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**A cognitive model for the representation of time  
in a man-machine dialogue.**

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**Abstract :**

This paper develops the foundations of a model for time representation in the framework of a man-machine dialogue system. While we analyse other approaches, especially Allen's interval calculus, we show how the relations that we commonly manipulate in everyday reasoning can in fact be reduced to two fundamental ones : succession and inclusion. By the way, we insist on the fact that a temporal model intended to reproduce some features of the human cognitive abilities shall include in a common representation linguistic information and conceptual objects.

We then present the main characteristics of our temporal model, introducing the concept of coherence zone, and how this one can be used to represent tense information in natural language. Finally, we briefly show the mechanisms that ensure temporal consistency when combining new temporal information to an existing structure, and present the main elements that allow learning and predicting mechanisms within this model.

**Area of reviewing:** (B3) cognitive modeling.

This paper has not been submitted to any other major conference in 1989.

## 1. Introduction.

Time has gained more and more importance in recent years research in Artificial Intelligence for it is the keystone of any system reasoning on a universe of moving objects or trying to understand natural language in a man-machine interaction for example. However, dealing with assembly lines or computer operating systems, which are rather deterministic, is not the same problem as conversing with a human being with all his uncertainties and inconsistencies. In an application such as administrative database questioning, where a potential user should not be told how to make the system work, any step of the interaction must be as natural as possible, especially when dealing with facts and events that the speaker easily handles. This points out the need for a model of time representation and reasoning that is not a mere instant or even interval calculus, but becomes part of a common representation of language and cognition.

Such models cannot be found in logical approaches as proposed by McDermott ([McDermott 82]) or Shoham ([Shoham 87]), which are based upon a formal system, too rigid and constraining to deal with commonsense reasoning, even if successive improvements finally solve some problems, such as non-monotonicity or persistence. As alternatives, many studies have been made within the scope of cognitive sciences, proposing new visions for knowledge representation and time understanding. Those rough models need to be revised in order to be implemented, but none of them shall be rejected without a real study of its possible contribution to the comprehension of cognition. For instance, in the early forties, Jean Piaget showed that the comprehension of time among young children, trying to recall an experience of emptying and filling water jugs, is essentially based on two aspects, namely, the succession of events, which lies at the root of the mechanism of causation, and the variable granularity of the analysis that can be made of a particular event, which appears to be split up, if the context requires it.

Beside this psychological approach, linguistics has endeavoured to explain the mechanisms that ensure the coherence of the tense system in many different languages. Here the problem is to link an utterance with the situation it describes, taking into account the specificity of aspect and modality which are crucial in speech communication.

The work presented here is an attempt to make a synthesis of these rather different trends, in order to propose a cognitive model of time representation that is both natural and computationally acceptable. We will first show the important work made by Allen to take into account the relative and hierarchic aspect of time in everyday use, and how his temporal relations can be put together in three homogeneous classes. Comparing this model with Hornstein's representation of tense, Yip yields some clues to a possible integrated notion of language and knowledge representation based on time. We shall then present a model that, we argue, can represent most problems bound to tense and aspect, but, over all, it seems an interesting way to explain some cognitive phenomena and implement them, particularly in the case of learning and predicting mechanisms.

## 2. Towards a unified vision of language and representation.

It is generally admitted that numbers are not a good tool for locating a situation in the course of time, since it gives at any moment an infinite precision and therefore makes it difficult to structure a space of events in a coherent way. Dealing with instantaneous objects or manipulating intervals with numerical boundaries lead to the same problem of determining those values which are typically unknown, or correspond to some other brief events whose location is not accessible either. The solution is then to consider the relations that several events establish with each other, that is, to deal with a temporal graph, rather than to try to situate those events precisely on a date line.

## 2.1 An advanced vision of time : Allen's intervals calculus.

Allen ([Allen 83]) develops a theory of time based on intervals between which thirteen different relations can hold (see figure 1). These relations represent the exhaustive links that may appear without considering whether those intervals are taken or not with their boundaries as would be the case when dealing with real dates. Allen proposes some algorithms in order to compute temporal constraints among a set of intervals, so that new information are easily integrated in an existing graph.

Relation	Symbol	Symbol for Inverse	Example
X before Y	<	>	XXX YYY
X equal Y	=	=	XXX YYY
X meets Y	m	mi	XXXYYY
X overlaps Y	o	oi	XXX YYY
X during Y	d	di	XXX YYYYY
X starts Y	s	si	XXX YYYYY
X finishes Y	f	fi	XXX YYYYY

Table 2.1 (from [Allen 83]).

However, it seems that he has not completely broken off with the traditional vision of time, as many of his relations present some redundancies, even if he sometimes introduces the relation *in*(i1, i2) equivalent to {*starts*(i1, i2) or *during*(i1, i2) or *finishes*(i1, i2)}, as a short cut when dealing with linguistic information. We can, in fact, display three main categories of relations in which we may classify Allen's :

- Succession, in which we can include *before* and *meets* .
- Inclusion, the one introduced by Allen for *starts*, *during*, *finishes* , to which we can add *equal*.
- *Overlaps*, which cannot be categorized in any of the preceding classes.

The first two classes appear when trying to specify the exact difference between, for example, before and meets. Apart from the case when you have to exactly synchronise two processes, it seems impossible to express that two events strictly meet. Their respective end and beginning may have occurred in a very tight space of time, but at the perception level, you cannot be sure of their exact concomitance. One way of experiencing this phenomenon is to imagine two events that seem to meet, let's say pushing a button and making an alarm ring. If you try to verify that these two events are adjacent, you can only explore the beginnings and ends of both to localize the very moments during which they happened. In fact, the same problem could be put forward for *starts*, *during* and *finishes*. Therefore, we may wonder if we shall distinguish those relations which cannot be easily differentiated. In the context of a dialogue between a man and a machine this seems all the more obvious, since we manipulate information that only come from natural language and thus, apart from the case when a date is explicitly mentioned, time is essentially seen as a set of situations upon which language only gives relative orders. So, shall we offer a precision to the user that he cannot even perceive?

The last relation, *overlaps*, is puzzling for several reasons. Unlike the others, it is not transitive (inside one single class of relations), and is difficult to perceive in a cognitive point of view. As a matter of fact, when you recall two events that overlap, you situate them by showing the very part common to both. You may mention the beginning of one of them in relation to the end of the other, but in any case, it is necessary to go deeper in the micro-structure of each event to explain this relation. In a first analysis, then, it is not obvious that this relation is fundamental when one deals with time representation.

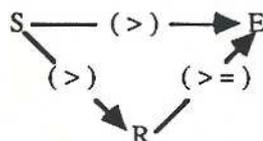
## 2.2 The need for tense interpretation.

Since the only source of temporal information in a dialogue comes from the utterances produced by the speaker, a system for reasoning about time shall include a tense interpreter. Hornstein ([Hornstein 81]) established the base for such a module, as he showed that, with three temporal points, namely S (the moment of speech), R (a temporal reference) and E (the situation described by the utterance), it was possible to construct a formula describing a combination of a matrix proposition, deictics and adverbial phrases. To locate these three moments, he introduces two relations, one that situates a point before the other, and another indicating an ordered proximity (symbolized respectively by ' $>$ ' and ' $>'$ '). For example, the simple past can be represented by the sequence : E,R\_S.

Two main critics can be made to this model of tense representation : First, it is difficult to give a clear meaning to the reference point as an actual temporal object. In fact, it seems that, before all, it is used as a tool for making inferences rather than as a real object. The second point is the relation of association between two time points which leads to some problems when combining several clauses [Yip 85], for it is not commutative, and it expresses a kind of precedence relation. Since this relation can be understood at a linguistic level to be important for being able to represent tense information, it is necessary to define it in another way, so that it will be possible to deal without any problem with such difficult things as the present tense (especially in french and other languages where no progressive form exists) which expresses situations in the near past as well as in the future. Finally, Hornstein's model is of conceptual importance, especially because it integrates the time of the utterance and the temporal meaning of this one into a single representation.

## 2.3 An attempt to combine time, tense and aspect.

As he analyses Hornstein's model for tense representation, Yip interprets it in relation to the interval calculus of Allen. To each sequence of the three points E, R and S he associates a graph of temporal relations between intervals representing those points. Since he reduces his temporal objects to instantaneous intervals, Yip obtains a simplified model to explain each constraint showed by Hornstein for combining utterances. For instance, the diagram obtained for the simple past is as follows :



Yip thus gives a vision of a temporal model for representing tense information that is not essentially based on natural language properties. As a matter of fact, this can be seen as a step towards a cognitive model of time which, if needed, can take into account information taken out of utterances. These principles lie at the root of our temporal model, of which we now present the main characteristics.

### 3. A simplified model of time interpretation.

We propose a model based on few relations and concepts that can however represent most phenomena bound to the design of a man-machine dialogue system. After defining the principal objects that we manipulate, we present the fundamental scheme of analysis based on coherence zones. These zones enable the model to deal with multiple input of information and to analyse utterances of a dialogue as objects among others in the system. Some examples of representation will be taken in the field of natural language understanding. In section 4, we will see how this model leads to a general analysis of temporal learning which is an important point for the understanding of causation.

#### 3.1 The concept of temporal zone.

We will call a temporal zone an object representing a situation that develops along the time dimension. This object is either an elementary zone, we might say a perception such as a phoneme, if we consider it as the finest granularity in a speech understanding system, or a more complicated one that can be defined in several ways. First, it may inherit from a more general class of zones. For example, the temporal zone representing a particular monday may inherit from the object that refers to all the mondays (all that can be called a monday). A temporal zone can also be defined, like in Allen's model, by the relations it establishes with other zones. For example, the day we were just speaking about can be defined approximatively as being the zone following the preceding sunday.

What is the origin of such zones? Two main sources can be observed from an understanding system point of view. A zone can be generated from the system's own subjective experience as a set of coherent data. For example, if it is connected to a vision system, a sequence of images of an object passing through its vision field can make a zone to be created. Even if the system is blind, a dialogue gives several levels along which temporal zones can be created. A word, a sentence, any sub-dialogue in the main dialogue course are coherent temporal zones that can appear in the representation space of a system. The other kind of zone that can be created comes from the understanding of utterances, to which elements of representation can be associated. For example, to the expression "yesterday" we can associate the zone representing the day before that of the utterance. The important point bound to the concept of zone is that time is not a dimension superimposed to a particular situation, but this latter is actually defined by a zone in a temporal point of view. As a consequence, no difference is made a priori between discourse elements (phonemes, words, structures...) and conceptual information (processes, states...) for their representation as temporal zones.

#### 3.2 Relations between zones.

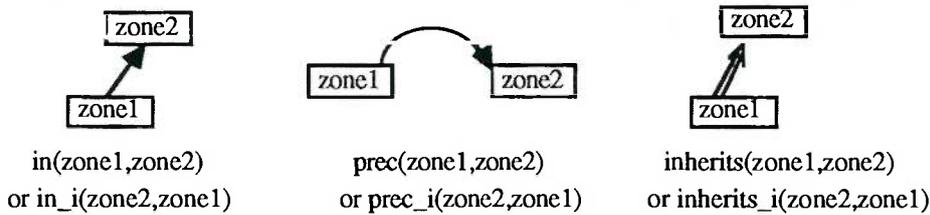
##### 3.2.1 Inheritance.

As we have seen it in the previous section, a zone may inherit some of its temporal properties from one or several other zones. The kind of relation thus defined allows the making of abstractions among zones that share common links. For example, syntactic structures can be seen as general relationships between classes of zones such as words or syntagmatic groups. We will see in section 4.2 how the inheritance relation lies at the root of the mechanism of comprehension.

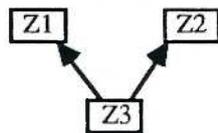
##### 3.2.2 Temporal relations.

We have seen that Allen's relations among intervals could be categorized in three main classes. In fact, we explained why we don't consider the *overlaps* relation as being fundamental in a cognitive point of view. Thus we shall define only two temporal relations

between zones, namely *succession* ( $\text{prec}(\text{Zone1}, \text{Zone2}) \leftrightarrow$  'zone1 precedes zone2') and *inclusion* ( $\text{in}(\text{Zone1}, \text{Zone2}) \leftrightarrow$  'zone1 is in zone2'). As we have mentioned it earlier, these relations are close to the mechanisms observed in human behavior, especially as they only define relative links between two situations to be studied. We will sketch those relations, the inheritance one and their inverses with the following figures :

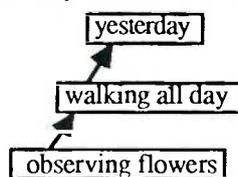


As an illustration, we can represent an extended situation where two zones Z1 and Z2 overlap, by stipulating a third zone Z3 that is included in both as follows :



The succession relation presupposes no range of distance between the two related zones. They can either be very close as in the case of two words in a single utterance, or separated by centuries when dealing with historical events. It only indicates that the information which specifies the location of a zone relatively to another has been considered by the system as being noticeable, either for causation reasons, or simply, because the sequence is recurrently met by the system.

The inclusion relation is complementary to the preceding one, for it gives the possibility of analysing any situation with a finer granularity, thanks to a set of zones to which it is related. This relation is not a simple temporal inclusion between two zones indicating that one situation occurred during another, but it may express the fact that an event is a component of another. Let's take an example : when you assert "Yesterday, I walked all day", you refer to a situation of walking the day before today. Still, you have not actually walked all day, but you stopped to refresh yourself or to observe flowers. This situation is schematized by the following figure where zones are labelled by the discourse elements by which they are referred to:

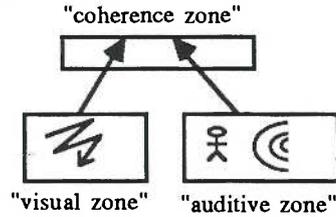


The action 'observing flowers' doesn't prevent you from saying that you walked all day. But, if you need to describe your wander at a lower level, you will have to mention it. Thus, reasoning about time requires such variable depth description of situations, whereas usual logical systems consider all events at the same level.

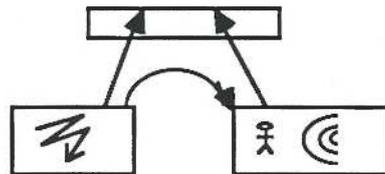
### 3.3 Associating intervals : the coherence zones.

We introduce here a way of seeing zones, not as simple time intervals, but as elements of analysis in a reasoning system. As a matter of fact, when you face several situations that appear in the same period, you feel some difficulties to locate them, unless you go into a precise study of these situations. At a high level of consideration, you can only say that they belong to a common temporal zone, which we will call a *coherence zone*. For example, consider the situation when you are under a stormy sky and you see a flash of

lightning in front of you. By the same time, you hear a loud sound reaching your attention. As they are nearly simultaneous, these two events can be joined under a single zone :



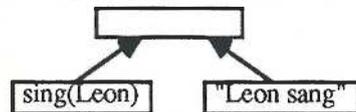
As an extension, two situations, not necessarily being in close temporal relation, can be linked, as soon as they are recognised as making a coherent whole. If you are far enough from the place where the lightning struck the ground, you may hear the sound after having seen it. Thus, some temporal relations may appear in the coherence zone that we have drawn:



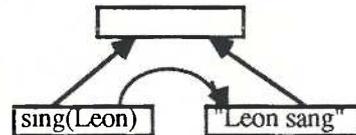
This corresponds to the general link that we could make between one concept and another which indicates it, that is a sign and its signification. A coherence zone is this association along a temporal scale, learned thanks to the system's experience. A special case of such a situation is the analysis of language, which can be seen as the association into a same coherence zone of a part of a linguistic exchange (a dialogue, a sentence, a word or even a phoneme) and a representation of its temporal meaning. We will see in the next section how this can be applied to actual cases.

### 3.4 Time representation in natural language.

Temporal information appear at different levels in a single utterance. Roughly, we can distinguish two kinds of elements that can produce such information. First, we observe locutions that refer to actual temporal zones, either in a lexicalized form as "yesterday", or when a predicate is mentioned which indicates the temporal zone along which it is valid. For example, to the utterance "Leon sang" we associate, through a coherence zone, the temporal zone that bears the predicate 'sing(Leon)' :



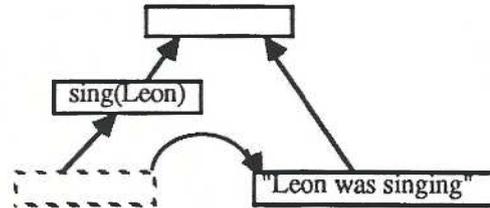
The second kind of information yields the relationships that generated temporal zones set up with the time of the utterance. Typically, tense markers specify the relative precedence between the predicate corresponding to the marked verb and the utterance. Thus, for the preceding example, we have :



A special case of this is the simple present in french, german or japanese, which points out no exact temporal relationship between the utterance and the situation described. It can be easily represented thanks to a coherence zone that simply indicates that the two events

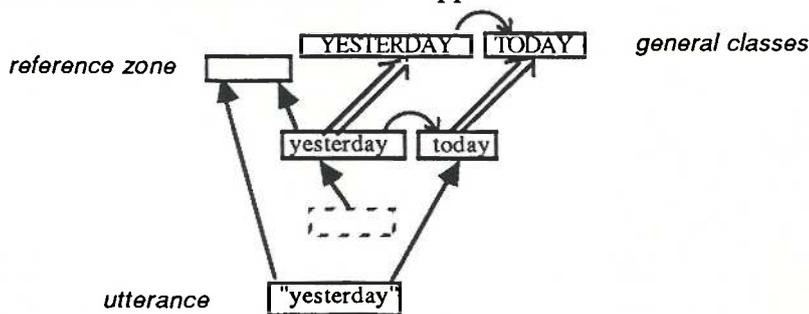
appear in a same situation. Afterwards, contextual information may be instanciated to precise this relationship.

At that point, a notion has to be mentioned which concerns the progressive aspect that is directly marked in English, but only appears in the french tense 'imparfait'. As the progressive form expresses the angle under which you look at a particular event, namely inside this event, it can be represented by the inclusion of a temporal zone in the predicated one. For instance, "Leon was singing" is represented by :

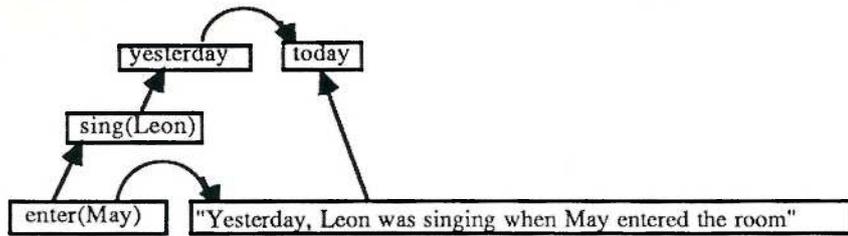


The dotted zone (we will call it an *attention zone*) expresses the fact that the system is waiting for something to happen at the place of this very zone. This means that "Leon was singing" must be followed by something like "...when May entered the room", whereas this was not the case in "Leon sang", which is an homogeneous sentence. We observe here that the temporal succession does not appear between the utterance and the predicate directly. As a matter of fact, the event 'sing(Leon)' may finish far after the time of the utterance.

The origin of the attention zone in the preceding example is a particular case of the general mechanism of instanciation within zones. As a temporal zone inherits relational properties from its ancestors, if one of them establishes a link with a particular zone, this latter is instanciated in the same operation. For example, when you instanciate the zone referred to by "yesterday" (labelled 'yesterday'), you usually attach to it the zone corresponding to the current day (labelled 'today'), where the utterance finds itself. This is shown in the following figure, in which we find again an attention zone since "yesterday" expresses a context where an event is to happen.



Finally, we show the representation obtained for the whole sentence : "Yesterday, Leon was singing when May entered the room", in which we have removed the coherence zones to clarify the drawing. Moreover, each attention zone that we have mentionned for sub-parts of this sentence has been supplied with other zones to take their place. Thus, the sentence is wholly interpreted. With the mechanisms that we will show in the next section, we see that it is possible to infer, in this sentence, that the action : 'sing(Leon)' ends before the time of the utterance.

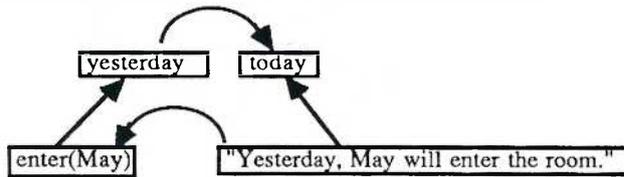


#### 4. Operations on temporal zones.

After having shown how it was possible to represent time information thanks to our temporal zones, it is important to draw the main lines of the kind of inferences that can be done within this model. Two kinds of operations can be distinguished. Those concerning temporal relations right away and assuring that no contradiction exists in the system, and the operations manipulating zones along the hierarchy built by the inheritance relation.

##### 4.1 Temporal coherence.

A contradiction between two incompatible relations can appear in our model when two zones are instantiated to build a new one, preserving the previous relations that each were holding before. As only one relation among the four that we have introduced can exist at a time between two zones, the presence of a contradiction can be detected by propagating the constraints along the intervals until such an incompatible couple of relations is met. For example, the interpretation of the sentence "Yesterday, May will enter the room" shall lead to an inconsistency as shown in the following figure :



A mechanism similar to Allen's is started at that moment to propagate constraints in the candidate for instantiation. However, some fundamental differences exist between the two methods. First, we can see that the reduced set of relations that we manipulate shortens drastically the number of computations that has to be done. We can sketch the associations that can be inferred from two relations  $r1$  and  $r2$  linking three zones  $a$ ,  $b$  and  $c$  thanks to figure 2, close to Allen's table of transitivity relationships.

$a \ r1 \ b \ \backslash \ \begin{matrix} b \ r2 \ c \\ \text{prec} \\ \text{prec}_i \\ \text{in} \\ \text{in}_i \end{matrix}$	prec	prec <sub>i</sub>	in	in <sub>i</sub>
prec	prec	no info	{prec,in} (1)	prec
prec <sub>i</sub>	no info	prec <sub>i</sub>	{prec <sub>i</sub> ,in} (2)	prec <sub>i</sub>
in	prec	prec <sub>i</sub>	in	no info
in <sub>i</sub>	{prec,in <sub>i</sub> } (3)	{prec <sub>i</sub> ,in <sub>i</sub> } (4)	{in,in <sub>i</sub> } (5)	in <sub>i</sub>

Table 4.1, possible deductions for two relations  $r1$  and  $r2$ .  
The numbered inferences are ambiguous (cf Appendix).

If a relation can be inferred, it is propagated, and the computation goes on. When no information is given by the couple of relation, then we do nothing, since no constraint has appeared. In the last case, a subset of the available relations is put forward. However, why shall we infer one or another, since the two graphs we would thus obtain, would

correspond to two completely different visions of the world described ? In fact, it turns out that if, through the course of computation, a relation appears that is not compatible with the restricted set, the following inferences will exhibit the contradiction further between two nodes (the complete proof of this last point may be seen in the appendix). So very few computations are necessary, and it allows quick verification of the possible associations of two sub-graphs of temporal zones.

#### **4.2 Learning and Predicting.**

We have seen that the hierarchy of temporal zones could provide us with general classes of zones that act as templates for their lineage. The generation of zones along the inheritance relations is based on one fundamental mechanism, that is, unification of temporal zones. As two zones are given, this unification tries to superimpose them, as well as the relations they keep up. The successive neighbours of the zones are thus recursively unified, until this process is stopped because no zone is left, or up to a number of zones to be explored when the size of the analysed graphs is too large. Concretely, as information is localized in the hierarchy thanks to coherence zones, the second issue seldom appears.

From the information gained after the unification process, we may either generate a common ancestor to the two groups of zones, which can be a way of abstracting information from experience, or we can create offsprings in the case when we want to increase the information known by the system.

The first case corresponds to the mechanism of learning structures along the temporal relations. This provides the system with the possibility of acquiring syntactic knowledge from the utterances that he has previously analysed, and more generally, it allows the system to memorize temporal sequences or decompositions, such as usual scripts or causation schemes, which need not be specially marked, but are naturally expressed thanks to coherence zones. The second mode for creating temporal zones corresponds to the instantiation of the schemes just mentioned, when the system construes an utterance, or simply when he tries to see the consequences of a particular state or event. This mechanism is done thanks to attention zones that we presented in section 3.4. These zones are predictions for elements that are candidate for instantiation. Such zones remain active until they can be replaced by new information coming from inputs or other inferences.

#### **5. Conclusions.**

We have shown here the main lines of a model for temporal representation, which can also be used, through its learning and predicting abilities, as a base for reasoning about time. This model is now under implementation in Flavors on a Sun workstation, in order to be used in two main applications, namely story understanding and man-machine dialogue system. The first results, especially when dealing with time consistency, proved the efficiency that comes out from using a reduced set of temporal objects and relations.

On a methodological point of view, we have tried to show that temporal problems could be treated with a cognitive approach, in order to put forward what relations are really fundamental when one wants to represent time information. With only inclusion and succession, most phenomena have proved to be representable in a natural way.

The second important thesis that we pointed out is that any system which communicates with a human in natural language shall include in one single representation linguistic structures and the world objects expressed through the user's utterances. This could be the base of a revised reflexion about the links between language and thought.

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## 8. Appendix.

In order to prove that no inconsistency may appear inside a triple of zones with our method, we produce the exhaustive list of the possible cases. So, let a,b and c be three temporal zones, and (r1,r2) a couple of relations, such that : a r1 b and b r2 c. We have seen (cf Table 4.1) that in five cases, there was an ambiguity, that is, only two relations out of four were forbidden between a and c. For each possible situation and for each rejected relation, we have put in table 8.1 the transitions that could be done if the relation appeared, and the incoherence that would thus be generated, given that only one relation can exist between two temporal zones. As a result, we see that our algorithm actually ensure that each triple of zones is coherent in relation to temporal succession and inclusion.

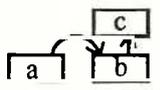
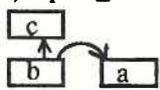
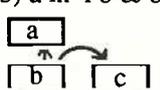
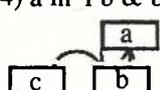
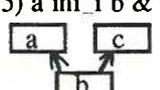
Ambiguous transition and situation scheme	Forbidden arc	Possible inference	Contradiction
1) a prec b & b in c 	a prec_i c  a in_i c	a prec_i c & c in_i b  a in_i c & c in_i b	a prec_i b ≠ a prec b  a in_i b ≠ a prec b
2) a prec_i b & b in c 	a prec c  a in_i c	a prec c & c in_i b  a in_i c & c in_i b	a prec b ≠ a prec_i b  a in_i b ≠ a prec_i b
3) a in_i b & b prec c 	a prec_i c  a in c	a prec_i c & c prec_i b  b in a & a in c	a prec_i b ≠ a in_i b  b in c ≠ b prec c
4) a in_i b & b prec_i c 	a prec c  a in c	a prec c & c prec b  b in a & a in c	a prec b ≠ a in_i b  b in c ≠ b prec_i c
5) a in_i b & b in c 	a prec c  a prec_i c	b in a & a prec c  b in a & a prec_i c	b prec c ≠ b in c  b prec_i c ≠ b in c

Table 8.1