 1001-1020.
- [28] Wildasin, D.E. (1994), Income Redistribution and Migration, *Canadian Journal of Economics* **27**, 637-656.