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ALGORITMO MECANÍSTICO SIMPLIFICADO PARA LA CUANTIFICACIÓN DEL EFECTO DE UN ACUÍFERO SOMERO SOBRE LA INFILTRACIÓN Y LA ESCORRENTÍA

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RESUMEN. La correcta cuantificación de los procesos hidrológicos requiere una descripción precisa de la infiltración del suelo a través de la zona no saturada. Modelos físicos como la ecuación Richards, basados en la aplicación de los principios de flujo en medio poroso y conservación de masa, pueden proporcionar soluciones exactas en medios ideales. Sin embargo, su aplicación a suelos naturales en condiciones dinámicas resulta a menudo costosa y en ocasiones problemática. Se han propuesto muchos métodos simplificados de cálculo de la infiltración, desde los empíricos cuya aplicación es limitada, hasta los de base física y más generales. Éstos últimos, por su generalidad y relativa rapidez y robustez de cálculo, a menudo representan una alternativa efectiva para el cálculo de la infiltración en modelos hidrológicos integrados complejos. La mayoría de estos métodos simplificados consideran infiltración en suelos profundos sin la presencia de un nivel freático somero. Esta presencia puede tener un importante efecto limitante sobre la infiltración en el suelo de gran consecuencia no sólo en la zona no saturada sino sobre los procesos superficiales de la escorrentía y el transporte de contaminantes en superficie hacia cauces y sistemas lacustres. En este trabajo se estudia la extensión de la ecuación de Green-Ampt para la infiltración de agua en presencia de un nivel freático somero estable bajo condiciones de lluvia natural, y su efecto sobre la redistribución de humedad en el suelo y la generación de exceso de lluvia (volumen de escorrentía superficial). Para ello, se propone la solución de forma temporalmente explícita de la ecuación integral implícita de Salvucci y Entekhabi (1995) para suelos encharcados, y su combinación con los conceptos de tiempo de encharcamiento del suelo y balance de agua durante un hietograma de lluvia natural. El modelo se contrasta favorablemente con la solución numérica de Richards para un rango representativo de suelos. Se resalta además la importancia que la descripción formal de las curvas características del suelo, y en especial del punto de burbujeo, tiene sobre las predicciones realizadas.

ABSTRACT. Salvucci and Enthekabi (1995) proposed an approximate, time-implicit integral solution to the infiltration case for ponded soils bounded by a water table (i.e. under a hydrostatic equilibrium bottom boundary condition). In this work, we further developed this solution to make it numerically explicit in time and to account for unsteady rainfall conditions.. The new infiltration component was validated against a numerical solution of Richards'equation. In general, the presence of a shallow water table was found to be more important for fine soils than coarse soils. Soils that exhibit a marked air entry (bubbling pressure) on their soil water characteristic curve introduce also a distinct behavior for very shallow water table depths.

1.- Introduction

Although Richards (1931) partial differential equation (RE) represents a fundamental equation to describe water infiltration and redistribution in soils, it does not have a general analytical solution and therefore must be solved numerically in many practical applications. The numerical solution of the equation can be computationally intensive and in some cases (coarse soils and highly dynamic boundary conditions) lead to mass-balance and instability errors (Celia et al., 1990; Paniconi and Putti, 1994; Miller et al., 1998; Vogel et al., 2001; Seibert 2003). Simpler, approximate physically based approaches have often been used for modeling soil infiltration (Jury and Horton, 2004; Singh and Woolhiser, 2002; Haan et al., 1993; Smith et al., 1993). In particular, the method of Green and Ampt (1911), modified for unsteady rain events (Mein and Larson, 1973; Chu, 1978), has been widely used in hydrologic modeling. Despite Green–Ampt's apparent limitations (assumptions of rectangular saturated piston flow and homogeneous isotropic soil with uniform initial content), the method produces good results in comparison with other approximate and numerical methods if it is effectively parameterized (Skaggs et al., 1969). In addition, Green–Ampt has the advantage that its parameters can be estimated directly based on physical measurements, or indirectly from soil textural classification (Rawls et al., 1982, 1983). Based on these advantages, Bouwer (1969) presented a case for the usefulness of the method when considering the trade-offs against the use of Richards numerical solution.
The Green-Ampt model has been extended beyond its initial assumptions to make it more applicable to real infiltration cases. In particular, the presence of a shallow water table can significantly influence the initial soil water content distribution from the Green-Ampt homogenous assumptions, and limit infiltration. As noted by Chu (1997), many authors (Childs, 1960; Duke, 1972; Holmes and Colville, 1970) have proposed an equilibrium hydrostatic condition to describe the soil water distribution above a shallow water table (Fig. 1).

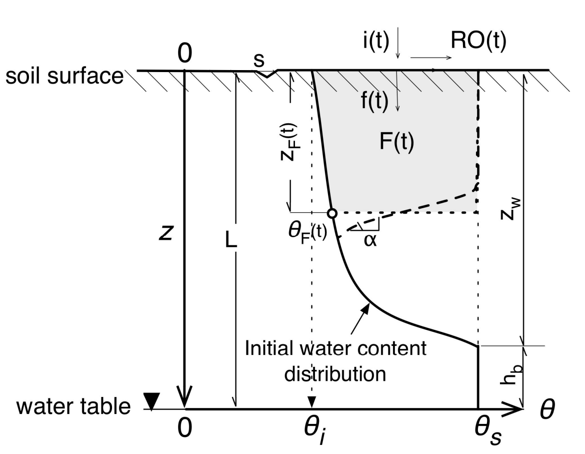

Fig. 1. Infiltration and reditribution hypothesis for soils with shallow water table in hydrostatic equilibrium/ Hipótesis de infiltración y distribución de humedad

This represents a linear relationship between the soil depth (z, [L]) and the soil matric potential (h, [L]), so that the soil water content of the soil (θ) is described by the soil water characteristic $\theta=\theta(h)$ (Jury, 1991) as,

$$h = L - z \Rightarrow \theta = \theta(L - z) \tag{1}$$

where L [L] is the shallow water table depth (distance from the surface).

Using hydrostatic equilibrium assumptions and Bouwer's (1969) implicit Green-Ampt formula for infiltration into non-uniform soil, Chu (1997) proposed an incremental infiltration calculation procedure for ponded soils with a shallow water table as,

$$t = \int_0^z \frac{\theta_s - \theta(L - z)}{q} \mathrm{d}z \tag{2}$$

with t [T] the time, θ_s [L^3L^{-3}] the saturation water content, θ [L^3L^{-3}] the soil water content (non uniform, function of the depth), q [L^3T^{-1}] the infiltration rate at surface, z [L] the depth of the wetting front, L [L] the depth of the water table. The cumulative infiltration at wetting front depth z can be obtained by integrating the profile to the depth of the wetting front (*Neuman*, 1976),

$$F = \int_0^z \theta_s - \theta(L - z)\mathrm{d}z \tag{3}$$

and the infiltration rate is obtained by integration of the hydraulic conductivity function K(h),

$$q = K_s + \frac{1}{z}\int_0^{L-z} K(h)\mathrm{d}h \tag{4}$$

Chu (1997) proposed assuming hydrostatic equilibrium during an infinitesimal time step $dt = t_i - t_{i-1}$. Using Eq. 1-3 the time can then be calculated explicitly at given increments of z,

$$t_i = t_{i-1} + dt = t_{i-1} + \frac{F_i - F_{i-1}}{0.5(q_i + q_{i-1})} \tag{5}$$

An initial value for F_1 and q_1 is obtained by assuming standard Green-Ampt conditions from the surface to the first dz, (piston flow), the value of moisture content at the surface (θ_i) and suction at the wetting front (S_{av}) calculated as (Bouwer, 1964),

$$S_{av} = \frac{1}{K_s}\int_0^L K(h)dh \tag{6}$$

The model was tested successfully against experimental data from Vachaud et al., (1974), although the experimental data and simulated results were not allowed sufficient time to test the model response when the wetting front arrives to the water table depth.

Salvucci and Entekhabi (1995) proposed an elegant approximate solution for infiltration under ponded and water table conditions for the Brooks and Corey (1964) representation of soil hydraulic properties (see Appendix). The solution uses Philip's (1957) approximation of Richards equation and provides the additional benefit of describing the soil water profile dynamics during the infiltration event and also de behaviour of the wetting front as it approaches the water table for long infiltration events. The solution estimates the slope of the wetting front during the infiltration process (α in Fig. 1) as a function of the wetting front position in time. The algorithm was tested succesfully against Richards numerical solution for three distinct soils, and against the experimental soil moisture profile data. However, the method relies on the particular Brooks and Corey description of soil water characteristics, considers only ponded conditions during the event, and like the original Green-Ampt equation and Chu (1997) method, the solution is implicit. This represents a limitation for coupling with common hydrological models with time-driven calculations of the water mass balance and fluxes during prescribed time steps.

The objective of this paper is to extend the practical applicability of the Green-Ampt based solution to non-ponding, unsteady rainfall infiltration in soils bounded with a shallow water table. This will be accomplished by proposing a generic (independent of the soil characteristic description) time-explicit solution procedure that considers the dynamics of the saturated wetting front as it approaches the water table, and handles non-ponded conditions during an unsteady rainfall event. New integral formulas for calculation of the singular times needed for the time-explicit solution for non-ponded soils are presented with the calculation procedure. The simplified method is tested against RE solution for accuracy of surface infiltration and water content predictions, and against previously published experimental data using a variety of soil water characteristics.

2.- Proposed explicit algorithm

2.1. Infiltration component

In general the infiltration rate in a soil bounded by a shallow water table with no ponding at the beginning of the event will present a profile similar to that depicted in Fig. 2a. For an explicit, time-based solution, two singular times need to be identified during the infiltration calculations: a) the time to reach ponding (t_p); and b) the time to column saturation (t_w) when the wetting front reaches the saturated (capillary fringe) region above the water table at depth z_w (Fig. 1).

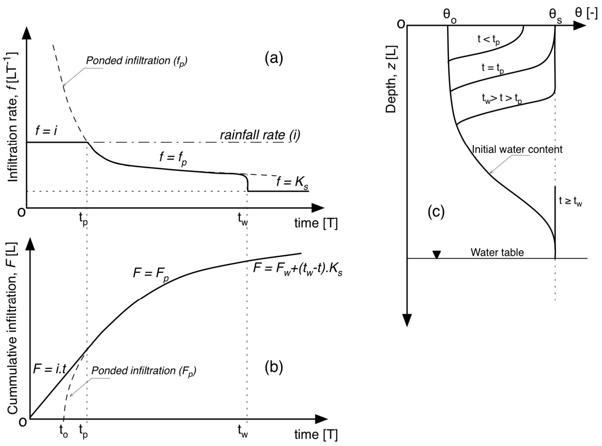

Fig. 2. Conceptual description of infiltration and water distribution for a soil bounded by a shallow water table and non-ponded initial conditions and rainfall rate (i).

The effective depth of saturation z_w depends on the position of water table depth (L) and the soil saturated capillary tension head or bubbling pressure (h_b), $z_w=L-h_b$ (Fig. 1). Note that h_b is usually set at 0 for soil characteristics other than Brooks and Corey functions (*i.e.* $z_w=L$). At t_w the soil column saturates and the infiltration rate abruptly drops to match the saturated hydraulic conductivity (Fig. 2a).

The time t_w also depends on the slope of the hydraulic conductivity function (Salvucci and Entekhabi, 1995). Thus, the infiltration rate (f) in the soil bounded by a shallow water table under a constant rainfall rate (i [LT^{-1}]) and non-ponded initial conditions is described by (Fig. 2a),

$$\left|\begin{array}{ll} f = i & ;0 < t \le t_p \\ f = f_p & ;t_p < t < t_w \\ f = K_s & ;t \ge t_w \end{array}\right. \qquad (7)$$

Following Mein and Larson (1973), t_p is the time when the rainfall rate (i) equals the soil infiltration capacity (f_p = i) (Fig. 2a), and corresponds to the time when the moisture content in the surface equals saturation (Fig. 2c). By definition the time to ponding t_p is the intersect of the two curves in Fig. 2a, and the equivalent wetting front depth at the ponding time (z_p),

$$z_p = \frac{1}{i-K_s}\int_0^{L-z_p} K(h)\mathrm{d}h \qquad (8)$$

Since the equation (8) is implicit in z_p, it can be solved in terms of time by defining the function G_p: : □→□ and its derivative dG_p/dz,

$$\begin{aligned} G_p(z_p) &= z_p - \frac{1}{i-K_s}\int_0^{L-z_p} K(h)\mathrm{d}h \\ \frac{dG_p(z_p)}{\mathrm{d}z} &= 1 + \frac{K_s}{i-K_s}\frac{K(L-z_p)}{K_s} \end{aligned} \qquad (9)$$

so that the root z_p of G_p =0 is the wetting front depth at time to ponding t_p. This minimization problem can be solved for example using a bracketed Newton-Raphson algorithm (Abramowitz and Stegun, 1972) that needs the expression of the function and its derivative to solve the problem by iteration for $z_p \in [0,z_w]$. Denoting k as the Newton-Raphson iteration level we obtain,

$$z_p^{k+1} = z_p^k - \frac{G_p(z_p^k)}{\frac{dG_p(z_p^k)}{dz}} \quad with \quad \left|z_p^{k+1} - z_p^k\right| < \varepsilon \qquad (10)$$

with ε the error tolerance for the Newton-Raphson algorithm, set to 10^{-8} here. Based on Fig. 2b, the cumulative infiltration at $t=t_p$ ($z=z_p$) is $F_p= i\cdot t_p$, and using eq. (3) for cumulative infiltration we finally obtain t_p as,

$$t_p = \frac{1}{i}\left(\theta_s z_p + \int_L^{L-z_p}\theta(h)\mathrm{d}h\right) \qquad (11)$$

Next, when the two lines intercept at t_p the potential cumulative infiltration curve yields F_p= 0 at time t-t_p (dashed line in Fig. 2b) and a shift time (t_o) (abscissa translation) must be applied to $\boldsymbol{F_p}$ so that the function computed at t_p equals $F_p= i\cdot t_p$ (*Mein and Larson*,1973). The time t_o is calculated explicitly by setting the condition $z=z_p$ on the infiltration eq. (2),

$$t_0 = \int_0^{z_p}\frac{\theta_s - \theta(L-z)}{f(z)}\mathrm{d}z \qquad (12)$$

Finally, the time to reach column saturation (t_w) while accounting for non-ponded initial condition is calculated by evaluating the integral eq. (2) at $z=z_w=L$-h_b (Fig. 1) and correcting for t_p and t_o,

$$t_w = t_p - t_0 + \int_0^{z_w}\frac{\theta_s - \theta(L-z)}{f(z)} \qquad (13)$$

From eq. (3) the cumulative infiltration at t_w is calculated as,

$$F_w = \theta_s z_w - \int_{h_b}^{L}\theta(h)dh \qquad (14)$$

Liu et al. (2011) also considered a non-dimensional time equivalent to t_w (X_c) to correct the piston flow approximation of Green-Ampt when the wetting front is close to the water table. However, in Liu et al. (2011) X_c is estimated through an empirical equation based on the comparison between Green-Ampt's standard uniform initial moisture equation and RE solution.

2.3 Infiltration capacity algorithm after surface ponding

To simplify the explicit solution of Salvucci and Entekhabi (1995) infiltration capacity equation under shallow water table conditions, we propose to set the slope of the wetting front to zero, *i.e.* a horizontal front (α=0) at depth z_F (Fig.1). As pointed by Salvucci and Entekhabi (1995), this effectively collapses their solution to make it similar to Eq. (2), used by Bouwer (1969) in his discussion on the applicability of Green-Ampt. When considering initial non-ponding conditions, the implicit infiltration capacity equation becomes,

$$t = t_p - t_0 + \int_0^{z_F} \frac{\theta_s - \theta(L-z)}{K_s + \frac{1}{z}\int_0^{L-z} K(h)\mathrm{d}h} \quad ;t_p < t < t_w \qquad (15)$$

Note that the deeper the wetting front moves into the soil profile, the larger α (steeper front slope) could be, depending of the soil type (more for fine soils). However, as the front nears the shallow water table, the available pore space is small, thus limiting the mass error in the calculations (Salvucci and Entekhabi, 1995). The validity of this assumption will be tested in the next section.

In order to solve equation (15) numerically explicit in time (*i.e.*, $z=z(t)$), we will define the function G: □ x□→□ and its derivative dG/dz, so that the root $z\in[z_{i-1}, z_w]$ for G=0 is the wetting front depth for a given time t,

$$G(z,t) = t - t_p + t_0 - \int_0^z \frac{\theta_s - \theta(L-z)}{K_s + \frac{1}{z}\int_0^{L-z} K(h)\mathrm{d}h}\mathrm{d}z \qquad z^{k+1} = z^k - \frac{G(z^k,t)}{\frac{dG(z^k,t)}{dz}} \qquad (16)$$

$$\frac{dG(z,t)}{dz} = -\frac{\theta_s - \theta(L-z)}{K_s + \frac{1}{z}\int_0^{L-z} K(h)\mathrm{d}h} \qquad \text{with} \quad \left|z^{k+1} - z^k\right| < \varepsilon$$

At each time step, the proposed algorithm calculates the depth of the wetting front, $z_{Fi}=z_i$ (Fig 1), which allows for the evaluation of the cumulative infiltration (eq. 3), and the infiltration rate (eq. 2). Note that the bracketing step in the Newton-Raphson algorithm is required because the function G is ill-behaved outside its physical range ($z_p<z<z_w$). Therefore, care should be taken to solve the equations within their prescribed boundaries. Although the proposed equations can be simplified by considering integrable soil hydraulic functions like Brooks and Corey (Salvucci and Entekhabi, 1995) or Gardner (1958), the generality of the method is preserved if numerical integration is used instead. In this study, we used 20-order Gauss-Quadrature Integration (Abramowitz and Stegun, 1972). The high order of the quadrature was needed to obtain accurate integral values due to the high non-linear and non-monotonic nature of the integrands of G and its derivative.

2.3. Unsteady rain infiltration without initial ponding for soils with a shallow water table

The runoff generated by infiltration excess (Hortonian overland flow) as well as the saturation excess overland flow (due to the presence of the shallow water table) are then calculated at each time step by the surface water balance (neglecting evaporation during the event) (Chu, 1978),

$$\Delta P = \Delta F + \Delta s + \Delta RO \qquad (17)$$

where ΔP, ΔF, Δs, and ΔRO [L] are the increments of cumulative precipitation (P), infiltration (F), surface storage ($0<s<s_{max}$) and excess rainfall (runoff volume, RO) for every time step. The surface storage term, when present ($s_{max}>0$), will act as a reservoir that needs to be filled ($s=s_{max}$) before runoff is produced (Chu, 1978; Skaggs and Khaleel, 1982). For unsteady storms described by a hyetograph with different rainfall intensities constant within each rainfall period j, i.e. $i=i_j$ for $t_j<t<t_{j+1}$, if surface storage becomes s=0 then t_p and t_0 will need to be recalculated for every new rainfall period. For rainfall periods other than the first one, i.e. t_j with j>1, the time to ponding will need to be adjusted to account for rainfall and excess rainfall happening before the current rainfall period (Chu, 1978) as,

$$t_p = \frac{1}{i_j}\left[\left(\theta_s z_p + \int_L^{L-z_p} \theta(h)\mathrm{d}h\right) - P(t_j) + RO(t_j)\right] + t_j \qquad (18)$$

Similarly, t_w has to be re-calculated each time t_p, and t_o are calculated. For the particular case when ponding is present from the beginning of the rain event, we have t_p= t_0=0. An illustrative unsteady calculation example is provided in the following section.

3- Numerical testing

The simplified algorithm was tested against RE solution on 3 soils (clay, silty loam, and sandy loam) representing a wide range of characteristics. The soils are described by the Brooks and Corey soil water characteristic and hydraulic conductivity curves (Table 1).

Table 1. Water Table Depth (L), Brooks and Corey parameters for the different soils and Nash and Sutcliffe coefficient of efficiency (C_{eff}) between the simplified model and Richards' finite differences results (CHEMFLO-2000).

Soil	L (m)	θ_r	θ_s	K_s ($\mathrm{m\cdot s^{-1}}$)	h_b (m)	λ	η	C_{eff} Richards
Silty Loam	1.5	0	0.35	3.40×10^{-6}	0.450	1.20	4.67	0.994
Clay	1.5	0	0.45	3.40×10^{-7}	0.900	0.44	7.54	0.999
Sandy Loam	1.5	0	0.25	3.40×10^{-5}	0.250	3.30	3.61	0.995

† h_b, λ, η are the Brooks and Corey parameters

The 3 soils contain a water table at 1.50m depth (Salvucci and Entekhabi, 1995). As numerical solution of RE we used the finite difference mass-conservative Celiat et al, 1990) CHEMFLO-2000 model (Nofziger and Wu, 2003). The soil initial condition corresponds to a linear matric potential distribution in hydrostatic equilibrium with a water table at the bottom of the soil (eq. 1). The boundary conditions were fixed matric potential on the bottom of the soil ($h(z=L)$=0) to represent the shallow water table, and on the surface a mixed type boundary condition with the flux density equal to the specified rainfall rate and the critical matric potential to represent the rainfall. The rainfall rates were selected based on the soil texture using a normalized ratio of i/K_s=6 for the fine soils (clay and silty loam), and i/K_s=2 for the other two coarser soils. This allowed for the development of distinct t_p and t_w values for testing.

Figure 3 presents the relative infiltration rate (f/K_s) comparison between RE (symbols) and the proposed algorithm (lines). The behavior in time is very close to the RE for all soils tested, with coefficients of efficiency (Nsh and Sutcliffe, 1970) (0.96>C_{eff} >0.9997, highest for clay). Salvucci and Entekhabi (1995) reported errors around 5% for the same clay soil for ponded conditions. This indicates that in spite of the simplification introduced here (α=0, horizontal wetting front), the integral solution (eq. 13) was able to predict infiltration rate well.

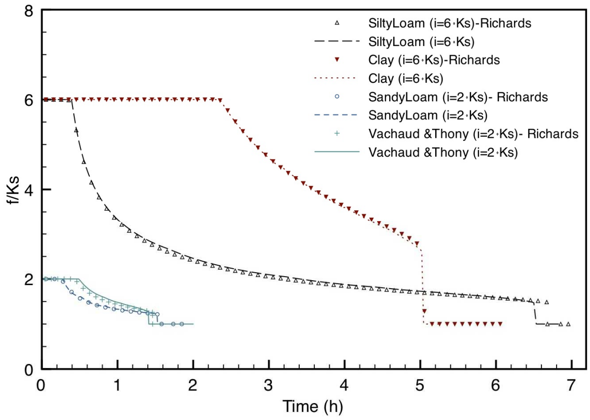

Fig. 3. Comparison of simplified model results (lines) against RE solution (symbols) for soils in Table 1 under non-ponding initial, conditions in the presence of shallow water table.

It is also interesting to notice the accurate time to ponding estimates obtained across the soils and normalized rainfall rates tested against RE solution. This limited testing indicates that the non-uniform integral equations (9-11) could effectively limit the t_p estimation issues found sometimes with the standard Green-Ampt formulation Barry et al., 1996).

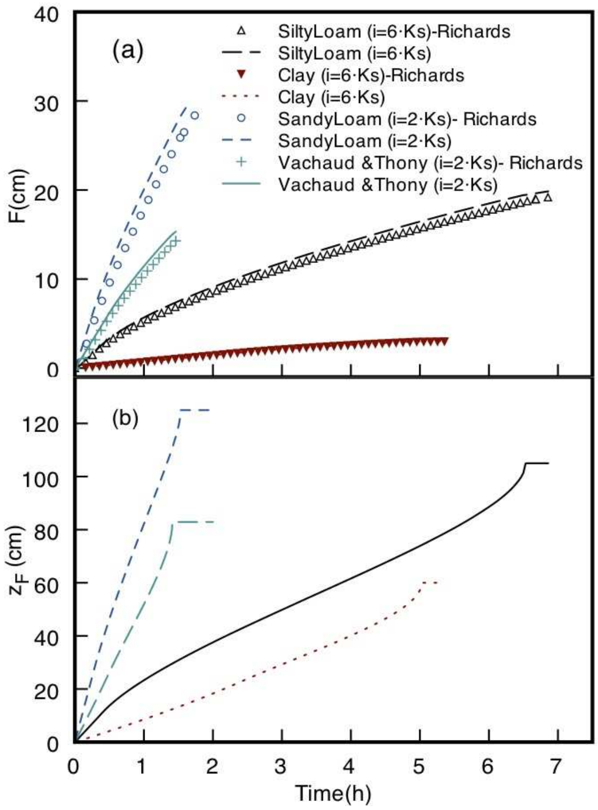

Fig. 4. Cumulative infiltration (F) and wetting front depth (z_F) calculated for soils in Table 1.

The cumulative infiltration of the different soils and the wetting front depth results calculated with eq. (19-20) for the cases in Table 1 are depicted in Fig. 4.

As with the infiltration rate curves, the values of the wetting front depth display a singularity (plateau) at t_w when reaching column saturation. Notice that this corresponds to a depth of $z_F = z_w = L - h_b$ (Fig. 1) and thus it is not the same as the water table depth (L=1.01 m for the fine sand and 1.50m for the rest).

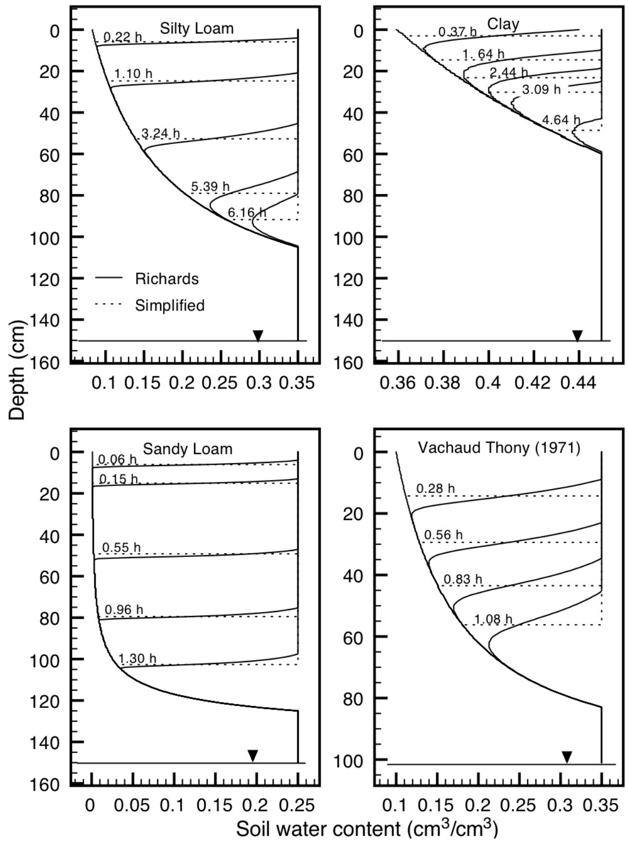

Fig. 5. Comparison of soil water content distribution predictions between RE solution (solid lines) and the simplified model (dashed lines) during infiltration under non-ponded initial conditions for soils in Table 1.

The ability of the simplified approach to predict the z_F allows also for the description of the soil water content redistribution during the infiltration event. Figure 5 quantifies the error in water content predictions by the simplified algorithm (dashed lines) against RE solution (solid lines) for the same non-ponding numerical testing cases. In general, the simplified model identifies well the depth of the wetting front mid point. In addition the horizontal wetting front (α=0) simplification produces accurate description of the wetting front position at early times for all soils, but degrades for later times near column saturation, except for the sandy soil that naturally exhibits near horizontal wetting front distribution. As discussed before, since the pore space is small near column saturation, the errors introduced by slopes different than 0 remain small. This can also be seen in the overall mass balance error, *i.e.* the cumulative infiltration at the end of the simulation (Fig. 4a), where errors are between 3 and 8%. These errors are deemed acceptable since they represent the summation of approximation errors (soil moisture distribution, infiltration rate) during the simulation.

4.- Conclusions

Analysis of shallow water table effects on soil infiltration and runoff is constrained by limitations in current modeling approaches. Simplified but realistic specialized algorithms coupled with existing hydrological models can help to assess the importance of ubiquitous shallow water table areas in the landscape. Previous work proposed Green-Ampt implicit integral formulae for infiltration into ponded soils by assuming initial soil hydrostatic equilibrium with the shallow water table. We tested a simplification of the existing method (assuming a horizontal front during the infiltration calculation) and developed a numerically explicit algorithm suitable for coupling with existing hydrological models. The proposed algorithm is generic because it handles any form of soil hydraulic functions and can be applied to ponded, non-ponded and unsteady rainfall conditions to calculate infiltration, excess rainfall (runoff) and soil water redistribution during the event.

The algorithm was tested against RE and the infiltration results showed good agreement with the reference numerical solution and data (*Ceff* from 0.91 to 0.99), with a good description of the soil water distribution during rainfall events with non-ponding initial conditions. The good predictions obtained indicate that the simplification introduced when considering a horizontal slope is acceptable for most practical applications. Limitations of the algorithm derive from its assumptions, in particular from the suitability of the homogeneous profile and horizontal wetting front for particular soils. Additional experimental testing of the model against measured data in natural conditions (especially with events long enough to saturate the soil by the bottom boundary) would also serve to ascertain the validity of the hydrostatic equilibrium assumptions and the proposed calculation of the singular points during the infiltration event.

Because the algorithm is fast, accurate and robust for the wide range of conditions tested, it is deemed suitable for coupling with existing hydrological models in order to evaluate the effect of a shallow water table areas on other landscape processes.

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6.- References

Abramowitz M, Stegun IA. Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables. National Bureau of Standards Applied Mathematics Series 55. 10th Printing;1972.

Barry DA, Parlange JY, Haverkamp R. Comment on « Variable-head ponded infiltration under constant or variable rainfall » by J. R. Philip. Water Resour. Res.

Bouwer H. Infiltration of water into nonuniform soil. J. Irrigat. Drain. Div. 1969;95(IR4):451–462.

Bouwer H. Unsaturated flow in ground-water hydraulics. Hydraul. Div. Amer. Soc. Civil Eng. 1964;90(HY5):121-144.

Brooks, RH, Corey AT. Hydraulic properties of porous media. Hydrol. Paper. 1964;3.

Celia MA, Bouloutas ET, Zarba RL. A general mass-conservative numerical solution for the unsaturated flow equation. Water Resour. Res. 1990;26(7):1483-1496.

Childs EC. The nonsteady state of the water table in drained land. J. Geophys. Res. 1960;65(2):780-782.

Chu ST. Infiltration during an unsteady rain. Water Resour. Res. 1978;14(3):461-466.

Chu ST. Infiltration model for soil profiles with a water table. Trans. ASAE. 1997;40(4):1041-1046.

Duke HR. Capillary properties of soils - influence upon specific yield. Trans. ASAE. 1972;15(4):688-691.

Gardner WR. Some steady-state solutions of the unsaturated moisture flow equation with application to evaporation from a water table. Soil Sci. 1958;85(4):228-232.

Green WH, Ampt GA. Studies on Soil Physics. J. Agr.Sci. 1911;4:1-24.

Haan CT, Barfield BJ, Hayes JC. Design hydrology and sedimentology for small catchments. San Diego, Calif.: Academic Press; 1994.

Holmes JW, Colville JS. Forest hydrology in a karstic region of Southern Australia. J. Hydrol. 1970;10(1):59-74.

Jury WA, Gardner WR, Gardner WH. Soil Physics, 5th Edition. John Wiley & Sons, New York;1991.

Jury, WA, Horton R. Soil Physics, 6th Edition. John Wiley & Sons, New York;2004.

Liu G, Craig JR, Soulis ED. Applicability of the Green-Ampt infiltration model with shallow boundary conditions. J. Hydrologic Eng. 2011;16:266.

Mein RG, Larson CL. Modeling infiltration during a steady rain. Water Resour. Res. 1973;9:384–394.

Miller CT, Williams GA, Kelley CT, Tocci MD. Robust solution of Richards' equation for nonuniform porous media. Water Resour. Res. 1998;34(10):2599–2610.

Nash JE, Sutcliffe JV. River flow forecasting through conceptual models, part I - A discussion of principles. J. Hydrol. 1970;10:282-290.

Neuman SP. Wetting front pressure head in the infiltration model of Green and Ampt. Water Resour. Res. 1976;12(3):564-566

Nofziger DL, Wu J. CHEMFLO-2000: Interactive software for simulating water and chemical movement in unsaturated soils. U.S.EPA, Cincinnati, Ohio: Rep. EPA/600/R-03/008. National Risk Management Research Laboratory Office of Research and Development; 2003.

Paniconi C, Putti M. A comparison of Picard and Newton iteration in the numerical solution of multidimensional variably saturated flow problems. Water Resour. Res. 1994;30(12):3357.

Philip JR. The theory of infiltration: 1. the infiltration equation and its solution. Soil Sci. 1957;83(5):345-358.

Rawls WJ, Brakensiek DL, Miller N. Green-Ampt infiltration parameters from soils data. J. Hydraul. Eng. 1983;109(1):62-70.

Rawls WJ, Brakensiek DL, Saxton KE. Estimation of soil water properties. Trans. ASABE. 1982;25(5):1316-1320.

Richards LA. Capillary conduction of liquids through porous mediums. Physics. 1931;1(5):318.

Salvucci GD, Entekhabi D. Ponded infiltration into soils bounded by a water table. Water Resour. Res. 1995;31:2751-2759.

Seibert J, Rodhe A, Bishop K. Simulating interactions between saturated and unsaturated storage in a conceptual runoff model. Hydrol. Process. 2003;17(2):379-390.

Singh VP, Woolhiser DA. Mathematical Modeling of Watershed Hydrology. J. Hydrologic Eng.. 2002;7(4):270-292.

Skaggs RW, Huggins LF, Monke EJ, Foster GR. Experimental evaluation of infiltration equations. Trans. ASAE. 1969;12(6):822-828.

Skaggs RW, Khaleel R. Infiltration in hydrologic modeling of small watersheds. ASAE Mon. St. Joseph, MI: Am. Soc. Agr. Engrs.; 1982.

Smith RE, Corradini C, Melone F. Modeling infiltration for multistorm runoff events. Water Resour. Res. 1993;29(1):133-144.

Vachaud G, Thony JL. Hysteresis during infiltration and redistribution in a soil column at different initial water contents. Water Resour. Res. 1971;7:PP. 111-127.

Vogel T, van Genuchten MT, Cislerova M. Effect of the shape of the soil hydraulic functions near saturation on variably-saturated flow predictions. Advances in Water Resources. 2001;24(2):133-144.

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