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GLOBAL INSTABILITY OF FLOWS ACROSS A JUNCTION

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1 Global instability in infinite domains

Hydrodynamic stability theory originally developed through the study of flows where the streamwise development of the flow could be neglected. Under this parallel-flow approximation linearized disturbance equations can be reduced to ordinary differential equations, like the Rayleigh or Orr-Sommerfeld equations, by representing an arbitrary disturbance by a superposition of waves. The results of these 'local' stability theories are assumed to be applicable close to the chosen point in the flow where the velocity profile has been extracted from the streamwise-developing flow, and made parallel by ignoring its streamwise-dependence. The issue of how these local stability results can be interpreted to predict the stability of the whole flow is referred to as the 'global' stability problem.

When the basic flow varies strongly in the streamwise direction nonparallel effects cannot be ignored, local results are meaningless, and the global stability problem must usually be approached by solving partial differential disturbance equations numerically. But when the basic flow only varies slowly in the streamwise direction, the local theory is the correct leading order approximation in a WKB theory that allows weak nonparallel effects to be included. In this case global stability properties can be inferred from local stability properties.

Global instability is related to local absolute instability, which describes the growth or decay of disturbances in the rest frame. Consideration of the upstream and downstream responses to periodic forcing at a point in a weakly inhomogeneous flow leads to the conclusion that the global mode is given at leading order by the absolute instability at a saddle point where $\partial\omega_0/\partial x = 0$, where ω_0 is the local absolute frequency and x the streamwise coordinate (this saddle is a turning point in the WKB theory), see [1], [2] and the review by [3]. If $\mathrm{Re}(\omega_0)$ does not depend on x, i.e. if there is no detuning, then the saddle $\partial\omega_0/\partial x = 0$ lies on the real x-axis, and the intuitively reasonable expectation that the x station with greatest absolute growth rate should act as the wave-maker for the flow holds. Less obvious is the fact that when there is detuning the saddle will lie at a complex value of x and then the global instability given by ω_0 at the saddle will have smaller growth rate than that of the most unstable absolute instability on the real x-axis.

2 Global instability in semi-infinite domains

[4] considered global instability in a semi-infinite domain in which the absolute growth rate decreases with downstream distance, as a model for wake flow, and they found that the global mode was given by ω_0 at the upstream boundary. [5] considered global instability in a domain in which the absolute growth rate increases with downstream distance, but the detuning effect is strong enough to maintain global stability. [6] argued that this global stabilization by detuning explained the global stability observed in the numerical solutions of the linearized Navier-Stokes equations for disturbances to the rotating-disc boundary-layer obtained in [7], and observed in experiments on the same flow by [8]. However, [5] showed that imposing a downstream boundary condition in this type of flow in a region of local absolute instability can create a global instability driven by the absolute instability at the downstream boundary. In rotating-disc flow the radial coordinate is the relevant streamwise coordinate, suggesting that the edge of the disc could be crucial in creating global instability.

3 Global instability in semi-infinite domains joined by a junction

The present work is motivated by these findings. They encourage us to explore the local stability properies near the edge of the disc in more detail. In particular, we seek to form a global mode in which the disturbance solution in the flow *over* the disc is matched to the disturbance solution in the flow *beyond* the edge of the disc.

Existing experiments on the stability of the rotating disc have not included measurements of velocity profiles for the flow beyond the edge of the disc. [9] used interactive boundary-layer theory to calculate the flow separating from the edge of a thin disc when there is von Kármán flow [10] over both the upper and lower surfaces of the disc. However, most stability experiments use a thick disc to reduce the effects of vibration, and the substantial mountings and motor arrangements below the disc are likely to prevent von Kármán flow from being established on the lower surface. Although a detailed study of the flow beyond the edge of a rotating disc used in a stability experiment would be desirable, in the present work we take a similar approach to that taken in [5], and explore instead the qualitative phenomena that arise when one considers the global instability of flow across a junction using the Ginzburg-Landau equation. Nonetheless, the stability and propagation properties of the flows on either side of the junction are taken from stability calculations for a simplified model of von Kármán flow over the disc, and for two simplified, but plausible, flows that could exist beyond the edge of the disc.

4 Results

Our most important finding is that in some circumstances the global mode can be more unstable than the local absolute instability on either side of the junction. For example, a stable flow upstream of a junction connecting to a downstream flow that is only convectively unstable can nonetheless be globally unstable, despite the absence of any local absolute instability. This arrangement can arise in a wide variety of practical applications, not just at the edge of a rotating disc, e.g. when stable flow in a nozzle becomes a convectively unstable jet flow after leaving the nozzle. The nozzle then acts as a junction between a channel (or pipe) flow and the jet. Another example is when a boundary layer separates from a surface: the separation point then acts as a junction between (possibly) stable attached flow, and convectively unstable separated flow. This applies both to the case of separation from a sharp edge and separation from a smooth surface. The origins of this unexpected behaviour will be explained.

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