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Marc Buffat, Lionel Le Penven, Anne Cadiou, Julien Montagnier

BY-PASS TRANSITION DESCRIPTION USING AN ORTHOGONAL DECOMPOSITION OF THE VELOCITY FIELD

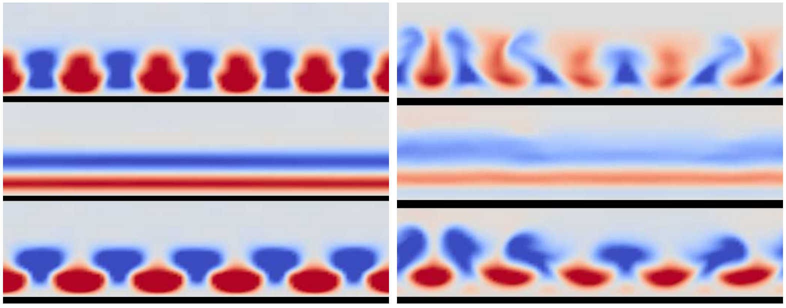
Marc BUFFAT, Lionel LE PENVEN, Anne CADIOU & Julien MONTAGNIER
Laboratoire de Mécanique des Fluides et d'Acoustique, Université de Lyon,
Université Claude-Bernard Lyon1/ CNRS/ Ecole Centrale de Lyon/ Insa de Lyon

In the last two decades, theory and computer simulations have led to significant advances in the understanding of boundary layer bypass transition induced by free-stream disturbances [5]. It has been shown that transition can be initiated by the growth of velocity perturbations taking the form of streaks elongated in the streamwise direction and modulated in the spanwise direction. As the streaks grow downstream, they become susceptible to high-frequency secondary instability and breakdown to turbulence finally occurs. The physical process explaining the emergence of streaks is known as the lift-up effect and is the result of interactions between streamwise vorticity and the boundary layer shear. Streaks can be generated by perturbations inside the boundary layer or by free-stream turbulence. The secondary instability can be initiated by free-stream turbulence, by selecting a high-frequency perturbation mode [8] or by streak interactions [2]. Numerical analysis of bypass transition can be found in [6], [7]. The main objective here is to study by numerical simulations (and to describe) boundary layer bypass transition between two parallel walls using an orthogonal decomposition of the velocity into two solenoidal fields.

The numerical simulations are performed using a spectral Galerkin projection method based on an orthogonal decomposition of the velocity into two solenoidal vector fields [3]. Since, by construction, the two orthogonal vector fields of the decomposition are associated with the Orr-Sommerfeld and the Squire modes of the linear stability theory respectively, the method makes it possible to evaluate kinetic energy transfers due to the coupling between these two modes and their interactions with the base flow. The code, developed in our team, is parallelized by an hybrid OpenMP/MPI approach and runs efficiently on ten thousand CPU cores. The studied problem is a plane channel flow with thin boundary layers and the computational domain is supposed to be far enough from the inlet section such that the potential effect of the leading edges can be neglected.

Different approaches can be used to select the disturbance that generates the streaks. Using the variational approach of [4], the optimal perturbation which sustains maximum temporal growth in the linear regime can be determined. This optimal perturbation consists of a spanwise wave with a streamwise wavenumber $\beta \approx 2/\delta$ inside the boundary layer and is zero outside the boundary layer. Such an optimal perturbation can be used to model disturbances inside the boundary layer. Free-stream turbulence can be described as a superposition of modes of the continuous spectrum of the Orr-Sommerfeld-Squire equations. To characterise the modes that generate streaks, Zaki et al [8] studied the forced Squire response to an Orr-Sommerfeld eigenfunction. They defined a coupling coefficient between an Orr-Sommerfeld eigenfunction and the Squire modes to characterize their interaction and the ability of the Orr-Sommerfeld mode to generate streaks.

To model the generation of streaks by free-stream turbulence, we follow a similar methodology adopted by [8], and study the transient response of the linearised equations to an Orr-Sommerfeld mode, chosen as initial condition. Using the same orthogonal decomposition of the velocity field as in the DNS code, the linear system of two equations is obtained using a weak formulation, and is equivalent to the classical Orr-Sommerfeld-Squire equations. Instead of using the two classical scalar fields: the normal velocity v and the normal vorticity η, this formulation allows us to analyse the solution in terms of two orthogonal velocity fields: the Orr-Sommerfeld (OS) $\boldsymbol{u}_{os}(v)$ and the Squire (SQ) $\boldsymbol{u}_{sq}(\eta)$ velocities, associated with two orthogonal velocity spaces W_{os} and W_{sq}. In our numerical approach, the eigenvectors are splitted into two distinct sets, depending on their orthogonality with the subspace W_{os}: the set $\{\boldsymbol{u}^-\} \subset W_{sq}$, associated with the Squire modes of the homogeneous Squire equation, and the complementary set $\{\boldsymbol{u}^+\}$. The projection $\boldsymbol{u}^+_{os}$ of $\boldsymbol{u}^+$ in W_{os} is associated with an Orr-Sommerfeld eigenmode and its projection $\boldsymbol{u}^+_{sq}$ in W_{sq} with the coupled Squire eigenmode. The ratio $r_\perp = ||\boldsymbol{u}^+_{sq}||/||\boldsymbol{u}^+_{os}||$ of these two projections characterize the non-orthogonality of an eigenvector $\boldsymbol{u}^+$ with the set $\{\boldsymbol{u}^-\}$, a large value of $r_\perp$ indicating an important nonorthogonality (non normality). By analysing the transient response with an initial condition equal to $\boldsymbol{u}^+_{os}$, we found that the most amplified $\boldsymbol{u}^+_{os}$ are associated to eigenvectors $\boldsymbol{u}^+_r$ with a normal to the wall wavelength (outside the boundary layer) of the order of 2δ. These eigenvectors $\boldsymbol{u}^+_r$ have large value of $r_\perp$, indicating an important non-orthoganlity with $\{\boldsymbol{u}^-\}$. An optimal problem can be solved for this transient problem, and the optimal initial condition is similar to the optimal perturbation calculated with the optimal growth theory. This optimal perturbation consists mainly of an OS velocity (streamwise vortices inside the boundary layer) at the initial time, transformed after the transient time into a streamwise SQ velocity. However, as this optimal perturbation is zero ouside the boundary layer, it cannot be chosen as a model of free-stream turbulence. By analysing the expansion of this optimal perturbation in the eigenvectors basis, the optimal modes are found to be a combination of eigenmodes around the eigenvectors $\boldsymbol{u}^+_r$ having the maximum



(a) $x = 4h$ (b) $x = 6.9h$

Figure 1: Instantaneous contours of streamwise velocity perturbation displayed using the orthogonal decomposition in two cross sections: (a) in the streaks region and (b) near the turbulent burst. From top to bottom: SQ velocity, OS velocity, total perturbation.

value of $r_\perp$. A remarkable finding is that the shape of the transient response is nearly identical for an initial condition equal to the optimal initial condition (or the optimal mode) and for an initial condition equal to the W_{os} projection of modes $\boldsymbol{u}_r^+$, and that only the amplitude of the transient response depends on the particular initial condition. For all these initial conditions, the transient response corresponds to SQ velocity or streaks characterised by a large peak of the streamwise velocity component inside the boundary layer. Based on this analysis, we use henceforth the ratio $r_\perp$ to select the inlet perturbation in our simulations.

In the simulations, the inlet boundary layers are thin compared to the distance $2h$ between the two parallel walls ($\delta/h = 0.05$). The inlet Reynolds number, based on the displacement thickness, is equal to 344 (corresponding to $Re_x = 40000$). The first perturbation is chosen as an oblique wave u_{os}^+ with $\beta_\delta = 2$ and $\alpha_\delta = 0.04$ having the largest value of $r_\perp$.To initiate the transition of the streaks, a second perturbation with a larger streamwise wavenumber $\alpha_\delta \approx 1$ and a smaller spanwise wavenumber $\beta_\delta \approx 0.3$ is introduced at the inlet as in [8]. This second perturbation is a vector $\boldsymbol{u}^+$ having a small value of $r_\perp$ Without external perturbations, the flow remains laminar and the classical Blasius skin friction coefficient is recovered. Using the two previous perturbations, by-pass transition is induced in the boundary layers. As the boundary layer thickness is small compared to the channel height, similar results are obtained to those presented by [8] and [7] for a boundary layer over a flat plate. Using the orthogonal decomposition of the velocity, specific information about the structure of streaks of finite length can be obtained (see Figure 1). Specifically, the streaks are found as the sum of two orthogonal contributions, one part is the SQ velocity field, which is a streamwise oriented contribution representing 60% of the total kinetic energy, the other part is the OS velocity field which is a spanwise invariant term with 40% of the kinetic energy. In the early steps of the transition, the secondary instability affects mainly the SQ streamwise velocity, whereas the OS streamwise velocity remains almost unaffected. In the transition region, strong oscillations of the streaks are observed in the spanwise direction. In that region, the plot of the streamwise perturbation (lower plot in Figure 1b) clearly shows spanwise oscillations of the low speed streaks on top of the boundary layer. Figure 1b shows that the instability mainly affects the SQ streamwise velocity, whereas the OS streamwise velocity remains almost unaffected. At the early stage, before breakdown, this instability occurs through perturbations of the SQ streamwise velocity and looks very similar to the transverse instabilities studied by [1].

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