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► To cite this version:

Jean-Luc Dion, Sylvie Le Moyne, Gaël Chevallier, Hamidou Sebbah. Gear impacts and idle gear noise: Experimental study and non-linear dynamic model. Mechanical Systems and Signal Processing, 2009, 23 (8), pp.2608-2628. 10.1016/j.ymssp.2009.05.007 . hal-00708792

HAL Id: hal-00708792

<https://hal.science/hal-00708792>

Submitted on 20 Jun 2012

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Gear Impacts And Idle Gear Noise: Experimental Study And Non Linear Dynamic Model

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Abstract

This paper develops an experimental and numerical study of dynamic phenomena involving gear impacts with one loose gear (non engaged gear pair) inside an automotive gearbox.

A dedicated test bench was designed for this study. Signal processing tools, based on the Order Tracking Method, were specially developed in order to clarify the underlying phenomena. A Particular attention was paid to the relationship between the drive shaft excitation and the energy and nature of impacts.

A topological model of contact is being proposed, based on a geometric description of solids in contact. The contact between gear meshes is defined using a Single Degree of Freedom, non linear, elastic and dissipative model. The chosen parameters of the model are not updated from measurements, but from the knowledge of the gear topology and tolerance class.

Simulations, compared with experimental results, confirm the accuracy of the model proposed.

Both model and experiments show, for particular excitation conditions, the emergence of repeated impacts on one side of the gear mesh.

Key words: Idle gear noise, hertzian contact, shock, backlash, non linear vibrations, signal processing

1 1 List of symbols

Definition	Variable
Angular position of the main shaft	θ_M
Angular position of the loose gear	θ_L
Definition	Parameters
Viscous coefficient introduced by lubricant between shaft and gear	C
Strutural damping of the L and M teeth in contact	C_{LM}
Angular damping of the contact	C_θ
Translational damping of the contact (along ξ –axis)	C_ξ
Momentum inertia of the loose gear	$I_{zz(k)}^k(P) = I_L$
Strutural elasticity of the L and M teeth in contact	K_{LM}^*
Angular elasticity of the contact	K_θ
Translational elasticity of the contact (along ξ –axis)	K_ξ
Radius of the loose gear	r_L
Radius of the main shaft	r_M
Geometric defaults on the tooth	ε
Exponent defined by the nature of the contact	α
Backlash	G
Pitch diameter on the loose gear	ξ_L
Pitch diameter on the main shaft	ξ_M

4 As the demand for acoustic comfort is constantly increasing, car
5 manufacturers are concerned by noise annoyance problems in ve-
6 hicles, and especially by the annoyance generated by gearboxes.
7 Among numerous gearboxes noises, idle gear noise is perceived
8 as particularly disagreeable and often wrongly associated with
9 an engine problem by consumers. This noise is generated by gear
10 impacts which can appear either in driving conditions ("rattle"
11 noise) or in neutral ("idle" noise or "neutral rattle" noise). As
12 neutral is the worst configuration, it is the focus of this paper.
13 The present paper presents an experimental and numerical study
14 of the gear impact. The aim of the study is to define a simple and
15 realistic model of contact able to describe the dynamic impacts
16 between gears with invariant parameters for several frequencies,
17 levels and types of excitations. To this end, the nonlinear Single
18 Degree of Freedom (SDOF) gear pair model with backlash, which
19 has often been investigated in the literature, has only included
20 few parameters for the gear topology and tolerance class. The
21 choice of the model to be developed was governed by a literature
22 study, but also by a preliminary experimental study.
23 The test bench designed for this study is composed of an off-the-
24 shelf simplified gear box (only one gear couple is kept). The choice
25 of measurement and signal processing techniques was governed
26 by the very transient character of the events being observed. This
27 experimental part of the study particularly focuses on the deter-
28 mination of relationships between the excitation kinematics and
29 the nature and the energy content of the impact. Energy contents
30 are analyzed globally and also in terms of frequency distribution.
31 The model thus formulated endeavors to predict the dynamic
32 behaviour of a gear box by associating a limited number of non-
33 linear SDOF. After an analytical description of the model, para-
34 metric identification techniques are presented.
35 The only model parameters that are identified with experimental
36 techniques are the dissipative coefficients. All other parameters

37 of the model are determined in accordance with technical design
38 data.

39 In the final section, before drawing our conclusions, numerical
40 simulations and experimental measurements will be compared.

41 3 Literature

42 Most previous studies of such phenomena propose analytical mod-
43 els for the vibroacoustic behaviour of the entire mechanical sys-
44 tem (gears, transmission chain). In [1], R.Singh et al. retain a
45 Lumped Model, non-linear, and find analytical solutions, derived
46 piecewise, section after section, from a linear analysis. In this par-
47 ticular case, the stiffness is described with functions which have
48 non continuous derivatives. In [2], the authors improved the latter
49 model by the introduction of friction. Moreover the authors de-
50 rive analytical solutions based on the Harmonic Balance Method.
51 Many investigations have been performed to define contact char-
52 acteristics of simple or multi-mesh gear trains, taking into ac-
53 count friction phenomena and the influence of backlash. The first
54 experiments focused on the detection and the analysis of non-
55 linear phenomena [3].

56 The instantaneous characteristics in the signals can be obtained
57 from the ridges and skeletons of the Wavelet Transform [4]. Such
58 non-linear systems can also be described with a Volterra Func-
59 tional Series which generalise the Superposition Principle and
60 allows Impulse Responses and Transfer Functions of various or-
61 ders to be obtained [5]. Other Detection Methods are based on
62 the speed variations, [6]; in this paper, the authors show that a
63 low cost conventional tachometer is sufficient for measuring the
64 induced speed variations due to backlash. They can also be pre-
65 dicted via simulation. For example, T. Tjahjowidodo et al., [4],
66 uses a detailed multibody simulation to develop and test the ef-
67 fectiveness of the proposed detection approach.

68 The idle gear noise is chaotic [7]. The chaotic behaviour gets
69 stronger with increasing backlash, see [8]. In [9], T. Tjahjowidodo

et al., quantify the chaotic responses and correlate them to the parameters of the non-linear system, in particular the magnitude of the backlash.

A.Al-shyyab et al. ([10] and M. Ajmi et al. [11]) proposed models to introduce non-linearity in the contact elasticity. They also take into account the structural elasticity of gears. M. Ajmi et al. [10] considers a continuous non-linear model concentrated on contact lines in gears with a Finite Element global description. A.Al-shyyab et al. [11] chose Lumped Models, non-linear with analytical solutions derived from decompositions in frequency domain. In a similar manner, J.P.Raclot et al. [12] built a combination of flexible bodies in contact with Spectral Methods for a wide excitation spectrum.

Various experimental studies, see for example ([13]), involve source location techniques by means of multiple Impulse Response Functions to detect sources of gearbox rattle using correlation between several excitations and several IRFs.

Other studies, of the complete gear box were performed taking into account the influence of manufacturing error on the rattle noise (see for example [14]). The degree of noise variability is assumed to be only induced by tolerances of geometric descriptions of gears. Mechanical models of solids are combined with acoustic models in a global statistical study. In The influence of the harmonic content of engine acyclism is analysed. As this is a main source of gear schoks, this work attempts to complement the work carried out by [15].

Models, such as the N.Barabanov et al. model [16], using Finite Element Methods are frequently employed for specific excitation sources: [17] introduces mechanical models for solids periodically excited by a static transmission error and incorporates acoustic models in the finite element method. Likewise, [18] conducted a study with several manufacturing errors (tooth spacing - pitch, eccentricities, misaligned gears) and variation in excitations using the model described in [10].

The vibroacoustic behaviour of the gear box is generally assumed to be linear and described by transfer functions following propa-

106 gation paths for solid transmission of vibration and vibroacoustic
107 coupling with fluid propagation (atmosphere) for acoustic trans-
108 mission ([19] [17]).

109 The only mechanical elements which cannot be described by lin-
110 ear models are bearings ([20], [21], [22], [23]) and gears in contact.

111 4 Test Bench

112 4.1 Gear Box

113 The test bench must show impacts induced by backlash between
114 two gears. Thus, all other sources of uncontrolled vibrations have
115 to be suppressed.

116 The solution adopted was to obtain an off-the-shelf gearbox and
117 modify it in order to keep only one gear couple (4th gear). As
118 a result of the gaps due to the removal of gears and the entire
119 command system, crossbars were added.

120 Principal gear data are the following:

121

- 122 • 4th helical gear on the main shaft:
 - 123 · pitch diameter: 57 mm
 - 124 · number of teeth: 28
 - 125 · face width: 20 mm
 - 126 · helix angle: 30°
- 127 • 4th loose helical gear:
 - 128 · pitch diameter: 74 mm
 - 129 · number of teeth: 37
 - 130 · face width: 20 mm
 - 131 · helix angle: 30°
- 132 • Contact ratio: 2.9
- 133 • individual pitch error : between 3 μm and 15 μm

134 4.2 Excitation System

135 The translational oscillations of an electrodynamic shaker rod
136 are transformed into angular oscillations of the main shaft through
137 two perpendicular transmission beams (Fig .1 and Fig .2). The
138 motion obtained is pure angular oscillations with an average an-
139 gular speed equal to zero. The authors chose to carry out the
140 experiment in this way in order to reproduce a gearbox in neu-
141 tral. Transmission beam dimensions were calculated in order to
142 respect the following conditions:

- 143 • The circular trajectory of the joint between the two transmis-
144 sion beams (point A) should not damage the shaker.
- 145 • The eigen frequencies of the excitation system should not per-
146 turb measurements.

147 The rigid transmission beam, between the flexible beam and the
148 axe of the main shaft, is 165 mm long.

149 The excitation reproduces actual engine torque oscillations which
150 are sources of the noise and vibration under study: sinusoidal ex-
151 citation from 30 Hz to 60 Hz with a maximal oscillation magni-
152 tude of 2000 rad/s².

153 4.3 Measurement Setup

154 The excitation is controlled by a piezoelectric accelerometer fixed
155 on the output of the shaker. In order to verify the non distortion
156 of the signal transmitted to the main shaft of the gearbox, a sec-
157 ond accelerometer is fixed at the end of the main shaft.

158 A rigid support is inserted in the loose gear under study, at 46
159 mm from the axe. This support is used for the fixation of a piezo-
160 electric accelerometer.

161 A hole was made in the gearbox house for accessibility and fixa-
162 tion of sensors (Fig .3 and Fig .4). The natural eigen frequency
163 of the support was calculated greater than 3200 Hz.

164 5 Experimental Study

165 5.1 Signal Processing And Analysis Method

166 The present method uses only instantaneous signals in both time
167 and frequency domains, and thus banishes averaging techniques.
168 Indeed, preliminary studies involving average signal in time do-
169 main, in order to eliminate the random part of the signals and
170 retain the deterministic part, led to significant signal leakage and
171 more particularly to an underestimation of the frequency content
172 of the signals.

173 After conducting a major experimental study, it was possible to
174 determine the most efficient signal processing representations for
175 those particular phenomena, in both time and frequency domains:

- 176 • Instantaneous temporal signals
- 177 • Periodograms representing the magnitude of the loose gear ac-
178 celeration, according to the period $N = t.F_{ex}$ where t is time
179 and F_{ex} the excitation frequency
- 180 • Three dimensional spectral representations of loose gear ac-
181 celeration magnitude versus frequency (or tracking order) and
182 excitation frequency.

183 Measures are converted from translation motion to angular mo-
184 tion in order to respect the kinematical properties of the system
185 (Fig .3).

186 The system is excited by a sine wave with a variable frequency,
187 the observed responses are then periodic but not harmonic. Ex-
188 perimental results are thus to be normalized to the excitation
189 frequency. A re-sampling technique based on FIR filter algorithm
190 is implemented to avoid signal distorsion.

192 One can first observe instantaneous time signals (Fig .5). Differ-
 193 ent types of behaviour appear, depending of the excitation level:

- 194 • for low excitation levels, no impact appears between the teeth
 195 of the gears,
- 196 • as the excitation increases, an impact occurs only on one side
 197 (F+) of the tooth. After the impact, the behaviour of the tooth
 198 leads to free vibrations (Fig .5c),
- 199 • for greater excitation, impacts occur on both sides of the tooth
 200 (F+ and F-). The impact level on F+ is greater and free vi-
 201 brations are observed only for this impact (Fig .5d),
- 202 • when the level of excitation increases, two impacts are observ-
 203 able in a very short time (less than 1ms): two pairs of teeth
 204 are in contact at the same time (Fig .5h),
- 205 • for the greatest excitation the impact on F- becomes stronger
 206 than the one on F+ (Fig .5g).

207 Those behaviours and their evolution were observed in the entire
 208 range of the excitation frequency ([30 Hz 60 Hz]).

209 It is particularly interesting to observe that impacts appear sev-
 210 eral times on the same side F+ before changing the side F-. In-
 211 deed, the first impact is not strong enough to push the tooth up
 212 to the other side. Two or three impacts then occur before chang-
 213 ing the side of impact. This phenomenon appears more clearly
 214 on periodgrams (Fig .6).

215 Acceleration signals of the loose gear show areas with a constant
 216 acceleration near zero (Fig .7 right). For these short sections, the
 217 velocity decreases slowly (Fig .7 left): the loose gear is in free
 218 motion with a small damping. During those sections of "free mo-
 219 tion", the decay of the velocity is proportional to the velocity:
 220 this energy dissipation can then be described with a classical vis-
 221 cous damping as an angular dashpot. This behaviour is due to
 222 the lubricant between the second shaft and the loose gear.

223 The analysis of the relative displacement on the pitch diameter

allows one to estimate the backlash between gears. When the level of excitation increases, this free motion gap presents an apparent increase (from $40\text{ }\mu\text{m}$ to $80\text{ }\mu\text{m}$): in fact, for the highest level, some parts of the mechanism can not be assumed to be rigid bodies, flexible behaviour has to be introduced. For example, a 1000 N force on the driving arm induces a deformation of $10\text{ }\mu\text{m}$. The actual backlash is in fact $80\text{ }\mu\text{m}$.

5.3 Frequency Analysis

Spectral analysis is represented in the frequency domain or with the Tracking Order Method. The three dimensional representations in Fig .8 show the evolution of the spectral signature of the teeth impacts in accordance with the excitation level for two excitation frequencies: 30 Hz and 60 Hz. For both frequencies a threshold of excitation shows a break-even point from which the impact spectrum extended strongly from 500 Hz to more than 5000 Hz. In all the experiments, the signal level strongly decreases around 3200 Hz: this is due to the sensor support plate on which measurements are performed, which plays the rule of dynamic absorber tuned at 3200Hz (Fig .8).

5.4 Parametric Analysis

With the aim of showing the influence of acceleration, velocity and displacement levels, three classes of experiments have been conducted with the excitation frequency between 30 Hz and 60 Hz:

- at a constant displacement magnitude,
- at a constant velocity magnitude,
- at a constant acceleration magnitude,

The excitation is controlled thanks to an accelerometer placed on the electro-dynamics shaker head.

253 These experiments were performed with several levels of excita-
254 tion.

255 The figure .5 represents periodgrams according to the excitation
256 frequency for each of the three classes of measurements.

257 Periodgrams with a constant magnitude of the excitation accel-
258 eration is the most invariant with excitation frequency (in term
259 of energetic level of impacts). According to a simple linear ap-
260 proach, forces can be described in terms of acceleration. So the
261 acceleration of the loose gear depends on the impact force and
262 consequently on the excitation level of the main shaft.

263 The number of impacts on a given side strongly depends on the
264 displacement level induced by the angular oscillations of the main
265 shaft: for a given main shaft displacement magnitude, the ener-
266 getic level of impacts increases with the excitation frequency, but
267 the number of impacts keep the same.

268 6 Model

269 6.1 Global Description

270 The model proposed in the present paper is based on a non linear
271 Single angular Degree Of Freedom (Fig .8) which can relate the
272 main shaft behaviour to the loose gear in terms of force, torque,
273 acceleration, speed and position in translation and rotation.

274 The non linear approach is obvious: for an excitation composed
275 of a single frequency, the dynamic response can be broken down
276 into several frequencies over ten times the excitation frequency.

277 The very first excitation is introduced by the behaviour of the
278 main shaft. The neutral gear dynamic rattle appears with im-
279 pacts between loose gears and the main shaft. The only possible
280 transmission path for vibrations is the shaft, which is supposed to
281 have a linear behaviour. Moreover, the main non linear behaviour
282 is concentrated between the main shaft and the loose gear.

283

284 For this paper several models were considered in order to pro-

pose the most appropriate one for the study. The very first tested models are lumped models described by [15] and [10]. These models are made with specific non linearity of elasticity and damping with the objective to performing analytical solutions section after section. The dynamic description of the contact and impacts is performed as a succession of sections (based on periodic solutions). In such models, the dynamic response between two temporal sections (large and discontinuous evolution of the stiffness for example) does not take into account the gradual evolution of the stiffness between the two states. Although, several studies ([24], [25]) had established that the dynamic response of solids in contact depends strongly on the progression of the stiffness function and more precisely on the geometric defaults (shape and micro geometry in terms of roughness). Simulations performed with these models overestimate the number and the levels of impacts and underestimate the duration of each impact (still using parametric identification obtained by updating the models with measurements). For these reasons this kind of Lumped Model was abandoned.

Other tested techniques ([13]) consist in building the solution in the frequency domain with a semi analytic method. This approach needs a specific analytical description of elasticity and damping (such as polynomial decomposition) and a large number of terms if realistic simulations are required. More over, this method could not describe an important phenomenon experimentally observed: several impacts occur on the same side of a tooth before changing sides. For these reasons this kind of method was not retained either.

Models based on Finite Element Method ([26]) are very heavy models in terms of computing time and memory. Moreover, it remains difficult to describe the complete geometry with specific defaults as roughness, which are important parameters in the dynamic of the shocks. However, this technique could be successfully used with the aim of identifying the structural stiffness of solids. For dynamic excitations this kind of model could not be used with reasonable computation time except for specific kinds

of excitation ([26]). In the present study, the roughness had to be taken into account and the excitation is not supposed to be known. This is the reason why this method has not been developed in this paper.

The proposed Lumped Model can be considered as a prolongation of classical Single Degree Of Freedom with non linear stiffness and damping. Models of stiffness and damping were chosen with the aim of taking into account the backlash, the geometric default of tooth (roughness) and the nature of the contact.

The elementary model for rotation on the z axis is defined by:

$$I_L \ddot{\theta}_L + K_\theta (\theta_M, \theta_L) + C_\theta (\dot{\theta}_M, \dot{\theta}_L, \theta_M, \theta_L) + C \dot{\theta}_L = B \quad (1)$$

with:

- B : external torque
- C : viscous coefficient for dissipative behaviour introduced by lubricant between the loose gear and the shaft.

The transmission error ξ on the curvilinear axis for translation displacement (on pitch diameters) is defined by:

$$\xi = \xi_L - \xi_M = r_L \theta_L - r_M \theta_M \quad (2)$$

where r_L and r_M are respectively the pitch radius of the loose gear and the main gear.

Impacts are conditioned by ξ along the pitch diameter, although the nonlinear dynamic equation for the elementary model is written for the angular variable.

$K_\theta (\theta_M, \theta_L)$ the angular elasticity of the contact and $C_\theta (\theta_M, \theta_L, \dot{\theta}_M, \dot{\theta}_L)$ the angular dissipation of the contact are simply and respectively linked to $K_\xi (\theta_M, \theta_L)$ and $C_\xi (\theta_M, \theta_L, \dot{\theta}_M, \dot{\theta}_L)$ the same function projected on the axis :

$$K_\theta (\theta_M, \theta_L) = r_L K_\xi (\xi_M, \xi_L) \quad (3)$$

$$C_\theta (\theta_M, \theta_L, \dot{\theta}_M, \dot{\theta}_L) = r_L C_\xi (\xi_M, \xi_L, \dot{\xi}_M, \dot{\xi}_L) \quad (4)$$

351 Five different cases are considered, depending on transmission er-
 352 ror value compared to geometric manufacturing errors and back-
 353 lash (see figure .11).

354 The explicit expression of the contact elasticity $K_\xi(\xi_M, \xi_L)$ is:

- 355 • when the gears are in contact on side F+ : $\forall \xi \leq -\varepsilon$

$$356 \quad K_\xi = K_{LM}^* (\xi_L - \xi_M) \left| \frac{\xi_L - \xi_M}{G} \right|^{\alpha-1} \quad (5)$$

- 357 • when the gears are near the contact on side F+ : $\forall -\varepsilon \leq \xi \leq 0$

$$358 \quad K_\xi = \frac{K_{LM}^*}{2} \left(1 - \left(\frac{\varepsilon}{2} + \xi_L - \xi_M \right) \sin \left(\frac{\pi}{\varepsilon} \right) \right) \left(\xi_L - \xi_M \right) \left| \frac{\xi_L - \xi_M}{G} \right|^{\alpha-1} \quad (6)$$

- 359 • when the gears are separated : $\forall 0 \leq \xi \leq G$

$$360 \quad K_\xi = 0 \quad (7)$$

- 361 • when the gears are near the contact on side F- : $\forall G \leq \xi \leq G + \varepsilon$

$$362 \quad K_\xi = \frac{K_{LM}^*}{2} \left(1 - \left(\frac{\varepsilon}{2} - (\xi_L - \xi_M - G) \right) \sin \left(\frac{\pi}{\varepsilon} \right) \right) (\xi_L - \xi_M - G) \left| \frac{\xi_L - \xi_M}{G} - 1 \right|^{\alpha-1} \quad (8)$$

- 363 • when the gears are in contact on side F- : $\forall G + \varepsilon \leq \xi$

$$364 \quad K_\xi = K_{LM}^* (\xi_L - \xi_M - G) \left| \frac{\xi_L - \xi_M}{G} - 1 \right|^{\alpha-1} \quad (9)$$

365 with :

- 366 • K_{LM}^* the equivalent static stiffness of contact between the teeth
 367 L and M ([24])
- 368 • α the contact-type coefficient (e.g. sphere to plane, cylinder to
 369 cylinder) ([27])(see figure .10
- 370 • ε the size of manufacturing errors on the tooth (e.g. shape
 371 defects and micro-geometric defaults) ([25])

372 • G the backlash ([28]) on the pitch diameter.

373 The equation (6 and 8) ensure the continuity of the function K_ξ
 374 and its first derivatives.

375 The dissipative coefficient of the contact is defined with the same
 376 existing conditions as for the elasticity function.

377 This kind of normal contact model is used in several applications:
 378 for particular applications such as the dynamic behaviour of the
 379 compressor valve contacts or non lubricated gears, a friction dis-
 380 sipative term in the dissipative function had to be introduced
 381 ([29]).

382 The explicit expression for the dissipative coefficient $C_\xi(\xi_M, \xi_L, \dot{\xi}_L, \dot{\xi}_M)$
 383 is:

$$\begin{aligned}
 384 & \bullet \xi \leq -\varepsilon \\
 385 & C_\xi = C_{LM} (\dot{\xi}_L - \dot{\xi}_M) \\
 386 & \bullet -\varepsilon \leq \xi \leq 0 \\
 387 & C_\xi = \frac{C_{LM}}{2} \left(1 - \left(\frac{\varepsilon}{2} + \xi_L - \xi_M \right) \sin \left(\frac{\pi}{\varepsilon} \right) \right) (\dot{\xi}_L - \dot{\xi}_M) \\
 388 & \bullet 0 \leq \xi \leq G \\
 389 & C_\xi = 0 \\
 390 & \bullet G \leq \xi \leq G + \varepsilon \\
 391 & C_\xi = \frac{C_{LM}}{2} \left(1 - \left(\frac{\varepsilon}{2} - (\xi_L - \xi_M - G) \right) \sin \left(\frac{\pi}{\varepsilon} \right) \right) (\dot{\xi}_L - \dot{\xi}_M) \\
 392 & \bullet G + \varepsilon \leq \xi \\
 393 & C_\xi = C_{LM} (\dot{\xi}_L - \dot{\xi}_M)
 \end{aligned}$$

394 Where C_{LM} denotes the associated damping of contact between
 395 the teeth L and M ([29])

396 7 Model Characterization

397 7.1 Aims And Methods

398 The aim of the proposed model is to simulate the source of dy-
 399 namic excitation in gear boxes.

400 The model proposed in the present paper is deterministic and
 401 preserves stationary properties. Each parameter is invariant and

402 independent of the excitation level and the frequency.
 403 All parameters, except for dissipative behaviour, were identified
 404 without dynamic measurements. Most of them had been pro-
 405 duced by Renault SA. Some of them, such as K_{LM}^* , K_θ and α were
 406 determined by using classical engineering gears methods ([28]).
 407 The choice of the model and its parameters had been strongly
 408 influenced by the possibility to identify the parameters with a
 409 greater degree of accuracy.
 410 Each parameter is determined by a simple analytic model based
 411 on the design of gear. But they can also be identified by a single
 412 dynamic measure on the studied test bench excited with har-
 413 monic signal.
 414 For each degree of freedom, 9 parameters had to be identified for
 415 the model:

- 416 • geometric parameters: I_L , r_L , r_M
- 417 • elastic parameters: K_{LM} , α , ϵ , G
- 418 • dissipative parameters: C_{LM} , C

419 These parameters are presented and explained in greater detail
 420 in paragraphs 7.2 to 7.4. Numerical values for each parameter
 421 are given in the case of the studied gear box. Corresponding sim-
 422 ulations are presented in paragraph 8, and compared with the
 423 experiments.

424 7.2 Geometric parameters

- 425 • I_L : Moment of Inertia for the loose gear. Normally, this pa-
 426 rameter is obtained numerically from the geometrical model
 427 and the density of the material. ($I_L=0.32(10^{-3})$ kg.m²)
- 428 • r_L and r_M : are respectively the pitch radius of the loose gear
 429 and the main gear ($r_L=37$ mm and $r_M=28.25$ mm)

430 7.3 Elasticity

- 431 • K_{LM} : the equivalent elasticity between the L and M teeth
 432 in contact. This can be obtained by a numerical identification

with a Finite Element study of teeth in contact or more easily by the retained method defined by Palmgren ([24]). To avoid coupling with α and G parameters and unit incompatibility, the parametric identification is performed on $K_{LM}^* = K_{LM} \cdot G^{(\alpha-1)}$. This method reveals a non-dimensional variable $\xi^* = \xi/G$ and independent parameters K_{LM}^* , α , G . ($K_{LM}^* = 3(10^6) \text{ N.m}^{-1}$; $K_{LM} = 1.03 \cdot (10^8) \text{ N.m}^{-4/3}$).

- α : This coefficient is defined according to the type of contact ([27]): a simple line contact leads to $\alpha=10/9$, a single point leads to $\alpha=3/2$. In fact, for spur or helical gears the actual contact is neither linear nor punctual. This parameter had then to be identified between these two cases and was used to minimize the discrepancy between experimental measures and numerical simulations (Fig .10). ($\alpha=4/3$). Generally speaking gear manufacturing requires that gear teeth are shaped to ensure a contact area limited to the central part of the tooth [28].
- ϵ : the size of manufacturing errors on the teeth (shape defects and micro-manufacturing errors) ([25]). This parameter transforms the point discontinuity into a continuous curve over an infinitesimal displacement interval thus the instantaneous stiffness increases from 0 to the equivalent solid stiffness K_{LM} over the deformation interval ϵ of manufacturing errors ($\epsilon=1 \cdot 10^{-6} \text{ m}$).
- G : the backlash ([28]) on the pitch perimeter (projected onto the ξ axis). This parameter is the main cause of idle gear noise. As this parameter is well known and statistically verified during the manufacturing process it cannot be excluded from consideration for technological reasons such as assembly or operating constraints. ($G=0.025 \text{ mm}$)

The complete elastic function is defined by 6 parameters (r_L , r_M , K_{LM} , α , ϵ , G) and 2 variables ξ_L and ξ_M .

$K_\xi(\theta_M, \theta_L)$ is exhaustively defined by 4 parameters (K_{LM} , α , ϵ , G). (Fig .11)

467 C_{LM} is the associated damping of the L and M teeth in contact
 468 ([24]). To avoid coupling with K_{LM} , I_L , α , G and rL parameters,
 469 the parametric identification is obtained from:

$$470 \quad C_{LM}^* = \frac{C_{LM}}{2\sqrt{K_{LM}^* \frac{I_L}{r_L^2}}} \quad (10)$$

471 This method reveals a non-dimensional and independent param-
 472 eter C_{LM}^* . This parameter cannot easily be predicted before ex-
 473 perimentation but it can be accepted as being in the band from
 474 1 to 5 percent.

475 For several industrial applications this dissipative model can be
 476 added or replaced by a friction model with an internal state vari-
 477 able ([30], [31], [16]). ($C_{LM}^*=0.04$; $C_{LM}=67 \text{ N.s.m}^{-1}$).

478 The model described in [30] was successfully tested but not re-
 479 tained in the final presented model in order to preserve the sim-
 480 plicity of the global model. More over, the accuracy needed for
 481 the dissipation function in this study does not justify the intro-
 482 duction of new parameters.

483 The complete dissipative function is defined by 5 parameters (r_L ,
 484 r_M , C_{LM} , ϵ , G) and 4 variables ξ_L , ξ_M , $\dot{\xi}_L$ and $\dot{\xi}_M$.

485 $C_\xi(\theta_M, \theta_L, \dot{\theta}_M, \dot{\theta}_L)$ is completely defined by 3 parameters (C_{LM} ,
 486 ϵ , G) and produces a dissipative force proportional to the velocity
 487 of the impact.

488 C is the viscous coefficient for dissipative behaviour introduced
 489 by a lubricant between the loose gear and the shaft. This coeffi-
 490 cient can be identified with an analytic fluid model between two
 491 infinite cylinders or with a simple measure of velocity when the
 492 loose gear is in free rotation (Fig .13).

493 With the intention of constructing independent parameters and
 494 to avoid coupling with I_L parameter, the parametric identifica-
 495 tion is led on $C^* = C/I_L$. This method reveals an independent
 496 parameter C^* and the time response for an initial velocity S_0 at

497 t_0 is:

$$498 \quad S(t) = S_0 e^{-C^*(t-t_0)} = S_0 e^{-\frac{C}{I_L}(t-t_0)} \quad (11)$$

499 $S(t)$ corresponds to computed dash-dot lines in red on Fig. .9
500 ($C^*=78 \text{ s}^{-1}$; $C=0.025 \text{ N.s.m}$). When these dash-dot lines are
501 compared with free motion episodes of the loose gear, they rep-
502 resent a good method to estimate the quality of the numerical
503 identification of C .

504 8 Numerical and Experimental Comparison

505 All simulations shown were performed with numerical values de-
506 fined in section 7 for each parameter of the model built.

507 In an effort to improve the graphic legibility and kinematical
508 analysis, measured and simulated signals are studied between 0
509 Hz and 1000 Hz using classical filtering techniques (10^{th} order
510 Cauer filter with phase compensations).

511 Three comparisons between the numerical simulations and the
512 experimental measures are shown. Each comparison is defined by
513 the excitation level of the main shaft and its frequency:

- 514 • 30 Hz, 300 Rad.s⁻² RMS (12 m.s⁻² amplitude peak on the
515 pitch diameter of the main gear)
- 516 • 60 Hz, 300 Rad.s⁻² RMS (12 m.s⁻² amplitude peak on the
517 pitch diameter of the main gear)
- 518 • 30 Hz, 500 Rad.s⁻² RMS (20 m.s⁻² amplitude peak on the
519 pitch diameter of the main gear)

520 Other comparisons were made for frequencies between 30 Hz and
521 60 Hz (by 2 Hz steps) and for several angular accelerations from
522 100 Rad.s⁻² RMS to 700 Rad.s⁻² . The presented comparisons
523 cover a large field of significant cases with a high level of idle gear
524 noise.

525 For each comparison, different kinds of kinematic results are
526 shown:

- 527 • acceleration on the pitch diameter of the loose gear
- 528 • velocity on the pitch diameter of the loose gear
- 529 • displacement on the pitch diameter of the loose gear
- 530 • transmission error in displacement between the main and loose
- 531 gears.

532 8.1 Initial Comparison: The Lower Limit Frequency Of Idle Gear Noise

533 The first comparison is undertaken at an excitation frequency
 534 of 30 Hz and an acceleration of 300 Rad.s⁻² RMS (12 m.s⁻²
 535 amplitude peak on the pitch diameter of the main gear)

536 The comparison based on angular position or displacement along
 537 the pitch diameter (ξ axis) (Fig .12) shows a very strong correla-
 538 tion between experimental measurements and numerical simula-
 539 tions. Often in cases of dynamic systems, observations involving
 540 this kind of kinematic variable display a close fit between mea-
 541 surements and models. The comparison of kinematic variables
 542 had to be conducted with greater order derivatives to confirm
 543 the models.

544 The simulated velocity describes with precision levels and events
 545 during the dynamic simulation: synchronization of the different
 546 steps during the excitation, shocks and free motions, are pre-
 547 dicted with strong accuracy (Fig .10).

548 Acceleration is the more convenient kinematic descriptor for im-
 549 pact identification and vibroacoustic prediction of complete struc-
 550 tures. Acceleration levels and the number of impacts are accu-
 551 rately predicted. Simulated peaks decrease in accordance with
 552 measurements (Fig .11).

553 Another way to identify impacts and dynamic behaviour of gears
 554 is to represent the relative displacement between the main gear
 555 and the loose gear. This displacement results from the backlash
 556 between teeth shown by the gap between the two horizontal dash-
 557 dotted lines on Fig .17, .19, .22.

559 The second comparison is performed at a frequency of 60 Hz and
 560 an acceleration of 300 Rad.s⁻² RMS (12 m.s⁻² amplitude peak
 561 on the pitch perimeter of the main gear).

562 The model's parameters are numerically the same as those used
 563 for the previous comparison and those identified in section 7.

564 The evolution of the frequency from 30 Hz to 60 Hz reduces the
 565 number of shocks from 4 or 5 to only 2 or 3 shocks each time on
 566 the same side. The level of acceleration does not change signifi-
 567 cantly as the frequency increases (Fig .13).

568 Transmission error clearly shows only two shocks on the same side
 569 of the tooth before changing sides (Fig .14). When the frequency
 570 increases the number of shocks on the same side is reduced and
 571 the length of free motion phases of the loose gear significantly
 572 shortens.

573 Velocity (Fig .15) is also correctly predicted in terms of magni-
 574 tude and events by the numerical simulation.

576 The third comparison is performed at a frequency of 30 Hz and
 577 an acceleration of 500 Rad/s² RMS (20 m.s⁻² amplitude peak on
 578 the pitch perimeter of the main gear).

579 The model's parameters are numerically the same as those used
 580 for the previous comparisons and those identified in section 7.

581 When the excitation magnitude increases, the number of shocks
 582 increases as does their acceleration magnitude (Fig .16).

583 Shock period and number can also be predicted from the trans-
 584 mission error diagram (Fig. .17). Shocks seem to happen over
 585 the interval G between teeth (red dash-dotted lines). In fact, this
 586 effect is due to the coupling between dynamic behaviour of the
 587 loose gear and the non linear stiffness of its teeth (progression
 588 depending on manufacturing errors ϵ).

589 When the excitation magnitude increases, the model maintains a

590 high level prediction for kinematic behaviour in terms of velocity
591 (Fig .18).
592 Another way to identify and quantify shocks is obtained in post
593 processing velocities with the intention of measuring the relative
594 velocity when shocks occur (Fig .19). This graph reinforces the
595 analysis of shocks and can also permit one to quantify the ve-
596 locity of each shock and thus, the kinematical energy of shocks
597 and its evolutions in Rayleigh (dissipative) and potential (elastic)
598 Energy.

599 9 Conclusion

- 600 • The experimental study outlines phenomena and parameters
601 responsible for the idle gear noise.
- 602 • The model built is a non linear Single Degree Of Freedom in
603 rotation composed of 9 independent and invariant parameters:
604 The identification of each parameter can be performed with
605 an analytical and numerical approach or by measurements on
606 a single dynamic test. To find the independence of each pa-
607 rameter and for a better accuracy, identifications presented in
608 section 7 are performed with reduced variables and parameters.
609 In several cases these reduced parameters are dimensionless.
- 610 • All simulations were carried out with the same set of param-
611 eters and reveal a very close fit with experimental measure-
612 ments. The kinematic of the impacts is well predictable in
613 terms of magnitude, duration, for number of one-sided impacts
614 as well as the decrease in the magnitude of each impacts.
- 615 • Both simulations and experimental measurements reveal the
616 following important points:
 - 617 · several impacts can often appear on one side of the gear mesh,
 - 618 · the number of impacts on one side strongly depends on the
619 angular displacement of the main shaft,
 - 620 · the energy level of impacts mainly depends on the accelera-
621 tion level of the main shaft oscillations,
- 622 • The excitation level at which impacts appear is identified, pre-
623 dicted and closely connected with the acoustic phenomenon
624 called idle gear noise.
- 625 • The next step of this study is to undertake a vibroacoustic
626 model of the gear box based on this study of a single couple of
627 gears. The aim of this work is to show the predictable vibroa-
628 coustic behaviour of a complete gear box with a small number
629 of (non linear) Degrees of Freedom.

630 Acknowledgment

631 The authors express their gratitude to Renault SA for the finan-
632 cial support of this study.

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