Inertia-gravity waves during the transition towards geostrophic turbulence within a baroclinic cavity

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INERTIA-GRAVITY WAVES DURING THE TRANSITION TOWARDS GEOSTROPHIC TURBULENCE WITHIN A BAROCLINIC CAVITY

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1 Introduction

The transition to disordered behaviour in the form of 'Baroclinic Chaos' provides an important prototypical form of chaotic transition in fluid dynamics. This is of particular geophysical relevance in the context of understanding the origins of chaotic behaviour and limited predictability in the large-scale atmospheres of terrestrial planets and in the oceans. For many years, the differentially-heated, rotating cylindrical annulus, "the baroclinic cavity", has proved a fruitful means of studying fully-developed, nonlinear baroclinic instability in the laboratory. Transitions within the regular wave regime follow canonical bifurcations to low-dimensional chaos [1], but disordered flow appears to emerge via a different mechanism involving small-scale secondary instabilities [2]. In previous laboratory experiments of baroclinic waves, such fluctuations have been associated with a flow regime termed *Structural Vacillation* which is regarded as the first step in the transition to fully-developed geostrophic turbulence. Subsequent development within this so-called 'transition zone' leads to the gradual and progressive breakdown of the initially regular wave pattern into an increasingly disordered flow, ultimately leading to a form of stratified 'geostrophic turbulence'. Here we present an analysis which focusses on the small-scale features.

2 The numerical model and the case discussed

The physical model is that of the experimental rig used at the university of Oxford, UK ([3], [4]), filled with a liquid defined by a Prandtl number $Pr = 16$. It is composed of two vertical coaxial cylinders of inner radius $a = 4.5cm$ and outer radius $b = 15.cm$, held at constant temperature difference, $\Delta T = T_b - T_a = 2K$ and enclosed by two horizontal insulating rigid lids separated by a distance $d = 26.cm$. The whole cavity rotates around the central axis with $\Omega = 1.25rad/s$. The Navier-Stokes and energy equations coupled via the Boussinesq approximation are solved using a pseudo-spectral collocation-Chebyshev Fourier method associated with a second order time scheme.

3 Results

3.1 Structural Vacillation

The results are consistent with the structural vacillation observed in the laboratory, characterized by a spatio-temporal behaviour. While the large scale of the flow persists, here with a dominant azimuthal wavenumber $m = 3$, the flow does show noticeable irregular fluctuations at small scales which are going to progressively destroy the regularity of these main cells. This is illustrated by the streaklines shown at different heights of the cavity in figure 1.

3.2 Inertia gravity waves

Unlike a previous study using air as working fluid [2] where the driven mechanism resulted from a radial Rayleigh-Bénard rotating convection due to high centrifugal accelerations, the dynamics of the observed structural vacillation is mainly governed by inertia-gravity waves developing on top of large-scale baroclinic instabilities [5]. The small-scale structures appear spontaneously as soon as the large-scale regular waves occur, and indeed satisfy the characteristic dispersion relation given in [6]. These fluctuations are found to behave like the travelling waves emitted by thermal boundary layer instability determined as inertia-gravity waves for the value of Prandtl number considered here, according to analytical studies [7]-[8]. Moreover, a recent similar numerical study carried out at the university of Oxford UK [9], but considering a higher Prandtl number fluid, confirmed that these small-scale fluctuations correspond to inertia-gravity waves. A space-time map of the temperature along the azimuth at fixed radius and height during about three fundamental periods of the large-scale motion is displayed in figure 2, showing the perturbations attached to the $m = 3$ large-scale structure.

Figure 1: Instantaneous streaklines at different heights of the cavity

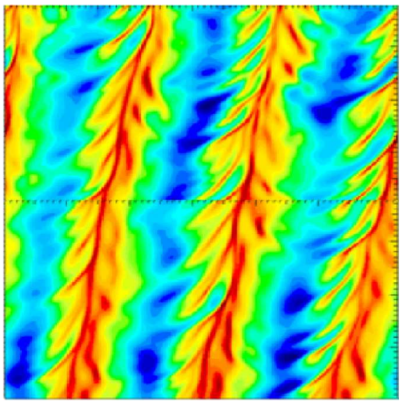

Figure 2: Space-time map of the temperature along the azimuth at fixed radius and height.

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