Design of Single-Layer Floating-Compression Tensegrities

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Résumé — Floating-compression tensegrities can conveniently be employed as deployable systems and as soft structures. Focusing on single-layer systems, we first identify the relevant parameters affecting their structural performances, by studying a simpler two-dimensional benchmark, then we derive new three-dimensional structures and present the design issues associated with them.
Mots clés — Tensegrity balloons, cable-strut polygons.

1 Introduction

Floating-compression tensegrities are prestressed systems composed by bars and cables such that bars are never connected to each other (see [1, 2] for an introduction on tensegrities and [3] about the origin of these systems). As in conventional trusses, we can distinguish between single-layer and double-layer systems : in the former case, by identifying a single surface with all the nodes of the system laying on it ; in the latter case, by identifying two closely spaced surfaces passing through all the nodes.

Since there are no bar-to-bar connections, these systems can be easily folded and it can be convenient to employ them as deployable structures [4, 2]. For the same reason, their stiffness is often lower than that of conventional trusses, so that they can preferably be employed as soft/flexible structures. For example, they have been proposed for marine cages in aquaculture applications [5]. The tensegrities we derive here can also be designed to be employed as domes and roofs, given the architectural appeal of floating-compression systems.

We present a study on single-layer system, such as two-dimensional or three-dimensional tensegrity balloons : systems where the characterizing surface is closed, with cables laying on the boundary of the convex hull of the structure, and bars inside. The term reflects the well-known balloon analogy by Fuller [6], which fully applies here. Cables and bars are analogous to the membrane in tension and the air under pressure in common balloons. In two dimensions, these systems have been referred to as *cable-strut polygons* by Connelly [7]. A couple of examples is shown in Figure 1 (left and center). Externally

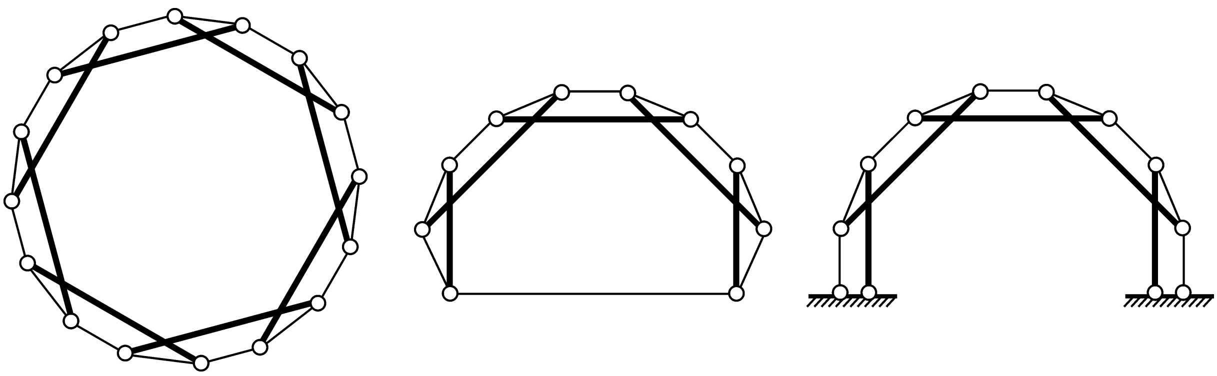

FIG. 1 – Tensegrity polygons (left and center). Externally constrained system obtained by sectioning a cable-strut polygon (right).

constrained structures can be easily obtained from tensegrity balloons by sectioning them in various ways, a two-dimensional example is shown in Figure 1 (right).

After discussing the stability of these systems, we show that there are two main parameters affecting their properties, namely the number of bars per unit area and the bars' overlap. In particular, buckling is more easily induced by external loads when the number of bars increases and when the overlap decreases. In order to prevent this effect, we evaluate the benefits of having additional internal bracing cables.

The study is continued by performing the design of two new types of tensegrity structures. The first type is derived from classical spherical tensegrities by modifying the typical layout of the set of cables, arranging them in a triangulated net. The resulting structures possess a higher stiffness if compared to classical ones. The second type is entirely original and it is obtained by a parallel juxtaposition of cable-strut polygons in three dimensions.

2 Modeling and computation

By modeling tensegrities as simple pin-connected bar frameworks, in the linear theory, the properties of solutions of the equilibrium and compatibility equations are determined from the fundamental subspaces of the equilibrium operator [8]. The system is characterized in terms of *self-stress states*, internal stresses balanced by null external loads, and *mechanisms*, nodal velocities causing no change in length of any member. We recall here the *extended Maxwell's rule* for a free-standing system, respectively, in two- and three-dimensions [8] :

$$2n_n - 3 - n_e = n_m - n_s \quad (2\text{D}) \quad ; \qquad 3n_n - 6 - n_e = n_m - n_s \quad (3\text{D})\,. \tag{1}$$

This rule relates the number of nodes, n_n, and members, n_e, with the number of independent mechanisms, n_m, and self-stress states, n_s.

Roth, Connelly and Whiteley were the firsts to develop the theory of *tensegrity frameworks* [9, 10], which are composed of rigid bars and inextensible, tension–only, cables. Tensegrity frameworks can be classified in terms of the numbers n_m and n_s [11]. If $n_s = 0$ then the system cannot be stable without the action of external forces causing tension in all cables. When $n_s > 0$ and $n_m = 0$, then the system is stable under no external forces if there exists a self-stress state for which all cables are in tension. When $n_s > 0$ and $n_m > 0$, then the system is stable under no external forces if the following condition holds for all mechanisms and self-stress states :

$$\sum_{\text{members}} \frac{t_{ij}}{l_{ij}} (v_i - v_j)^2 > 0\,. \tag{2}$$

In this condition, which is referred to as *prestress-stability*, t_{ij} and l_{ij} are the axial force and the length of the member connecting nodes i and j, while v_i and v_j are the velocities of these nodes.

When the members of a tensegrity system are considered to be linear elastic, then the above stability conditions are not enough to guarantee the actual physical stability of the system. A configuration will be stable if the total potential energy is at a minimum value. In other words, the tangent stiffness operator needs to be positive definite. We recall here that this operator can be decomposed additively into its material and geometric parts,

$$\mathbf{K}_T = \mathbf{K}_M + \mathbf{K}_G\,, \tag{3}$$

the former depending on members' material properties, the latter depending on members' stresses. When a system under no external loads is stable regardless of its material properties and level of self-stress it is called *super-stable* [12] (cf [13]).

Finding the stable configuration(s) of a tensegrity system constitutes the most important task when dealing with kind of structures. In the literature, this is always referred to as *form-finding problem*, which can be formulated in many non-equivalent ways (see [14]) and it has been solved with a large variety of methods [15, 16]. Here, we solved the form-finding problem for linear elastic tensegrity balloons by shortening cables in a large-displacement elastic analysis, and we employ a Newton-Raphson procedure. A similar conceptual strategy, the *cocoon method*, has been proposed and demonstrated in [17] by making use of physical models. In that method, a deformable envelop is stretched against elongating bars, to find stable configuration according to the balloon analogy.

We employ the same Newton-Raphson procedure to perform static analyses. We remark that there are many other possibility for the choice of form-finding and static analysis methods, which can be more effective in many diverse situations. However, due to the smooth behavior of tensegrity balloons, this simple procedure was sufficient for our purposes.

3 A two-dimensional parametric benchmark

In this section we study cable-strut polygons, whose mechanical properties can be useful to understand more complicate three-dimensional cases. An important theoretical result have been established by Connelly about these and other similar polygonal systems [10] (cf [12]), proving that *any self-stressed convex polygon, having a positive stress in external members and negative (or zero) otherwise, is super-stable* (Polygon theorem [12]).

A n-bars symmetric cable-strut polygon, with $n > 2$, possesses a n-fold axis of cyclic-symmetry, which is orthogonal to the plane of the polygon, and n axes of reflection symmetry, which belong to that plane (Figure 2, left). Other than the number of bars and the polygon *radius*, i.e. the radius of the

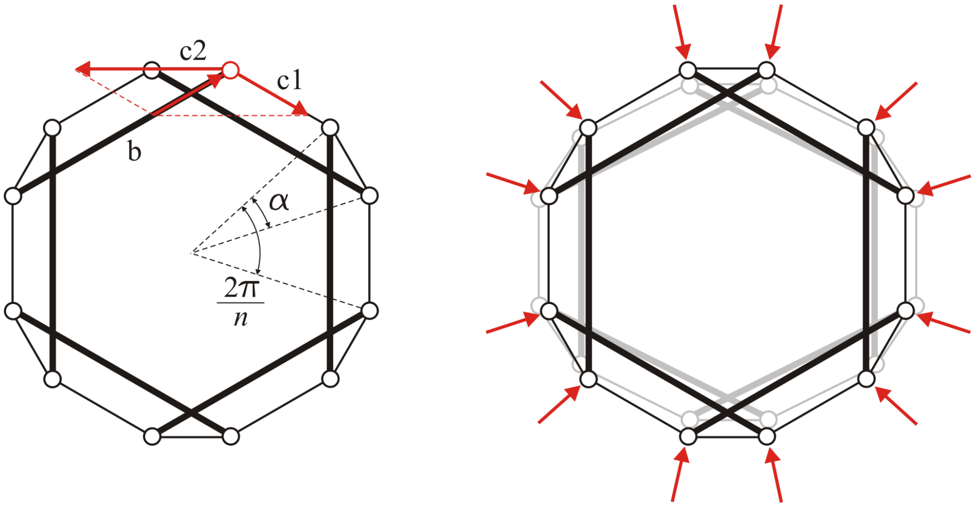

FIG. 2 – Overlap angle and self-stress of a cable-strut polygon (left). Buckling under a radial load (right).

circle circumscribed to the polygon, the geometry of this system can be uniquely identified by the angle at center α depicted in Figure 2 (left). This angle measures the *overlap* between bars, a crucial structural parameter.

Symmetric cable-strut polygons always possess a single states of self–stress, $n_s = 1$. Since there are $3n$ members and $2n$ nodes, by the extended Maxwell's rule there are $n_m = n-3$ independent mechanisms. The self-stress state it is easily computed from the graphic representation of axial forces. We have

$$t_b = -t_{c1} \ , \quad t_{c2} = 2\cos\left(\frac{\pi}{n}\right) t_{c1} \ , \tag{4}$$

where t_b is the force in bars, t_{c1} and t_{c2} are the forces in cables (Figure 1). Notice that the self-stress does not depend on α.

Although cable-strut polygons are super-stable, it can be seen that they can be affected by buckling under an external loading. To test their resistance against buckling, we performed numerical computations by loading these systems radially, as shown in Figure 2 (right). Results can be summarized as follows :

– when α (the overlap) increases, the buckling load increases ;
– when n increases, then the buckling load decreases ;
– when the self-stress level increases, the buckling load increases.

Intuitively, the first two effects can be explained geometrically since the distance between a bar and the parallel cable close to it decreases, resulting in a 'thinner' structure. We also verified that any 'weak' cable-strut polygon can be significantly strengthened with additional bracing cables, as shown in Figure 3. Notice that these cables need only to be slightly in tension to be effective. Notice also that a high tension in these bracing cables would affect and lower the tension in the other external cables.

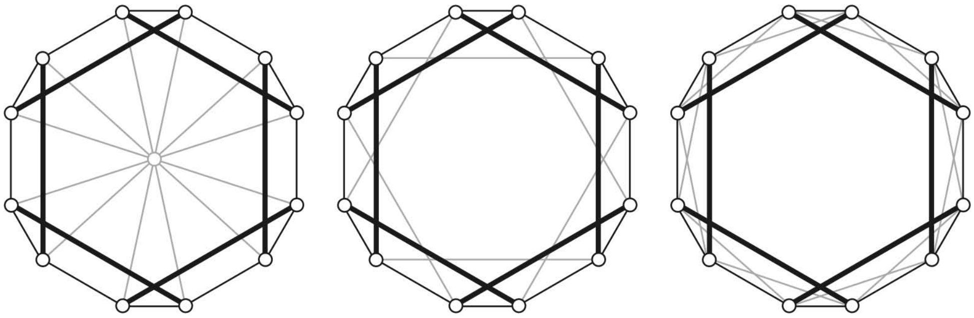

FIG. 3 – Possible arrangements for bracing cables.

4 Three-dimensional examples

4.1 ‘Spherical’ systems

We can obtain spherical systems from a net of cables with the layout similar to that of a geodesic sphere, and then by adding bars to it following the arrangement found in classical tensegrities by Fuller (Figure 4). As introduced earlier, by taking linear elastic elements, we progressively decreased the rest length of cables until a self-stressed system is obtained. To improve structural performances, it is beneficial to avoid cables with low self-stress, since they can be slackened by the action of external loads. For this reason we iteratively shortened the rest lengths of low stressed cables until a more uniform self-stress state is obtained. Figure 5 shows the result of this procedure.

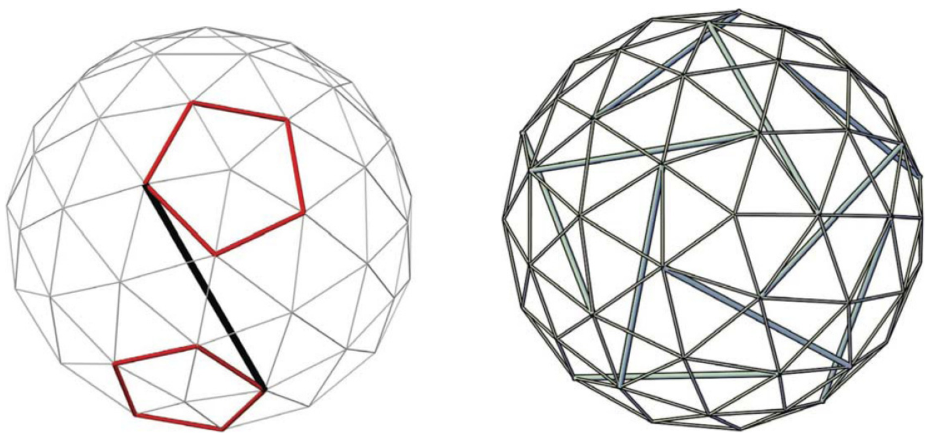

FIG. 4 – Layout of cables and bars for an improved spherical system.

The structure obtained in this way can be further modified by changing the layout of cables. In Figure 6 we replaced a ‘star’ of six cables, and the common node connected to them, with a set of three cables, with no significant geometrical modification of the rest of the structure.

At this point, we tried obtained domes from these structure by sectioning a part of it and by adding external constraints. This operation can be done in different ways. For example, we can section the members with a plane and pin the intersection points to ground, as shown in Figure 7 (left), or we can ‘cut’ along the cables, section the bars, and add external constraints in a similar way, as shown in Figure 7 (right).

Both domes may look too complicate with respect to their foundations, as in the first we have a large number of anchorages and in the second the foundations are not horizontal. We also tested a system where the starting layout of the cable-net is that of a geodesic dome, then we added bars to it and performed the form-finding procedure. However, this way we obtained poor results, since a suitable uniform self-stress state was much more difficult to achieve. In the cases we presented, the high symmetry of the system allowed us to easily obtain structures whose properties are intrinsically closer to the optimal ones.

Figure 8 shows the system obtained from a finer geodesic subdivision of the sphere. The structural performances of spherical systems are quite good. The interlocked layout of bars make these systems quite strong against overall buckling and the triangulated net of cables provides a higher stiffness with

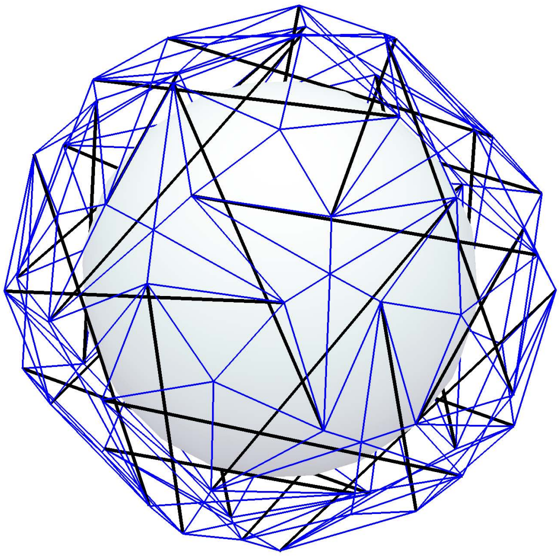

FIG. 5 – Form-finding results : self-stressed system obtained from the layout in Figure 4. A sphere has been plotted inside the structure for a better visualization of the members.

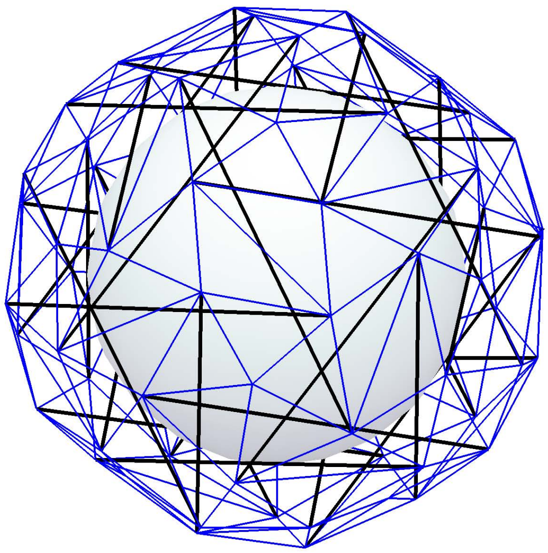

FIG. 6 – Form-finding results : system obtained from a slightly modified arrangement of cables. A sphere has been plotted inside the structure for a better visualization of the members.

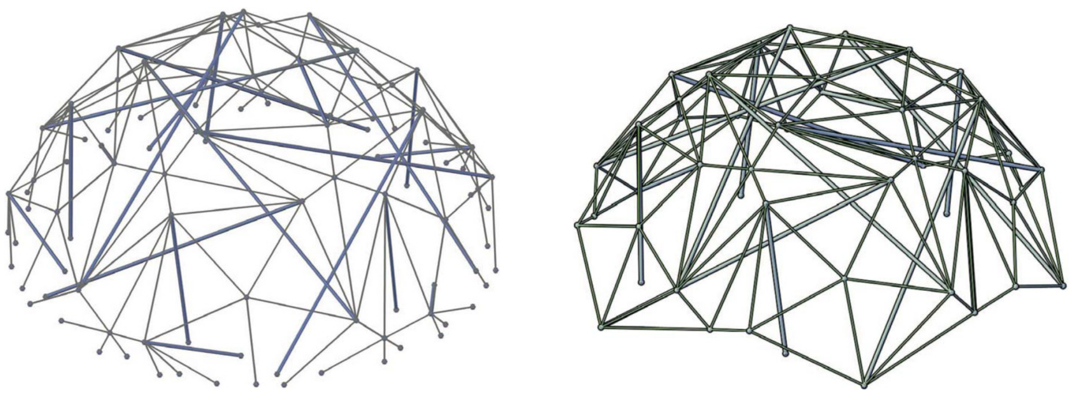

FIG. 7 – Two different ways of obtaining a dome from the system on the left in Figure 5.

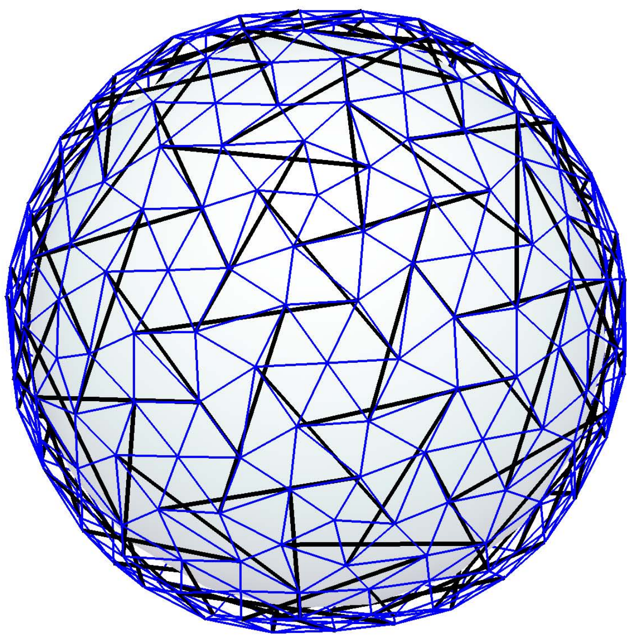

FIG. 8 – System obtained from a finer geodesic subdivision of the sphere, with the same kind of modification performed in the system in Figure 6 (right). A sphere has been plotted inside the structure for a better visualization of the members.

respect to classical tensegrities. Moreover, these structures are highly redundant, i.e. they have a large number n_s of self-stress states, so that breaking a cable would not cause the collapse of the entire system. On the other side, during construction, to precisely establish the desired self-stress state it is necessary to monitor the tension of at least n_s members.

4.2 A new system

The idea for a new single-layer system originated from the seed structure in Figure 9 (left). By looking at this system in projection (Figure 9, right) we recognized a cable-strut polygon. Figure 10 shows a system where such a polygon is visible in a clearer way. Bars lay alternatively on different planes, and these planes are orthogonal to a central longer bar. By extending this concept further it is possible to obtain a whole family of structures, like the one in Figure 11. Figure 12 shows a dome obtained with the operation previously described. The stability properties of these systems are not as good as those of the spherical ones, hence, we needed to add a set of bracing cable in order to strengthen them.

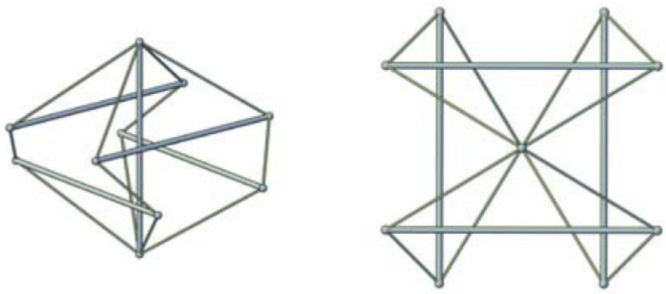

FIG. 9 – The seed structure. Perspective view (left), top view (right).

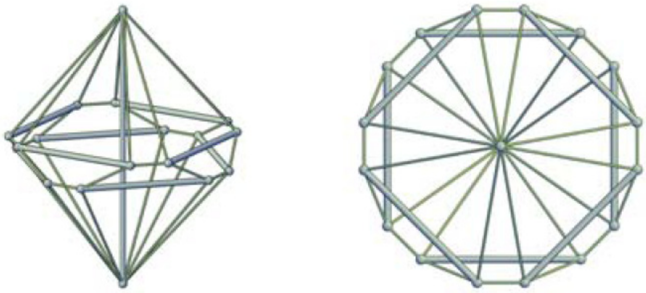

FIG. 10 – A small system derived from the seed structure. Perspective view (left), top view (right).

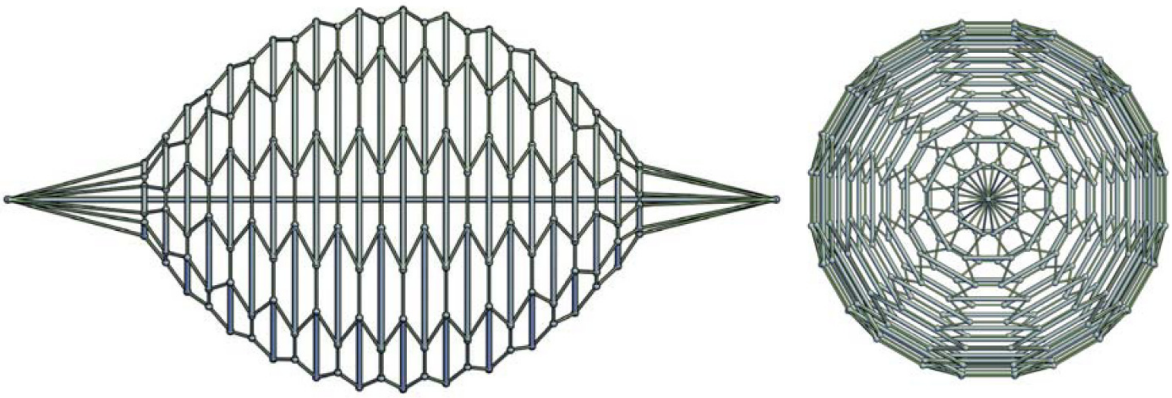

FIG. 11 – Two views of the new system obtained by parallel juxtaposition of cable-strut polygons.

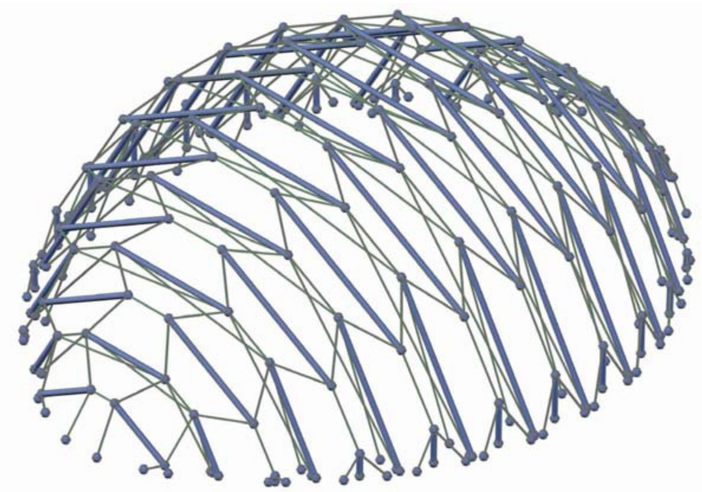

FIG. 12 – A dome obtained from the system in Figure 11.

5 Discussion and conclusions

From the analysis performed in this study, the design of single-layer floating-compression tensegrities results from the balance of different factors. It is clear that it is not possible to increase indefinitely the number n of bars in a cable-strut polygon, since this would increase the chance of buckling under external load. On the other side the number n cannot be too small to avoid an excessive bar length and to gain internal space. Due to buckling as well, the overlap parameter α cannot be too small and, at the same time, to avoid an excessively short length of cables, cannot be too large. Regarding the self-stress state, it should be designed to be at a level guaranteeing both the tension in cables under the envisaged load conditions and an adequate buckling strength, without causing excessive compression in bars. Lastly, the tension in bracing cables, when they are necessary, should be low to not affect the rest of the cables.

In this study we devised a methodology for the form-finding, the analysis and the design of single-layer tensegrities, by identifying the relevant parameters which are typical of these systems and by presenting two new types of systems. This methodology can be easily applied to discover and realize new systems, both regular and irregular. Several variation are possible, involving connectivity operations and numerical procedures. Future work can regard the automatic generation and optimization of these systems, with the aid of the criteria outlined here. Another important issue is the establishment of stability results in the three-dimensional case.

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