A unified criterion for the centrifugal instability of vortices and swirling jets

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A UNIFIED CRITERION FOR THE CENTRIFUGAL INSTABILITY OF VORTICES AND SWIRLING JETS

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It is well known that swirling jets can become centrifugally unstable like pure vortices but with a different azimuthal wavenumber selection. The Leibovich and Stewartson [1] criterion is a generalization of the Rayleigh criterion to swirling jets: it is a sufficient condition for instability with respect to perturbations with both large axial and azimuthal wavenumbers. We have relaxed the large azimuthal wavenumber assumption in this criterion and obtained a new criterion that is valid whatever the azimuthal wavenumber and whatever the magnitude of the axial flow: from zero (pure vortex) to finite values (swirling jets). The new criterion recovers the Leibovich-Stewartson criterion when the azimuthal wavenumber is large and the Rayleigh criterion when the azimuthal wavenumber is finite [2]. The criterion is confirmed by comparisons with numerical stability analyses of various classes of swirling jet profiles. In the case of the Batchelor vortex, it provides more accurate results for perturbations with finite azimuthal wavenumbers than the Leibovich-Stewartson criterion (figure 1). The criterion shows also that a whole range of azimuthal wavenumbers are destabilized as soon as a non-zero axial velocity component is present in a centrifugally unstable vortex.

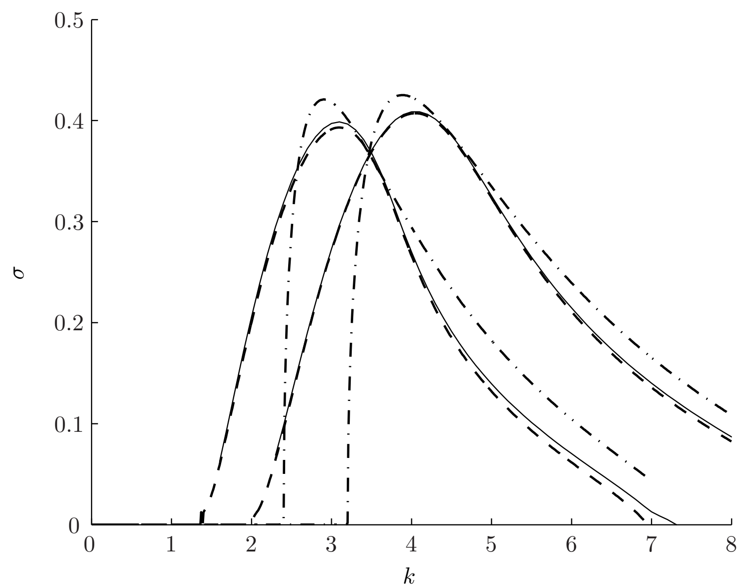

Figure 1: Growth rate as a function of the axial wavenumber k for the Batchelor vortex for the swirling parameter $q = 0.8$ and the azimuthal wavenumbers $m = -6$ and $m = -8$ (from left to right). Solid line: exact result, dashed line: present criterion, dashed-dotted line: Leibovich & Stewartson criterion.

References

[1] S. Leibovich & K. Stewartson A sufficient condition for the instability of columnar vortices. *J. Fluid Mech.* **126**, 335–356 (1983).

[2] P. Billant & F. Gallaire Generalized rayleigh criterion for non-axisymmetric centrifugal instabilities. *J. Fluid Mech.* **542**, 365–379 (2005).