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A VIBROACOUSTIC ENERGY DENSITY FIELD APPROACH FOR AN AUTOMATIC SUB-STRUCTURING METHOD OF COMPLEX STRUCTURES IN LOW- AND MEDIUM-FREQUENCY RANGES.

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An energy density field approach for the vibroacoustic analysis of complex structures is proposed. This method is developed for an uncertain vibroacoustic computational model. An important result of using this method is the construction of an automatic sub-structuring method for complex structures which is applicable at low- and medium-frequency ranges. Though the proposed energy method is inspired from the Statistical Energy Analysis (SEA) method, it is quite different. Mainly, in the stochastic context in which the method is applied the statistical properties of the FRF were observed even in the low- and medium-frequency ranges which makes the method applicable in these frequency ranges.

In the used method random dimensionless FRF are obtained from the usual dimensional FRF projected on the local coordinates system defined by the principal direction of mean mobility. These FRF are shown to be less dispersed than the dimensional FRF and are independent of the direction of excitation. Based on these results a simplified model is constructed. This simplified model showed that, starting at about 70Hz, at degrees of freedom lying in geometrically close zones, the random dimensionless FRF have very close values. From which the idea of sub-structuring became evident. In order to perform sub-structuring and to better specify the low-frequency limit of our method, the theory of information is used. An application of this work on automotive vehicle structures is presented.

In a first part of the paper, we present the mean reduced computational model. In a second part we present a probabilistic modelling of uncertainties. The development of the energy method and the construction of the simplified model are presented in a third part. The sub-structuring method is presented in a fourth part. Finally, results and conclusions are presented.

1. Introduction

The vibroacoustic performance of industrial structures such as automotive vehicle structures has become an important choice criteria. In order to cope with such tendency, industrials aim to continually improve the conception process. In order to do so while keeping a strategy of cost reduction and robustness of the process the need for numerical simulation is increasing and thus vibroacoustic numerical models became very sophisticated and in general possessing a very large number of Degrees of freedom (DOF). Thus, the analysis of such structures is a very complex problem. This complexity arises not only from the complex nature of the structure but also from the presence of multiple acoustic and vibration sources which increases a lot the number of Frequency Response Functions (FRF) that have to be treated.

Another element adding to the complexity of the analysis is the inaccuracy of the numerical forecasts causing a gap between the experimental measures and the numerical results. This inaccuracy is due to the presence of uncertainties not only in the physical system parameters, but also in the numerical model itself.

The present work deals with the problem of the vibroacoustic analysis of complex structures by proposing a model simplification based on a sub-structuring method. There exist several methods dedicated to the simplification of vibroacoustic models as well as the source-structure interaction specially in the medium-frequency ranges. These methods are mainly energy-based or statistically-based methods. Sub-structuring methods are considered as one of the most efficient ways to simplify the vibroacoustic problem. An example of existing vibroacoustic analysis simplification methods are, the Statistical Energy Analysis (SEA) and its derivations (see (1; 2; 3; 4; 5; 6; 7)), mobility methods (see (8; 9; 10)), wave component analysis and its derivations (12), mode-based approaches (13; 14), direct-dynamic stiffness method (15) and energy flow models (16). Most of these methods deal with sub-structures of built-up systems. Other methods dedicated to sub-structuring can be found in the references (17).

The sub-structuring method presented in this paper is based on an energy approach (see (18)). The energy approach is based on a stochastic vibroacoustic computational model constructed using the non parametric probabilistic approach of model and system-parameters uncertainties((19; 20; 21)). The interest of using a probabilistic model is to take into account both model and system-parameters uncertainties and thus, constructing a simplified model which is robust regarding those uncertainties. The parameters of the energy approach presented in reference (18) were chosen such that they are less sensitive to uncertainties which joins the probabilistic model in solving the problem of the inaccuracy of numerical predictions.

A second solution to the complexity of the vibroacoustic problem proposed by the energy approach is the construction of an energy model whose FRF are dimensionless, independent of the directions of excitation or observation, less variable as a function of frequency and finally independent of the precise position of the observation point. This last characteristic of the dimensionless FRF leads to the idea of sub-structuring presented in this paper.

The simplified sub-structured model is obtained using averaged values obtained from ensemble averaging over independent Monte Carlo realizations of the parameters of the energy model rather than frequency or space averaging which are used in most of the existing methods treating the same problem. Based on this averaged model a sub-structuring parameter is obtained. The sub-structuring method then consists in the evaluation of the sub-structuring parameter for all the chosen DOF of the system, based on which the an initial sub-structuring is obtained. Then the sub-structuring parameter is reevaluated for each sub-structure in order to confirm the limits of each sub-structure.

The work is presented in five parts. First, The energy approach is explained briefly. Then the construction of the simplified model is shown. The automatic sub-structuring method is then presented in a third part. Finally, the results of an application on an automotive vehicle structure and the conclusion are given in the last part.

2. Presentation of the energy approach

The energy approach was presented in (18). In this section only a quick review of the approach is presented. The Energy approach is constructed based on a probabilistic model constructed using the The non-parametric probabilistic approach (19; 20; 21) in order to take into account both parameter uncertainties and model uncertainties. the implementation of this method to an automotive vehicle structure can be found in reference (23). The response of the probabilistic model is represented by the following random equations

$$\mathbf{U}^s(\omega) = \Psi \mathbf{Q}^s(\omega) \quad , \quad \mathbf{P}^f(\omega) = \Phi \mathbf{Q}^f(\omega) \quad , \quad (1)$$

in which $\mathbf{Q}^s(\omega)$ and $\mathbf{Q}^f(\omega)$ are the random generalized coordinates verifying the random matrix equation

$$\begin{bmatrix} \mathbf{A}^s(\omega) & \mathbf{C} \\ \omega^2 \mathbf{C}^T & \mathbf{A}^f(\omega) \end{bmatrix} \begin{bmatrix} \mathbf{Q}^s(\omega) \\ \mathbf{Q}^f(\omega) \end{bmatrix} = \begin{bmatrix} \mathbf{f}^s(\omega) \\ \mathbf{f}^f(\omega) \end{bmatrix} \quad , \quad (2)$$

This Random equations are solved using the Monte Carlo method and the random response of each realization is obtained on the physical degrees of freedom. Since the number of physical DOF is very high only a set of DOF is chosen for the construction of the energy approach.

Let $\mu = n^s + n^f$ be the total number of DOF. One will only use the subset $\{j_1, \dots, j_\alpha, \dots, j_\nu\}$ of the ν observed and excited DOF of the vibroacoustic system with $\nu \ll \mu$.

Let $\mathbb{Z}(\omega)$ be the $(\mu \times \mu)$ complex random matrix such that

$$\mathbb{Z}(\omega) = \begin{bmatrix} \Psi & 0 \\ 0 & \Phi \end{bmatrix} \begin{bmatrix} \mathbf{A}^s(\omega) & \mathbf{C} \\ \omega^2 \mathbf{C}^T & \mathbf{A}^f(\omega) \end{bmatrix}^{-1} \begin{bmatrix} \Psi^T & 0 \\ 0 & \Phi^T \end{bmatrix} \quad , \quad (3)$$

which exists for all ω in B . Let $\mathbf{Z}(\omega)$ be the $(\nu \times \nu)$ complex random matrix such that, for all α and β in $\{1, \dots, \nu\}$, one has

$$\mathbf{Z}_{\alpha\beta}(\omega) = \mathbb{Z}_{j_\alpha j_\beta} \quad . \quad (4)$$

For all ω fixed in B , let $\mathbf{T}(\omega)$ be the $(\nu \times \nu)$ complex random matrix defined by

$$\mathbf{T}(\omega) = i\omega \mathbf{Z}(\omega) \quad . \quad (5)$$

The function $\omega \mapsto \mathbf{T}(\omega)$ is called the matrix-valued random FRF related to the excited and the observed DOF. It should be noted that $\mathbf{T}(-\omega) = \overline{\mathbf{T}(\omega)}$. Let $\mathbf{V}^\alpha(\omega)$ be the ν complex random vector of the velocity responses for the observed DOF $\{j_1, \dots, j_\nu\}$. One then has

$$\mathbf{V}^\alpha(\omega) = \mathbf{T}(\omega) \mathbf{f}^\alpha(\omega) \quad . \quad (6)$$

It can be shown that the system represented by Eq. (6) can be represented in terms of input and output power density functions as follows,

$$\boldsymbol{\pi}^R(\omega) = \mathcal{E}(\omega) \boldsymbol{\pi}_{in}(\omega) \quad . \quad (7)$$

The random input power density vectors can be represented in terms of spectrum functions of the input force and of the response velocity using the input and output point mobility matrices such that,

$$\boldsymbol{\pi}_{in}(\omega) = 2\mathbf{Y}(\omega)\mathbf{s}^f(\omega) \quad , \text{ and } \quad , \mathbf{s}^v(\omega) = \mathbf{Y}(\omega)\boldsymbol{\pi}^R(\omega) \quad , \quad (8)$$

where $\mathbf{s}^f(\omega)$ and $\mathbf{s}^v(\omega)$ are the spectrum vectors of the input forces and of the output velocity and where $\mathbf{Y}(\omega)$ is the diagonal mobility matrix defined by,

$$\mathbf{Y}_{\alpha\beta}(\omega) = \begin{cases} \text{Re}(\mathbf{T}_{\alpha\alpha}(\omega)) & \text{if } \alpha = \beta \\ 0 & \text{if } \alpha \neq \beta \end{cases} \quad . \quad (9)$$

Also in Eq. (7) $\mathcal{E}(\omega)$ is the dimensionless FRF which can be defined as a normalisation of the usual FRF with respect to the input and output mobilities of the system (24) such that,

$$\mathcal{E}(\omega) = \mathbf{Y}(\omega)^{-1}\mathbf{H}(\omega)\mathbf{Y}(\omega)^{-1} \quad , \quad (10)$$

where $\mathbf{H}(\omega)$ is the term to term square of the FRF such that $\mathbf{H}_{\beta\alpha}(\omega) = |\mathbf{T}_{\beta\alpha}(\omega)|^2$.

The spectral density function of the output velocity can then be related to the spectral density function of the input forces in terms of mobility matrix and dimensionless FRF and using the following equation

$$\mathbf{s}^v(\omega) = 2\mathbf{Y}(\omega)\mathcal{E}(\omega)\mathbf{Y}(\omega)\mathbf{s}^f(\omega) \quad , \quad (11)$$

it should be noted that the mobility matrix is a diagonal matrix due to the assumption of uncorrelated forces. This hypothesis might lead to loss of exactness in case of correlated forces. Thus, in order to avoid this loss of information the system is projected on the local coordinates system defined by the mean local mobilities. In these local coordinates the mobility matrix is diagonal by definition. To calculate the local coordinates one solves the eigenvalue problem of the mean value of the complex (3×3) $\mathbf{T}_p(\omega)$ of the translational DOF of the random FRF at a given point p of the structure (note that the rotational DOF are not considered here). The mean value of $\mathbf{T}_p(\omega)$ over all the independent Monte Carlo realizations is defined by $E\{\mathbf{T}_p(\omega)\}$, with E denoting the mathematical expectation. Let $\mathbb{T}_p(\omega)$ be the symmetric real (3×3) matrix such that $\mathbb{T}_p(\omega) = \Re\{E\{\mathbf{T}_p(\omega)\}\}$. The representation of the random matrix $\mathbf{T}_p(\omega)$ in the local coordinates attached to the given point and defined by the principal direction of the mean local mobility, is the random matrix denoted by $\mathbf{T}_p^{loc}(\omega)$ and is written as

$$\mathbf{T}_p^{loc}(\omega) = \mathbb{X}_p(\omega)^T \mathbf{T}_p(\omega) \mathbb{X}_p(\omega) \quad . \quad (12)$$

One can now consider the Eq. (10) in the local coordinates for all the local DOF of the structure at points p and all the the global DOF of the acoustic cavity all together which can then be rewritten

$$\mathcal{E}^{loc}(\omega) = \mathbf{Y}^{loc}(\omega)^{-1} \mathbf{H}^{loc}(\omega) \mathbf{Y}^{loc}(\omega)^{-1} \quad . \quad (13)$$

All other equations of the energy method still hold true in the local coordinates. Thus, these equations are going to be used in what follows with a subscript or a superscript *loc* to refer to values in these coordinates.

3. Construction of the simplified sub-structured model

The construction of the simplified model is based on the mean values of the energy method parameters. The mean value of a parameter is calculated using statistical averaging which is defined as the mathematical expectation of this parameter. In what follows an underlined parameter represents a mean value. Taking the mathematical expectation of Eq. (11) in the local coordinates leads us to

the calculation of what we call the reference mean value of the spectral density function of the output velocity calculated without any approximation,

$$\underline{\mathbf{s}}_{loc}^v(\omega)^{ref} = 2 E\{\mathbf{Y}^{loc}(\omega)\mathcal{E}^{loc}(\omega)\mathbf{Y}^{loc}(\omega)\mathbf{s}_{loc}^f(\omega)\} \quad (14)$$

Now, let J and O be the set of excitation and observation DOF respectively such that $J = \{k_q, q = 1, \dots, \mu\}$ and $O = \{j_p, p = 1, \dots, \nu\}$, where μ and ν are the number of excitation and number of observation DOF respectively.

Assuming that the excitation and observation DOF J and O are sufficiently distant from each, let e_{OJ} be the real number such that for each ω in the frequency band B , One can assume that,

$$(\underline{\mathcal{E}}^{loc}(\omega))_{j_pk_q} \simeq e_{OJ}(\omega) \quad (15)$$

In order that this last assumption be verified, the quantity $e_{OJ}(\omega)$ is computed using the reference mean value of the output velocity such that,

$$e_{OJ} = \frac{\sum_{p=1}^{\nu} (E\{\mathbf{s}_{loc}^v(\omega)\})_{j_pk_q}}{E\{\pi_{in}^{loc}\} \sum_{p=1}^{\nu} (\mathbf{Y}^{loc}(\omega))_{j_pk_q}} \quad (16)$$

The associated error due to this hypothesis can then be evaluated by defining $\varepsilon_{\mathcal{E}}(\omega)_{j_pk_q}$ such that

$$\varepsilon_{\mathcal{E}}(\omega)_{j_pk_q} = |\text{dB}(\omega)_{j_pk_q} - \text{dB}(\omega)_{j_pk_q}^{app}| \quad (17)$$

$$\text{dB}(\omega)_{j_pk_q} = 10 \log_{10} \underline{\mathcal{E}}^{loc}(\omega)_{j_pk_q} \quad , \quad \text{dB}(\omega)_{j_pk_q}^{app} = 10 \log_{10} \mathbb{E}_{OJ}(\omega)_{j_pk_q}^{app} \quad (18)$$

where $\mathbb{E}_{OJ}(\omega)_{j_pk_q}^{app} = e_{OJ}(\omega)$ and which measures the accuracy of the calculation of $\mathbb{E}_{OJ}(\omega)_{j_pk_q}^{app}$.

4. Numerical application

The proposed method was applied to a numerical model of a production vehicle. The frequency band of analysis was chosen to be $B = [50, 350]$ Hz. Only translational DOF were considered for the structure. Forces were applied to twenty eight points of the structure including the motor support and the front suspension while twelve acoustic sources were placed in the acoustic cavity for a total of 96 DOF. Observation points were assigned to each excitation point in order to obtain a square FRF matrix. Six hundred realizations are necessary for the convergence of the Monte Carlo method for both the structure and the acoustic cavity. Fig. 1 shows the mean values of all the Monte Carlo realizations of the usual FRF and the dimensionless FRF corresponding to an excitation in the three local directions at one point of the structure and observation at a DOF on another point.

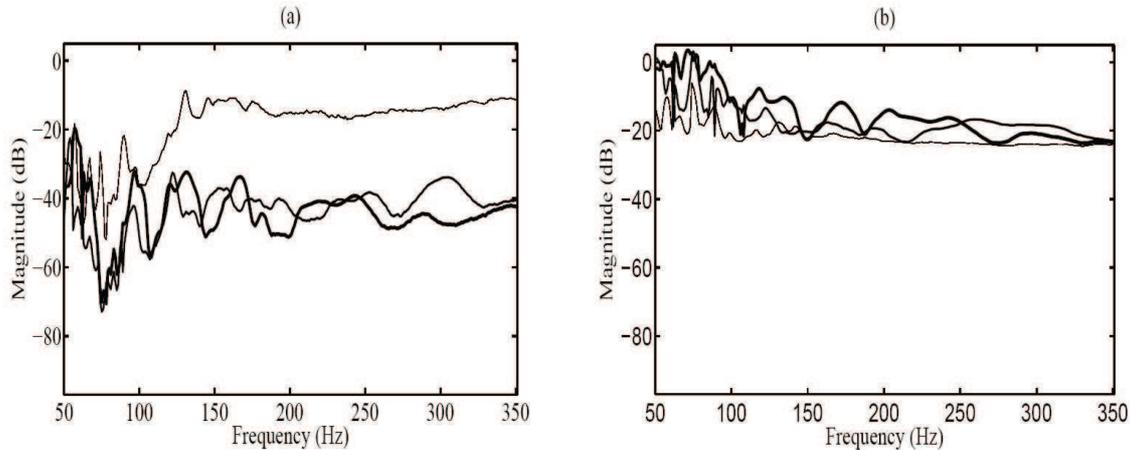


Figure 1. Mean values of the usual FRF (a) and dimensionless FRF (b). Excitation: at 3 structural DOF of the right engine mount, observation: maximum mobility direction on the roof.

It can be seen that the dimensionless FRF show less fluctuations versus frequency than the usual FRF. Moreover, when exciting in the three local DOF the resulting dimensionless FRF have very close values and tend to converge towards an asymptotic value, which is not the case for the usual FRF. This shows that, above a given frequency, these dimensionless FRF are slightly dependent of the direction of excitation. Reciprocally, they are also nearly independent of the observation direction. This property could lead to a significant reduction of the number of FRF to be treated for such a complex model. In addition, the dimensionless FRF have been shown to be much less sensitive to uncertainties than the usual FRF as shown in Fig. 2 The proposed formulation is thus more robust regarding the model and parameters uncertainties.

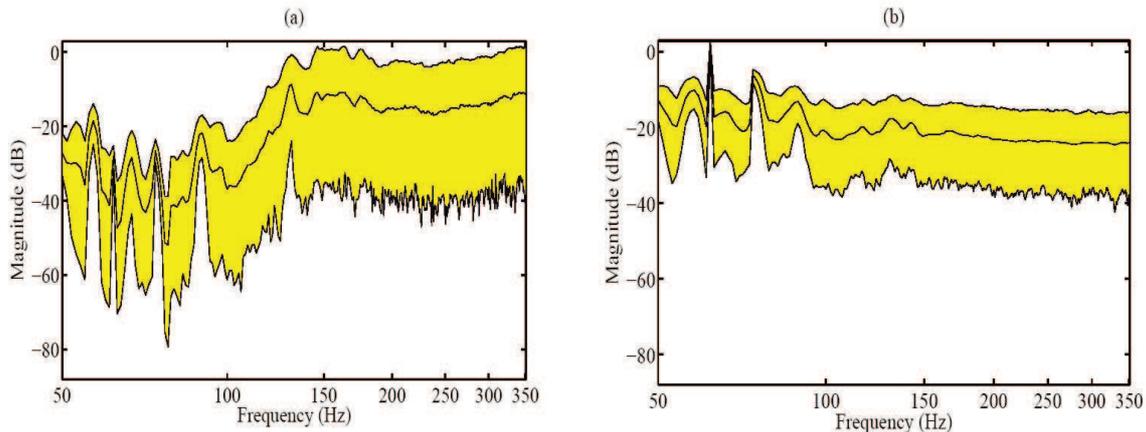


Figure 2. Confidence regions of the dimensional FRF (a) and dimensionless FRF (b) Excitation: right engine mount in its maximum mobility direction, observation: single point on the roof in its maximum mobility direction.

One may say that the vibrational FRF between two points may be separated into local effects described by the diagonal mobility matrices, and a global robust effect characterized by the dimensionless FRF matrix. Similar results are observed for acoustic and vibroacoustic FRF. The error function introduced by Eq. (18) was studied for the whole set of DOF. On observing the values of this error function for all DOF it has been noticed that DOF lying not far from each others had similar error values. Thus based on this primary observation an initial sub-structuring was obtained, which corresponded to functional parts of the vehicles structure (roof, windshield, etc.). The error function was then reevaluated inside each of the initial sub-structures in order to determine whether all the DOF are part of this sub-structure or not and thus determine the exact limits of each substructure. Fig. 3 shows the error matrix calculated at 60 and 90 Hz for the principal DOF lying on the roof and corresponding to an excitation on the front suspension. It can be observed that all the DOF chosen on the roof belong to this sub-structure starting at about 90 Hz.

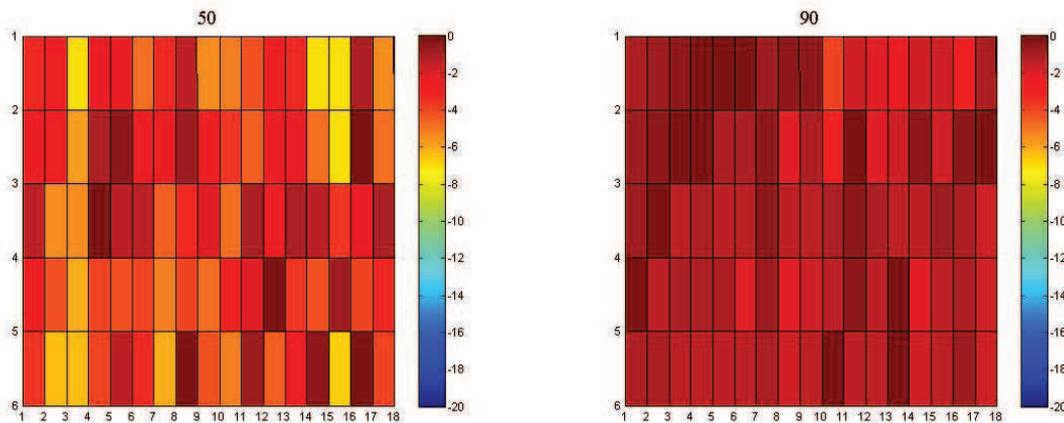


Figure 3. Color plots of $\varepsilon \mathcal{G}(\omega)$ for principal observation DOF on the roof and excitation DOF on the front train at (a) 60 Hz, (b) 90 Hz

5. Conclusion

In this paper, a sub-structuring method based on an energy approach has been presented. The energy approach is derived based on a non-parametric probabilistic computational model. In the energy approach the random vibroacoustic system is represented using input and response power density functions related using a dimensionless FRF. This dimensionless FRF is independent of the excitation or observation directions, less dependant on frequency and less dispersed than the usual FRF. It has been seen that these dimensionless FRF behaves similarly for neighbor DOF's lying in the same zone. A simplified model was constructed and a formula for obtaining a positive scalar considered as an indicator for sub-structuring was obtained. Using this sub-structuring indicator and the error induced by the model simplification the boundaries of each zone are obtained.

References

- ¹ R.H. Lyon, *Statistical Energy Analysis of Dynamical Systems*, San Diego, MIT Press, 1975.
- ² R.H. Lyon and R.G. DeJong, *Statistical Theory and application of statistical energy analysis*, Boston, Butterworth-Heinemann, 1995.
- ³ F.J. Fahy and A.D. Mohammed, A study of uncertainty in applications of SEA to coupled beam and plate systems, partI: Computational experiments, *Journal of Sound and Vibration*, 1992, **158**(1), 45-67.
- ⁴ L. Maxit and Guyader J-L, Extention of SEA model to subsystems with non-uniform modal energy distribution, *Journal of Sound and Vibration*, 2003, **265**(1), 337-358.
- ⁵ R.S. Langley, V. Cotoni, Response variance prediction in the statistical energy analysis of built up systems. *Journal of Acoustical Society Of America*, 2004, **115**(2), 706-718.
- ⁶ R.S. Langley and P. Bremner, A hybrid method for the vibration analysis of complex structural-acoustic systems, *Journal of Acoustical Society of America* 1999, **105**(3), 1657-1671.
- ⁷ N. Totaro and J.L. Guyader, SEA substructuring using cluster analysis: The MIR index, *Journal of Sound and Vibration*, 2006, **290**(1), 264-289.

- ⁸ P.J Shorter and R.S. Langley, Vibroacoustic analysis of complex systems, *Journal of Sound and Vibration*, 2005, **288**(1), 669-699.
- ⁹ G. Orefice, C. Caccilati and J.L. Guyader, The energy mobility. *Journal of Sound and Vibration*, 2002, **254**(2), 269-295.
- ¹⁰ L. Ji and B.R. Mace, R.J. Pinnington, A power mode approach to estimating vibrational power transmission by multiple sources. *Journal of Sound and Vibration*, 2002, **265**(2), 387-399.
- ¹² E.C.N. Wester and B.R. Mace, Wave component analysis of energy flow in complex structures - Part I: a deterministic model, *Journal of Sound and Vibration*, 2005, **285**(2), 209-227.
- ¹² E.C.N. Wester and B.R. Mace, Wave component analysis of energy flow in complex structures - Part I: a deterministic model, *Journal of Sound and Vibration*, 2005, **285**(2), 209-227.
- ¹³ L. Ji and B.R. Mace, R.J. Pinnington, Estimation of power transmission to a flexible receiver from a stiff source using a power mode approach, *Journal of Sound and Vibration*, 2003, **268**(2), 525-542.
- ¹⁴ L. Ji and B.R. Mace, R.J. Pinnington, A mode based approach for the mid-frequency vibration analysis of coupled long- and short-wavelength structures, *Journal of Sound and Vibration*, 2006, **298**(2), 148-170.
- ¹⁵ R.S. Langley, Analysis of power flow in beams and frameworks using the direct-dynamic stiffness method, *Journal of Sound and Vibration*, 1990, **136**(3), 439-452.
- ¹⁶ B.R. Mace and P.J Shorter, Energy flow models from finite element analysis, *Journal of Sound and Vibration*, 2000, **233**(3), 369-389.
- ¹⁷ D.J. Rixen, A dual Craig-Bampton method for the dynamic substructuring, *Journal of Sound and Vibration*, 2004, **168**, 383-391.
- ¹⁸ M. Kassem, C. Soize and L. Gagliardini, Energy-density field approach for low- and medium-frequency vibroacoustic analysis of complex structures using a statistical computational model. *Journal of Sound and Vibration*, 2009, **323**, 849-863.
- ¹⁹ C. Soize, Maximum entropy approach for modeling random uncertainties in transient elastodynamics, *Journal of Acoustical Society of America*, 2001, **109**(5), 1979-1996.
- ²⁰ C. Soize, A comprehensive overview of a non-parametric probabilistic approach of model uncertainties for predictive models in structural dynamics, *Journal of Sound and Vibration* 2005, **288**(1), 623-652.
- ²¹ C. Soize, Random matrix theory for modeling uncertainties in computational mechanics, *Computer Methods in Applied Mechanics and Engineering* 2005, **94**(12-16), 1333-1366.
- ²² R. Ohayon and C. Soize, *Structural Acoustics and Vibrations*, San Diego, Academic Press, 1998.
- ²³ J.F. Durand, C. Soize and L. Gagliardini, Structural-acoustic modeling of automotive vehicles in presence of uncertainties and experimental identification and validation, *Journal of Acoustical Society of America*, 2008, **24**(3), 1513-1525.
- ²⁴ L. Gagliardini, L. Houillon, G. Borello, L. Petrinelli, Virtual SEA- FEA based modeling of mid-frequency structure-borne noise, *Journal of Sound and Vibration*, Vol. 39, No. 1, Academic Press (2005), pp. 22-28.