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# “Leftist”, “Rightist” and Intermediate Decompositions of Poverty Variations with an Application to China from 1990 to 2003.

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## Abstract

*This paper investigates the influence of invariance axioms in the decomposition of observed poverty variations into growth and inequality effects. After a complete and critical review of the invariance axioms suggested in the literature, we show that few information is needed for the ordering of the effects respectively obtained through scale, translation and intermediate invariance. Using Chinese data for the period 1990-2003, we find that some commonly observed results of the decomposition are contingent to the invariance axiom choices whilst other are robust to changes in ethical preferences.*

**JEL classification:** I32, D63, D31.

**Keywords:** Poverty, inequality effect, growth effect, decomposition, scale invariance, translation invariance, intermediate invariance, China.

## Introduction

Does multiplying the incomes of each members of a population by the same scalar increases, decreases or leaves income inequality unchanged? Does adding the same absolute amount of income to each member of a population increases, decreases or leaves inequality unchanged? It is very interesting to note that people may give very different answers to questions related to axiomatic choices, and thus express so heterogeneous feelings about how inequality should be defined and measured. Using questionnaires with large samples of students (generally undergraduate students in economics), Amiel and Cowell (1992, 1997, 1999, 2001) noticed that very few respondents were likely to support most of the core traditional axioms used in the inequality and poverty measurement literature. In particular, the majority of the respondents was not in agreement with the classical opinion that doubling each income in a distribution does not change the degree of inequality. Such a reaction against this scale invariance axiom is not really surprising since there is no unanimous approval of this axiom among economists. For instance, many

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famous scholars like Dalton (1920) or Kolm (1976a) also expressed heterodox views about how additional incomes should (or could) be divided among individuals so as to preserve the degree of inequality.<sup>1</sup>

Questioning the desirability of the properties of any inequality or poverty measure is not a trivial exercise since it may have a direct impact on policy decisions. International income inequalities, that is inequality among countries, are a good illustration of the importance of the heterogeneity of feelings about inequality and its consequences. Many individuals will focus on the increasing absolute differences between mean incomes while others will just consider the decreasing relative differences between nations. The first ones may certainly conclude that inequalities have risen during the last decades, while the second would support the opposite point of view. The consequence is that very different policies, in particular aid and development policies, could be recommended on the basis of such heterogeneous interpretations of observed trends.

However, the subject of the present paper is not international income inequalities but presents similar interpretation issues. Here we would like to emphasize the importance of axiomatic choices on the analysis of poverty variations. Since the pioneering developments of Jain and Tendulkar (1990); Kakwani and Subbarao (1990) and Datt and Ravallion (1992), the decompositions of poverty variations into growth and inequality effects have become very popular in empirical studies since it is a very elegant way of estimating the relative contribution of the increase in mean income and of the changes in the relative distribution of incomes. In the present studies we stress the crucial role of ethical preferences involved in the general conception of inequality since it defines the frontier between what can be considered as “pure” growth, that is growth without inequality change, and “pure” redistribution, that is change in the relative distribution with a constant mean income. This remark is particularly relevant for some poverty measures like the headcount index that are compatible with many rival axioms and thus that leave room for personal judgments. Consequently, the same variation of poverty may be mostly attributed to “pure” growth or “pure” redistribution depending of individual tastes, a result that may lead to great misunderstandings and inefficient policy recommendations if researchers do not explicitly explain the axiomatic basis involved in their decomposition of observed poverty trends.

In the present paper, we first review the different techniques used for the decomposition of poverty spells (section 1) and then the different inequality views which have been presented and formalized in the inequality and poverty measurement literature (section 2). More precisely, we focus on inequality views that are attributed to “rightist” and “leftist” political opinions according to Kolm (1976a). A “rightist” is based on the opinion that inequality does not change when incomes grow at the same rate as mean income through the curse of economic development whereas “leftist” individuals feel that the degree of inequality is constant when economic agents’ incomes increase by the same amount as mean income does.<sup>2</sup> Our review also includes interme-

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<sup>1</sup> Concerning the opposition between scale invariance and translation invariance that will be treated in the next sections, Kolm (1976a, p. 419) argue that “*it is no less legitimate to attach the inequality between two incomes to their difference than to their ratio.*”

<sup>2</sup> The “leftist” and “rightist” labels are linked to the french political context and the ideological differences between left-wing and right-wing opinions. Kolm (1976a) introduces these expressions with a reference to debates that occurred for the Grenelle agreements in 1968 which decreed the same proportional increase in wages for all employees. Kolm reports (p. 419) that “*the Radicals felt bitter and cheated; in their view, this widely increased incomes inequality.*” In

mediate views that constitutes compromises between these two polar cases and for which we provide a new definition. Among the formalized intermediate views that are presented in the paper, we argue that the non-linear view suggested by Krtscha (1994) and Yoshida (2005) is the sole relevant way of expressing an intermediate inequality view for poverty decompositions.

Considering the theoretical implications of invariance related axiom changes on the growth-inequality decompositions of poverty spells (section 3), we show that few information is needed so as to order the different growth and inequality effects based on these “rightist”, “leftist” and intermediate views in the case of the headcount index. Consequently, we can easily predict which interpretation differences should be observed within this set of inequality conceptions for the same observed poverty variations.

An application on Chinese data during the period 1990-2003 is then provided in section 4. The conclusions that regularly appear in empirical studies (Fan et al., 2002; Wan and Zhang, 2006; Chen and Ravallion, 2007) are that growth is the main contributor to poverty reduction and that relative distribution changes hampers the beneficial effects of growth on poverty. Our aim is thus to test if these conclusion are robust to changes in inequality views. Our results show that upholders of the “leftist” view will consider that inequality changes have contributed to the increase of the number of poor people in China during the whole period whilst those who believe in a “rightist” view will support the opposite conclusion. An other important consequence is that considering rival inequality views may reverse conclusions when comparing the value of the estimated effects between many subperiods. In the case of China, these methodological considerations should be seen as crucial since it modifies conclusions that concerns about one fifth of the world population. On the other hand, some traditional conclusions are strengthened like the need to improve redistribution so as to fight extreme poverty.

Finally, section 5 concludes with some remarks about economists’ practices. In particular, we argue that economists should, at least, be aware of the normative implications of the tools that they use for purely positive analysis of observed economic phenomena.

## 1 The decomposition of poverty spells

In the present paper, our attention is confined to absolute poverty measures  $\Theta$ , which can be fully characterized by a poverty line  $z$ , the mean income  $\mu$  and a vector of inequality measures  $\pi$  that account for all inequality features of the observed distribution.<sup>3</sup> Thus, poverty at time  $t$  is given

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section 2, we show that the “leftist” view implies a more egalitarian way of sharing additional incomes among individuals than the “rightist” view. As the expressions of “leftist” and “rightist” views are common in the invariance-related literature and do not yield confusions like the terms absolute and relative (see note 3), we will make an intensive use of them throughout the paper. However, we are conscious that these expressions may not correspond to the reality of political doctrines and movements, especially outside the french context. For a justification of the differences between “leftist” and “rightist” inequality views in terms of utility functions, see note 15.

<sup>3</sup> In the present study, absolute poverty refers to the use of an absolute poverty line which is only defined by the amount needed to satisfy some “basic” needs (see Sen, 1983, 1985, for further details). So it contrasts with relative poverty in which the poverty line is set with respect to the observed distribution of income. Sometime, absolute (relative) poverty corresponds to poverty views which comply with translation (scale) invariance axioms (cf. section 2). The adjectives absolute and relative are also used in the context of inequality measurement and refers to indices that are respectively defined as differences and ratios of mean income with the corresponding equally distributed equivalent income (Atkinson, 1970; Kolm, 1976*a,b*), *i.e.* the per capita income which if equally shared among the population

by:

$$\Theta_t = \Theta(z_t, \mu_t, \boldsymbol{\pi}_t). \quad (1.1)$$

In order to compare values of  $\Theta$  at different points of time,  $z_t$  is held constant (we assume income are measured in real terms). So  $z_t = z$ . Consistent with this assumption and equation (1.1) is the intuition that variations of  $\Theta$  can be decomposed into different components that can be attributed to growth and variations in inequality. In mathematical terms, our intention is to get:

$$\Theta_{t+k} - \Theta_t = G_{t,t+k} + D_{t,t+k} \quad (1.2)$$

where  $G$  and  $D$  are respectively the growth and inequality components of poverty.<sup>4</sup> The growth component is the variation of the poverty measure that is only due to change in mean income, that is when inequality is held constant. Similarly, the inequality component is the variation of the poverty measure that can be attributed to variations of the elements of  $\boldsymbol{\pi}$ . This technique was initiated by Jain and Tendulkar (1990); Kakwani and Subbarao (1990) and Datt and Ravallion (1992) and is now standard in the poverty literature. It should be acknowledged that this decomposition is a purely statistical decomposition and differs from the econometric analysis, like Chen and Ravallion (2007), in the sense that it does not account for the correlations between growth and variations of the degree of inequality (whatever the direction of the causality, if it does exist).

The decomposition of poverty spells can be carried in different ways, depending on whether initial or final values are used for the fixed element of each component. In the present paper, we choose to focus on the two most widely used decomposition techniques namely the one suggested by Datt and Ravallion (1992) and the Shapley decomposition developed by Shorrocks (1999) and Kakwani (2000). The Datt and Ravallion (1992) procedure is characterized by the use of the initial values as references for the computation of each effects and thus by the presence of a residual term. The growth and inequality effects are then defined by the following equations:

$$G_{t,t+k} = \Theta(z, \mu_{t+k}, \boldsymbol{\pi}_t) - \Theta(z, \mu_t, \boldsymbol{\pi}_t), \quad (1.3)$$

$$D_{t,t+k} = \Theta(z, \mu_t, \boldsymbol{\pi}_{t+k}) - \Theta(z, \mu_t, \boldsymbol{\pi}_t). \quad (1.4)$$

In the context of a multi-period analysis, this technique proves to be time-transitive when the same distribution is used as reference for the computation of the effects for each period.<sup>5</sup> For recent application of this technique, see Fan et al. (2002) for urban China from 1992 to 1998, Contreras (2003) for Chile during the period 1990-1996, Kappel et al. (2005) for Uganda from 1992 to 2002 and Ferreira et al. (2006) for Brazil during the period 1981-2004.

However, this method has been heavily criticized since it generally does not provide a perfect decomposition of  $\Theta_{t+k} - \Theta_t$ . The evidence shows that the residual component of this decomposition is generally important and cannot be easily interpreted. To avoid this shortcoming, Shorrocks

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would yield the same total welfare as the observed income distribution (for a short review of the links between absolute indices, relative indices, scale invariance and translation invariance, see Fleurbaey, 1996). In order to avoid confusion, we will not make use of these expressions throughout the rest of the paper.

<sup>4</sup> In the case of the Datt and Ravallion (1992) approach, the decomposition is not exact and a residual term should be added.

<sup>5</sup> This property is called sub-period additivity in Datt and Ravallion (1992).

(1999), using the Shapley-value from the cooperative game theory, and Kakwani (2000), using an axiomatic approach, provide a decomposition framework, so that *i*) the decomposition is exact (there is no residual components), *ii*) the variation of the poverty measure is positive (negative) when both the growth and the inequality components are positive (negative), and *iii*) the value of the growth (inequality) component between  $t$  and  $t+k$  is the opposite of its value between  $t+k$  and  $t$ . According to this Shapley decomposition, we get the following values for each component:

$$G'_{t,t+k} = \frac{(\Theta(z, \mu_{t+k}, \pi_t) - \Theta(z, \mu_t, \pi_t)) + (\Theta(z, \mu_{t+k}, \pi_{t+k}) - \Theta(z, \mu_t, \pi_{t+k}))}{2}, \quad (1.5)$$

$$D'_{t,t+k} = \frac{(\Theta(z, \mu_t, \pi_{t+k}) - \Theta(z, \mu_t, \pi_t)) + (\Theta(z, \mu_{t+k}, \pi_{t+k}) - \Theta(z, \mu_{t+k}, \pi_t))}{2}. \quad (1.6)$$

Recent illustrations of this decomposition technique include Kolenikov and Shorrocks (2005) for Russia in the mid 90s, Baye (2006) for Cameroon during the period 1984-1996 and Wan and Zhang (2006) for rural China during the period 1988-2000.<sup>6</sup> Despite its attractiveness, the Shapley decomposition is not the panacea since it can be proved that the estimated effects are not time transitive.<sup>7</sup> However, since our objective is to question economists current practices, it does not matter which particular decomposition technique is the right one.<sup>8</sup> This explains why this paper focuses on the decompositions corresponding to equations (1.3) to (1.6).

## 2 Invariance and the decomposition of poverty variations

From a technical point of view, the estimation of the growth and inequality components of the poverty spells implies the computation of intermediate, or counterfactual, values for the chosen poverty measure, that is the values that would be reached by the poverty measure if only  $\mu$  or  $\pi$  changed between the dates  $t$  and  $t+k$ . The design of these intermediate values requires an explicit formulation of what inequality means, in particular which ethical values are involved in the concept of inequality used for the analysis. Of particular interest for the decomposition exercise is the concept of invariance that will be extensively discussed through the next paragraphs.

Consider an income distribution  $X$  of size  $n \geq 2$  with  $n \in \mathbb{N}^*$ . Incomes are defined on the set  $D_\alpha : [\alpha, +\infty)$ . Each distribution  $X$  is then drawn from the set  $\mathcal{D}_\alpha = \bigcup_{n \in \mathbb{N}^*} D_\alpha^n$ . Sometime  $\mathcal{D}_\alpha$  is

<sup>6</sup> Kolenikov and Shorrocks (2005) prime interest is not dynamic but regional decomposition of the variations of poverty.

<sup>7</sup> In order to get growth and inequality components that respect this property, Kakwani (2000) suggests using the following formula for  $G_{t,t+k}$  and  $I_{t,t+k}$ :

$$\tilde{G}_{t,t+k} = \frac{1}{s} \sum_{j=1}^s (G'_{t,j} + G'_{j,t+k}),$$

$$\tilde{D}_{t,t+k} = \frac{1}{s} \sum_{j=1}^s (I'_{t,j} + D'_{j,t+k}).$$

when the total observed period is 1 to  $s$  with  $1 \leq t \leq t+k \leq s$ . However, even if these effects are time transitive and yield a perfect decomposition, they present the undesirable feature of being path-dependent since they also depend of the income distributions during the periods  $1 \dots t-1$ ,  $t+1 \dots t+k-1$  and  $t+k+1 \dots s$ . So two economies with the same income distributions in  $t$  and  $t+k$  may present different values of  $\tilde{G}_{t,t+k}$  and  $\tilde{D}_{t,t+k}$  if they do not share the same evolution during the period of analysis. To our knowledge, Kakwani (2000) is the sole application of these formula.

<sup>8</sup> For more critics of the aforementioned decomposition techniques, see Muller (2006).

restricted to the nonnegative or strictly positive orthant of the  $n$ -dimensional Euclidean space  $R^n$  with the origin deleted. Such sets will be respectively noted  $\mathcal{D}^+$  and  $\mathcal{D}^{++}$ . Each vector  $X$  is ordered so that  $x_1 \leq x_2 \dots \leq x_n$ . An inequality index  $\Psi$  is a mapping of  $\mathcal{D}_\alpha$  into  $\mathbb{R}^+$  such that  $\Psi(X_1) < \Psi(X_2)$  implies that  $X_1$  is considered as less unequal than  $X_2$ .

For the sake of simplicity, a traditional assumption is  $\Psi(\mu I) = 0 \forall \mu \in \mathbb{R}^{++}$  with  $I$  being a  $n$ -vector of 1.<sup>9</sup> We also impose as minimum requirements the respect of the core anonymity, continuity and population axioms.<sup>10</sup> In the following paragraphs, we will make use of the Pigou-Dalton principle of transfers such that progressive (regressive) transfers lower (increase) inequality.<sup>1112</sup> The respect of the anonymity axiom and of the Pigou-Dalton principle of transfers imply that  $\Psi$  is S-convex (Dasgupta et al., 1973).<sup>13</sup> However, this principle of transfers can be debated (see for instance Amiel and Cowell 1992 or Chateauneuf and Moyes 2005) and will sometime conflict with other axioms. So, though it will be considered as a desired property,  $\Psi$  may sometime not respect the Pigou-Dalton principle of transfers.

Invariance is the property of any inequality measure  $\Psi$  such that:

$$\Psi(\Phi(X)) = \Psi(X), \tag{2.1}$$

where  $\Phi$  is a continuous increasing function  $\Phi : \mathcal{D} \rightarrow \mathcal{D}$ . Such an axiom is necessary for the comparison of income distributions with different means.<sup>14</sup> So invariance can be seen as the way of sharing an additional income in order to leave the judgment on inequality unchanged.<sup>15</sup> Re-

<sup>9</sup> In the case of the inequality measure defined by Alonso-Villar and del Rio (2007a), this condition may not be respected since its domain generally does not include distributions where incomes are equally shared. The underlying invariance axiom is presented in section 2.3.1.

<sup>10</sup> Chakravarty (1999) is a fairly comprehensive review of the most common axioms used in the inequality measurement related literature. Anonymity, also called symmetry, horizontal equity or equal treatment of equals, means that  $\Psi(PX) = \Psi(X)$  with  $P$  being any permutation matrix of size  $n \times n$ . Continuity implies that marginal variations of any element of  $X$  do not cause large variations of the measure  $P$ . Finally, a measure respects the population axiom, also called replication invariance axiom, if a  $m$ -replication of  $X$  exhibit the same degree of inequality as  $X$ , whatever  $X \in \mathcal{D}$ .

<sup>11</sup> A transfer is progressive (regressive) if it increases (decreases) the income of an individual at the expense of (in favour of) a richest individual without changing their relative position in the distribution. A weaker version of the principle version would require regressive (progressive) transfers not to increase (lower) the value of  $\Psi$ .

<sup>12</sup> This property is called “rectifiante” in Kolm’s (1976a) seminal paper.

<sup>13</sup> For any bistochastic  $n \times n$ -matrix  $B$ , that is a square matrix which contains only positive elements and which columns and rows sum to one, a function is S-convex if  $\Psi(BX) \leq \Psi(X)$ . If strict S-convexity is required, then we should observe  $\Psi(BX) < \Psi(X)$  for all bistochastic matrices except permutations matrices.

<sup>14</sup> Ebert (2004) stressed that invariance only defines relations between distributions for which we feel indifferent with respect to inequality. So it is of no help for ranking distributions that are not in the same iso-inequality set.

<sup>15</sup> An other justification for the various invariance axioms presented here can be found in the normative approach of inequality measurement. Since Kolm (1969); Atkinson (1970) and Sen (1973), inequality measures are often derived from social evaluation functions  $W : \mathcal{D}_\alpha \rightarrow \mathbb{R}$  which provide a quasi-ordering of income distributions from the set  $\mathcal{D}_\alpha$ . In other words,  $W$  reflects the opinions of the social evaluator (the observer) in terms of distributive justice. Let  $a$  and  $b$  be some negative constant parameters. Kolm (1969, theorems 13 and 14) shows that social evaluation function that complies with anonymity and Independence — a social evaluation function fulfills Independence if it can be expressed as  $\sum_i f(y_i)$  with  $f : \mathbb{R} \rightarrow \mathbb{R}$  being an increasing function —, are of the form:

$$W = \sum_{i=1}^n a y_i^b \quad \text{or} \quad W = \prod_{i=1}^n y_i^a,$$

if the social evaluator believes in a “rightist” view and:

$$W = \sum_{i=1}^n a e^{b y_i},$$

cently, Zheng (2004) has shed light on an axiom which is closely linked to invariance, namely the unit-consistency axiom.<sup>16</sup> Unit-consistency requires the inequality ordering to be invariant with respect to changes in the common unit of measure adopted to evaluate the distributions. So we have to observe  $\Psi(\lambda X) = \Lambda(\Psi(X)) \forall X \in \mathcal{D}_\alpha$  where  $\lambda$  is a positive scalar and  $\Lambda$  is a continuous monotone function from  $\mathbb{R}^+$  to  $\mathbb{R}^+$ . In other words, two income distributions should be ranked in the same manner according to  $\Psi$  when incomes are measured in euros or in dollars. As we will see in the next sections, unit-consistency is necessary when considering inequality views that do not rely on scale invariance.

Most of these axioms, even slightly modified, are shared by poverty measures. For instance, continuity is generally replaced by restricted continuity such that  $\Theta$  is a left continuous function of  $x$  for all  $x < z$ . The main addition is the focus axiom which states that the only relevant information related to the non-poor members of the population is their number.<sup>17</sup> So a poverty measure  $\Theta$  is not affected by any increment of the income of non-poor person. This explains why poverty measures are often considered as a restriction of inequality measures on the subset  $X^p$  of the income distribution such that each element of  $X^p$  is not greater than the poverty line  $z$ . As a consequence, the following expressions are perfect substitutes  $\Theta(z, \mu, \pi) = \Theta(z, X) = \Theta(z, X^p, n)$ . An additional requirement is the weak monotonicity axiom which imposes on a poverty measure not to decrease if a poor person's income decreases. Finally,  $\Theta$  should be non decreasing in  $z$ .<sup>18</sup>

In the following sections, we now details some particular versions of the invariance axiom and present their implementation for the calculation of growth and inequality effects of poverty variations.

## 2.1 Scale invariance

The most widely used invariance axiom is the scale invariance axiom, such that:

$$\Psi(\lambda X) = \Psi(X) \quad \forall \lambda > 0. \quad (2.2)$$

The scale invariance axiom means that doubling each income of the observed distribution if the social evaluator's preferences are in accordance with the "leftist" view. Applying the famous results of Arrow and Pratt, Atkinson (1970) emphasizes that the first two functional forms reflects a constant relative inequality (or risk) aversion and the third one a constant absolute inequality (or risk) aversion. The function  $W$  is used to define the equally distributed equivalent income  $\tilde{x}$ , that is the level of per capita income which, if equally distributed, would provide the same level of social welfare as the observed distribution. Then the natural form of scale and translation inequality indices is respectively:

$$\Psi^r = 1 - \frac{\tilde{x}}{\mu},$$

$$\Psi^a = \mu - \tilde{x}.$$

According to Kolm (1976a),  $\Psi^r$  should be considered as a measure of inequality "per pound" and  $\Psi^a$  as a measure of inequality "per person".

<sup>16</sup> This property was already detailed in Aczél and Moszner (1994). Kolm (1995) and Zoli (2003) also considered this desired property and called it respectively "unit invariance" and "weak currency-independence".

<sup>17</sup> For mathematical convenience, a weak definition of poverty — an individual is poor if his income is strictly inferior to the poverty line — is generally preferred (see for instance Donaldson and Weymark, 1986).

<sup>18</sup> If we put forward that  $\Theta$  is strictly increasing in  $z$ , monotonicity is implicitly assumed. For more details about the different poverty axioms and their interrelations, see Zheng (1997).

does not affect inequality as measured by  $\Psi$ . In mathematical words,  $\Psi$  complies with scale invariance if it is homogeneous of degree zero.<sup>19</sup> With the measurement of poverty, a markedly modified version of the scale invariance axiom has to be invoked, that is:<sup>20</sup>

$$\Theta(\lambda z, \lambda X^P, n) = \Theta(z, X^P, n) \quad \forall \lambda > 0 \quad (2.3)$$

which is less restrictive than the first version since the sole condition imposed on the income of the non-poor is to remain greater than  $z$ . Most inequality (*e.g.* Gini coefficient, Atkinson index, generalized entropy indexes) and poverty (*e.g.* Watts index, Sen index, Foster, Greer and Thorbecke indexes) measures used in empirical analysis rely on these scale invariance axioms. The same property holds for the traditional Lorenz curve. In the context of the decomposition of poverty spells, scale invariance is often used in an implicit manner for the computation of the intermediate values of the poverty measure since these one are generally defined with respect to the Lorenz curve.<sup>21</sup> In the two period case, the respect of the scale invariance implies equations (1.3) and (1.4) to be computed as follow:

$$G_{t,t+1}^S = \Theta(z, \lambda_{t,t+1} X_t^P) - \Theta(z, X_t^P), \quad (2.4)$$

$$D_{t,t+1}^S = \Theta(z, \lambda_{t+1,t} X_{t+1}^P) - \Theta(z, X_t^P). \quad (2.5)$$

where  $\lambda_{t,t+k} = \frac{\mu_{t+k}}{\mu_t}$ .<sup>22</sup> Extension to equations (1.5) and (1.6) is straightforward.<sup>23</sup>

Scale invariance is frequently seen as a desirable feature for an inequality measure so as its value does not depend on the unit used for the measurement of incomes. Many authors like Zheng (2004) argue that this is a rather strong requirement for an inequality or a poverty measure and that one only need the ranking of different distributions to be preserved when income are expressed in a different measuring unit.<sup>24,25</sup> This unit-consistency axiom is weaker than scale invariance since it allows for different way of thinking inequality while keeping the sole desirable characteristic of scale invariance. So, if inequality is considered from an ordinal point of view,

<sup>19</sup> In Kolm (1969), inequality measures that fulfills scale invariance are called "intensive".

<sup>20</sup> Mitra and Ok (1995) are dubious about the usefulness of invariance axiom in the context of poverty since it would be a non-sense from a practical point of view to compare  $\Theta(\Phi(X^P), \Phi(z))$  with  $\Theta(X^P, z)$ . "Since the value of a poverty index is explicitly a function of the poverty line, it does not make sense to compare the poverty levels of two income distributions with two different poverty lines." We advocate that the knowledge of the properties of poverty measures are essential when linking poverty to inequality. The nonsense would be to make use of inequality and poverty studies that are not based on the same ethical ground. In the context of poverty spells decompositions between growth and inequality components, our feeling is that the knowledge of the invariance axiom underlying the chosen poverty measure is essential since it predetermines the relative contribution of growth and redistribution to poverty reduction. Moreover, invariance properties of the poverty measures are crucial for relative poverty measures, that is when the poverty line depends on the observed income distribution.

<sup>21</sup> Most of the time, poverty measures respect a sole invariance axiom. So an explicit formulation of which invariance axiom is used may be considered as secondary.

<sup>22</sup> In the present section, we will assume  $n$  to be constant. Thus it can easily be dropped in order to save space. However, results do not change when  $n$  varies through time.

<sup>23</sup> In order to save space, we do not report the corresponding formula for the Shapley decomposition but use them in the application developed in section 4.

<sup>24</sup> The unit-consistency axiom is further analyzed for inequality and poverty measurement in Zheng (2007a, 2005, 2007b,c).

<sup>25</sup> The argument that changes in the unit of measurement do not necessarily have the same effects that changes of size, was earlier presented by Aczél and Moszner (1994) in the more general context of economic indices.

scale invariance is not the unique way of thinking inequality any more.

However, one should note that the scale invariance axiom may find little support in presence of negative incomes since it induces a failure of the Pigou-Dalton principle of transfers.<sup>26</sup> Moreover, ethical values associated with the measurement of inequality and poverty are not unanimously shared, even within welfare economists. In particular, there is no unanimous agreement on the invariance axiom that should be used. For instance, Dalton (1920) argued that applying the same positive rate of growth to all income decreases the degree of inequality of the income distribution. Using questionnaires on samples of undergraduate students, Amiel and Cowell (1992; 1999; 2001) and Harrison and Seidl (1994*b*) observed that scale invariance was generally not supported for inequality analysis by a majority of the respondents and that many rival invariance axioms were preferred by some respondents.<sup>27</sup> A major implication of these studies is that inequality measurement tools should reflect the heterogeneity of feelings and moral judgments about inequality since one cannot discriminate between values without ethical, yet subjective, arguments. As the design of poverty-reducing policies requires the use of tools that are consistent with policy makers' ethical values, one should be cautious of a systematic use of indexes based on the scale invariance axiom. Thus, we have to examine rival versions of the invariance axiom and their implications for the decomposition of poverty spells into growth and inequality components.

## 2.2 Translation invariance

The first rival invariance axiom that is commonly treated in the literature is the translation invariance axiom which, according to Kolm (1976*a*) is associated to a "leftist" view of inequality (in Kolm's words scale invariance corresponds to a "rightist" view). An inequality measure is said to respect the translation invariance axiom if:

$$\Psi(X + \delta I) = \Psi(X) \quad \forall \delta \in \mathbb{R} \quad (2.6)$$

which implies that any equal increment or decrement of each income of the distribution leaves the inequality index unchanged.<sup>28,29</sup> The less restrictive version of the translation invariance ax-

<sup>26</sup> For instance, it may be difficult to argue that the distributions  $X_1 = \{-2, 20\}$  and  $X_2 = \{-4, 40\}$  exhibit the same degree of inequality. An acceptance of the statement  $\Psi(X_1) = \Psi(X_2)$  would imply a failure of the Pigou-Dalton transfers principle. For instance a progressive transfer of 2 units would lead to the distribution  $X_3 = \{-2, 38\}$  such that  $\Psi(X_3) \leq \Psi(X_2)$ . The respect of both scale invariance and transfer principle would lead to the hardly justifiable conclusion that  $\Psi(X_3) \leq \Psi(X_1)$ . Zoli (2003) shows that the sole invariance axiom which is compatible with both S-convexity and incomes defined on  $\mathbb{R}$  is the translation invariance axiom for  $n \geq 3$ , a result that was already observed by Kolm (1976*a*) in the context of the "centrist" inequality view.

<sup>27</sup> Primary interest of Amiel and Cowell (2001) is difference of inequality and risk perceptions, but the authors choose to focus on invariance perceptions. Amiel and Cowell (1997) also performed an empirical investigation of students' agreement about axioms commonly used in the poverty measurement related literature but did not examine compliance with invariance axioms.

<sup>28</sup> For a detailed examination of inequality indices based on translation invariance, see Blackorby and Donaldson (1980). However, we can mention the variance as a widely used translation-invariant inequality measure. Other absolute poverty indices are suggested in Mitra and Ok (1995) and Zheng (2007*c*).

<sup>29</sup> Harrison and Seidl (1994*a*) argue that one can easily find some inequality measure that is based on a combined

iom that is suitable for poverty analysis is:

$$\Theta(z + \delta I, X^p + \delta I, n) = \Theta(z, X^p, n) \quad \forall \delta \in \mathbb{R}. \quad (2.7)$$

In this case,  $\Theta$  should not be defined any more with respect to the Lorenz curve. The counterpart of the traditional Lorenz curve for translation invariant inequality measures is the absolute Lorenz curve  $L^a$  (Moyes, 1987). Thus a translation invariant poverty measure can be written as  $\Theta(\mu, L^a)$ . Under this axiom, the estimation of the growth and inequality components of poverty spells will differ from the one corresponding to scale invariant poverty measures.<sup>30</sup> In the two period case,  $G_{t,t+k}$  and  $D_{t,t+k}$  now become:

$$G_{t,t+k}^T = \Theta(z, X_t^p + \delta_{t,t+k} I) - \Theta(z, X_t^p), \quad (2.8)$$

$$D_{t,t+k}^T = \Theta(z, X_{t+k}^p + \delta_{t+k,t} I) - \Theta(z, X_t^p), \quad (2.9)$$

where  $\delta_{t,t+k} = \mu_{t+k} - \mu_t$ .

### 2.3 Intermediate invariance

The set of invariance axioms is not restricted to scale and translation invariances, and many rival axioms, the so-called intermediate invariance axioms, have been developed during the last decade. Intermediate views are based on the intuition that an equiproportional addition to all incomes should increase inequality while an equal-increment to all incomes should reduce inequality. The first reason of considering an intermediate view is of course that it may be the way some people feel inequality should be defined. In the context of poverty analysis, it will also be useful to consider families of intermediate inequality views when the decompositions based on scale and translation invariance do not yield the same conclusions. As in the context of inequality orderings, it may be wise to use intermediate inequality views so as to find cut-off values of the ethical parameters involved in the definition of each intermediate view such that conclusions change when this particular value is crossed. Thus it can be seen as a way of assessing the robustness of a conclusion obtained through scale or translation invariance.

view which satisfies both scale and translation invariance like:

$$\Psi(X) = \Psi\left(\frac{X - \min\{x_i | i = 1 \dots n\} I}{n(\mu - \min\{x_i | i = 1 \dots n\})}\right)$$

This view differs from Kolm; Kolm's (1969; 1976b) "synthetic" solution which suggests using inequality measures that are scale invariant in their relative form and translation invariant in their absolute form (see note 15 for the definition of the relative and absolute forms).

For the sake of simplicity, we will suppose that the different invariance views are rival and thus should not be respected simultaneously. Such an hypothesis is standard in the related literature but would merit a further examination since empirical evidence shows that some individuals may feel in accordance with both scale and translation invariance, a finding that most authors see as the results of mistakes (see for instance Amiel and Cowell 1992).

<sup>30</sup> Duclos and Wodon (2004) also considered translation invariance in the context of the social evaluation of "pro-poor" character of growth, an issue that is closely linked to the decomposition of poverty spells into growth and inequality effects. However, the authors did not investigate the implications of a change in the chosen invariance axiom, nor do they illustrate their approach with an empirical application.

Kolm (1976*a,b*) was the first to give a formal treatment to inequality indices based on an intermediate axiom.<sup>31</sup> His “centrist” view is defined by the relation:

$$\Psi(\beta(X - I\varepsilon) + I\varepsilon) = \beta\Psi(X) \quad \forall \beta > 0, \varepsilon \in ]-\infty, 0]. \quad (2.10)$$

In these case, note that invariance is implicitly defined since the transformed income distribution of equation (2.10) is  $\beta$  times as unequal as the original distribution. Thus this inequality view is poorly operational for the decomposition of poverty spells.

In order to avoid such an undesirable feature, many authors formulated intermediate inequality views which explicitly defines the iso-inequality set of distributions that corresponds to any distribution  $X$ . Before reviewing the various intermediate inequality views suggested in the literature, it may be useful to state precisely what is meant by intermediate inequality views. The question is not trivial since the concept is given different meanings by authors of the field. Generally, an inequality view is intermediate if an equiproportional increase in all incomes raises the degree of income inequality, whereas an equal increment decreases it.<sup>32</sup> In the present paper we will consider classes of intermediate views with the help of a general parametrized definition which states that the equally unequal income distributions ought to be expressed as weighted means of the corresponding transformed distributions under scale and translation invariance.<sup>33</sup> Hence, we suggest using a definition based on the following lemma:

**Lemma 1.** *An inequality view is said intermediate if the transformed distribution  $\Phi^I(X, \mu_Y)$  which is considered as exhibiting the same degree of inequality as  $X$  and with mean income  $\mu_Y$ , respects the following condition:*

$$\Psi(\Phi^I(X, \mu_Y)) = \Psi\left(u(\mu_X, \mu_Y)X \frac{\mu_Y}{\mu_X} + (1 - u(\mu_X, \mu_Y))(X + (\mu_Y - \mu_X)I)\right) = \Psi(X) \quad (2.11)$$

with  $u(\mu_X, \mu_Y) \in [0, 1] \forall \mu_Y \in \mathbb{R}^{++}$ .

*Proof.* We know that whatever the chosen inequality view, every transformed income distribution  $\Phi(X)$  is located on the two-dimension sub-space  $\mathcal{S}_X$  defined by the vectors  $I$  and  $X$ . On the other

<sup>31</sup> In the related literature, it is common to focus on invariance axioms that are bounded by translation and scale invariance. Amiel and Cowell (1992) and Harrison and Seidl (1994*b*) studies show that there is (little) support for extreme “rightist” — multiplying incomes by the same constant  $\lambda > 1$  decreases inequality — and extreme “leftist” — adding the same amount  $\delta > 0$  to all incomes increases inequality — views. However, there is currently little formal treatment of such views. An exception is the “ultra rightist” reference-point inequality view defined by Ebert (2004). However, it can be proved that this generalization of Bossert and Pfingsten (1990) is not unit-consistent.

<sup>32</sup> In Bossert and Pfingsten’s (1990) and Zoli’s (2003) words, this property is respectively called “compromise condition” and “compromise inequality equivalence”.

<sup>33</sup> An other definition, which is derived from Bossert and Pfingsten (1990) and Zheng (2007*c,a*), requires the formulation of an intermediateness axiom which implies that the intermediate inequality (poverty) ordering of two distributions should be the same as the ordering corresponding to “leftist” and “rightist” views when their inequality (poverty) ordering of the two distribution is identical. According to this definition, an inequality (poverty) index is intermediate if it respects the intermediateness axiom and includes some vector of parameters such that scale invariant poverty measures and translation invariant inequality (poverty) measures are obtained for some limiting specific combinations of these parameters. In fact, this parametrized definition does not define an intermediate view, but a class of intermediate views. Moreover, it is generally possible to extend the domain of the values of these parameters so as to include extreme views.

hand, the set of all income distributions with mean income equal to  $\mu_Y$  is on the hyperplane defined by the equation  $\sum_{i=1}^n y_i = n\mu_Y$ .

As this hyperplane is defined by the normal vector  $I$  which is by definition included in the subspace  $\mathcal{S}_X$ , its intersection with  $\mathcal{S}_X$  is non-empty and defines a unique ray  $\mathcal{L}$ . By definition,  $\mathcal{L}$  passes through the distributions  $X \frac{\mu_Y}{\mu_X}$ ,  $X + (\mu_Y - \mu_X)I$  and  $I\mu_Y$ , and, more generally, includes all distributions  $\Phi(X, \mu_Y)$  with mean income  $\mu_Y$ . Moreover, since it is a straight line, every distribution  $\Phi(X, \mu_Y)$  can be expressed as a linear combination of the distributions  $X \frac{\mu_Y}{\mu_X}$  and  $X + (\mu_Y - \mu_X)I$ . It also deserves to be stressed that the closer a distribution  $\Phi(X, \mu_Y)$  is to the distribution  $I\mu_Y$ , the more equal will it be considered.

The most general definition of an intermediate inequality view is that an inequality view is intermediate when an equiproportional addition (subtraction) to all incomes increases (decreases) inequality while an equal-increment (decrement) to all incomes reduces (increases) inequality. Consequently, any distribution  $\Phi^I(X, \mu_Y)$  derived from an intermediate inequality view is necessarily located between the points  $X \frac{\mu_Y}{\mu_X}$  and  $X + (\mu_Y - \mu_X)I$  on  $\mathcal{L}$ . Thus, we can use the following expression of  $\Phi^I(X, \mu_Y)$ :

$$\Phi^I(X, \mu_Y) = u(\mu_X, \mu_Y) X \frac{\mu_Y}{\mu_X} + (1 - u(\mu_X, \mu_Y)) (X + (\mu_Y - \mu_X)I) \quad (2.12)$$

with  $u(\mu_X, \mu_Y) \in [0, 1] \forall \mu_Y \in \mathbb{R}^{++}$ . Plugging equation (2.12) into equation 2.1 give a general definition of intermediate invariance axioms through equation (2.11).  $\square$

In the case of the intermediate invariance axioms that will be reviewed in the next paragraphs, the weighing term  $u$  can be expressed as  $u(\mu_X, \mu_Y, \boldsymbol{\rho})$ ,  $\boldsymbol{\rho}$  being some set of parameters. For some combinations of these parameters,  $u(\mu_X, \mu_Y, \boldsymbol{\rho}) = 1$  ( $=0$ ) and  $\Phi^I(X, \boldsymbol{\rho})$  becomes the equally unequal distribution with mean income  $\mu_Y$  corresponding to scale (translation) invariance. One can also note that, for a given initial distribution  $X$ ,  $u$  may depend on the value of the mean of the final distribution. In this case,  $u$  is not constant and the intermediate view may tend to “leftist” or “rightist” views inequality as mean income increases.

Figures 1 illustrates this property of intermediate views in the case of a three-person distribution  $X = \{x_1, x_2, x_3\}$ . Perfect equality is represented by the straight line through the points  $O$  and  $M$ . All distributions with mean equal to  $\mu_Y$  are on the plane defined by the points  $A$ ,  $B$  and  $C$ . If incomes are non-negative, the set of distributions with mean equal to  $\mu_Y$  is restricted to the surface  $ABC$ . All equally unequal distributions issued from distribution  $X$  are on the subspace defined by the vectors  $\overrightarrow{OX}$  and  $\overrightarrow{OM}$ . The ray through the points  $X$  and  $O$  is the iso-inequality line corresponding to scale invariance. It intersects the surface  $ABC$  at  $X^S$ . The translation invariance iso-inequality ray is the straight line through  $X$  and supported by  $\overrightarrow{OM}$ . The projection of  $X$  according to this “leftist” view on the surface  $ABC$  is the point  $X^T$ . It can be easily seen that any transformation of  $X$  with mean  $\mu_Y$  is on the segment  $LM$ , that is the intersection of surface  $ABC$  and the subspace defined by  $\overrightarrow{OX}$  and  $\overrightarrow{OM}$ .<sup>34</sup> The point  $M$  is the one corresponding to an equal distribution while  $L$  represents the most unequal distribution with mean  $\mu_Y$  that can be

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<sup>34</sup> We do not consider the whole part of this intersection since points along the line through the points  $L$  and  $M$  but closer to  $A$  than  $M$  are just permutations of the income distributions observed on the segment  $LM$ .



may follow Zheng (2007a) and rewrite (2.13) as:

$$\Psi\left(\frac{X - \mu_X I}{\eta(\mu_X - 1) + 1} + \eta\right) = \Psi\left(\frac{Y - \mu_Y I}{\eta(\mu_Y - 1) + 1} + \eta\right) \quad (2.14)$$

where  $Y$  is the transformed distribution. If we intend to transform the distribution  $X$  in order to get an income distribution with mean  $\mu_Y$  without changing inequality, equation (2.14) implies  $\varphi = \frac{\mu_Y - \mu_X}{\eta(\mu_X - 1) + 1}$ . Using the general form of equation (2.11), it can be shown that:

$$u(\mu_X, \mu_Y, \rho) = u(\mu_X, \eta) = \frac{\eta \mu_X}{\eta(\mu_X - 1) + 1}. \quad (2.15)$$

Many authors like Zheng (2004) have stressed that such an intermediate transformation tends to behave like a “rightist” transformation as mean income increases when  $\eta > 0$ . This can be seen from equation (2.15) since  $\lim_{\mu_X \rightarrow +\infty} u(\mu_X, \eta) = 1 \forall \eta > 0$ .

Using the intermediate invariance axiom defined by equation (2.13) and a given value of  $\eta$  yields the following relation for the measurement of poverty :

$$\Theta\left(z + \varphi(\eta z + 1 - \eta), X^p + \varphi(\eta X^p + (1 - \eta)I), n\right) = \Theta(z, X^p, n) \quad \forall \varphi > -1/\eta \quad (2.16)$$

and thus the respective expressions of the growth and inequality components of poverty:

$$G_{t,t+k}^I = \Theta\left(z, X_t^p + \varphi_{t,t+k}(\eta X_t^p + (1 - \eta)I)\right) - \Theta(z, X_t^p), \quad (2.17)$$

$$D_{t,t+k}^I = \Theta\left(z, X_{t+k}^p + \varphi_{t+k,t}(\eta X_{t+k}^p + (1 - \eta)I)\right) - \Theta(z, X_t^p), \quad (2.18)$$

where  $\varphi_{t,t+k} = \frac{\mu_{t+k} - \mu_t}{\eta(\mu_t - 1) + 1}$ . However, Zheng (2004) demonstrated that any inequality measure based on the view developed by Bossert and Pfingsten (1990) violates unit-consistency. Thus, it should not be used for the measurement of inequality and poverty since it may lead to non-robust conclusions.

In order to get a linear family of transformations that do not tend to behave like the “rightist” view, Pfingsten and Seidl (1997) have proposed the so-called ray-invariance axiom. An inequality view respects a ray-invariance axiom if equally unequal distributions are along a ray which includes the observed income distribution. This ray is defined by a vector drawn from the  $n$ -simplex and which has to respect the following conditions: *i*) the vector Lorenz-dominates the original distribution  $X$ ; *ii*) the vector reflects an unequal distribution ( $\neq n^{-1}I$ ).<sup>36</sup> This view differs from the one described through equation (2.13) inasmuch as the part of the incremental income that is not equally shared between each income receivers, is not necessarily distributed in proportion of each income’s share in the initial distribution. Ethical preferences are then described through a  $n$ -vector that unfortunately cannot be easily interpreted.<sup>37</sup> A particular case of Pfing-

<sup>36</sup> A general expression of ray-invariance was also provided by Krtscha (1994) under the name of “weak relative inequality”. The author argued that this is not a suitable way of thinking inequality since there is no reason that a large additional amount should be shared in the same manner that a smaller one.

<sup>37</sup> Moreover, Zoli (2003) observes that individuals with the same level of income may receive a different amount. A different treatment of identical income-receivers can be seen as a violation of the anonymity axiom. The same remark seems to hold for the inequality view defined through equation (2.19). However, conditions imposed on  $V$  in Alonso-

sten and Seidl (1997) ray-invariance is the  $(V, v)$ -invariance described by del Rio and Ruiz-Castillo (2000) and generalized by Alonso-Villar and del Rio (2007a), which imposes the use of a reference distribution  $V$  of size  $n_V$  and the following relation for inequality measurement:

$$\Psi\left(X + \tau\left(v\frac{V_X}{\mu_V} + (1-v)I\right)\right) = \Psi(X) \quad \forall \tau \in \mathbb{R} \text{ s.t. } X + \tau\left(v\frac{V_X}{\mu_V} + (1-v)I\right) \in \mathcal{D}_\alpha. \quad (2.19)$$

where  $v \in [0, \iota]$  reflects ethical preferences and  $V_X$  is the projection of the reference distribution  $V$  into the subspace  $\mathcal{S}_X$  defined by the vectors  $X$  and  $I$ . For  $v = 0$ , the “leftist” inequality view is obtained whereas  $v = \iota$  corresponds to the “rightist” view.<sup>38</sup> In appendix A, we demonstrate that a valid equation for the computation of  $V_X$  is:

$$\frac{V_X}{\mu_V} = \frac{1}{\iota} \frac{X}{\mu_X} + \left(1 - \frac{1}{\iota}\right) I. \quad (2.20)$$

with:

$$\iota = \sqrt{\frac{n^{-1} \sum_{i=1}^n \left(\frac{y_i}{\mu_Y} - 1\right)^2}{n_V^{-1} \sum_{i=1}^{n_V} \left(\frac{v_i}{\mu_V} - 1\right)^2}}. \quad (2.21)$$

In order to simplify we can chose  $X$  as the reference distribution. In this case, the link with equation (2.11) is straightforward since  $u(\mu_X, \mu_Y, \rho) = v$ .<sup>39</sup> With any other regular distribution  $V$ , it can easily be proved that lemma 1 still holds under certain conditions (*cf.* appendix B).<sup>40</sup>

Contrary to Bossert and Pfingsten’s (1990) intermediate view, one should note that the value of the parameter  $v$  is contingent to the choice of a reference distribution. If we consider two distributions  $X$  and  $Y$  with different means, one can easily demonstrate that the transformation of  $Y$  into  $X$  using  $Y$  as the reference will require a value  $v'$  that is different from the one corresponding to a transformation of  $X$  into  $Y$  using  $X$  as the reference except for  $v = 1$  and  $v = 0$ . Otherwise, we observe (del Rio and Ruiz-Castillo, 2000, proposition 1)  $v' = \frac{v\mu_Y}{(1-v)\mu_X + v\mu_Y}$ .<sup>41</sup>

Villar and del Rio (2007a) ensure anonymity to be respected (see note 40).

<sup>38</sup> In del Rio and Ruiz-Castillo (2000) and Alonso-Villar and del Rio (2007a), the parameter  $v$  is defined on the unit interval. However, for any distribution  $X$ , it can be shown that  $v = 1$  corresponds to an “ultra-rightist” view until  $V_X$  is equal to  $X$  up to a scale factor.

<sup>39</sup> Zheng (2004, proposition 2.4) argued that inequality measures based on Pfingsten and Seidl (1997) tend to behave like translation invariant measures as mean income increases when  $v < 1$ . Our result that  $u(\mu_X, \mu_Y, \rho)$  is constant invalidates these proposition.

<sup>40</sup> In Alonso-Villar and del Rio (2007a), ray-invariant inequality views are characterized by substituting the reference distribution  $V$  (or  $X$ ) by the Euclidean distance  $\chi \in \left[0, \sqrt{n \sum_i \left(\frac{x_i}{n\mu_X} - \frac{1}{n}\right)^2}\right]$  between the chosen vector of increments  $v\frac{V}{\mu_V} + (1-v)I$  and the one corresponding to equal increments  $\frac{I}{n}$ . The vector  $(\chi, v)$  defines a unique intermediate view since a unique vector  $v\frac{V}{\mu_V} + (1-v)I$  is associated with each two-dimension subspace for given values of  $\chi$  and  $v$ . This view implicitly introduces a new (and maybe controversial) axiom for the measurement of inequality that suggests that two distributions with the same mean are equally unequal if their size-normalized Euclidean distance from the vector of perfect equality is the same.

In practice, such a generalization of del Rio and Ruiz-Castillo’s (2000) view will be helpful to compare the different values of  $v$  when the decomposition of poverty spells is realized for many subperiods (*cf.* appendix A).

<sup>41</sup> If the size of distributions  $X$  and  $Y$  is respectively  $n$  and  $m$  with  $n \neq m$ , the relation between  $v'$  and  $v$  becomes:

$$v' = \frac{vm\mu_Y}{(1-v)n\mu_X + vm\mu_Y}.$$

The related invariance axiom for poverty measurement of this inequality view is:

$$\Theta \left( z + \tau \left( v \frac{V_{X,b}}{\mu_V} + 1 - v \right), X^p + \tau \left( v \frac{V_X^p}{\mu_V} + (1 - v)I \right), n \right) = \Theta(z, X^p, n). \quad (2.22)$$

where  $V_X^p$  is the bottom part of  $V_X$  so that  $X^p$  and  $V_X^p$  are of the same size, and  $V_{X,b}$  is the  $b$ -th element of  $V_X$  so that  $V_{X,b-1} \leq z < V_{X,b}$ . The parameter  $\tau$  is restricted in the same way as in equation (2.19). With such kind of inequality view, we get the following formula for the computation of  $G_{t,t+k}$  and  $D_{t,t+k}$ :

$$G_{t,t+k}^R = \Theta \left( z, X_t^p + \tau_{t,t+k} \left( v \frac{V_t^p}{\mu_V} + (1 - v)I \right) \right) - \Theta(z, X_t^p), \quad (2.23)$$

$$D_{t,t+k}^R = \Theta \left( z, X_{t+k}^p + \tau_{t+k,t} \left( v \frac{V_{t+k}^p}{\mu_V} + (1 - v)I \right) \right) - \Theta(z, X_t^p), \quad (2.24)$$

with  $\tau_{t,t+k} = \mu_{t+k} - \mu_t$  and  $V_t$  and  $V_{t+k}$  being the respective projections of  $V$  in the two-dimension subspaces including  $X_t$  and  $X_{t+k}$  and defined through equation 2.20.<sup>42</sup> It can easily be proven that del Rio and Ruiz-Castillo's (2000) inequality view complies with unit-consistency (*cf.* appendix C) and so is suitable for poverty analysis.<sup>43,44</sup>

However, a major issue with equations (2.23) and (2.24) is that the decomposition of poverty variations may not provide the results corresponding to scale invariance in a multiperiod analysis. As the value of the parameter  $\iota$  vary with from a distribution to an other, we have to adopt its minimal value for all comparisons in order to avoid "ultra-rightist" views. When using this particular value  $\iota^*$ , we will then obtain intermediate decompositions for some periods and "rightist" decompositions for the periods which initial or final distribution is the one that defines  $\iota^*$ . This result is puzzling since it would be a non-sense to compare on the basis of the same  $(V, \iota^*)$ -intermediate view intermediate effects for a period with "rightist" effects for other periods. Consequently, we argue that the view developed by del Rio and Ruiz-Castillo (2000) is not suitable for poverty analysis.

Moreover, linear intermediate invariance axioms may not be an appropriate way of modelling individual' tastes and feelings. Amiel and Cowell (2001) results suggest that many people may

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It can be easily shown that  $v' > v$  if  $\mu_Y > \mu_X$ .

<sup>42</sup> Alternatively, if  $X_t$  is always chosen as the reference distribution, an alternative formulation for equation (2.24) is:

$$D_{t,t+k} = \Theta \left( z, X_{t+k}^p + \tau_{t+k,t} \left( v \frac{X_t^p}{\mu_t} + (1 - v)I \right) \right) - \Theta(z, X_t^p).$$

From a practical point of view, it may be easier to use equation (2.24), since observed distributions  $X_t$  and  $X_{t+k}$  are not necessarily of the same size.

<sup>43</sup> An alternative way of proving unit-consistency (Alonso-Villar and del Rio, 2007a) is to define an inequality measure based on the expressed inequality view and which respects basic inequality axioms and then to demonstrate that the measure is unit-consistent.

<sup>44</sup> Zheng (2007a) recently demonstrates that the sole unit-consistent intermediate inequality view which Lorenz-criterion can be expressed as a quasilinear weighted mean of the relative and absolute Lorenz curves is the non-linear inequality view proposed by Krtscha (1994) and Yoshida (2005). This result is consistent with our findings since one can prove that the intermediate Lorenz curve corresponding to del Rio and Ruiz-Castillo's (2000) view is not intermediate in the sense of Zheng (2007a) and thus can generally not be expressed as a quasilinear weighted mean of the relative and absolute Lorenz curves.

think inequality in a way that involves non-linear invariance axioms. Such views are presented in the next section.

### 2.3.2 Non-linear intermediate invariance

In the preceding section, we considered inequality views such that distributions that are considered as equivalent to distribution  $X$  from an inequality point of view, are aligned on a unique ray through  $X$ . In the present section we focus on non-linear intermediate views. The difference with linear intermediate views is that the complete sequence of equally unequal distributions of size  $n$  defines a curve through  $X$ .<sup>45</sup>

Recently, Zoli (2003) and Yoshida (2005) developed some non-linear intermediate view of inequality that do not break with basic desirable properties like the linear transformations presented above. Zoli (2003) first suggested a “flexible inequality equivalence” transformation such that:

$$\Psi\left(\left(\frac{\omega\mu + \kappa}{\mu + \kappa}\right)^\sigma (X - \mu I) + \omega\mu I\right) = \Psi(X) \quad \forall \omega \in \mathbb{R}^+ \text{ s.t. } \left(\frac{\omega\mu + \kappa}{\mu + \kappa}\right)^\sigma (X - \mu I) + \omega\mu I \in \mathcal{D}^+ \quad (2.25)$$

where  $\sigma$  and  $\kappa$  are ethical preference parameters respectively defined on the unit interval and on  $\mathbb{R}^+$ . For  $\sigma = 1$ , we get equation (2.13) with  $\kappa = \frac{1-\eta}{\eta}$  and  $\omega = \frac{\varphi(\eta(\mu-1)+1)}{\mu} + 1$ . Bossert and Pfingsten (1990) intermediate inequality is a particular case of Zoli (2003) non-linear inequality view. In the spirit of Krtscha’s (1994) fair compromise inequality view, a single-parameter version of this general invariance axiom, the  $\sigma$ -invariance axiom, is suggested by Yoshida (2005) with  $\kappa = 0$ . Zheng (2007a) demonstrates that Zoli (2003)’s “flexible inequality equivalence” can be used to define inequality measures that respect the unit-consistency axiom only if  $\kappa = 0$ . Thus, we only focus in the present study on the  $\sigma$ -invariance defined through equation (2.26):<sup>46</sup>

$$\Psi(\omega^\sigma X + (\omega - \omega^\sigma)\mu I) = \Psi(X) \quad \forall \omega \in \mathbb{R}^+. \quad (2.26)$$

As for the preceding intermediate inequality views, it can be useful to express the equally unequal income vector with mean  $\mu_Y$  and corresponding to distribution  $X$  using equation (2.11).

<sup>45</sup> Hagenaars (1987) was apparently the first to define a poverty measure that does not comply with scale or translation invariance. Her famous measure,  $\Theta^H(x, z) = \frac{1}{n} \sum_{i=1}^q 1 - \frac{\log x_i}{\log z}$  where  $q$  is the length of the vector  $X^p$ , implicitly relies on the following non-linear intermediate invariance axiom:

$$\Psi(X^v) = \Psi(X) \quad \forall v \in \mathbb{R}^{++}, X \in \mathcal{D}_1.$$

which corresponds to an ultra-rightist view for any positive rate of growth ( $x_j > 1 \forall j$  and  $v > 1$ ). This inequality view is considered by Ebert (2004) as non-coherent since a sequence of a progressive transfer and an increase in mean income that does not change the degree of inequality, does not yield the same distribution as the converse sequence. However, we can question if this “transfer-consistency” axiom is really desirable.

On the other hand, it should be stressed that this inequality view is not suitable for poverty and inequality measurement since unit-consistency is not respected for all  $\lambda \in \mathbb{R}^{++}$ . For instance if  $\Theta^H(X, z) > \Theta^H(Y, z)$ ,  $H(\lambda X, \lambda z)$  will be greater than  $H(\lambda Y, \lambda z)$  if and only if  $\lambda > \frac{1}{z}$ . Thus unit-consistency is violated for  $\lambda \in \left(0, \frac{1}{z}\right)$ . Moreover, it seems that no ultra-rightist inequality or poverty measure can comply with unit-invariance. In fact, ultra-rightist views require incomes to be defined on the set  $\mathcal{D}_\alpha \subset \mathcal{D}^{++}$ . Since there always exist some strictly positive scalar  $\lambda$  such that  $\lambda X \notin \mathcal{D}_\alpha$ , unit-consistency cannot be respected.

<sup>46</sup> Zoli (2003) also considered this special case and called it “proportional inequality equivalence”.

Using Yoshida (2005) original version of equation (2.26), we derive the following expression of the weighing function:

$$u(\mu_X, \mu_Y, \rho) = u(\mu_X, \mu_Y, \sigma) = \frac{\left(\frac{\mu_Y}{\mu_X}\right)^\sigma - 1}{\frac{\mu_Y}{\mu_X} - 1}. \quad (2.27)$$

This  $\sigma$ -invariance axiom implies that, in order to keep inequality unchanged, any incremental income should be divided into infinitesimal amounts that are sequentially shared such that 100 $\sigma$  percent are distributed in proportion of the income relative shares and 100(1 -  $\sigma$ ) percent equally among income receivers. Alonso-Villar and del Rio (2007*b*) note that for  $\sigma > 0$ , this inequality view tends to behave like a “leftist” view as the initial mean income increases. Here, we would like to stress that this statement depends on the assumption made about the relation between  $\mu_X$  and  $\mu_Y$ . If we consider a constant difference between the initial and final mean incomes, we have to recognize that  $\lim_{\mu_X \rightarrow +\infty} u(\mu_X, \mu_Y, \sigma) = 0 \forall \sigma < 1$ . On the other hand, for a given positive growth rate  $g = \frac{\mu_Y - \mu_X}{\mu_X}$ , it can be seen from equation (2.27) that  $\lim_{\mu_X \rightarrow +\infty} u(\mu_X, \mu_Y, \sigma) = \frac{(1+g)^\sigma - 1}{g} \forall \sigma$ .<sup>47</sup> In other words, the  $\sigma$ -invariance axiom keeps being intermediate if we consider constant growth rates. It can also be seen from equation 2.27 that the value of  $\sigma$  such that the intermediate counterfactual distribution is the arithmetic mean or the counterfactual “leftist” and “rightist” income distributions, is  $\tilde{\sigma} = \log^{-1}\left(\frac{\mu_Y}{\mu_X}\right) \log\left(\frac{1}{2}\left(\frac{\mu_Y}{\mu_X} + 1\right)\right) \simeq 0.5$  in most cases. For instance if mean income increases by 10% over the period of interest, the value of  $\tilde{\sigma}$  is approximately equal to 0.51. Thus, even if we compare the results of intermediate decompositions over many periods with different growth rates, it is reasonable to accept the same value of  $\sigma$  for each period as standing for the same intermediate inequality view.

The weaker counterpart of equation (2.26) in poverty analysis is:

$$\Theta(\omega^\sigma z + (\omega - \omega^\sigma)\mu, \omega^\sigma X^p + (\omega - \omega^\sigma)\mu I, n) = \Theta(z, X^p, n) \quad \forall \omega \in \mathbb{R}^+. \quad (2.28)$$

and the corresponding value of  $G_{t,t+k}$  and  $D_{t,t+k}$  are:

$$G_{t,t+k}^K = \Theta\left(z, \omega_{t,t+k}^\sigma X_t^p + (\omega_{t,t+k} - \omega_{t,t+k}^\sigma)\mu_t I\right) - \Theta(z, X_t^p), \quad (2.29)$$

$$D_{t,t+k}^K = \Theta\left(z, \omega_{t+k,t}^\sigma X_{t+k}^p + (\omega_{t+k,t} - \omega_{t+k,t}^\sigma)\mu_{t+k} I\right) - \Theta(z, X_t^p), \quad (2.30)$$

with  $\omega_{t,t+k} = \frac{\mu_{t+k}}{\mu_t}$ .

### 3 Invariance, the measurement of poverty and the decomposition of its variations

#### 3.1 The headcount index

Most poverty indexes respect a unique invariance axiom. As a consequence there is no uncertainty about the invariance axiom that should be adopted for the decomposition of variations

<sup>47</sup> An interesting feature is that the value of  $u(\mu_X, \mu_Y, \sigma)$  does not depend of the monetary unit chosen for the measurement of incomes. Consequently, for a given value of  $\sigma$ , we will get the same growth and inequality effects if incomes are measured in dollars or in thousand dollars.

of such measures. However it can be easily shown that the most widely used poverty index, the headcount index, is the sole measure that is consistent with all the invariance axioms presented through the preceding lines.

For a given income distribution  $X$ , we know that the headcount index  $h$  is simply:

$$h(z, X) = \frac{\Xi(X|x_i < z)}{\Xi(X)} \quad (3.1)$$

where  $\Xi$  is a function returning the length of the specified vector. A particular feature of the headcount index among the traditional poverty measures is presented in the following proposition:

**Proposition 1.** *Continuous increasing functions of the headcount index are the sole poverty measures that respect both scale, translation and intermediate invariance axioms.*

*Proof.* In the first paragraphs of section 2, an invariance axiom is given a general definition with the help of a continuous increasing function  $\Phi$  on  $D_\alpha$ . This precludes the use of extreme “leftist” views, since the class of functions  $\Phi$  ought to be restricted to rank preserving functions. So whatever the specific form of  $\Phi$ , we should observe  $\Phi(x_p) \leq \Phi(z) \leq \Phi(x_{p+1})$  for  $x_p \leq z \leq x_{p+1}$ ,  $\Phi(x_i) < \Phi(x_p) \forall i < p$  and  $\Phi(x_i) > \Phi(x_p) \forall i > p$ . Consequently  $\Xi(\Phi(X)|\Phi(x_i) < \Phi(z)) = \Xi(X|x_i < z)$  and  $h(\Phi(z), \Phi(X)) = h(z, X)$ . We can conclude that the headcount index complies with all invariance axioms which imply transformations of incomes that are included between those induced by scale and translation invariance axioms.

To prove that the headcount is the sole traditional poverty measure that is compatible with the various invariance axioms presented earlier, we can make use of the results of Zheng (1994, proposition 2) which states that the sole poverty measures that respect both scale and translation invariance axioms are the headcount-related poverty indexes, *i.e.* poverty indexes that are defined as continuous increasing functions of the size of the distribution and the number of poor.<sup>48</sup>  $\square$

Proposition 1 means that the decomposition of variations of the headcount index into growth and inequality components can be handled in many ways. So, the couple of equations (2.4,2.5), (2.8,2.9), (2.17,2.18), (2.23,2.24) and (2.29,2.30) are all consistent with the axiomatic of the headcount index. The choice of a particular decomposition framework relies entirely on individual perceptions and tastes about inequality. However, for reasons that have been already detailed in the preceding lines, we argue that researchers should make use of the sole “rightist”, “leftist” and non-linear intermediate decompositions, and then use the sole couple of equations (2.4,2.5), (2.8,2.9) and (2.29,2.30).

### 3.2 Implications for the decomposition of poverty variations

In the following paragraphs, we try to sketch the consequences on the estimated growth and inequality effects of a move from a “rightist” to a “leftist” view. It can easily be shown that we do not

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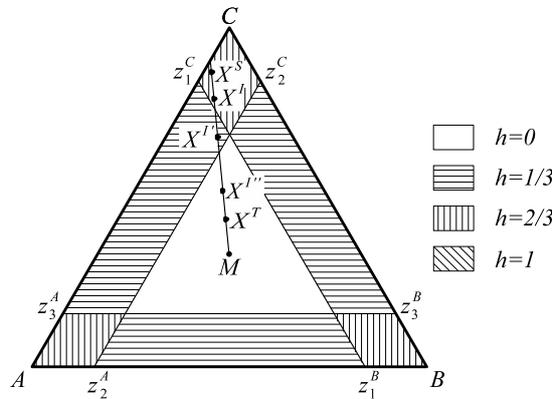
<sup>48</sup> According to Zheng (1994) a less restrictive definition of the family of headcount-related poverty indexes can be adopted if we do not impose the respect of the population, weak monotonicity and subgroup consistency axioms.

need to consider explicitly intermediate invariance to define an ordering of “leftist”, intermediate and “rightist” effects since the first and the last defines the range of the second. This result is summarized in proposition 2.

**Proposition 2.** *The value of any intermediate growth (inequality) effect of observed variations of the headcount index is always comprised between the values of the “leftist” and “rightist” growth (inequality) effects.*

*Proof.* The demonstration is a direct implication of lemma 1. As the intermediate equally unequal distributions  $\Phi^I(X, \mu_Y)$  are weighted mean of the “leftist” an “rightist” counterfactual distributions  $X \frac{\mu_Y}{\mu_X}$  and  $X + (\mu_Y - \mu_X)I$ , its cumulative distribution function (CDF) is bounded between the CDFs of  $X \frac{\mu_Y}{\mu_X}$  and  $X + (\mu_Y - \mu_X)I$ . As a result, the effects obtained through the equations (1.3) to (1.6) under scale and translation invariance are bounds for the effects obtained through intermediate invariance axioms.  $\square$

Figure 2 illustrates this proposition in the case of a three-person (or three-group) distribution. The triangle  $ABC$  is the same as in figure 1 but can be reduced to a simplex for the sake of simplicity. In this case, the distance of point with respect to the points  $A$ ,  $B$  and  $C$  respectively indicates the share of each individual in total income. The points  $z_1^B, z_1^C, z_2^A, z_2^C, z_3^A$  and  $z_3^B$  are the projections of the poverty line  $z$  for each individual along the axis  $AB$ ,  $BC$  and  $AC$  (see figure 2). Then, the poverty status of each member of the population depends of the position of the distribution with respect to the lines  $z_1^B z_1^C, z_2^A z_2^C$  and  $z_3^A z_3^B$ . In this example, we consider that  $\mu_Y$  is larger than  $z$ . Then the first individual fall into poverty if the point distribution is on the right of the line  $z_1^B z_1^C$  and non-poor otherwise. If  $X^T$  and  $X^S$  are the corresponding equally unequal distributions to  $X$  with mean  $\mu_Y$ , we know that every intermediate distribution  $\Phi^I(X, \mu_Y)$  will be located on the segment  $X^T X^S$ . Points  $X^I, X^{I'}$  and  $X^{I''}$  are potential counterfactual distributions corresponding to intermediate transformations of  $X$ . Whatever the location of  $\Phi^I(X, \mu_Y)$ , we can see that the value of the headcount index is always comprised between  $h(z, X \frac{\mu_Y}{\mu_X})$  and  $h(z, X + (\mu_Y - \mu_X)I)$ .



**Figure 2: Invariance and poverty variations in the three-person simplex.**

### 3.2.1 “Leftist” vs “Rightist” growth effects

In the following paragraphs, we show that the computation of the growth and inequality effects of variations of the headcount index under each invariance axiom are most of the time not necessary if one just intend to compare the magnitude of the effects. Proposition 3 states that a move from a “rightist” to a more “leftist” inequality view is not likely to change the sign of the estimated growth effect.

**Proposition 3.** *Whatever invariance axiom is considered, the sign of the growth effect is the same as the observed growth rate.*

*Proof.* The proposition is just the result of the application of the weak monotonicity axiom when every income  $x_i$  is increased.  $\square$

It deserves to be emphasized that when continuous distributions of income are considered and the probability of observing an income equal to  $z$  is non zero, the growth effect is always different from zero.<sup>49</sup>

**Proposition 4.** *In the context of the Datt and Ravallion (1992) decomposition of the headcount index, the “leftist” growth effect is lower than the “rightist” growth effect if and only if the observed growth rate is positive (negative) and the final mean income is above (below) the poverty line.*

*Proof.* Using the Datt and Ravallion (1992) approach, the relation between the different growth effects uniquely depends on the sign of the observed rate of growth and the relative position of the final mean income and the poverty line. Thus, we have to consider the following different four cases situations:

*i)* Let’s consider first the most common case of a positive rate of growth ( $\mu_Y > \mu_X$ ) and  $\mu_Y > z$ , with  $X$  and  $Y$  being respectively the initial and final income distributions. If mean incomes are higher than the poverty line, poor individuals gain less from “pure” growth under scale invariance than under translation invariance, *i.e.*:

$$x_j + (\mu_Y - \mu_X) > x_j \frac{\mu_Y}{\mu_X} \quad \forall j \in \{1, \dots, p\}. \quad (3.2)$$

If  $\mu_X < z$ , then:

$$\begin{cases} x_j + (\mu_Y - \mu_X) \geq x_j \frac{\mu_Y}{\mu_X} & \forall j \in \{1, \dots, s | x_{s-1} < \mu_X \leq x_s\}, \\ x_j + (\mu_Y - \mu_X) < x_j \frac{\mu_Y}{\mu_X} & \forall j \in \{s+1, \dots, p | x_{s-1} < \mu_X \leq x_s\}. \end{cases} \quad (3.3)$$

In this last case, we are only interested in individuals which rank are in the set  $\{1, \dots, s | x_{s-1} < \mu_X \leq x_s\}$  since all the other poor individuals become non-poor when their income are increased by proportional or equal increments. As a consequence, whatever the respective position of  $\mu_X$  and  $z$ ,  $h(z, X + (\mu_Y - \mu_X)I) \leq h\left(z, X \frac{\mu_Y}{\mu_X}\right)$ . Hence  $G^T \leq G^S$  as  $G$  is an increasing function of  $h(\Phi(X), z)$ .

<sup>49</sup> An analytical demonstration for marginal changes of mean income using scale invariance can be found in Kakwani (1993).

*ii)* On the other hand if the growth rate is negative ( $\mu_Y < \mu_X$ ), we have to consider the evolution of non-poor income, or more precisely the income of the non-poor that would become poor after the “pure” growth effect. The comparison of these counterfactual incomes with scale and translation invariance yields:

$$\begin{cases} x_j + (\mu_Y - \mu_X) \leq x_j \frac{\mu_Y}{\mu_X} & \forall j \in \{p, \dots, s | x_{s-1} < \mu_X \leq x_s\}, \\ x_j + (\mu_Y - \mu_X) > x_j \frac{\mu_Y}{\mu_X} & \forall j \in \{s+1, \dots, n | x_{s-1} < \mu_X \leq x_s\}. \end{cases} \quad (3.4)$$

since  $z < \mu_Y < \mu_X$ . As individuals from the set  $\{s+1, \dots, n | x_{s-1} < \mu_X \leq x_s\}$  do not cross the poverty line, we can focus on the first line of equation (3.4). Thus we observe  $h(z, X + (\mu_Y - \mu_X)I) \geq h(z, X \frac{\mu_Y}{\mu_X})$  and conclude  $G^T \geq G^S$ .

*iii)* For a positive rate of growth but  $\mu_Y < z$ , the income of the poor individuals increase according to equation (3.3). This time, only individuals which rank is in the set  $\{s+1, \dots, p | x_{s-1} < \mu_X \leq x_s\}$  can cross the poverty line. Consequently,  $h(z, X + (\mu_Y - \mu_X)I) \geq h(z, X \frac{\mu_Y}{\mu_X})$ . Hence  $G^T \geq G^S$ .

*iv)* Considering the last situation of a negative growth rate with  $\mu_Y < z$ , we know that income of the non-poor become:

$$x_j + (\mu_Y - \mu_X) > x_j \frac{\mu_Y}{\mu_X} \quad \forall j \in \{p+1, \dots, n\}. \quad (3.5)$$

if  $\mu_X < z$  and:

$$\begin{cases} x_j + (\mu_Y - \mu_X) \leq x_j \frac{\mu_Y}{\mu_X} & \forall j \in \{p, \dots, s | x_{s-1} < \mu_X \leq x_s\}, \\ x_j + (\mu_Y - \mu_X) > x_j \frac{\mu_Y}{\mu_X} & \forall j \in \{s+1, \dots, n | x_{s-1} < \mu_X \leq x_s\}. \end{cases} \quad (3.6)$$

otherwise. In the second case, all members from the set  $\{p, \dots, s | x_{s-1} < \mu_X \leq x_s\}$  become poor whatever invariance axiom is considered. Thus results differ only with respect to the evolution of the richest part of the non-poor population. Consequently, we should observe  $G^T \leq G^S$  since  $h(z, X + (\mu_Y - \mu_X)I) \leq h(z, X \frac{\mu_Y}{\mu_X})$ . □

Figure 3 gives some insight about the rationale underlying proposition 4 for a two-person (or two-group) distribution and a positive rate of growth.<sup>50</sup> Starting from the point  $X$  with coordinates  $(x_1, x_2)$ , the equally unequal distributions corresponding to scale and translation invariance are respectively represented by the lines  $XX^S$  and  $XX^T$ . For a final distribution with mean income  $\mu_Y$ , the corresponding counterfactual incomes can be found at the points where each of these curves cross the line  $X^S X^T$  which represents the set of income distributions with mean  $\mu_Y$ . Thus, we obtain the two distributions  $(x_1^S, x_2^S)$  and  $(x_1^T, x_2^T)$ . If the poverty line is set to  $z < \mu_Y$ , only one individual is considered as poor in the initial distribution. Whereas the scale invariance transformation of incomes does not change the value of the headcount index (the first individual is still poor), the sharing out of the additional income under translation invariance lowers poverty

<sup>50</sup> The case of negative growth rate can be easily derived from figure 4 if distribution  $Y$  (point  $E$ ) is chosen as the initial distribution.



ii) Considering the situation with  $\mu_X > z > \mu_Y$ , the income of the non-poor individual in distribution  $X$  change according to equation (3.6) and  $h(z, X + (\mu_Y - \mu_X)I) \leq h(z, X \frac{\mu_Y}{\mu_X})$ . As growth is negative, we have to focus on the evolution of the income of the poor in distribution  $Y$ . These counterfactual incomes exhibit the following relation:

$$\begin{cases} y_j + (\mu_X - \mu_Y) \geq y_j \frac{\mu_X}{\mu_Y} & \forall j \in \{1, \dots, r | y_{r-1} < \mu_Y \leq y_r\}, \\ y_j + (\mu_X - \mu_Y) < y_j \frac{\mu_X}{\mu_Y} & \forall j \in \{r+1, \dots, q | y_{r-1} < \mu_Y \leq y_r\}. \end{cases} \quad (3.9)$$

As every individual  $j \in \{r+1, \dots, q | y_{r-1} < \mu_Y \leq y_r\}$  is not poor anymore whatever invariance axiom is considered, only members from the set  $\{1, \dots, r | y_{r-1} < \mu_Y \leq y_r\}$  matter. We conclude that  $h(z, Y + (\mu_X - \mu_Y)I) \leq h(z, Y \frac{\mu_X}{\mu_Y})$ .

Whatever the ordering of the mean income of the initial and final distributions, developments of cases i) and ii) yields the same expression of the difference between the “rightist” and “leftist” growth effects:

$$G'^S - G'^T = \frac{1}{2} \left( \underbrace{h\left(z, X \frac{\mu_Y}{\mu_X}\right) - h\left(z, X + (\mu_Y - \mu_X)I\right)}_{>0} + \underbrace{h\left(z, Y + (\mu_X - \mu_Y)I\right) - h\left(z, Y \frac{\mu_X}{\mu_Y}\right)}_{<0} \right). \quad (3.10)$$

Thus, according to equations (3.10), the ranking of  $G'^S$  and  $G'^T$  cannot be known until the intermediate values  $h(z, \Phi(X))$  and  $h(z, \Phi(Y))$  are computed. For  $G'^T > G'^S$ , rearranging the second term of equation (3.10) yields equation (3.7).  $\square$

For convenience, the combined results of propositions 3, 4 and 5 are summarized in table 1.

**Table 1: Comparison of the growth effects under scale and translation invariance.**

Condition		Decomposition technique	
Growth	Poverty line	Datt and Ravallion (1992)	Shapley
$\mu_Y > \mu_X$	$\mu_Y > z$	$G^T \leq G^S \leq 0$	$G'^T \leq G'^S \leq 0$
			or <sup>a</sup> $G'^S \leq G'^T \leq 0$
$\mu_Y > \mu_X$	$\mu_Y < z$	$G^S \leq G^T \leq 0$	$G'^S \leq G'^T \leq 0$
$\mu_X > \mu_Y$	$\mu_Y > z$	$G^T \geq G^S \geq 0$	$G'^T \geq G'^S \geq 0$
$\mu_X > \mu_Y$	$\mu_Y < z$	$G^S \geq G^T \geq 0$	$G'^S \geq G'^T \geq 0$
			or <sup>b</sup> $G'^T \geq G'^S \geq 0$
a: if $z > \mu_X$ and $h\left(z, X \frac{\mu_Y}{\mu_X}\right) - h\left(z, Y \frac{\mu_X}{\mu_Y}\right) \geq h\left(z, X + (\mu_Y - \mu_X)I\right) - h\left(z, Y + (\mu_X - \mu_Y)I\right)$ .			
b: if $\mu_X > z$ and $h\left(z, X \frac{\mu_Y}{\mu_X}\right) - h\left(z, Y \frac{\mu_X}{\mu_Y}\right) \geq h\left(z, X + (\mu_Y - \mu_X)I\right) - h\left(z, Y + (\mu_X - \mu_Y)I\right)$ .			

In most cases, the value of the poverty line is below those of the initial and final mean income. As a consequence, we should expect the “leftist” growth effect to be inferior (superior) to the “rightist” growth effect for positive (negative) observed growth rates.

### 3.2.2 “Leftist” vs “Rightist” inequality effects

Now, we turn to the “leftist” and “rightist” inequality effects of headcount index variations. Contrary to growth effects, the sign of these inequality effects cannot be derived neither from the sole comparison of the poverty line and the initial and final mean incomes, nor from the use of inequality measures. For instance, a decrease in inequality according to any scale invariant inequality measure, do not necessarily implies that the corresponding inequality effect is negative.<sup>51</sup> Moreover, no Lorenz dominance criterion can be used for the purpose of headcount index comparisons. In the context of the Datt and Ravallion (1992) decomposition framework, the sole ordering criterion that may be helpful is the first-degree stochastic dominance condition (Atkinson, 1987) between the initial distribution and the counterfactual distribution derived from the final distribution. In other words, the only way of getting the sign of  $D$ , whatever invariance axiom is chosen, is to compute its value.

**Proposition 6.** *Using the Datt and Ravallion (1992) decomposition of the headcount index, the “leftist” inequality effect is larger than the “rightist” inequality effect if and only if the observed growth rate is positive (negative) and the initial mean income is above (below) the poverty line.*

*Proof.* As for the comparison of the different growth effects using the Datt and Ravallion (1992) decomposition framework, the ordering of the “leftist” and “rightist” growth effects only depends on the sign of the growth rate and the relative position of  $\mu_X$  and  $z$ . Consequently, the four following cases must be separately treated:

*i)* Suppose first that  $\mu_Y > \mu_X$  and  $\mu_X > z$ . For the computation of the inequality effect using the Datt and Ravallion (1992) decomposition technique, the focus ought to be put on the transformation of non-poor individuals income from distribution  $Y$  in the context of a positive rate of growth. The comparison of the transformed incomes is given by equation (3.8). Since only non-poor individuals whose incomes are lower than  $\mu_Y$  are susceptible to cross the poverty line, we are only interested in the first line of equation (3.8). As a result,  $h(z, Y + (\mu_X - \mu_Y)I) \geq h(z, Y \frac{\mu_X}{\mu_Y})$ , and then  $D^T \geq D^S$ .

*ii)* For a negative growth rate but  $\mu_X$  still larger than  $z$ , we have to consider two different cases, depending on the relative position of  $\mu_Y$  and  $z$ . If  $\mu_Y > z$ , the comparison of the counterfactual incomes is given by:

$$y_j + (\mu_X - \mu_Y) > x_j \frac{\mu_X}{\mu_Y} \quad \forall j \in \{1, \dots, q\}. \quad (3.11)$$

On the other hand, if  $\mu_Y < z$ , the ranking is given by equation (3.9). Since all members from the set  $\{r + 1, \dots, q\} | y_{r-1} < \mu_Y \leq y_r$  cross the poverty line, only the very poorest will make the difference for the comparison of the “leftist” and “rightist” effects. In both situations, we thus find  $h(z, Y + (\mu_X - \mu_Y)I) \leq h(z, Y \frac{\mu_X}{\mu_Y})$  and conclude  $D^T \leq D^S$ .

*iii)* Considering the situation of a positive growth rate and  $\mu_X < z$ , the value of the inequality effects depends on the way non-poor income in distribution  $Y$  change. Two different cases can be met. With  $\mu_Y > z$ , the situation is described by equation (3.8). Since the income of all members

<sup>51</sup> Bresson (2007) shows in the context of the analytical derivation of a class of inequality elasticities of poverty that an increase in inequality may just as well result in an increase or a decrease of the level of poverty whatever the respective position of mean income and the poverty line.

of the set  $\{q, \dots, r | y_{r-1} < \mu_Y \leq y_r\}$  fall below the poverty line, only the second line of equation (3.8) can be considered.

Assuming  $\mu_Y < z$  gives:

$$y_j + (\mu_X - \mu_Y) > x_j \frac{\mu_X}{\mu_Y} \quad \forall j \in \{q, \dots, n\}. \quad (3.12)$$

In both situations,  $h(z, Y + (\mu_X - \mu_Y)I) \leq h(z, Y \frac{\mu_X}{\mu_Y})$  and we find  $D^t \leq D^S$ .

iv) Finally, for a negative growth rate and  $\mu_X < z$ , the ordering of poor individual incomes is given by equation (3.9). As only those from the set  $\{r+1, \dots, q | y_{r-1} < \mu_Y \leq y_r\}$  may cross the poverty line, we find  $h(z, Y + (\mu_X - \mu_Y)I) \geq h(z, Y \frac{\mu_X}{\mu_Y})$  and conclude  $D^t \geq D^S$ . □

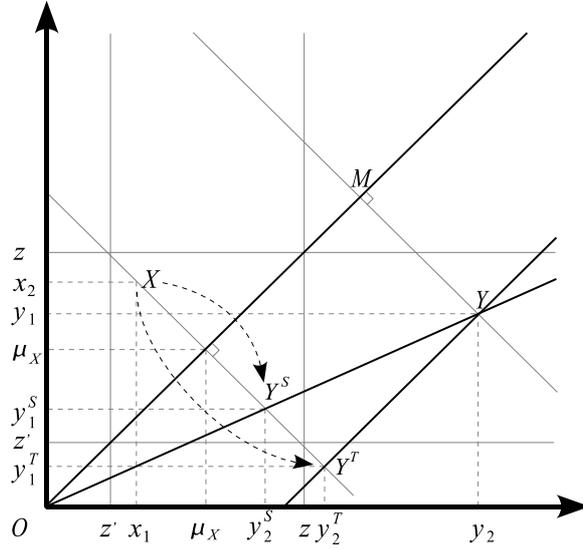
Figure 4 is the counterpart of figure 3 for the computation of the inequality effects using the Datt and Ravallion (1992) decomposition framework. In this case, the final distribution  $Y$  is reported as it is needed to find the corresponding equally unequal distributions with mean income  $\mu_X$ . In order to improve the readability of the figure, the coordinates  $(y_1, y_2)$  are permuted, but this modification is of no consequence for our purpose. If the poverty line is set to  $z$ , both individuals are considered as poor in the original distribution (point  $X$ ). With scale invariance, the counterfactual distribution  $Y^S$  which exhibit the same degree of inequality as  $Y$  does not change the value of the poverty index since every income remains below the poverty line. On the contrary, the translation invariance transformation  $Y^T$  of distribution  $Y$  yields a counterfactual distribution with only half of the population being poor since  $y_2^T > z > y_1^T$ . Thus, we find  $D^T < D^S = 0$ . On the other hand, with a poverty line  $z'$  that is larger than the initial mean income, the initial value of the headcount index is zero and does not change if we adopt a “rightist” view. With the “leftist” inequality view, the number of poor increases as the first individual income falls below the poverty line. As a consequence, we conclude  $D^T > D^S = 0$ .

**Proposition 7.** *With the Shapley decomposition of the headcount index, the “leftist” inequality effect is higher than the “rightist” inequality effect if and only if the observed growth rate is positive (negative) and the final mean income is above (below) the poverty line, except if the poverty line lies between the initial and final mean incomes, and:*

$$h\left(z, X \frac{\mu_Y}{\mu_X}\right) - h\left(z, Y \frac{\mu_X}{\mu_Y}\right) < h(z, X + (\mu_Y - \mu_X)I) - h(z, Y + (\mu_X - \mu_Y)I). \quad (3.13)$$

*Proof.* In the case of the Shapley decomposition, the demonstration is trivial since we know that  $D^l = \Delta h - G^l$ . As  $\Delta h$  remains the same whatever invariance axiom has been adopted, the difference between  $D^{lS}$  and  $D^{lT}$  is simply the opposite of the difference between  $G^{lS}$  and  $G^{lT}$ . □

A noticeable feature of propositions 6 and 7 is that the ordering of the inequality effects under scale and translation invariance does not depend of the sign of these effects. Generally the value of the poverty line is below the observed mean values of the initial and final income distributions. In this situation, we should expect the “leftist” inequality effect to be superior (inferior) to the “rightist” inequality effect for positive (negative) observed growth rates. Considering the relative contribution of growth and redistribution to variations of the headcount, moving from a “rightist”



Note: in order to improve the readability of the figure, incomes of the two individuals are permuted with respect to distribution X.

Figure 4: “Leftist” vs “rightist” inequality effects with a positive rate of growth.

Table 2: Comparison of the inequality effects under scale and translation invariance.

Condition		Decomposition technique	
Growth	Poverty line	Datt and Ravallion (1992)	Shapley
$\mu_Y > \mu_X$	$\mu_X > z$	$D^T \geq D^S$	$D'^T \geq D'^S$
$\mu_Y > \mu_X$	$\mu_X < z$	$D^T \leq D^S$	$D'^T \leq D'^S$
			or <sup>a</sup> $D'^T \geq D'^S$
$\mu_X > \mu_Y$	$\mu_X > z$	$D^T \leq D^S$	$D'^T \leq D'^S$
			or <sup>b</sup> $D'^T \geq D'^S$
$\mu_X > \mu_Y$	$\mu_X < z$	$D^T \geq D^S$	$D'^T \geq D'^S$
$a: \text{if } \mu_Y > z \text{ and } h\left(z, X \frac{\mu_Y}{\mu_X}\right) - h\left(z, Y \frac{\mu_X}{\mu_Y}\right) \geq h(z, X + (\mu_Y - \mu_X)I) - h(z, Y + (\mu_X - \mu_Y)I).$			
$b: \text{if } z > \mu_Y \text{ and } h\left(z, X \frac{\mu_Y}{\mu_X}\right) - h\left(z, Y \frac{\mu_X}{\mu_Y}\right) \geq h(z, X + (\mu_Y - \mu_X)I) - h(z, Y + (\mu_X - \mu_Y)I).$			

to a “leftist” inequality view increases the contribution of growth to the reduction of poverty with respect to redistribution for a positive rate of growth.

This result has major implications for the evaluation of “pro-poor” growth. If growth is said “pro-poor” when observed poverty reduction is higher than the reduction that would occur under distribution neutrality (Kakwani and Pernia, 2000), leaving scale invariance for intermediate and “leftist” inequality views makes generally the occurrence of “pro-poor” growth more scarce when mean income increases. At the other hand, negative growth is more likely to be deemed “pro-poor” under the translation invariance axiom than under the scale invariance axiom.

## 4 An application to poverty in China, 1990-2004

Most of the time, empirical studies related to income inequality and poverty are (implicitly) based on the prior that inequality and poverty should be analyzed through scale invariant tools. This may reflect the mainstream view in economics but not necessarily the dominant view of policy-makers and citizens. As stated earlier, the heterogeneity of inequality perceptions is a relevant justification for analyzing the sensibility of results to ethical preferences in poverty studies.<sup>52</sup> Empirical studies that do not rely on scale invariance are scarce. Such studies include del Rio and Ruiz-Castillo (2001) on the evolution of inequality in Spain from 1980 to 1991 and Atkinson and Brandolini (2004) on international and global income inequalities in the last century.

In the present section, we want to illustrate the importance of a choice of a particular axiom for the decomposition of poverty variations into growth and inequality components using Chinese data. Considering China is of prime importance: recent publications (Bhalla, 2004; Sala-i Martin, 2004, 2006) related to the evolution of the world income distribution have stressed how their results were sensitive to changes in the Chinese distribution. Moreover many authors (Besley and Burgess, 2003; Chen and Ravallion, 2004) have emphasized the crucial role of China in the achievement of the global objective of halving extreme poverty during the period 1990-2015. Such an important contribution to global poverty reduction is generally attributed to the impressive economic performances of China during the last decade (Chen and Ravallion, 2007). In the following paragraphs, we emphasize that this conclusion is contingent to axiomatic choices and may not hold when moving from a “rightist” to a “leftist” view.

### 4.1 Data

The data used in this paper stem from the 1990, 1996, 1999 and 2003 rounds of the China Health and Nutrition Survey (CHNS). The CHNS is an ongoing longitudinal survey that covers nine provinces (Guangxi, Guizhou, Henan, Hubei, Hunan, Jiangsu, Liaoning, Heilongjiang and Shangdong). Although the survey is not nationally representative, these provinces were selected to provide significant variability in geography, economic development and health indicators, so that they may be considered to be roughly representative of the whole population of the country.

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<sup>52</sup> One should have in mind that the use of different measures like the Theil and the Gini coefficient in a given empirical study also implies axiomatic changes and so involves a mix of different ethical preferences.

A multistage random-cluster sampling procedure was used to draw the sample from each of the provinces. Counties in the eight provinces were stratified by income (low-, middle- and high-income groups) with per capita income figures from the State Statistical Office, and a weighted sampling scheme was used to select four counties randomly in each province (one low income, two middle income, and one high income). A probability-proportional-to-size sampling was then chosen to select the sample from these units. In addition, urban areas that were initially not within the county-strata were later incorporated by including the provincial capital and a low-income city from each province. Within each county, the township capital was added and three villages were chosen randomly. Within each city, urban and sub-urban neighborhoods were randomly picked out. The same random selection procedure was used to choose the neighborhoods for townships and villages.

Income data are divided between incomes issued from agriculture, business, paid activities, subventions and remittances. The agricultural incomes come from fishing, farming, crops growing, gardening and rearing. Business incomes are related to handicraft and small businesses. Paid activities represent all the jobs for which individuals are wage earners (including work in agricultural and business activities) and include bonuses received all along the year. Subventions are distributed by enterprises or the State for housing, food, energy, childbearing, childcare, health. . .<sup>53</sup> Finally, remittances represent money sent back by children to their parents or financial help from friends or relatives. For self-employment in agriculture or business, we construct net income defined as the income generated by the products sold plus the monetary value of products kept by the household, minus the costs engaged for the production. We do not consider observations for which informations related to costs or incomes were missing.

The aggregation of all kinds of revenue constraints us to consider yearly income for the whole household and consequently to assume that total income is equally shared between each members. However, we do not use the direct number of household members to obtain the individual income, but assume some possible economies of scale in the household. Consequently, we use the methodology suggested by (Deaton, 1997) and normalize the incomes by dividing them by  $n^a$  where  $n$  is the number of the household members and  $a$  is an equivalence factor. In our empirical application, we will use a value of  $a = 0.8$ , a value than was chosen by Wan and Zhang (2006) for their estimations on the same CHNS data. To get real incomes, we use the consumer price indexes (CPIs) provided the Chinese National Bureau of Statistics. We consider provincial CPIs, with a distinction between urban and rural areas, for all the years considered, with the 1990 year as the reference. In order to account for the spacial price differences in the reference year, incomes are adjusted using the provincial (rural and urban) deflators constructed by Brandt and Holz (2006).

In table 3 are presented the values of the headcount index for the different periods of observation. In this paper, we consider the traditional US\$1.08 and US\$2.16 (latter mentioned as US\$1 and US\$2 for convenience) per day poverty lines in 1996 PPP. In the context of China, the definition of a relevant poverty line is the object of great debates (Fan et al., 2002; Hanmer et al., 2004; Gregory et al., 2005; Chen and Ravallion, 2007). Some authors argue that it is important to distin-

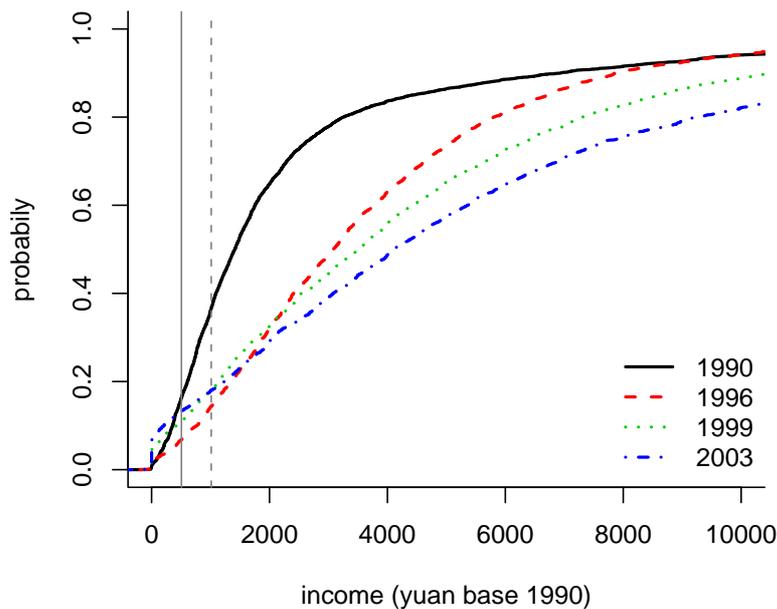
<sup>53</sup> For the 1991 survey, food coupons received by households are isolated from the subventions. We consequently include them in the subvention to make database comparable.

guish the rural and urban areas (Chen and Ravallion, 2007), and even consider specific poverty lines corresponding to adequate consumption baskets for each area (Gregory et al., 2005). Nevertheless, we choose the commonly used US\$1 and US\$2 lines as we realize a general analysis of poverty in China and as these two measures are the ones used in the context of the Millennium Development Goals. The bottom part of the cumulative distribution functions for each survey are reported in figure 5.

**Table 3: Values of the headcount index in China during the period 1990-2003.**

Year	1990	1996	1999	2003
US\$1	16.2 (0.004)	6.8 (0.003)	11.2 (0.005)	13.3 (0.004)
US\$2	36.5 (0.004)	14.3 (0.004)	17.9 (0.005)	18.0 (0.004)

Note: standard errors in parentheses using a bootstrap procedure with 200 replications.



Note: The solid and dashed vertical lines represents the US\$1 and US\$2 per day poverty lines.

**Figure 5: The distribution of incomes in China from 1990 to 2003.**

Considering the 3, we note that after a drop in poverty between 1990 and 1996, a slight increase occurs in China since the end of the 1990's. This seems surprising as we know that China has experienced a huge growth since the beginning of the 1980's. On the other hand, our figures are not totally supported by other studies related to poverty in China and which tend to demonstrate a constant decrease of poverty since the movement of reforms initiated by the end of the 1970's. Nevertheless, some other authors underline short episodes of increase in the headcount index. For instance, Chen and Ravallion (2007) for the whole China and Wan and Zhang (2006)

for the rural areas note a slight increase in 2000. The differences we can stress between our results and other studies ones are essentially due to the data structure and our definition of income. The CHNS database is highly detailed and many rival hypothesis can be done concerning which income and costs must or not be considered. This can have important impact on the results. With this caveat in mind, our conclusions concerning China's poverty need to be taken with caution. However, as this is not the central goal of our paper, problems concerning data will not influence the major results concerning the differences between decompositions done with scale invariance and the ones based on translation invariance.

We can have a closer look at the level of poverty looking at the incomes distribution given on figure 5. Thanks to this figure, we clearly see that there has been a huge decrease in poverty between 1990 and 1996 using the US\$2 poverty line but that after this date, and for the poorest individuals, no significant evolution can be drawn. These distributions emphasize a decrease of inequalities for the highest quartile but not for the lowest ones. This is in coherence with the evolution of the distribution of wealth in China as we see since few years the development of a new middle class which begins to balance with the enrichment of a narrow share of the Chinese population.

#### 4.2 “Leftist” vs “rightist” effects

In this paragraph, we focus on the comparisons of the differences of the estimated effects obtained through the two limiting views presented in the preceding sections, that is those based on scale and translation invariance. Tables 4 and 5 respectively present the estimations using the Datt and Ravallion (1992) and the Shapley decomposition techniques for the period 1990-2003 and the sub-periods 1990-1996, 1996-1999 and 1999-2003.<sup>54</sup> The figures included in these tables give the total variations of the headcount index, the growth and inequality effects in percentage points as well as their relative contribution (trade-off) to poverty reduction. For instance, looking at the results based on scale invariance in table 4 for the period 1990-2003 and for the US\$1 poverty line, we can observe that poverty has decreased by about 2.9 percentage points. Decomposing this evolution into growth and inequality effects, we find that poverty would have decreased by 8.9 percentage points thanks to growth if inequalities had remained stable during the period. In parallel, if the growth rate had been null, the evolution of inequalities would have increased poverty by 0.7 percentage points. The same interpretations hold for the Shapley decomposition results given in table 5.

As noted earlier, our objective is not to promote any of the two techniques, but to emphasize the judgment differences involved by a change in the conception of inequality. By and large, the two decompositions techniques yield the same conclusions, the only salient difference being with the “leftist” inequality effect for the period 1990-1996 using the US\$1 poverty line (2.55 percentage points for the Shapley decomposition versus  $-1.63$  with the Datt and Ravallion (1992) approach).

At first, it is important to stress that the theoretical results summed up in tables 1 and 2 are confirmed by the empirical results presented in tables 4 and 5. We are in the case of a positive

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<sup>54</sup> Estimations and figures are obtained with R 2.5.1 (R Development Core Team, 2007). Scripts are available upon request.

**Table 4: “Rightist” and “leftist” decompositions of poverty spells in China 1990-2003 using the Datt and Ravallion (1992) technique.**

Axiom	Scale invariance				Translation invariance			
	1990-1996	1996-1999	1999-2003	1990-2003	1990-1996	1996-1999	1999-2003	1990-2003
<i>US\$1 poverty line</i>								
Total	-9.44	4.44	2.04	-2.96	-9.44	4.44	2.04	-2.96
(% point)	[-10.2, -8.67] (0.004)	[3.65, 5.16] (0.004)	[1.18, 2.96] (0.005)	[-3.93, -1.99] (0.005)	[-10.2, -8.67] (0.004)	[3.65, 5.16] (0.004)	[1.18, 2.96] (0.005)	[-3.93, -1.99] (0.005)
Growth	-2.7	-1.36	-1.39	-8.98	-16.2	-6.85	-11.3	-16.3
(% point)	[-3.54, -1.52] (0.005)	[-1.59, -1.17] (0.001)	[-1.65, -0.97] (0.002)	[-9.74, -8.07] (0.004)	[-16.9, -11.4] (0.015)	[-7.32, -6.4] (0.002)	[-11.9, -10.7] (0.003)	[-17, -15.7] (0.003)
Inequality	-8.13	5.91	2.98	0.76	-1.63	19.1	13.5	24.2
(% point)	[-9.21, -7.29] (0.005)	[5.1, 6.64] (0.004)	[2.11, 3.82] (0.005)	[-0.31, 1.73] (0.005)	[-5.64, 1.88] (0.019)	[17.3, 20.7] (0.009)	[11.3, 15] (0.01)	[21.8, 26.3] (0.012)
Residual	1.39	-0.11	0.46	5.26	8.35	-7.84	-0.19	-10.9
(% point)	[0.85, 2.02] (0.003)	[-0.40, 0.38] (0.002)	[0.10, 0.69] (0.002)	[4.58, 5.93] (0.003)	[4.85, 8.73] (0.01)	[-9.57, -5.83] (0.009)	[-1.73, 2.06] (0.01)	[-13.1, -8.38] (0.012)
Trade-off	0.33	-0.23	-0.46	-11.8	9.91	-0.35	-0.83	-0.67
(G/D)	[0.17, 0.45] (0.073)	[-0.29, -0.18] (0.027)	[-0.68, -0.28] (0.103)	[-128, 117] (5335)	[-108, 97.7] (213)	[-0.40, -0.32] (0.022)	[-1.01, -0.73] (0.071)	[-0.75, -0.60] (0.038)
<i>US\$2 poverty line</i>								
Total	-22.2	3.56	0.072	-18.6	-22.2	3.56	0.072	-18.6
(% point)	[-23.2, -21.2] (0.005)	[2.58, 4.46] (0.005)	[-0.95, 1.14] (0.005)	[-19.7, -17.5] (0.006)	[-23.2, -21.2] (0.005)	[2.58, 4.46] (0.005)	[-0.95, 1.14] (0.005)	[-19.7, -17.5] (0.006)
Growth	-4.93	-3.83	-2.39	-17.9	-21.1	-12.2	-17.9	-36.6
(% point)	[-6.91, -3.34] (0.009)	[-4.28, -3.12] (0.003)	[-2.78, -1.65] (0.003)	[-19.1, -16.3] (0.007)	[-29.7, -12.7] (0.044)	[-14.8, -11.1] (0.012)	[-18.6, -12.3] (0.017)	[-37.4, -35.7] (0.004)
Inequality	-19.7	7.82	2.01	-10.4	-13.4	18.3	13.1	9.04
(% point)	[-21.3, -18.1] (0.008)	[6.79, 8.74] (0.005)	[1.11, 3.15] (0.005)	[-11.8, -8.95] (0.007)	[-17.2, -9.76] (0.019)	[16.6, 19.9] (0.009)	[10.3, 14.6] (0.01)	[6.7, 10.9] (0.011)
Residual	2.47	-0.43	0.45	9.73	12.3	-2.47	4.92	8.99
(% point)	[1.93, 3.3] (0.004)	[-1.05, 0.10] (0.003)	[-0.26, 0.66] (0.003)	[8.73, 10.5] (0.005)	[7.51, 17.4] (0.025)	[-3.05, -0.41] (0.007)	[1.74, 6.12] (0.011)	[7.1, 11.5] (0.012)
Trade-off	0.25	-0.48	-1.19	1.72	1.58	-0.67	-1.37	-4.05
(G/D)	[0.16, 0.37] (0.056)	[-0.58, -0.39] (0.048)	[-2.14, -0.59] (0.386)	[1.45, 2.08] (0.161)	[0.75, 3.04] (0.576)	[-0.80, -0.62] (0.051)	[-1.54, -1.1] (0.107)	[-5.48, -3.33] (0.586)

Note: 95% confidence intervals in brackets and standard errors in parentheses using a bootstrap procedure with 500 replications.

**Table 5: “Rightist” and “leftist” decompositions of poverty spells in China 1990-2003 using the Shapley technique.**

Axiom	Scale invariance				Translation invariance			
	1990-1996	1996-1999	1999-2003	1990-2003	1990-1996	1996-1999	1999-2003	1990-2003
<i>US\$1 poverty line</i>								
Total	-9.44	4.44	2.04	-2.96	-9.44	4.44	2.04	-2.96
(% point)	[-10.2, -8.67]	[3.65, 5.16]	[1.18, 2.96]	[-3.93, -1.99]	[-10.2, -8.67]	[3.65, 5.16]	[1.18, 2.96]	[-3.93, -1.99]
	(0.004)	(0.004)	(0.005)	(0.005)	(0.004)	(0.004)	(0.005)	(0.005)
Growth	-2	-1.42	-1.16	-6.35	-12	-10.8	-11.4	-21.7
(% point)	[-2.6, -1.08]	[-1.6, -1.16]	[-1.35, -0.84]	[-6.9, -5.58]	[-13.8, -7.75]	[-11.7, -9.77]	[-12.3, -10.2]	[-22.9, -20.3]
	(0.004)	(0.001)	(0.001)	(0.003)	(0.016)	(0.005)	(0.005)	(0.006)
Inequality	-7.44	5.86	3.21	3.39	2.55	15.2	13.4	18.8
(% point)	[-8.59, -6.56]	[5.09, 6.54]	[2.32, 4]	[2.38, 4.24]	[-1.81, 4.4]	[14.2, 16.1]	[12.2, 14.3]	[17.5, 19.9]
	(0.005)	(0.004)	(0.004)	(0.005)	(0.016)	(0.005)	(0.005)	(0.006)
Trade-off	0.26	-0.24	-0.36	-1.87	-4.71	-0.70	-0.84	-1.16
(G/D)	[0.12, 0.37]	[-0.29, -0.19]	[-0.50, -0.23]	[-2.6, -1.48]	[-29.9, 33.3]	[-0.75, -0.66]	[-0.90, -0.78]	[-1.22, -1.1]
	(0.066)	(0.025)	(0.067)	(0.295)	(259.8)	(0.023)	(0.032)	(0.028)
<i>US\$2 poverty line</i>								
Total	-22.2	3.56	0.072	-18.6	-22.2	3.56	0.072	-18.6
(% point)	[-23.2, -21.2]	[2.58, 4.46]	[-0.95, 1.14]	[-19.7, -17.5]	[-23.2, -21.2]	[2.58, 4.46]	[-0.95, 1.14]	[-19.7, -17.5]
	(0.005)	(0.005)	(0.005)	(0.006)	(0.005)	(0.005)	(0.005)	(0.006)
Growth	-3.7	-4.04	-2.17	-13	-15	-13.5	-15.5	-32.1
(% point)	[-5.39, -2.3]	[-4.5, -3.39]	[-2.6, -1.67]	[-14.2, -11.7]	[-21.1, -8.87]	[-15.5, -12.2]	[-16.4, -11.2]	[-33.2, -30.6]
	(0.008)	(0.003)	(0.003)	(0.006)	(0.031)	(0.01)	(0.013)	(0.006)
Inequality	-18.5	7.61	2.24	-5.54	-7.23	17	15.5	13.5
(% point)	[-20.2, -16.7]	[6.64, 8.47]	[1.3, 3.23]	[-6.85, -4.2]	[-13.2, -1.16]	[15.8, 19]	[11.5, 16.4]	[12.3, 14.5]
	(0.009)	(0.005)	(0.005)	(0.007)	(0.031)	(0.009)	(0.012)	(0.006)
Trade-off	0.2	-0.53	-0.96	2.35	2.07	-0.79	-0.99	-2.37
(G/D)	[0.11, 0.31]	[-0.62, -0.44]	[-1.7, -0.63]	[1.76, 3.34]	[0.66, 17.1]	[-0.85, -0.74]	[-1.06, -0.92]	[-2.56, -2.23]
	(0.052)	(0.045)	(0.276)	(0.394)	(8.37)	(0.029)	(0.036)	(0.087)

Note: 95% confidence intervals in brackets and standard errors in parentheses using a bootstrap procedure with 500 replications.

growth rate ( $\mu_Y > \mu_X$ ) with initial and final mean incomes that are both above the poverty line ( $\mu_Y > z$  and  $\mu_X > z$ ). Consequently, we find in all cases that the negative growth effect is more important in the translation invariance case than in the scale invariance one. Moreover the translation inequality effect is larger than the effect based on scale invariance. When the values are negative, a “leftist” observer would then attribute a lower contribution of inequality changes to variations of the headcount index than a “rightist” observer. On the other hand, with a positive inequality effect, he would think that inequality changes hampers more poverty alleviation than the “rightist” one. Of course, these results hold for both the Datt and Ravallion (1992) and the Shapley decomposition techniques.

What is also interesting is the differences relative to the trade-offs between the growth and the inequality effects. They can be found in the tables 1 and 2 and are measured by the ratio  $G/D$ . We clearly see that the weights given to growth relative to inequalities in the explanation of observed poverty trends are highly different between the “leftist” and “rightist” views. The same phenomenon happens whatever poverty line and decomposition approach are chosen. For example, for the US\$1 poverty line, the Shapley growth effect measured with the scale invariance is approximately four times less important than the inequality effect when we consider the evolution of poverty between 1990 and 1996, whereas it is 4.71 times more important than the inequality effect once we move to the translation invariance case. For the same period and the same poverty line, the Datt and Ravallion trade-offs are 0.33 for the scale invariance and 9.91 for the translation one. No clear relationship appears between the type of invariance chosen and the more or less high trade-offs that are observed. This underlines even more the need for a sensitivity analysis of the poverty decomposition to invariance preferences.

To illustrate the importance of the axiomatic choice, let’s have a look at the results of the Datt and Ravallion decomposition for the period 1990-2003 and for the US\$1 poverty line. We note that the growth effect for the translation invariance ( $G^T = -16.3$ ) is nearly two times higher than that for the scale invariance ( $G^S = -8.98$ ). Considering the inequality effect, the difference is even larger, the “leftist” effect ( $D^S = 24.2$ ) being more than 30 times higher than the “rightist” one ( $D^T = 0.76$ ). To test the statistical significance of these results, we computed 95% confidence intervals for each effect using a bootstrap procedure with 500 replications and resampling at the household level. In most cases, we find that these differences are significant as interval crossings are rarely noted.

The most impressive consequence of invariance axiom changes is that a modification of ethical preferences may induce a change in the sign of the inequality effect. Comparing the results of the scale and translation invariance, we observe opposite (and significantly different) signs for the inequality effects for the period 1990-2003 with the US\$2 poverty line whatever decomposition technique is chosen. The same phenomenon is observed in table 5 for the sub-period 1990-1996 and the US\$1 poverty line but only with the Shapley decomposition approach. But as can be seen thanks to the confidence interval, the positive “leftist” effect is not statistically different from zero. Consequently for this sub-period and this poverty line, moving from a “rightist” to a “leftist” point of view implies that the effect of the changes in the relative distribution of incomes on poverty is not significantly different from zero anymore.

Concomitantly to this result, another major observation can be made about the dependency of the ordering of effects to invariance choices for many sub-periods. For instance, the Shapley growth effect for the US\$1 poverty line is higher during the 1996-1999 period when using the scale invariance ( $G'_{96-99}{}^S = -1.42 < -1.16 = G'_{99-03}{}^T$ ) and during the 1999-2003 when the translation invariance is chosen ( $G'_{96-99}{}^S = -10.8 > -11.4 = G'_{99-03}{}^T$ ). The same kind of observations can be made on the same subperiods, for both the Shapley growth effects at the US\$2 poverty line and the Datt and Ravallion growth effects at the US\$1 poverty line. Consequently, it seems that moving from a “rightist” to a “leftist” point of view implies a different perception of the range of the impact of growth or inequality on poverty. However, it is important to stress that differences between these estimated growth effects are not statistically significant except for the scale invariance inequality effects obtained with the Shapley decomposition technique at the US\$2 poverty line.

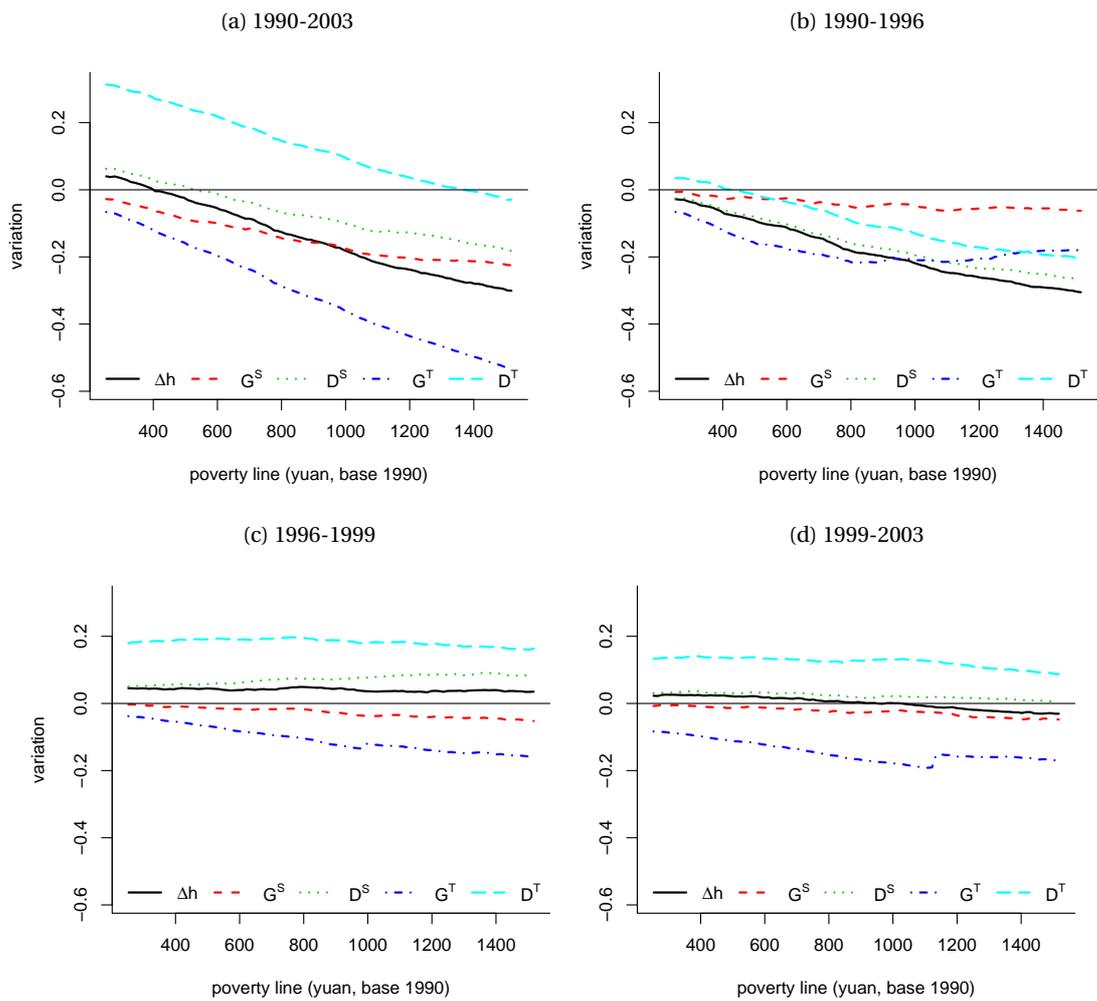
An other important fact is that all these results crucially depend on the level of the poverty line. Figures 6 and 7 present the value of the different estimated effects as well as observed poverty variations as a function of the poverty line. At first, these figures confirm the meaningful differences that we find between the scale and the translation invariance decompositions. As the range of observed values for the translation invariance is wider, the curves relative to  $G^T$  and  $D^T$  are respectively below and above the ones for  $G^S$  and  $D^S$ . Most of the time, the evolutions of the effects for the two types of invariance are parallel. Nevertheless, we clearly see on the figure 6a for the 1990-2003 period and on the figure 6c for the period 1996-1999 a divergence of the growth effects between the scale and the translation invariances as the poverty line increases. Therefore, it is important, as stressed in the poverty ordering literature (Atkinson, 1987), to analyse the sensibility of results to the level of the poverty line.<sup>55</sup>

### 4.3 Intermediate effects

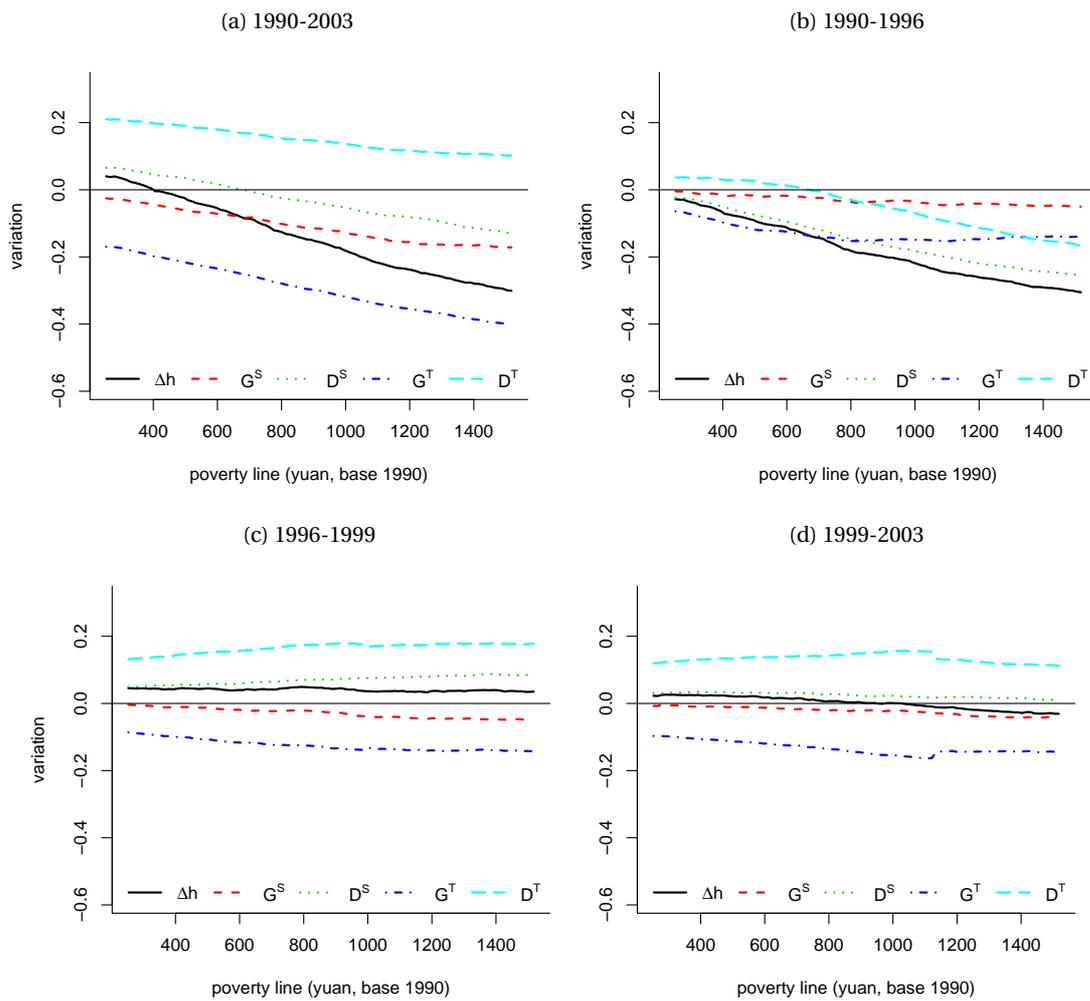
In the preceding section, we have shown that choosing the scale invariance as the sole relevant inequality view in the context of the decomposition of poverty variations provide conclusions that may not be shared by individuals which preferences are closer to views based on translation invariance. In the following paragraphs, we introduces intermediate invariance in the empirical analysis so as to get a more subtle and deeper analysis of the effects of changes in ethic preferences related to invariance.

First, we have to remind that the intermediate invariance axiom we are using is the one of Yoshida (2005) since it is the sole described in section 2.3 that is suitable for poverty decompositions. Consequently, the parameter which is determinant to this analysis is  $\sigma$ , as it describes position of individuals between the “rightist” and “leftist” views. The estimated effects corresponding to intermediate positions are reported in figure 8 for the Datt and Ravallion (1992) decomposition technique and in figure 9 when using the Shapley decomposition approach. It can be seen from each of these figures that proposition 2 is respected since curves are monotonically (weakly) increasing or decreasing. However, it is particularly interesting to note that the estimated effects are sometimes stable on some significant portions of the definition interval of the parameter  $\sigma$ . This

<sup>55</sup> For a comprehensive survey of poverty orderings, see Zheng (2000).

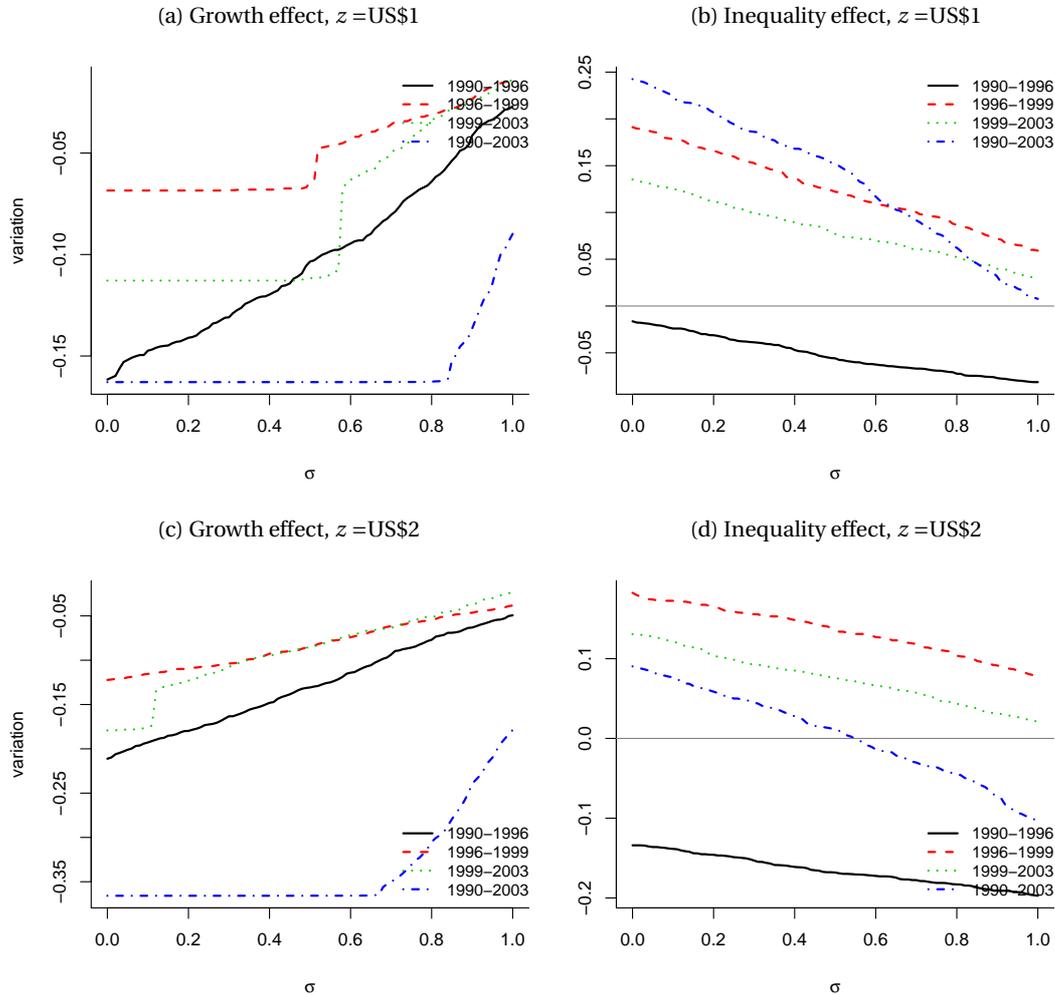


**Figure 6: “Rightist” and “leftist” decompositions of poverty spells in China 1990-2003 using the Datt and Ravallion (1992) technique.**



**Figure 7: “Rightist” and “leftist” decompositions of poverty spells in China 1990-2003 using the Shapley technique.**

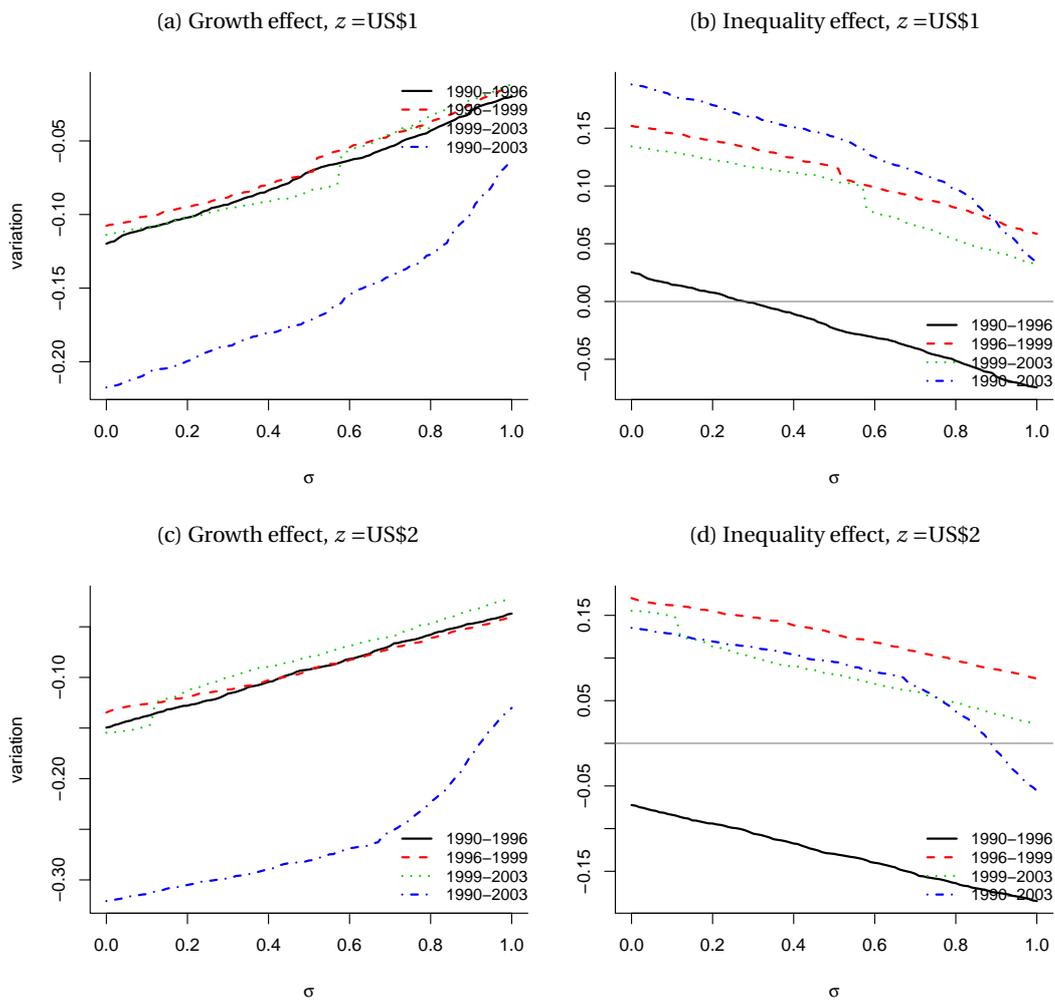
is in particular the case in figure 8a for the growth effects obtained through the Datt and Ravallion (1992) decomposition technique on the periods 1990-2003, 1996-1999 and 1999-2003.



**Figure 8: Intermediate (Yoshida, 2005) decomposition of poverty spells in China 1990-2003 using the Datt and Ravallion (1992) technique.**

We have noted in the preceding paragraph two major differences once we adopt translation invariance: a change in the sign of the inequality effect for the period 1990-2003 and 1990-1997, and an inversion of the ordering of the growth effects on poverty variations between the periods 1996-1999 and 1999-2003. These results translates differences between two opposite ethic preferences. Thanks to the intermediate methodology, we are able to use a continuum of ethic preferences and consequently determine the levels of  $\sigma$  which correspond to a reversal of the conclusions.

Concerning the change of sign underlined for the inequality effect related to the whole period, the sensibility of the results can be appreciated from figures 8d and 9d. They describe the evolution of the different effects with the parameter  $\sigma$ . We clearly see that the changes of sign occur for a value of  $\sigma = 0.5$  for the Datt and Ravallion decomposition and around 0.9 for the Shapley decomposition. When looking at the figure 10a, which adds the confidence intervals to inequality effects



**Figure 9: Intermediate (Yoshida, 2005) decomposition of poverty spells in China 1990-2003 using the Shapley technique.**

for the whole period, interesting differences appear between two considered decomposition approaches. For both of them, we stress values of  $\sigma$  for which the inequality effect is sometimes positive, neutral or negative. Using the Datt and Ravallion (1992) decomposition technique, the positive impact is found for  $\sigma$  inferior to 0.42, the neutral one for  $\sigma$  comprised between 0.42 and 0.62, and the negative one for  $\sigma$  superior to 0.62. With the second technique, the cut-offs are respectively 0.82 and 0.91. Consequently, the observation of a positive Shapley inequality effects ( $D' > 0$ ) which translate the perception of an harmful impact of inequality variations on poverty during the period 1990-2003, occurs until higher values of  $\sigma$  than for the Datt and Ravallion ones. If we consider that invariance preferences are uniformly distributed along the values of  $\sigma$ , this implies that with the Shapley methodology, more people will tend to have a “leftist” view of the inequality impact than those who will have a more “rightist” position. Moreover, we note that the confidence intervals do not overlap, suggesting that the differences between the two methodologies are also significant.<sup>56</sup>

For the period 1990-1996 with the Shapley decomposition technique, we also previously noted a change of sign. This is confirmed by the figure 9b as the corresponding curve for the inequality effect crosses the x-axes for  $\sigma = 0.3$ . However, we have noted in table 5 that the “rightist” inequality effect obtained through the Shapley decomposition technique is not significantly different from zero. Figure 10b focuses on the curve we are interested in, and gives the confidence intervals. We emphasize now clearly that individuals whose ethical preferences are below  $\sigma = 0.53$  consider that inequalities changes had a non significant impact on the evolution of poverty during the period 1990-1996 and that individuals whose preferences are above this value may feel that it significantly contributes to poverty alleviation. It is also interesting to note that the inequality effects  $D$  are never significantly different from those corresponding to the Shapley decomposition.

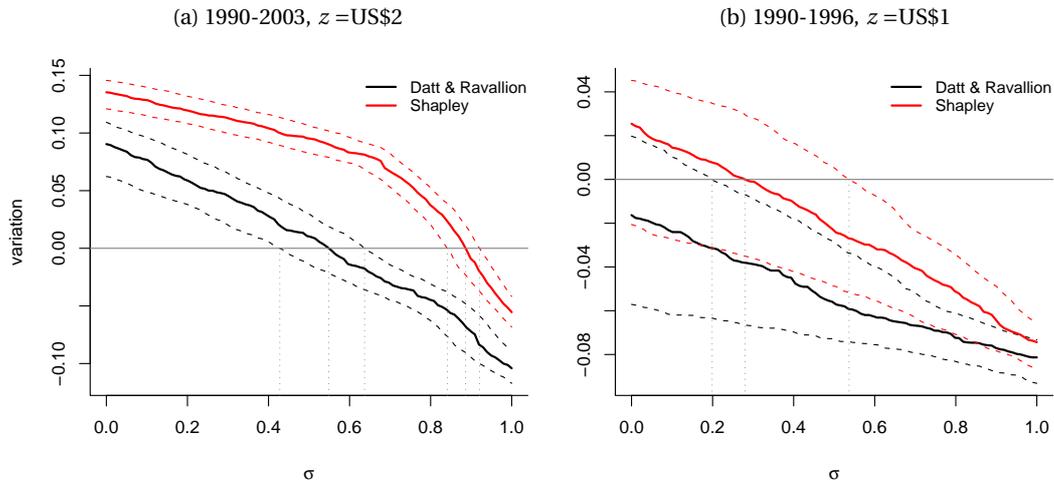
As noted earlier, ethical preferences changes may reverse the ordering of the effects between many periods. If we take a look at the growth effects issued from both Datt and Ravallion (1992) and Shapley decompositions techniques, we find that with the US\$2 poverty line, the 1996-1999 growth effect is roughly equal to the 1999-2003 one for respective values of  $\sigma$  that are approximately comprised between 0.4 and 0.72. For values that are lower than 0.4, individuals consider that the growth reducing effect has been larger between 1996 and 1999 than between 1999 and 2003. The converse conclusion holds for  $\sigma > 0.72$ .

#### 4.4 Some more words about growth and redistribution in China

Coming back to the Chinese context, we can draw important conclusions on the evolution of poverty and the role played by growth and relative distribution changes.

First, if we look at the whole period 1990-2003, we underline a decrease in poverty that is mostly due to the high growth rates that were observed during this period. This result is particularly robust since it is consistent with all inequality views and poverty lines considered in the

<sup>56</sup> These are only observations as our goal is not to promote one or the other methodology. However it is important to stress these differences as they give rise to opposite conclusions in some cases. For instance, when we consider the value of  $\sigma$  comprised between 0.62 and 0.82, we clearly see that the inequality effect is positive considering the Shapley decomposition and negative once we move to the Datt and Ravallion's one. Consequently, this underlines the potential need for a clear choice between the two methodologies, or the definition a third procedure.



Note: The area between the dashed curves represents the 95% confidence interval obtained through a bootstrap procedure with 500 replications.

**Figure 10: Sensibility of the inequality effects to ethical preferences parameter  $\sigma$ .**

analysis. On the other hand, the effects of inequality changes on the variation of poverty are always positive when we consider the US\$1 poverty line but depends on the ethic preferences in terms of invariance when we look at the results for the US\$2 poverty line. In the later case, inequalities tend to decrease poverty considering the “rightist” view but increase it once we adopt the “leftist” point of view. It is well known that China has experienced a huge growth since the reforms movement initiated at the end of the 1970’s. But in parallel to this economic development, inequalities raised dramatically. The harmful impact of inequalities on poverty for the period 1990-2003, stressed in the US\$1 case and in the US\$2 case only for the translation invariance decomposition, are thus not surprising. These results are consistent with those found in studies related to Chinese poverty. For instance, the articles of Chen and Wang (2001); Fan et al. (2002); Hanmer et al. (2004); Gregory et al. (2005); Wan and Zhang (2006) and Chen and Ravallion (2007) all demonstrate the positive role of growth in decreasing poverty in China but the negative role of increasing inequalities. Explanations of this phenomenon can be found in the rural late development (Hanmer et al., 2004; Gregory et al., 2005), or in the evolution of the labor market (Fan et al., 2002), but our analysis cannot give any support to these hypotheses.

With a closer look at the evolution of poverty through time, we note a very important decrease between 1990 and 1996 but then a slight increase between 1996 and 2003. This unstable evolution of poverty has been previously emphasized in some studies on poverty in China (Chen and Ravallion, 2007; Gregory et al., 2005). Considering the recent increase of poverty, we see that its main cause is the positive impact of inequalities, a result that does not depend on the chosen inequality view. This means that whatever decomposition procedure, invariance preferences and poverty line are chosen, growing inequalities in China have worsened the situation of the poorest population. In a “pro-poor” growth analysis *à la* Kakwani and Pernia (2000), both “leftist”, intermediate and “rightist” observers would then consider that growth can be deemed “anti-poor” on the periods 1996-1999 and 1999-2003, and surely more when moving from a “rightist” to a “leftist” point

of view. The political recommendation issued from these conclusions join the ones previously done by other researchers: there is an important need to associate growth with a more “pro-poor” redistributive policy if the Chinese government wants to succeed in alleviating extreme poverty.

## 5 Concluding remarks

In these few lines, we will not draw policy recommendations but methodological ones. From our point of view, the issues illustrated in the present paper could lead to the three following attitudes:

- i)* standardization of the practices,
- ii)* consistency with personal ethical preferences,
- iii)* sensibility analysis to ethical preferences.

Attitude *i)* consists in the definition of a standard view that should be used by every economist. The main argument in favor of this strategy is that it defines a common analyzing framework and helps to make different studies comparable. Moreover, we should recognize that it corresponds to the attitude that presently seems to prevail since most economists implicitly feel in accordance with inequality views based on scale invariance. However, we would like to stress that economists should be aware of the normative implications of this particular axiomatic choice and of the potential discrepancy between the ethical preferences reflected by scale invariance and their own personal preferences. At least should they clearly express on which axioms are based their analyses when chosen measures are compatible with many rival axioms.

The second strategy reflects the opposite strategy. It implies economists to make use on the sole measures that are consistent with their own personal ethical preferences. A major problem is that knowing oneself, or at least his own feelings, and expressing these preferences through rigorous mathematical properties is a difficult task.<sup>57</sup> Kolm (1995, p.301) observes that “*the view concerning the comparative justice of covariations in incomes depends on the setting of the question, and, of course, on the political reading of this setting. It depends on the levels of the real incomes, and in particular on the average level and on the levels of the lowest and of the highest; on the conceived solidarity or duty of solidarity; of course on the origin of these transformations; on past and expected history; on the fact that the considered variation is an increase or a decrease; and so on.*” Of course this attitude raises the problem of the comparability of researchers’ works since most individuals are not likely to speak the same “language”.

Finally, attitude *iii)* consists in not choosing for the reader which inequality view he should adopt, and presenting a sensibility analyses of the results to axiomatic changes so as the reader can find which results fit his own conception of inequality.<sup>58</sup> As illustrated by our application on

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<sup>57</sup> It is particularly interesting to note that after hours and endeavours devoted to the review of the different invariance axioms suggested in the literature and their implications, the authors of the present paper are still not able to express precisely their own feelings on this precise subject.

Moreover, economists’ tastes may be to some extent endogenous as noted by Amiel and Cowell (1992, p. 22): “*Wideranging policy decisions can be influenced by ideas about inequality; these ideas are, in turn, influenced by the way individuals are trained to think about the issues.*”

<sup>58</sup> Kolm (1969, p.148) advocates that the economist “*is an observer of citizens’ value judgements and opinions, as he is an observer of their tastes concerning consumers’ goods.[...] Useful normative economics is therefore a positive science since its basis is the objective observation of subjective opinions.*” Thus citizens’ preferences should be given the pre-eminence over the economist’s tastes.

Chinese data, considering different inequality views can reverse conclusions but may also improve the robustness of some results, and thus give more convincing arguments for policy recommendations. This corresponds to a traditional attitude in welfare economics, in particular for the description of poverty and inequality trends. For instance, when comparing different distributions, it is common to make use of many different measures like the Atkinson's (1970) inequality measures, which are based on classical von Neumann and Morgenstern utilitarianism, and Gini indices which are derived from rank-dependant expected utility models (see Gajdos, 2001, for a review). In the same spirit, it is common to find studies that use both Foster et al. (1984) and Sen's (1976) poverty measures.<sup>59</sup>

Concerning the specific subject of the decomposition of observed poverty variations, attitudes *ii*) and *iii*) call for the development of appropriate inequality and poverty measures. In the present paper, we focus on the headcount index since it is the sole known poverty measure (*cf.* proposition 1) that simultaneously complies with all the aforementioned invariance axioms, and thus leaves room to individual preferences for the interpretation of its variations. Indeed, the headcount index is considered by most authors as a poor measure of poverty since it does not account for the intensity and inequality dimensions of poverty. As a consequence, many distribution-sensitive poverty measures have been proposed by (Watts, 1968; Sen, 1976; Kakwani, 1980; Clark et al., 1981; Foster et al., 1984; Hagenaaers, 1987), but each one is consistent with a unique invariance axiom so that comparisons of their "leftist", "rightist" and intermediate decompositions are not possible. As a result, the evaluation of the relative contribution of growth and redistribution to poverty alleviation using distribution-sensitive measures for various inequality views can only be performed with the help of classes of invariance-sensitive poverty measures, that is poverty measures which features some parameters that reflect invariance preferences. Recent propositions by Zheng (1997) are to our knowledge the sole tentative to provide such tools and should inspire further research.<sup>60</sup>

## Appendices

### A The $(V, v)$ -invariance axiom in practice

The major concern with the implementation of the  $(V, v)$ -invariance axiom is the presence of the reference distribution  $V$  that generally cannot be directly used to find the counterfactual incomes distributions needed for the computation of the growth and inequality effects. For convenience, suppose that  $V$  is on the two-dimension subspace  $\mathcal{S}_X$  defined by the vectors  $\frac{X}{\mu_X}$  and  $I$ . We also

<sup>59</sup> In these two examples, one should note that ordering criterions have been developed so as to determine in which cases different inequality and poverty indices would respectively yield the same conclusions. Consequently we suggest that further research could be devoted to the findind of ordering conditions for many invariance axioms.

<sup>60</sup> The indices suggested by Zheng (2007*c*) are not included in the present paper because they are not yet well documented. More precisely, it can be demonstrated that the proposed Krtscha-type poverty index does not comply with the non-linear invariance axiom presented in section 2.3.2. Moreover the Dalton-Hagenaaers index developed by the author relies on an unknown invariance axiom. Without explicit formulation of the transformations that preserves inequality, the computation of the counterfactual incomes cannot be done. Consequently, this precludes a decomposition of the variations of this measure.

consider that  $Y$  is of size  $n_Y$  that may be different from  $n$ . In the original version of the axiom (del Rio and Ruiz-Castillo, 2000), a distribution  $Y$  can be directly compared with a distribution  $X$  only if it belongs to  $\mathcal{S}_X$ . For the purpose of decomposing poverty spells,  $V$  can generally not be used for the computation of the inequality effect  $D$  since the transformed distribution would not be on the same subspace  $\mathcal{S}_Y$  as  $Y$ . In order to make the comparison feasible with each distribution  $Y \in \mathcal{D}_\alpha$ , one needs to find the distribution  $\frac{V'}{\mu_{V'}}$  that corresponds to the projection of the reference distribution  $\frac{V}{\mu_V}$  into the subspace  $\mathcal{S}_Y$ . In the spirit of (Alonso-Villar and del Rio, 2007a), we can define  $\frac{V'}{\mu_{V'}}$  as the distribution in  $\mathcal{S}_Y$  which exhibits the same Euclidean distance from perfect equality as  $\frac{V}{\mu_V}$ . Thus,  $V'$  must be chosen so as to respect:

$$\sqrt{n^{-1} \sum_{i=1}^n \left( \frac{v_i}{\mu_V} - 1 \right)^2} = \sqrt{n_Y^{-1} \sum_{i=1}^{n_Y} \left( \frac{v'_i}{\mu_{V'}} - 1 \right)^2}. \quad (\text{A.1})$$

To get a unique distribution  $V'$ , we have to add some constraints to equation (A.1). If we normalize the distributions  $V'$  and  $Y$  by their mean value, we know that  $V'$  has to meet the following condition:

$$\frac{V'}{\mu_{V'}} = \zeta \frac{Y}{\mu_Y} + (1 - \zeta)I \quad \zeta \in \mathbb{R}^{++}. \quad (\text{A.2})$$

As a consequence:

$$\sqrt{n_Y^{-1} \sum_{i=1}^{n_Y} \left( \frac{v'_i}{\mu_{V'}} - 1 \right)^2} = \sqrt{n_Y^{-1} \sum_{i=1}^{n_Y} \left( \zeta \frac{y_i}{\mu_Y} + (1 - \zeta) - 1 \right)^2}, \quad (\text{A.3})$$

$$= \zeta \sqrt{n_Y^{-1} \sum_{i=1}^{n_Y} \left( \frac{y_i}{\mu_Y} - 1 \right)^2}. \quad (\text{A.4})$$

Rearranging equation (A.4) and using equation (A.1), we obtain:

$$\zeta = \frac{\sqrt{n^{-1} \sum_{i=1}^n \left( \frac{v_i}{\mu_V} - 1 \right)^2}}{\sqrt{n_Y^{-1} \sum_{i=1}^{n_Y} \left( \frac{y_i}{\mu_Y} - 1 \right)^2}}. \quad (\text{A.5})$$

Concerning the value of  $\iota$ , scale invariance is obtained when:

$$\iota \frac{V'}{\mu_{V'}} + (1 - \iota)I = \frac{Y}{\mu_Y}. \quad (\text{A.6})$$

Consequently:

$$\sqrt{n_Y^{-1} \sum_{i=1}^{n_Y} \left( \iota \frac{v'_i}{\mu_{V'}} + (1 - \iota) - 1 \right)^2} = \sqrt{n_Y^{-1} \sum_{i=1}^{n_Y} \left( \frac{y_i}{\mu_Y} - 1 \right)^2}, \quad (\text{A.7})$$

which yields, using equation (A.1):

$$\iota = \frac{1}{\zeta}. \quad (\text{A.8})$$

## B The $(V, v)$ -invariance axiom and lemma 1

If the projection  $V_X$  of chosen reference distribution is not equal to  $X$  up to a scale factor, we have:

$$\frac{V_X}{\mu_V} = \zeta_1 \frac{X}{\mu_X} + \zeta_2 I. \quad (\text{B.1})$$

In order to get a transformed distribution  $\Phi(X, \mu_Y)$  which mean value is equal to  $\mu_Y$ , the two parameters must respect the following condition  $\zeta_2 = 1 - \zeta_1$ . Moreover, since  $V_X$  should Lorenz dominate the distribution  $X$ , we observe  $\zeta_1 > 1$ . Consequently, we get:

$$\Phi(X, \mu_Y) = X + (\mu_Y - \mu_X) \left( v \frac{V_X}{\mu_V} + (1-v)I \right), \quad (\text{B.2})$$

$$= X + (\mu_Y - \mu_X) \left( v \left( \zeta_1 \frac{X}{\mu_X} + (1-\zeta_1)I \right) + (1-v)I \right), \quad (\text{B.3})$$

$$= v\zeta_1 X \frac{\mu_Y}{\mu_X} + (1-v\zeta_1) (X + (\mu_Y - \mu_X)I). \quad (\text{B.4})$$

Comparing with equation (2.12), we can conclude that lemma 1 will be fulfilled if and only if  $V$  is chosen so that  $\zeta_1 \leq v^{-1}$ . Another important conclusion is that the transformed distribution corresponding to scale invariance can be obtained from  $X$  only if  $V_X$  is equal to  $X$  up to a scale factor. On the other hand, no condition is imposed for the value of  $\zeta_1$  so as to obtain the transformed distribution that would correspond to translation invariance.

## C The $(V, v)$ -invariance axiom and unit-consistency

In a recent paper, Zheng (2004) argued that any inequality measure based on the  $(V, v)$ -inequality (del Rio and Ruiz-Castillo, 2000) view violates unit consistency. His demonstration is based on the corresponding intermediate Lorenz criterion which is defined by:<sup>61</sup>

$$L(X, j, v) := \frac{1}{n} \sum_{i=0}^j v \frac{x_i}{\mu_X} + (1-v)(x_i - \mu_X + 1). \quad (\text{C.1})$$

and is weighted mean of the relative and absolute Lorenz curves.<sup>62</sup> To compare the distributions  $X$  and  $Y$ , we have to draw the corresponding intermediate Lorenz curve for  $Y$ . Following the rationale of equation (C.1), Zheng (2004) gets:

$$L(Y, j, v) := \frac{1}{n} \sum_{i=0}^j v \frac{y_i}{\mu_Y} + (1-v)(y_i - \mu_Y + 1). \quad (\text{C.2})$$

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<sup>61</sup> The definition of the adequate Lorenz criterion is quite easy since one only need the use the equation of the invariance axiom on the generalized Lorenz curve defined by Shorrocks (1983) so as to normalize mean income to unity.

<sup>62</sup> This is a slightly modified version of the absolute Lorenz curve since original version by Moyes (1987) is:

$$L(X, j) := \frac{1}{n} \sum_{i=0}^j x_i - \mu_X.$$

In order to prove that unit-consistency is not respected,  $X$  and  $Y$  must be chosen so as there exists a value  $v^* \in [0, 1]$  such that the two distributions can be considered as exhibiting the same degree of inequality. In other words, one distribution should be relative-Lorenz dominated by the other which absolute-Lorenz dominates the former. Let  $X$  relative-Lorenz dominates  $Y$ . So one can find  $v^*$  such that:

$$\frac{1}{n} \sum_{i=0}^j v^* \frac{x_i}{\mu_X} + (1 - v^*)(x_i - \mu_X + 1) = \frac{1}{n} \sum_{i=0}^j v^* \frac{y_i}{\mu_Y} + (1 - v^*)(y_i - \mu_Y + 1) \quad (C.3)$$

If a smaller unit of income is then used (each income is scaled up by the same constant, the respect of unit-consistency implies that we should still feel that the two distributions are equally unequal for  $v = v^*$ . However, multiplying the vector  $X$  and  $Y$  by  $\lambda > 1$  yields:

$$\frac{1}{n} \sum_{i=0}^j v^* \frac{x_i}{\mu_X} + (1 - v^*)(\lambda x_i - \lambda \mu_X + 1) \leq \frac{1}{n} \sum_{i=0}^j v^* \frac{y_i}{\mu_Y} + (1 - v^*)(\lambda y_i - \lambda \mu_Y + 1) \quad (C.4)$$

since we have supposed that  $X$  is absolute-Lorenz dominated by  $Y$ , that is to say:

$$\frac{1}{n} \sum_{i=0}^j x_i - \mu_X + 1 \leq \frac{1}{n} \sum_{i=0}^j y_i - \mu_Y + 1 \quad (C.5)$$

Zheng (2004) then concludes on the basis of (C.3) that unit-consistency is violated. However, this result is due to a misunderstanding of del Rio and Ruiz-Castillo's (2000) approach. The author assumes that the part of the incremental income that is not equally shared between income receivers must be distributed in proportion of their respective relative contribution to total income.<sup>63</sup> In fact, this part must be distributed with respect to income shares of a reference distribution which should be the same when comparing  $X$  and  $Y$  so as a unique value of  $v$  can be used. If  $X$  is chosen as the reference-distribution, the real intermediate Lorenz curve corresponding to  $Y$  is :

$$L(Y, j, v) := \frac{1}{n} \sum_{i=0}^j y_i + (1 - \mu_Y) \left( v \frac{x_i}{\mu_X} + 1 - v \right). \quad (C.6)$$

Unit-consistency is not violated if the differences  $L(\lambda Y, j, v) - L(Y, j, v)$  and  $L(\lambda X, j, v) - L(X, j, v)$  are equal for  $v = v^*$ . We observe:

$$L(\lambda Y, j, v^*) - L(Y, j, v^*) = (\lambda - 1) \frac{1}{n} \sum_{i=0}^j y_i - \mu_Y \left( v^* \frac{x_i}{\mu_X} + 1 - v^* \right) \quad (C.7)$$

$$L(\lambda X, j, v^*) - L(X, j, v^*) = (\lambda - 1) \frac{1}{n} \sum_{i=0}^j x_i - \mu_X \left( v^* \frac{x_i}{\mu_X} + 1 - v^* \right) \quad (C.8)$$

Adding  $(\lambda - 1) \frac{1}{n} \sum_{i=0}^j v^* \frac{x_i}{\mu_X} + 1 - v^*$  to each member yields:

$$L(\lambda Y, j, v^*) - L(Y, j, v^*) + (\lambda - 1) \frac{1}{n} \sum_{i=0}^j v^* \frac{x_i}{\mu_X} + 1 - v^* = (\lambda - 1) L(Y, j, v^*) \quad (C.9)$$

<sup>63</sup> The same mistake can be observed in Zoli (2003).

$$L(\lambda X, j, v^*) - L(X, j, v^*) + (\lambda - 1) \frac{1}{n} \sum_{i=0}^j v^* \frac{x_i}{\mu_X} + 1 - v^* = (\lambda - 1)L(X, j, v^*) \quad (\text{C.10})$$

Since we supposed  $L(Y, j, v^*) = L(X, j, v^*)$ , equations (C.9) and (C.10) lead to the conclusion that  $L(\lambda Y, j, v^*) = L(\lambda X, j, v^*)$ , QED.

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