STUDY OF RAINFALL INTENSITIES FOR MICRO-SCALES IN A SEMI-ARID ZONE (TUNIS) BY FIF MODEL

By

Hanen GHANMI[1, 2] Cécile MALLET[2] , Sébastien VERRIER[2] Laurent BARTHES[2] and Zoubeida BARGAOUI[1]

[1] *Laboratoire de Modélisation en Hydraulique et Environnement, Ecole Nationale d'Ingénieurs de Tuni,s Université Tunis El Manar,BP 37 Ecole Nationale d'Ingénieurs de Tunis, Le Belvédère 1002 Tunis (hanen.ghanmi@gmail.com; zoubeida.bargaoui@laposte.net)*

[2] *Université de Versailles Saint-Quentin-; CNRS/INSU, LATMOS-IPSL Laboratoire Atmosphères Milieux, Observations Spatiales,.Adresse : Quartier des Garennes, 11 Boulevard d'Alembert, 78280 Guyancourt, France. (cecile.mallet@latmos.ipsl.fr)*

ABSTRACT

This work focuses on studying the rainfall intensities of high frequencies (less than a few hours). Multifractal fractionally integrated model (FIF) which is a universal multifractal model modified, based on a multiplicative cascade and having three parameters is used. The intensity of rain derived from recorded rainfall data at a resolution 5 minutes, collected between 2007 and 2010 at three stations belonging to the Grand Tunis are the subject of this study. The spectral analysis and the study of the structure function, have allowed detecting the scaling regime corresponding to micro-scales which stops at time scales for around 2h30min. It has proved that the process of rain is not conservative. Multifractal fractionally integrated model (FIF) is then adjusted. However, a problem of bias due to the high rainfall intermittency appears when estimating its parameters. Analysis of the results reveals that the increase of intermittency leads to overestimate the model parameter the codimension C_1, which measures the average heterogeneity of the process and to underestimate the model parameter α which measures the degree of multifractality. To solve this problem, we select continuous sequences, on which we have applied the FIF model, which results in parameters almost identical to those obtained in the literature for very different rainfall patterns.

Keywords: multiplicative cascade, multifractal fractionally integrated model (FIF) , biais, intermittence. Semi-arid Mediterranean climate; rainfall time series; scale-invariance; on-off intermittency; multi-fractal.

1 INTRODUCTION

For two decades, several authors have focused on precipitation in terms of spectral analysis and study of scale invariance as well as the application of the cascade model (Tessier et al., 1993; Ladoy et al., 1993; Fraedrich and Lardner 1993, Hubert et al., 1993, Olsson et al., 1995, Olsson 1996, Tessier et al., 1996, Schmitt et al., 1998; De Lima and Grasman 1999; Pathirana et al, 2003; de Montera et al, 2009; De Lima and De Lima 2009; Gires et al., 2011; Verrier et al., 2010 and 2011). Multifractal models reflect the extreme variability of geophysical fields extending over large ranges of scales (Schertzer 2004). To reduce the number of parameters to a finite number, Schertzer & Lovejoy., (1987) proposed a multifractal model with two parameters, this model is called multifractal universal (MU) model. It is based on a multiplicative cascade whose logarithm generator (known as weight) follows a Levy law. To extend the application of this model to non-conservative process a third parameter, which quantifies the deviation of the phenomenon of a conservative process, is necessary (Bernerdara P. et al 2007). Multifractal Universel (MU) formulation of Schertzer and Lovejoy (1987) has shown its relevance for the rainfall data. However, as highlighted by de Montera et al. (2009) this model is not built to model the intermittency rain- no rain. Unfortunately this is the main problem to solve in our case study because it does not rain for 98% of the time in the Tunis arean with semi-arid climate. In this context, we followed the works of Schmitt et al. (1998), de Montera et al. (2009), Gires et al. (2009) and Verrier et al. (2010, 2011), which showed the effect of intermittent rain non-rain on the estimation of model parameters MU to estimate the unbiased parameters of the model.

2 CONCEPTS ET NOTIONS

2.1 Multifractal Analysis

The multiplicative cascade model suppose that a quantity $\Phi_{\lambda 0}$ (also called "flow" in reference to the turbulence) is transferred through successive iterations, defined on a scale external of width *T* (here we are interested in a temporal process of dimension 1) in a uniform manner. From one level to another, the initial quantity $\Phi_{\lambda 0}$ multiplied by multiplicative increments (which are random variables whose distribution does not depend on the scale) is redistributed in the interval [0, T] to λ_1 (usually two) sub-interval of the same lengths that succeed. The multiplicative increments of passage from one stage to another are subject to a canonical conservation (Mandelbrot, 1974) which ensures the conservation of the mean: $\forall \lambda, \langle \Phi_\lambda \rangle = M$, where *M* is a constant and <.> denotes the statistical average operator. If we densify the cascade, in other terms, we tend to infinity the number of multiplicative steps, to move from one level to another, without changing the largest and the finest scale, we can obtain a continuous cascade. The multiplicative increment between two different scales is then an infinite product of random variables *iid* (identically and independently distributed). Assuming that the cascade is conservative we accomplished the following relation (for further details see de Montera 2009):

$$\forall q, \langle \Phi_\lambda^q \rangle \approx \lambda^{K(q)} \qquad \text{Eq. 1}$$

Where <.> denotes the statistical average operator, *q* is the order of moment and where *K (q)* is called scaling function of moments and it is convex.

The multiplicative cascade generally tends towards a universal class of processes called universal multifractal (UM) (Schertzer and Lovejoy, 1987, Tessier et al., 1993, Lovejoy and Schertzer, 1997; De Montera et al., 2009). If the process is conservative ($H = 0$), its moment function *K (q)* can be described by only two parameters α and C_1:

$$K(q) = \frac{C_1}{\alpha - 1}\left(q^\alpha - q\right) \qquad \text{Eq. 2}$$

Where C_1 (≥ 0) is the fractal codimension which measures the heterogeneity of the field ($C_1 = 0$ for a homogeneous field). The multifractality index α ($0 \leq \alpha \leq 2$) measure the degree of multifractality. $\alpha = 0$ for a process uni / mono-fractal (*β* model). The case in which $\alpha = 2$, corresponds to the maximum multifractality for a log-normal model. The scaling function *K (q)* is a key tool to detect the character of a series multifractal. It is linear when the corresponding process is monofractal, it is determined only by the value of *H*, and otherwise if it is nonlinear, the process is multifractal. Given that most of the geophysical processes are not stationary, Schertzer and Lovejoy (1991) have proposed a modified version of MU for non-conservative fields called fractionally integrated flux (FIF). They added a third parameter, which is the order of fractional integration *H*, the following relationship for the rate process similar in form to the Kolmogorov law for turbulence (de Montera et al., 2009):

$$\left|\Delta R(\Delta t)\right| \approx \Phi_\lambda \left|\Delta t\right|^H \qquad \text{Eq. 3}$$

Where $\Delta R(\Delta t)$ is the average of the absolute value of the gradient, about a time interval of length t (*T/λ*), the parameter H, quantifies the deviation phenomenon vis-à-vis a conservative process. Φ_λ is a conservative cascade, which refers to FIF (see De Montera et al., 2009). The moment of order q of the structure function of rain rate is written as follows:

$$\left\langle \left| \Delta R(\Delta t) \right|^{q} \right\rangle \sim \lambda^{-\xi(q)} \qquad \text{Eq. 4}$$

$\xi(q)$ is known as the exponent of the structure function of order q which has the following form:

$$\xi(q) = qH - K(q) \qquad \text{Eq. 5}$$

2.2 Spectral Analysis

Spectral analysis is a standard technique of treatment and study of the temporal signal of stochastic process with nonlinear variability (Fraedrich and Lardner1993). It constitutes a preliminary step of the multifractal analysis and allows an assessment of the scale invariance (Seuront et al., 1999, Macor 2007). Indeed, a field is characterized by scale invariance if its energy spectrum *S(f)*, which is a representation of the characteristics of order 2 of the series in the frequency space, follows a power law over a wide range of wavenumbers *f*, where β is the spectral exponent which is the negative slope of *S(f) in* log-log graphr (Fraedrich & Lardner, 1993; Bernardara., et al., 2007):

$$S(f) \propto f^{-\beta} \qquad \text{Eq. 6}$$

Schertzer and Lovejoy (1993), showed that the power form of $S(f) \propto f^{-\beta}$ Eq. 6, can be deduced from **Erreur ! Source du renvoi introuvable.** in the case of q = 2. The spectral analysis also allows the characterization of the multifractal process by estimating the parameter *H* presenting the deviation from the field conservation. According to the Wiener-Khintchine theorem (Montera, 2009), the slope of the power spectrum is equal to β (Schertzer and Lovejoy, 1993; de Lima, 1998):

$$\beta = 1 + 2H - K(2) \qquad \text{Eq. 7}$$

If H> 0 we have β> 1 and the process is not stationary, it corresponds to integrated field. Otherwise, if H = 0 and since *K (2)* is always positive, we find a β <1, indicating of a stationary process.

Because sometimes spectral analyzes give a β slopes very close to 1 (upper limit indicative of a conservative process) and that the limits of scaling regimes are linked to subjective visual assessment, it would be interesting study the structure function of the first order which is written as follows::

$$\left\langle \left| R_\lambda(t+\Delta t) - \right| R_\lambda(t) \right\rangle = f(\Delta t) \propto \left| \Delta t \right|^{H} \qquad \text{Eq. 8}$$

By plotting $\langle |\Delta R(\Delta t)| \rangle$ based on Δ*t*, in logarithmic coordinate, we can deduce the *H* parameter, and the increment from which the phenomenon becomes conservative.

3 DATA SETS

The data used for this study come from the database of the General Directorate of Water Resources (DGRE) of the Ministry of Agriculture of Tunisia. The study sites are those of Tunis-Manoubia Mornag and Sidi Thabet. They belong to the network SYCOTRAC

(Collection System real-time hydrological measurement and flood warning for Tunisians wadis). This system was implemented following major damage to the hydrometric network of Medjerda during floods in 2002 and 2003. The data were collected between 1 January 2008 and summer 2010, time series are length of two and a half years. The study sites are equipped with automatic tipping bucket rain gauges. The capacity of a bucket is 0.1 mm of rain. The amount of rainfall is measured by the number of tipping performed by the buckets for a period of 5 minutes, it follows necessarily that the observed data are quantified. As we shall see later, given the non-conservative process, multifractal analysis of the data to the finest scales [5mn-1day] is based on analysis of rainfall series differentiated. Under these conditions the quantification of data is problematic, especially for periods when precipitation amounts measured varied little, the series of volumes of water being quantified, it is then constant 1.2 mm / h in 5 min (a single tipping per period). The differentiation introduces in this case an additional number of zero values. This problem of quantification in the presence of very low rainfall may be compounded by the fact that some water remains 'glued' to the walls and is not measured. A previous study performed on the same data with the box counting method to support data one dimension for time increments lower than 2 days of 0.44.

Table I - Characteristics of the studied stations and data

Station	Long.	Lat.	Alt. (m)	Begin	End	Resolution
Tunis-Manoubia	8Gr, 7060	40Gr, 8711	66	31/12/2007	05/08/2010	5 minutes
Mornag	8Gr, 8500	40Gr, 6600	35	31/12/2007	31/07/2010	5 minutes
Sidi Thabet	8Gr, 5580	41Gr, 0040	20	31/08/2007	14/05/2010	5 minutes

4 DETERMINATION OF SCALING REGIMES, AND THE FACTOR OF NON-CONSERVATION

4.1 Application of spectral analysis on available data

Spectral analysis of complete sets of three stations size of 2.5 years at 5-minutes time step, is preliminary stage intended to observe regimes of scale invariance and the conservative (or not conservative) nature of these regimes. The influence of zeros on the results of the analysis already mentioned in the introduction has led us to use several analysis strategies. In a first time complete sets were analyzed (Table II a), in a second time the analysis was performed on the relatively wet period (except during summer) (Table II b). We selected a series of about eight months (2^{16} * 5 min,). As the rainy season begins practically in September, we chose September the 1st as start date of selected sequences if available data permit. We note that even for so-called rainy periods the percentage of zero is very important which probably explains the similarity of the results obtained with the two approaches. Average spectra (combination of stations) (Figure I) were also analyzed for the series of 2.5 years, and for those lengths of 8 months. The exponent of the spectrum, β (Eq. 11) is estimated for micro-scale range for which the property of scale invariance is identified.

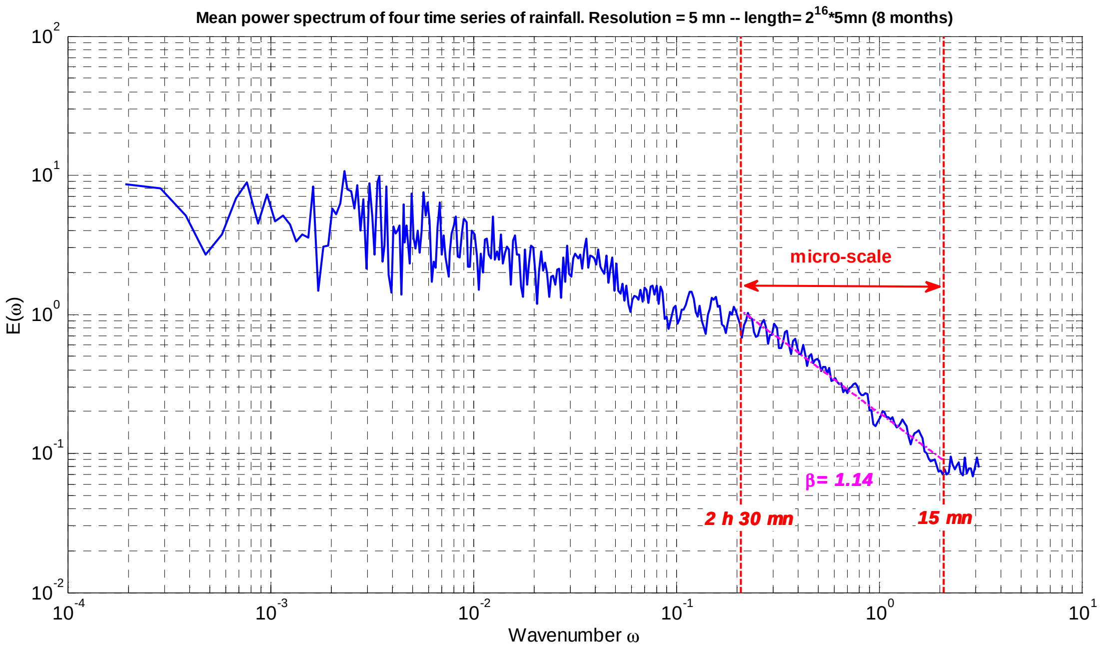

Figure I- Averaged power spectrum of four series of rainfall length of about 8 months, measured at 5 minutes time step

Table II - Spectral slopes β obtained for series at 5 minutes resolution
a) spectra obtained on all available data (2.5 years) b) spectra obtained on eight months of the rainy season (except during summer)

a- Stations	number of observations	micro-scale	Slope	Start date	End date	% of zero
Manoubia	2^{18}	14mn-2h48mn	0,67	31-Dec-2007-07 :00	28-Jun-2010-09 :50	99,17
Mornag	2^{18}	17mn-3h30mn	1,09	31-Dec-2007-07 :00	28-Jun-2010-09 :50	98,99
Sidi Thabet	2^{18}	14mn-2h18mn	1,06	31-Aug-2007-07 :00	26-Feb-2010-10 :50	99,03
Mean spectre	2^{18}	15mn-2h40mn	0,98			

b- Stations	number of observations	micro-scale	Slope	Start date	End date	% of zero
Manoubia 2009	2^{16}	10mn-2h30mn	0,93	01-Sep-2008-07:00	16-Apr-2009-12:40	98,79
Mornag 2009	2^{16}	10mn-2h30mn	0,98	01-Sep-2008-07:00	16-Apr-2009-12:40	98,96
Mornag 2010	2^{16}	15mn-2h	1,27	01-Sep-2009-07:00	16-Apr-2010-12:40	98,24
Sidi Thabet 2008	2^{16}	17mn-2h	1,33	01-Sep-2007-07:00	15-Apr-2008-12:40	98,61
Spectre moyen	2^{16}	15mn-2h30mn	1,14			

According to Figure I, we identify a break of scales located around 2.5 hours and variying from one spectrum to another as shown in Table II. Thus, we distinguish a regime of scaling of micro-scale [5-min 2h30]. Flattening behavior at the end of the spectrum (high frequencies) is observed for the majority of studied sequences (sizes 2.5 years and 8 months) as shown in Figure I. It may be explained by quantization noise mentioned in the preceding paragraph. Most spectra have a spectral slope β of the first regime, greater than 1, indicator of non-conservation process for high frequency. By comparing the spectra related to the series of 2.5

years and those of length of eight months, we notice that the values of the spectral slope β significantly increased. Rupture recorded for a time scale around 2h30mn (from 1 hour to 3.5 hours) reminiscent of Fraedrich & Lardner (1993) observed at 2.4 hours. This scale could be the boundary between the frontal system of convective system, characterized by $\beta > 1$ index of non-stationary. The position of the break as well as the value of the slope differs significantly according to the strategy of analysis used. For series of eight months which excludes the summer, the percentage of zero is only slightly lower, but we observe that the break off scale seems to appear earlier (between 1 and 2.5 hours) still with a slope greater than 1 on average, while on the complete set, break is observed between 2 hours 20 minutes and 3 hours 30 minutes. The mean slope is 0.98. This confirms the possibility mentioned in the work of De Montera et al., (2009) that this break is not due to physical reasons but artificially due to the presence of null values.

4.2 The structure function calculating (parameter *H*)

Since on the micro-scale regime, the spectral analyzes give slopes β very close to 1, upper limit indicator of a non-conservative process, and the limits of scaling regimes are subjective and related to the visual assessment, a study of the structure function of the first order is necessary to infer more precisely the parameter *H*. This parameter quantifies the degree of non-conservation of process. The study results of the parameter H, quantifying the deviation phenomenon compared to a conservative process are illustrated in Figure III in logarithmic coordinates. The slope of the curve corresponds to the exponent H (Equation 13):

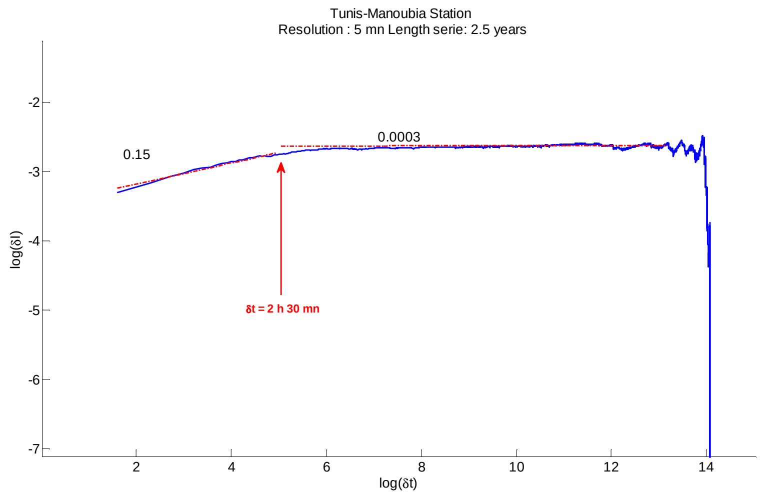

Figure II. Parameter estimate H for Δt = 5mn (λ = 1)

Table III - Summary values H, depending on the resolution for micro-scale

Station	Manoubia	Mornag	Sidi Thabet
Micro-scale=[5mn -2h30 mn]	0,15	0,15	0,13

According to Figure III related to the station Tunis Manoubia the slope of the curve showing $log\left\langle\left|\Delta R(\Delta t)\right|\right\rangle$ versus *log(δt)*, corresponding to *H*, still growing up δt = 2 h 30 min and then stabilizes at zero. Thus, we have two different regimes of both sides of the value corresponding to 2 h 30 min. We are interesting in the first one which correspond to high frequencies and which is not conservative and corresponds to a FIF (H> 0). This result is important because it means that rain rate is not a pure multiplicative cascade but should be (fractionally) integrated to reconstruct the cascade itself. According to Table III, which summarizes the parameter estimates *H* for the three stations, they behave in the same way. These results are generally consistent with those found in the spectral analysis of rainfall data series. For multifractal analysis that follows, the necessity of working only with a number of samples that is a power of 2 will lead us to consider the break at 2h40min, the length of the sequence is $2^5 * 5$ minutes. As we have just seen, these results confirm that there is a regime of micro-scale rain process, which is not conservative as reflected by a spectral slope greater than 1 and a non-conservation parameter *H* nonzero that is in agreement with the work of Fraedrich and Lardner (1993). They found, a spectral slope of 1 for increment of scale repectively lower then 2.4 hours and 1 hour.

5 ESTIMATION OF PARAMETER OF UNIVERSAL MULTIFRACTAL MODEL

Since the process is not conservative on finer scales, to obtain K(q), we have applied the fractionally integrated flux model (FIF), which is a modified version of MU for non-conservative fields. In this case we use the methodology described by De Montera et al., (2009), which consists in modeling the differentiated series. As a first step, we check the relevance regimes of scaling obtained by spectral analysis by checking the linearity of the relationship between $\log\left\langle\left|\Delta R\left(\Delta t\right)\right|^{q}\right\rangle$ and $\log\left(\lambda\right)$ for different values of q. Then we estimate the parameters of the function K(q) obtained by estimating the different slopes of the linear relationship. The coefficients of determination for linear regressions are calculated and a test ensures that they are well above 0.85. As in the preceding paragraph, and given the sensitivity of the estimate to the presence of null values, the study is performed firstly by using sets of two and a half years and secondly using the series 8 months excluding the dry period. To simplify the writing, the results are not significantly different, only those corresponding to the complete series are presented. Once the moment scaling function experimental K(q) is obtained, two estimation methods are implemented for the settings. The first method, called optimization method consists in, directly adjust, the theoretical shape of the pairs (*q*, *K (q)*) obtained by a method of least squares minimization. The second, called derivation method consists in using the first and second derivatives of K(q) for the parameters (Lovejoy et al., 1995):

$$C_1 = K'(1), \quad \alpha = C_1 / K''(1) \qquad \text{Eq. 9}$$

To simplify the writing, the results obtained with both methods are presented only if they are significantly different. Table IV summarizes the results of the series of two and a half years for the three stations.

Table IV - Parameter Estimation of universal multifractal model for series of lengths 2.5 years, with a resolution 5 min for Tunis Manoubia stations, and Mornag Sidi Thabet (rain + no rain) (here we have only considered the method of optimization)

Parameters	C_1			α		
	Manoubia	**Mornag**	**Sidi- Thabet**	**Manoubia**	**Mornag**	**Sidi-Thabet**

5 min -2h40 min	0.53	0.46	0.46	0.06	0.00	0.00
mean	0.48			0.02		

We therefore obtain C_1 equal to 0.48 and α practically zero for micro-scales (5min – 2h40min). The value of C_1 is close to previous studies especially those for which the effect of intermittency has not been taken into account. However, comparing the found value of α to those found by Schmitt et al. (1998), de Lima and Grasman (1999), Pathirana et al, 2003, Montera et al. (2009) and Verrier et al. (2011) allows us to say that our estimate seems not good. This may be explained the one hand by a percentage of null values extremely high due to climate of the region of Tunis and on the other hand given the problem of quantifying the data described in section 3, which artificially increases the number of zero values of discrete series. This results in obtaining an null value for α characteristic of a monofractal process. To overcome the errors introduced by the problems described above, a new analysis of micro-scale regime is performed by selecting shorter series but without zero. Given the necessary differentiation of the series for the study of this regime, the impact of the quantification of the data is very important for low intensity events because it creates artificial zeros, which are added to existing zeros (not rain), which intensify the problem of bias in the estimation of parameters. Since the micro-scale regime can be considered as that which governs the rainfall variability within events, five sequences of continuous rain, sufficiently long and relatively intense therefore were selected. Figure III illustrates the analysis performed from an event, lasting 2h40min, at station Mornag station 23-May-2010. This plot suggests the multifractal process at scales between 5 minutes and 2h40min.

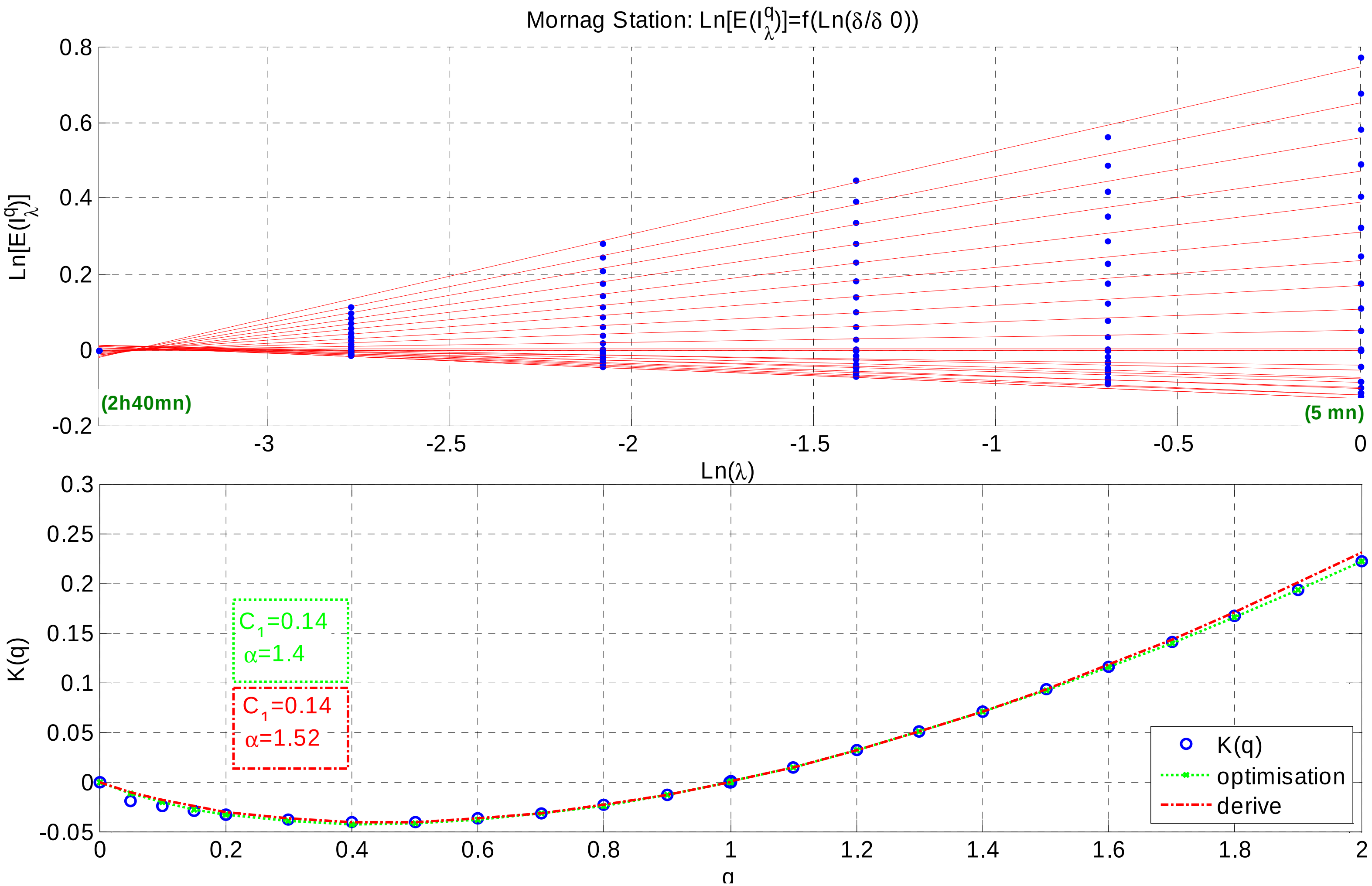

Figure III - The analysis of the scaling function of the sequence of recorded rainfall station Mornag recorded on 23-May-2010 at 13:15 and lasted 2 hours 40 minutes. (Top) The statistical moments of order q = {[0: 0.1: 2] + 0.05 +1.0001 +0.15 0999 (bottom-up)} and (bottom) the moment scaling function K (q) .

Table V - Universal multifractal model parameters estimated on sequences of continuous rain with a time step of 5 minutes

				optimization method		derivation method	
Station	**Start date**	**H_{max}(*) (mm)**	**L**	***C_1***	***α***	***C_1***	***α***
Sidi Thabet	24-Sep-2007 17:00	8.63	2^5	0.28	1.03	0.27	1.36
	13-Jan-2009 02:05	5.59	2^4	0.12	0.85	0.10	1.22
Manoubia	12-Jan-2009 15:20	1.4	2^4	0.08	1.42	0.08	1.50
Mornag	23-May-2010 13:15	9.2	2^5	0.14	1.40	0.14	1.52
	13-Jan-2009 03:40	4.00	2^4	0.13	1.27	0.12	1.46
mean				0.15	1.3	0.14	1.4

*: H_{max} is the maximum height of rain per sequence

The parameters α and C_1 summarized in Table V show an increase of α equal to 1.3 and a decrease of C_1 equal to 0.15 instead of 0.02 and 0.48 previously obtained by analyzing the sequences of rainfall containing more than 98% of null values. These results are consistent with those of Pathirana et al. (2003), de Montera et al. (2009) and Verrier et al., (2011) following the use of sequences of continuous rain. From our study, it appears that the quality of fit is correlated with the maximum heights of rain, which evolves in the same way. This could be explained because when we have rainfall amounts high enough to be free from the problem of quantification and sequence becomes more multifractal. However the variations of C_1remain incomprehensible. This method is difficult to implement in practice, because there is little long events, so little scale range available for continuous events. In addition, there are not enough events strong enough to override the discretization problem, among these some long events. Finally we use very few events, despite the availability of 2.5 years of data, hence the need to propose an alternative.

6 CONCLUSIONS

According to the spectral study and the study of the structure function, scale break is retained for time increments of 2.5 hours. The results for the rainfall series with a resolution of 5 min are close to those of Fraedrich & Lardner (1993) in terms of spectral slope β. For micro-scale(<3 hours), β is generally greater than 1. The application of universal multifractal FIF model on datasets of rainfall intensity in the region of Grand-Tunis characterized by a semi-arid climate resulted in a very intermittent rain (percentage of non rain exceeding 98%) gives a biased estimate of the parameters: overestimation of C_1, the codimension of the singularity that provides the largest contribution to the mean process and underestimation of the α parameter which describes the speed of variation of the codimension according to the singularity. The study of series of rainfall outside summer period showed the extreme sensitivity of the parameter α to the intermittency. To overcome this situation was to select sequences of continuous rain. This method is good but difficult to implement because the scarcity of events witch ara enough strong to override the problem. Finally, we have very few events useable despite 2.5 years of data, so it is necessary to propose an alternative method of investigation. As future work, we think to link the parameters biased of the FIF model to the percentage of zero contained in each sequence to estimate rainfall in late the unbiased parameters of the model.

7 REFERENCES

Bernardara.P.Michel.L.Sauquet.E.Schertzer.D.Tchiguirinskaia.L. (2007). Analyse multifractale en hydrologie. Versailles Cedex: Edition Quae.

DeLima. (1998). Multifractals and the temporal structure of rainfall. PhD Thesis, Wageningen Agricultural University, 229 pp .

DeLima.M.I.P.and.DeLima.J.L.M.P. (2009). Investigating the multifractality of point precipitation in the Madeira archipelago. Nonlin. Processes Geophys , 16, 299–311.

DeLima.M.I.P.and.Grasman.J. (1999). Multifractal analysis of 15-min and daily rainfall from a semi-arid region in Portugal. J.Hydrol.Volume 220, Issues 1-2 , 1-11.

De Montera L., Barthes L., Mallet C and Golé P. (2009) The effect of rain-no rain intermittency on the estimation of the Universal Multifractal model parameters, J. of Hydrometeorology,10,493-506.

Fraedrich.K.and.C.Lardner. (1993). Scaling regimes of composite rainfall time series. Tellus , A (45), 289-298.

Friggit.F.Hubert.P.Carbonnel.J.P. (1991). Caractérisation fractale de la succession des jours pluvieux au Burkina Faso. Utlilisation rationelle de l'eau des petits bassins versants en zone aride , AUPELF-UREF. John Libbey Eurotext. Paris 1991. 43-49.

Gires.A. (2009). Intercomparaison multi-échelle des précipitations du modèle Méso-NH avec des données radar. (Vol. M.S. thesis). Marne-la-Vallée,France.: Ecole des Ponts ParisTech, .

Hubert.P.et.Carbonnel.J.P. (1993). Analyse multifractale et précipitations extremes. IAHS (213), 67-74.

Hubert.P.et.Carbonnel.J.P. (1989). Dimensions fractales de l'occurrance de pluie en climat soudano-sahélien. Hydrol. continent., vol. 4, n° 1 , 3-10.

Hubert.P.Friggit.F.Carbonnel.J.P. (1995). Multifractal structure of rainfall occurrence in West Africa. New Uncertainty Concepts in Hydrology and Water Resources , 109-113.

Kolmogorov.A.N. (1962). A refinement of previous hypotheses concerning the local structure of turbulence in a viscous incompressible fluid at high Reynolds number. J. Fluid Mech , 13, 82–85.

Ladoy.P.Schmitt.F.Schertzer.D.Lovejoy.S. (1993). Variabilité temporelle des observations pluviométriques à Nimes. Comptes Rendues Acad. des Sciences , 317, 775-782.

Lovejoy.S.Pinel.J.Schertzer.D. (2011). The Global space-time Cascade structure of precipitation: satellites, gridded gauges and reanalyses . Advances in Water Resources, (in press).

Macor.J. (2007). Développement de Technique de Prévision de Pluie Basées sur les Proprietés Multi-échelles des Données Radar et Satellites. Thèse de Doctorat: ÉCOLE NATIONALE DES PONTS ET CHAUSSÉES.

Mandelbrot.B. (1974). Intermittent turbulence in self-similar cascades: divergence of high moments and dimension of the carrier. J. Fluid Mech. , 62, 331-350.

Mandelbrot.B. (1977). The Fractal Geometry of nature. Freeman Edition.

Olsson.J. (1996). Validity and applicability of a scale-independent, multifractal relationship for rainfall. Atmos. Res. , 42, 53-65.

Olsson.J.Niemczynowicz.J.Berndtsson.R.and.Larson.M. (1992). An analysis of the rainfall time structure by box counting-some parctical implications. Journal of Hydrology, 137 , 261-277.

Parisi.G.et.Frisch.U. (1985.). A multifractal model of intermittency. Turbulence and predictability. Geophysical fluid dynamics and climate dynamics , 84, North-Holland, New-York.

Pathirana.A.Herath.S.Yamada.T. (2003). Estimating rainfall distributions at high temporal resolutions using a multifractal model. Hydrology and Earth System Sciences , 7, 668-679.

Saito.Y. (1992). Log-gamma distribution model of intermittency in turbulence. J. Phys. Soc. , 61 (2), 403–406.

Schertzer.D. (2004). Quelques réflexions sur turbulences, intermittence et géophysique, les enjeux associés pour le nouveau GDR. Retrieved 11 3, 2011, from http://gdr-turbulence.pmmh.espci.fr/Nov2004/rap_gagne_nov2004.pdf.

Schertzer.D.and.Lovejoy.S. (1993). Nonlinear variability in geophysics: scaling and multifractal processes. Lecture notes. Institut d'études scientifiques de Cargese.

Schertzer.D.and.Lovejoy.S. (1987). Physically based rain and cloud modeling by anisotropic, multiplicative turbulent cascades. J. Geophys. Res , 92, 9692–9714.

Schertzer.D.Lovejoy.S.Hubert.P. (2002). An introduction to stochastic multifractal fields. Mathematical Problems in Environmental Science and Engineering A. Ern and Liu Weiping (eds.), Series in Contemporary Applied Mathematics, vol.4, Higher Educati , 106-179.

Schmitt.F.Vannitsem.S.and.Barbosa.A. (1998). Modeling of rainfall time series using two-state renewal processes and multifractals. J. Geophys. Res. , volume 103, pages 23,181–23,193.

Seuront.L.Schmitt.F.Lagadeuc.Y.Schertzer.D.Lovejoy.S.Frontier.S. (1996). Multifractal analysis of phytoplankton biomass and temperature in the ocean. . Geophys. Res. Lett. , 23:3591–3594.

She.Z.S.Waymire.E. (1995). Quantized Energy Cascade and Log-Poisson Statistics in Fully Developed Turbulence. Phys. Rev. Lett , 74, 262–265.

Tessier.Y.Lovejoy.S.Hubert.P.Schertzer.D.Pecknold.S. (1996). Multifractal analysis and modelling of rainfall and river flows and scaling, causal transfer functions. J. Geophys. Res , 101, 26427–26440.

Tessier.Y.Lovejoy.S.Schertzer.D. (1993). Universal Multifractals: Theory and obsevations for rain and clouds. Journal of applied meteorology , Volume 32, 223-250.

Verrier.et.al. (2011). Multiscaling properties of rain in the time domain, taking into account rain support biases. J. of Geophysical Research .

Yaglom.Y. (1966). The influence of fluctuations in energie dissipation on the shape of turbulence characteristics in the inertial interval. Sov. Phys. Dokl , 11, 26-29.