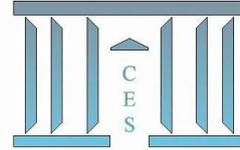




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## **The optimal short-term management of flexible nuclear plants in a competitive electricity market as a case of competition with reservoir**

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# The optimal short-term management of flexible nuclear plants in a competitive electricity market as a case of competition with reservoir.

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## Abstract

In many countries, the electricity systems are quitting the vertically integrated monopoly organization for an operation framed by competitive markets. It therefore questions how nuclear plants should be operated in an open market framework. We address the medium-term horizon of management to take into account the fluctuations of demand according to the seasons of year. A flexible nuclear set (like the French) could be operated to follow a part of the demand variations. Since nuclear plants have to stop periodically to reload their fuel (every 12 or 18 months), we can analyze the nuclear fuel as a stock behaving like a reservoir. The flexible operation of the reservoir permits to get different nuclear fuel allocations according to the different levels of the seasonal demand. We then analyze it within a general deterministic dynamic framework with two types of generation: nuclear and thermal non-nuclear. We study the optimal management of the production in a perfectly competitive market. In this paper, we focus on the optimal short-term (monthly) production behaviour before moving to a yearly or multi-annual optimization. This constitutes a prudent research strategy of a flexible nuclear set leaving the monopoly organization and exploring how to reach a market equilibrium in a competitive market. Then, we set up a simple numerical model (based on data from the French market) given that the nuclear production set is managed in a flexible manner in order to follow the variations in demand (like the French nuclear set actually does).

The marginal cost of nuclear production being (significantly) lower than the one of non-nuclear induces a discontinuity of producer's short-term profit. The problem of discontinuity makes the resolution of the optimal short-term production problem extremely complicated and even leads to a lack of solutions. That is why it is necessary to study an approximate problem (continuous problem) that constitutes a "regularization" of our economical problem (discontinuous problem). The simulations show why future demand has to be anticipated to manage the current use of the nuclear fuel reservoir. Moreover, to ensure the equilibrium between supply and demand, the management of the nuclear set has to take into account the thermal non-nuclear generation capacity.

**Key words:** Electricity market, nuclear generation, competition with reservoir, short-term optimal reservoir operation, electricity fuel mix, price discontinuity.

**JEL code numbers:** C61, C63, D24, D41, L11.

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## Résumé

Dans de nombreux pays, les systèmes électriques sont en train de quitter l'organisation de monopole verticalement intégrée pour une opération encadrée par des marchés concurrentiels. On s'interroge donc sur la façon dont les centrales nucléaires doivent être utilisées dans un cadre de marché ouverte. Nous nous plaçons dans un horizon "moyen terme" de la gestion afin de prendre en considération les fluctuations de la demande selon les différentes saisons de l'année. Un parc nucléaire flexible permet de suivre une partie des variations de la demande. Dans ce cadre, le stock de combustible nucléaire peut être analysé comme un réservoir puisque les centrales nucléaires s'arrêtent périodiquement (tous les 12 ou 18 mois) pour recharger leur combustible. La gestion flexible de ce réservoir permet d'obtenir de profils différents d'usage de combustible nucléaire selon les différents niveaux de la demande saisonnière. On se place dans un cadre dynamique déterministe général avec deux types de production: nucléaire et thermique non-nucléaire. Nous étudions la gestion optimale de la production dans un marché parfaitement concurrentiel. Dans cet article, nous nous concentrons sur le comportement optimale de la production à court terme (mensuel) avant de passer à une optimisation annuelle ou pluriannuelle. Il s'agit d'une stratégie prudente de recherche d'un parc nucléaire flexible qui quitte l'organisation monopolistique et explore la façon d'atteindre un équilibre de marché dans un marché concurrentiel. Ensuite, nous construisons un modèle numérique très simple (basé sur les données du marché français) étant donné que la production nucléaire est gérée de manière flexible afin de suivre les variations de la demande (comme le parc nucléaire français).

Le coût marginal de production nucléaire, étant (significativement) inférieur à celui de production non-thermique, induit une discontinuité de profit à court-terme des producteurs. Le problème de la discontinuité rend la résolution du problème d'optimisation de la production à court terme extrêmement compliquée et conduit même à un manque de solutions. Pour cette raison, il est nécessaire d'étudier un problème d'optimisation approximative (problème continu) qui constitue une "régularisation" de notre problème économique (problème discontinu). Nos simulations expliquent pourquoi il faut anticiper la demande future pour gérer la production actuelle du parc nucléaire. Il faut de plus pour assurer l'équilibre offre - demande prendre en compte les capacités thermiques non nucléaires dans la gestion du parc nucléaire.

**Mots clés:** Electricité, production nucléaire, concurrence avec réservoir, gestion optimale des réservoirs à court-terme, discontinuité des prix.

**JEL :** C61, C63, D24, D41, L11.

# 1 Introduction

Nuclear generation differentiates itself from other technologies by its important fixed cost and low marginal cost (DGEMP and DIDEME (2003), DGEC (2008), MIT (2003, 2009), European Economic and Social Committee (2004)). Consequently, nuclear is deemed to serve “baseload”<sup>1</sup> to target a constant ribbon of consumption at the bottom of the yearly demand and produce at its own maximum capacity. This helps to cover its fixed costs (e.g. United Kingdom, Pouret and Nuttall (2007)). Nevertheless, this mode of operation is not for all countries. An important participation of nuclear in the generation of a country can lead to a different operation. An example of this is France where nuclear generation accounts for 80% of generation. The high share of nuclear in the national mix asks nuclear to not behave as rigid baseload but to adjust its production to (partially) follow the variations of the energy demand (daily and seasonal). This load-following ability of nuclear is crucial in order to ensure the equilibrium of the electricity system.

Furthermore, in numerous countries, electricity systems are quitting the vertically integrated monopoly organization for an operation framed by competitive markets (e.g. European Union). This reopens -both empirically and theoretically- the question of nuclear operation. Economic reasoning supports that in a changing environment, the choice and operation of generation may also change (Glachant and Finon (2003), Chevalier (2004)). Consequently, a question arises: what could be the optimal management of a nuclear set in such a competitive setting? Within this new competitive framework, we address the medium-term horizon (1 to 3 years) of management to take into account the fluctuations of demand according to the seasons of year. In the medium-term, the manager of a large nuclear set (like the French) has to set its seasonal variation of output according to the demand level. We emphasize two stylized seasons: a season of high demand and a season of low demand. In continental Europe, it corresponds respectively to winter and summer. In this medium-term horizon, a core feature of market based nuclear is that the nuclear fuel works as a “reservoir” of energy - partly similar to a water reservoir of hydro energy. Therefore, we need an economic analysis of the operation of nuclear fuel as a “reservoir” to answer this question. A nuclear “reservoir” contains a limited and exhaustible amount of fuel which has to be allocated between winter (season with high demand) and summer (season with low demand). The reservoir nature of nuclear is based on the discontinuous reloading of the reactor. Nuclear units<sup>2</sup> stop only periodically (from 12 to 18 months) to reload their fuel. Then a new period of production (“campaign”<sup>3</sup> of production) starts. The potential length of each campaign of production is related to the quantity of nuclear fuel stored in the “reservoir” at the beginning of the campaign.

When nuclear generation is very high in the energy mix of a country (e.g. France), nuclear ensures the most important part of the domestic demand. Consequently, the manager of the nuclear set is the dominant generator in the national market and has a visible responsibility to ensure market equilibrium. For this reason, reinforcing the flexibility of the nuclear set ameliorates its ability to follow the variations of demand. To decrease the market dominance, the

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<sup>1</sup>Baseload plants are the production facilities used to meet some or all of a given region’s continuous energy demand (called baseload demand), and produce energy at a constant rate, usually at a low cost relative to other production facilities available to the system.

<sup>2</sup>A nuclear unit is the set that consists of two parts: the reactor which produces heat to boil water and make steam and the electricity generation system in which one associates: the turbine and the generator. The steam drives the turbine which turns the shaft of the generator to produce electricity. A nuclear power plant may contain one or more nuclear reactors and hence one or more nuclear units (e.g. Flamanville Nuclear Power Plant) (Source: SFEN).

<sup>3</sup>The length of a campaign (of production) is determined by the maximum number of days during which a nuclear unit produces until exhaustion of its fuel of reloading.

nuclear operator may also choose to decentralize the management of its nuclear units, giving rise to a large number of subsidiary generation units (London Economics, Global Energy Decision (2007), Green and Newbery (1992)). Each subsidiary generation unit operates with a certain amount of nuclear capacity and responds to the total demand. Within this structure, called “divisional”, the general manager of the nuclear set is in charge of the general management and coordination of the company while the generation unit managers are autonomous, having their own operational decisions and aiming to maximize their “costs versus revenue” margin (Chandler (1977)). The “divisional” form has several advantages which justify the choice of this management structure. First, the decentralized production decisions can better satisfy market compliance for electricity. Second, the existence of a general manager that coordinates all divisions contributes to the attainment of other key objectives like nuclear security compliance.

The introduction of competition in the electricity markets may motivate the choice of the nuclear operator to decentralize the management of its generation. The nuclear manager introduces a “virtual” competition in production through its voluntary decentralization. This decentralization leads to the creation of an internal virtual market composed of a number of subsidiary generations units which maximize their profits when meeting customer requirements for electricity.

A constraint to take into account for the optimal operation of market based nuclear plants is the equality between supply and demand at any time. This is due to the following factors: (i) The important size of the nuclear set makes the nuclear operator the principal generator of electricity and hence responsible for the global equilibrium of the electricity system, (ii) The nuclear operator has a state shareholder (in the case of EDF<sup>4</sup> (Électricité de France)). As a consequence the supply-demand equilibrium constraint is also rooted in social welfare. This has to be considered in the management of market based nuclear. In view of these remarks, in the medium-term, the nuclear fuel reservoirs of flexible market based nuclear units have to be managed so that the equality between supply and demand is respected throughout a campaign of production.

We study the optimal management of the fuel reservoir of a flexible nuclear unit given the decentralization of the nuclear generation. We assume the existence of two flexible types of generation: nuclear and non-nuclear thermal. The optimization of the management of the nuclear fuel reservoir is built in two steps. In this paper, we focus on the first step towards the monthly optimization of flexible nuclear plants in a market based electricity system while the second step of yearly or multi-annual optimization will be studied in our next paper (Lykidi, Glachant, Gourdel (2010, 2011)). At the beginning, the nuclear units managers do not know immediately how to reach a market equilibrium in the medium term (all over a fuel campaign). For this reason the subsidiary units reduce their management horizon to that portion of the market being easier to anticipate: the monthly horizon. At this early stage of decentralization of nuclear generation, each subsidiary unit playing in a competitive setting intends to determine a production profile that: (i) respects the constraints imposed by the flexible operation of a nuclear unit and the non-nuclear thermal generation capacity for each month (minimum/maximum generation capacity constraints), (ii) respects the constraints imposed by the inter-temporal management of the nuclear fuel stock over the entire time horizon of production (storage constraints), (iii) respects the constraints induced by the equality between supply and demand at every moment over each month, (iv) maximizes the value of profit for each month. In the second step, the company and hence its subsidiary units being now acquainted with the operation of competitive

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<sup>4</sup>EDF specialises in electricity, from engineering to distribution. The company’s operations include: electric generation and distribution; power plant design, construction and dismantling; energy trading; transport. The company is characterized by the dominance of nuclear power in its production segment. It operates a set of 58 nuclear reactors in France.

electricity markets and nuclear fuel reservoirs on a monthly basis may be interested in increasing their horizon to one or more campaigns of production (typically 36 months). This would permit them to build a long-term optimal management of the reservoir's nuclear fuel stock.

We start in section 2 with the study of the operation of nuclear power plants. We focus on the management of the nuclear generation set in France which is distinct from other countries (like the UK or Sweden) because of the far higher share of nuclear generation in the energy mix (IEA (2008)). This leads to an operation (called load-following operation) very different from the baseload operation since according to it, the amount of electricity supplied by a generating system at any given time (load) follows the predicted evolutions of the energy demand. Then, we proceed with an analysis of nuclear fuel as a "reservoir".

In section 3, we build a model to study the optimal short-term operation of flexible nuclear units in a perfect competitive<sup>5</sup> setting (Ventosa et al. (2005), Smeers (2007)). We start with a description of the model principal assumptions and we then proceed with the modelling of different parameters regarding the electricity demand and the time horizon. In addition, we look at the modelling of the generating units, the production costs, the nuclear fuel reservoir and we determine the constraints resulting from the flexible operation of nuclear units, the non-nuclear thermal generation capacity and the nuclear fuel storage ((i), (ii)).

In section 4, we study the optimal short-term production behaviour. From a purely mathematical point of view, the complexity of our model requires simplifying it through the reduction of its optimization variables. Starting from the original economy with  $N \geq 2$  producers, we can build an alternative economy with a unique producer resulting from the "aggregation" of the  $N$  producers. We show that under the assumption that each producer has the same amount of non-nuclear thermal capacity, an equilibrium of the economy with an aggregate producer can be decentralized as an equilibrium of the economy with  $N$  producers. Conversely, the "aggregation" of an equilibrium of the economy with  $N$  producers leads to an equilibrium of the economy with a single producer. This mathematical "equivalence" of equilibrium between the economy with  $N$  producers and the economy with one aggregate producer enables us to simplify our model by determining an equilibrium within the economy with a unique producer instead of the economy with  $N$  producers. Therefore, we assume the existence of an aggregate producer that holds the total capacity (nuclear and non-nuclear thermal). Thanks to this mathematical proposition, we reduce the number of the optimization variables by working in the case of a single producer (which is very important for the numerical model). Under the assumption of symmetry of the non-nuclear thermal capacities, the equilibrium that we determine in the "aggregate" economy constitutes an equilibrium of the decentralized economy. We should note that logically (mathematically) these simplifications are not basic redefinitions of the treated economical issues. They are only simplifications realized in order to decrease the level of difficulty in resolving these issues.

Then, we present two different approaches to calculate an equilibrium of the optimal short-term management of flexible nuclear plants in a competitive electricity market.

In a first approach, we study the behaviour of supply with respect to the merit order<sup>6</sup> price. We show that, within our model, it is not possible to compute a feasible production vector (it satisfies (i), (ii)) every month that respects the equality between supply and demand by looking

<sup>5</sup>In a perfectly competitive framework, a large number of firms that constitute the supply do not have any affect upon prices. They are considered as "price takers" which means that they "take" the price as it emerges from the equilibrium between supply and demand. Thus, they have no market power.

<sup>6</sup>The merit order is a way of ranking the available technologies of electricity generation in the same order as their marginal costs of production. This ranking results in a combination of different generation technologies to reach the level of demand at a minimum cost. The price in the market is therefore determined by the marginal cost of the "last technology" used to equilibrate supply and demand. This technology is also called marginal technology.

at the value of the merit order price in order to maximize profit. This is because nuclear supply is a correspondence<sup>7</sup> and not a function of the merit order price; several feasible production levels (and not a sole feasible production level) may be associated with the merit order price during a month, when the price is given by the constant marginal cost of nuclear. Thus, every subsidiary unit decides on its nuclear production level among a continuum of feasible production levels to maximize its profit. This ambiguity in the individual choice of a single feasible output level to maximize its own profit leads to the necessity of a coordination of the generation units in order to achieve the equilibrium between supply and demand each month.

In a second approach, we propose a short-term optimization of the production under production constraints imposed by generation capacity, fuel storage and the equality between supply and demand ((i), (ii), (iii)). This optimization problem, called optimal short-term production problem, consists in determining the level of production that maximizes the profit in the course of a month, given the optimal production realized in the previous months. Then, the price is determined by the equality between supply and demand in this month (merit order price rule). The level of stock of the next month is determined by the nuclear supply of the current month. This approach permits us to see how to optimize production on a monthly horizon of operation that is easier to apprehend before proceeding with an inter-temporal optimization of the production. Within this approach, we distinguish two different tactics of resolving the optimal short-term production problem: according to the first tactic, a producer determines an equilibrium of the optimal short-term production problem by taking into consideration the equality between supply and demand only during the current month of optimization without being concerned about this constraint in future months. This tactic simplifies the resolution of this optimization problem since the supply-demand equilibrium constraint is considered only for a month (current month). However, following this tactic, we see (through a numerical example) that when a producer becomes “short-sighted” with respect to future demand, it can not equilibrate supply and demand in future months respecting at the same time the production constraints ((i), (ii)). Consequently, this way to compute an equilibrium of this problem appears empirically as a dead end. For this reason, we carry on with a second and final tactic to calculate an equilibrium of the optimal short-term production problem. According to this tactic, a producer takes into account the supply-demand equilibrium constraint in the optimal short-term management of the reservoir over the entire time horizon of our model.

The marginal cost of nuclear production is (significantly) lower than the one of non-nuclear thermal production which implies a discontinuity of the merit order price on a mathematical level. We prove that this discontinuity of the merit order price leads to a discontinuous profit. In particular, we show that the decrease of market price during months when nuclear turns into marginal technology induces a decrease in the producer’s current monthly profit.

Finally, in section 5, we analyze the production and the nuclear fuel storage decisions of our last approach within a simple numerical model. The problem of discontinuity mentioned above makes the numerical resolution of the optimal short-term production problem extremely complicated and our theoretical approach proves that it even may lead to a lack of solutions. That is why it is necessary to study an approximate problem (continuous problem) that constitutes a “regularization” of our economical problem (discontinuous problem).

In order to collect some basic data to feed our model, we did significant bibliographic research. However, in order to get a complete data set suitable for the numerical modelling, we proceed with an interpolation of the missing data. Finally, we run numerical tests of the model with that set of data using Scilab.

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<sup>7</sup>Correspondence is a mathematical concept that extends the notion of function. With each element of the set of departure, we associate a possibly empty subset of the set of arrival. Numerical optimization of a general correspondence is not possible.

## 2 The nuclear fuel as a “reservoir”

There is no previous theoretical analysis of the operation of flexible nuclear plants in a competitive market while the difficulties of modelling are numerous. It is obvious that gas or coal power stations operate a load follow-up, which implies a variable fuel consumption and supply. This is not the case with nuclear power. The existing economics of nuclear estimate that nuclear plants should always run at full capacity to cover their extremely high fixed costs (Charpin (2000)). In a competitive market, such nuclear plants should roughly be price-taker. This is why nuclear technology is assumed to resemble to the hydro run-of-river<sup>8</sup> because the latter does not try to make any follow-up of load. In the French case, nuclear generation is not of that kind. France is distinct from other countries like UK or Sweden because its far higher generation of nuclear power implies not to run nuclear plants strictly as rigid base load units.

In view of the high fixed costs of nuclear, operators would want their nuclear plants to run constantly at full capacity in order to amortize their fixed costs. Furthermore, within the electricity market, the low fuel and low variable operation and maintenance costs, place nuclear at the bottom of the merit order behind the fossil fuel technologies (i.e. coal, gas, oil, etc.) making nuclear “economically suitable” to operate at full-load (NEA/IEA (2005)). On the contrary, coal, gas and oil having greater variable costs are better suited economically for follow-up load. Consequently, we deduce that nuclear operators would not prefer their units to take part in load-following operations but to be operated as baseload units.

However, when a system has a very large proportion of nuclear power, nuclear power plants must inevitably load-follow. France for instance, constitutes an interesting case very different from other countries like the UK (15.7% of electricity generated by nuclear) or Sweden (38.1% of electricity generated by nuclear) because of the significantly high participation of nuclear in its energy mix. In France, there exist 58 nuclear reactors operated by Electricity of France (Électricité de France (EDF)), being the main producer of electricity, with a total capacity of 63 GW. The share of nuclear in the total electricity generation is around 80% which makes nuclear the main electricity generation technology compared to other countries. Consequently, most nuclear plants have to operate occasionally at semi-base load that corresponds to less than 5000 hours of operation per year and responds to a part of the variable demand. In addition, some plants must be sufficiently flexible (capable to increase or decrease electricity output quickly to follow demand’s variations (daily or intra-daily)) if the French grid operator (RTE) asks them to in order to balance supply and demand and therefore ensure the equilibrium of the electricity grid (Nuttall and Pouret (2007), Bruynooghe et al. (2010)).

From a technical point of view, nuclear reactors of modern design (the third generation and its evolution III+) are capable of a flexible operation (Nuttall and Pouret (2007)). The key to the flexible operation of nuclear power plants is the ability to adjust electricity output quickly, but evenly; that is to say, to adjust output power without overly disturbing the neutron flux distribution within the reactor core (e.g. PWR reactor design (Guesdon et al. (1985))). In fact, this flexibility is primarily due to the new types of fuels which affect the constraints that determine the speed of increase and decrease of production. This type of constraint (called ramping rate constraints) binds the change of operation level of a unit between two successive periods. In principle, all nuclear reactors might reasonably be regarded as having some capacity to follow load. In practice, however, the ability to meet grid needs efficiently and safely is restricted to a certain set of design types (for technical engineering, safety and licensing reasons). The new reactor EPR (European Pressurized Reactor), which is an evolution

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<sup>8</sup>The run-of-river hydro plants have little or no capacity for energy storage, hence they can not co-ordinate the output of electricity generation to match consumer demand. Consequently, they serve as baseload power plants.

of the pressurized water reactor (PWR) with a capacity of 1600 MW, is an example of a III+ generation<sup>9</sup> nuclear reactor which is designed to accommodate load-following operation (Source: AREVA (2005)). In the case of EPR, a load-following operation can be implemented between 25% of nominal capacity (technical minimum) and 100% of nominal capacity (Source: CEA (2009)). In particular, two profiles are provided for load-following: (i) A “light” load-following, between 60 % of nominal capacity and 100% of nominal capacity at the maximum speed<sup>10</sup> of 5% of nominal capacity per minute (ramping rate), (ii) A “deep” load-following between 25% of nominal capacity and 60% of nominal capacity at the maximum speed of 2.5% of nominal capacity per minute (ramping rate).

The monitoring report realized by the French energy regulator (CRE) in 2007 gives an illustration of the operation of the French nuclear set (Regulatory Commission of Energy (2007)). It illustrates the way that the nuclear generation set is managed in France, focusing on the flexibility and the load-following ability which characterize it.

As stated in the report of CRE, the nuclear fleet has been operated at baseload (share of constant consumption throughout the year) and partly at semi-base load (part of the variable consumption): it followed the modulation of supply between seasons and intraday. In view of the significant share of nuclear energy in the total electricity generation in France, the allocation of the shutdowns of nuclear units between high demand seasons (winter) and low demand seasons (summer) during a year is an important issue for the French operator. Therefore, the majority of shutdowns for the reloading of the fuel in the reactors was programmed during the summer season because of the low levels of demand. This permitted the release of the essential nuclear capacity in the winter, when demand was high. In this way, the nuclear industry has contributed, with cogeneration<sup>11</sup>, to respond to the seasonal variations of demand during the year. They also observed that during periods of low demand, the non-nuclear thermal generation technologies have reduced their production to the minimum (especially at night) and thus nuclear frequently became the marginal means of production of the French set. As a result, the marginal cost of nuclear determined the market price during periods of low demand and this led to low electricity prices.

At this point, we assume that we have to distinguish two time horizons of operation of nuclear plants: the short-term and the medium-term. The short-term horizon of operation is related to daily or intra-daily variations of demand while the medium-term horizon of operation is connected to the seasonal variations of demand. The core point of the short-term operation is the daily to intra-daily flexibility of nuclear generation. Can the plant manager adjust its power daily or intra-daily to follow the demand in order to maximize its “costs versus revenue” margin? Of course the nuclear output flexibility depends on the ramping rate constraints that bind the variation of output between two steady production periods. The short-term horizon is therefore organized around a “hard” technological constraint: the operational flexibility of a

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<sup>9</sup>Generation II refers to a class of commercial reactors designed to be economical and reliable for a typical operational lifetime of 40 years (Goldberg and Rosner (2012)). The pressurized water reactor (PWR) is a prototypical Generation II reactor. Generation II systems began operation in the late 1960s. Generation III nuclear reactors are essentially Generation II reactors with evolutionary, state-of-the-art design improvements. These improvements are in the areas of fuel technology, thermal efficiency, modularized construction, safety systems (especially the use of passive rather than active systems), and standardized design for reduced maintenance and capital costs. Improvements in Generation III reactor technology have aimed at a longer operational life, typically 60 years of operation. Their operation period started in the late 1990s. Generation III+ reactor designs are an evolutionary development of Generation III reactors, operating by the year 2010, offering significant improvements in safety over Generation III reactor designs certified by the NRC (Nuclear Regulatory Commission) in the 1990s. The European Pressurized Reactor (EPR) is a Generation III+ reactor.

<sup>10</sup>This is the maximum rate of change of the operation level of an EPR.

<sup>11</sup>The production of electricity and other energy jointly, especially the utilization of the steam left over from electricity generation to produce heat.

given nuclear reactor technology. However, different nuclear reactor technologies have different operational flexibilities. In fact, in France that short-term flexibility is quite high for a nuclear set.

Nevertheless we do believe that the second time horizon - the medium-term - deserves more attention than the short-term. While the short-term horizon is capped by a straight technological constraint, being the operational flexibility of nuclear output, the medium-term horizon appears to be a “pure” question of economic strategy. The core point of the medium-term operation is the seasonal flexibility of nuclear generation. In the medium-term, the nuclear manager has to set its seasonal variation of output according to the demand level. Within this medium-term horizon, the nuclear fuel works as a “reservoir” of energy, partly similar to a water reservoir for hydro energy. In order to understand why nuclear can be viewed as a “reservoir” of energy in the medium-term, it is necessary to know how fuel loading of nuclear reactors is done.

From a technical point of view, the heart of a French-like nuclear reactor consists of a bunch of nuclear fuel bars controlled through neutralizing graphite bars called control rods. The control rods are used to control the rate of fission of uranium and plutonium and thus the rate of heat released from nuclear fission. These reactors stop periodically to reload their fuel and neutralizing bars with an opening of the heart of the reactor. The reloading of a nuclear reactor lasts about 30 days (EDF (2010)). After reloading, a new period (named “campaign”) of nuclear generation starts (see Figure 1). A campaign of a nuclear unit consists of transforming

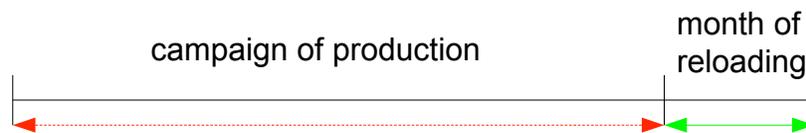


Figure 1: Campaign of production

the potential energy contained in the uranium bars into electricity. The length of a campaign is given by the maximum number of days during which a nuclear unit produces until exhaustion of its fuel of reloading. It generally takes between 12 and 18 months. The regular length of a campaign depends on many factors: technical specificities of the reactor, size, age, management decisions to reload the reactor’s heart per a third or quarter of its full capacity (which implies a length of campaign equal to 18 or 12 months, respectively (Source: EDF)), type of nuclear fuel put into the fuel bars, forecasted average rate of use of the reactor, regulatory constraints issued by safety inspectors, etc. (World Nuclear Association (2006), CEA (2007 and 2008), Bertel and Naudet (2004)).

Reloading of reactors is to be avoided when the level of demand is high (which is winter in France). For operational reasons, the normal duration of a campaign of a nuclear unit is determined in advance in order to get a general scheduling of reloading. The reloading of a nuclear reactor requires the intervention of many qualified persons external to the nuclear operator. It has also to be consistent with the scheduling of all the 58 reactors of the French set. As a result, a nuclear unit has a given horizon to manage its fuel stock.

In view of the operation of reloading of a nuclear reactor, in the medium-term horizon, the nuclear plant manager is allocating a limited and exhaustible amount of nuclear fuel between the different seasons of a campaign, each season having different demand and pricing characteristics. More precisely, we distinguish a season with high demand (respectively high prices) and a season with low demand (respectively low prices). In the European continent, it is correlated respectively with winter and summer. Here a key feature of nuclear fuel as a “reservoir” is based

on the discontinuous reloading of the nuclear reactor. Nuclear units stop only periodically to reload their fuel. Then managers have to determine the length of each campaign of production and hence the amount of nuclear fuel which will be reloaded and stored in the “reservoir” at the beginning of the campaign.

Assuming that nuclear energy has to be sold in the wholesale market, we bet that it will be sold like a stream of “energy blocks”. Energy blocks are fixed quantities sold over a very short period of time at a price determined by the market at each period (then a “spot price”<sup>12</sup> i.e. a market clearing price). The French market has periods of half an hour, which means 48 prices per day, 17520 prices per year. Such spot prices are very volatile from day to day, during the day and throughout the year (see Figure 2, Source : Reuters EcoWin). These 17520 prices are essentially determined by three characteristics (hour, work day as opposed to the weekend or at holidays, monthly components). There is a strong seasonal variation described by the monthly components.

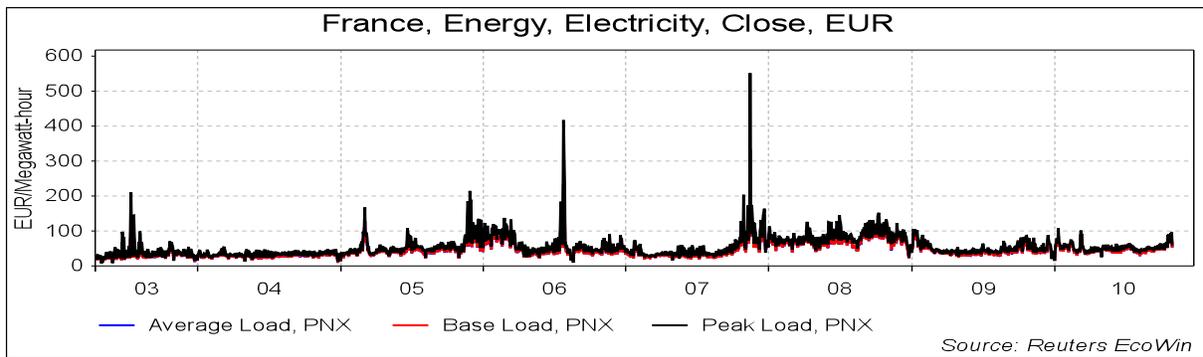


Figure 2: Spot prices on the French market during the years 2003 – 2010.

Of course, the total value of the electricity produced during a campaign of nuclear fuel reservoir depends in a crucial way on the temporal profile of generation and how it can respond to the variation of demand and of market price. In a market based electricity industry, the goal should be the maximization of the value of electricity production. Can the nuclear producer allocate the nuclear fuel of the reservoir to follow the seasonal demand in order to maximize the value of its profit? To answer this question, we can benefit from an analogy with a hydro producer managing its reservoir<sup>13</sup> and having to allocate the water of its basin between different periods of generation. To analyze the management of the reservoir of nuclear fuel, we can draw from the existing literature on the optimal management of hydro reservoirs.

The study of the optimal management of a reservoir in an hydro-electric system could be used for the economical analysis of the operation of the nuclear fuel “reservoir” in market based electricity systems. Articles related to this subject provide us with models that capture the dynamic effect of the hydro-reservoir via a multi-period optimal production problem. An hydro-producer allocates the water (fuel) stored in the reservoir across periods with different demand and pricing characteristics to maximize its profit. Through the different cases (monopoly/duopoly/oligopoly/perfect competition) considered by the authors we can see the

<sup>12</sup>The spot electricity market is actually a day-ahead market, as trading typically terminates the day before delivery. This is due to the fact that the (transmission) system operator (TSO, SO) needs advanced notice to verify that the schedule is feasible and lies within transmission constraints.

<sup>13</sup>We refer to hydroelectric units which dispose a reservoir to store water. These units run only during periods of high demand (peak load demand) for electricity; the reservoir gives them the possibility to modify the electricity production to balance consumer demand.

behaviour of a producer who operates its reservoir either strategically to affect the market price or as a price-taker.

Arellano (2004) and Bushnell (1998) analyze the optimal management of an hydro reservoir and the effects of storage in monopolistic and competitive markets. They quantify the inherent advantages that hydro can have in a competitive market because of storage.

First, we study the general modelling of a mixed hydro-thermal electric system (e.g. market structure, assumptions of the model, exogenous variables, etc.). Arellano (2004) looks at the question of the optimal hydro-scheduling decisions in a monopolistic and a duopolistic power industry and Bushnell (1998) analyzes this question in a deregulated oligopolistic electricity market. The perfect competitive case is also examined by both authors through the existence of small producers acting as price-taking suppliers. Their principal assumption is that the water (fuel) inside the reservoir permits production during a period of time (similar to a campaign of nuclear production). At the end of this period, the water is entirely used and production stops. The hydro-thermal capacities are exogenous variables in their models. Then, we are interested in the construction of the optimization problems and in particular we regard the production constraints which are taken into consideration by the manager of the hydro-reservoir and the methods used to resolve these optimization problems. In both articles, a producer determines an optimal level of hydro-production at each time period of the game by maximizing its inter-temporal profit over the entire time horizon of the model under minimum and maximum hydro-thermal generation capacity constraints and a hydro (fuel) storage constraint. According to this constraint, the aggregate hydro-production realized during the entire time period of the model equals the total quantity of water (fuel) stored in the reservoir. The authors apply the Karush-Kuhn-Tucker<sup>14</sup> (KKT) conditions in order to determine a solution of the optimal inter-temporal production problem in each case (Mas Colell et al. (1995)). Scott and Read (1996) also propose a modelling of the optimization of hydro reservoir operation in a deregulated market with both hydro and thermal capacity. Ambec and Doucet (2003) study the effects of monopolistic behaviour and of decentralized decision-making in competition on the management of hydro-ressources within a dynamic model.

Another approach that is not related to the framework of our study but could be considered in the future is the one presented by Crampes and Moreaux (2001, 2008). The authors focus on the question of how competition in electricity markets works when hydro and thermal units belong to separate owners. They explore the first-best dispatching and the monopoly and the duopoly equilibrium in an economy where the two technologies (hydro - thermal) compete.

There are however differences between the nuclear plants and the hydro storage stations with respect to the characteristic of the “reservoir”. An important point of differentiation is the timing of reloading of the “reservoir”. In the case of nuclear, the timing and the frequency of reloading of the reservoir depends on the producer since it is responsible for the allocation of shutdowns of the nuclear units between high and low demand seasons and thus for the scheduling of fuel reloading. On the contrary, hydro reservoir stations cannot choose when and how much to reload; only capricious rain will do it when enough has fallen (they have a very typical “seasonal reloading”). Another difference is that during the time of reloading of the nuclear “reservoir”, a unit does not produce. This is because the heart of the nuclear reactor opens during the reloading in order to replace some of the used nuclear fuel bars with new ones. Nevertheless, hydro reservoir stations do not stop during the reloading of the reservoir.

A seasonality of reloading has also to be considered in the nuclear case. A “good” seasonal allocation of the shutdowns of nuclear units consists of avoiding shutdowns in high demand (win-

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<sup>14</sup>In mathematics, the Karush-Kuhn-Tucker (KKT) conditions are first order necessary for a solution in nonlinear programming to be optimal, provided that some regularity conditions (or constraint qualifications) are satisfied (Bazaraa et al. (1993), Leonard and Van Long (1992)).

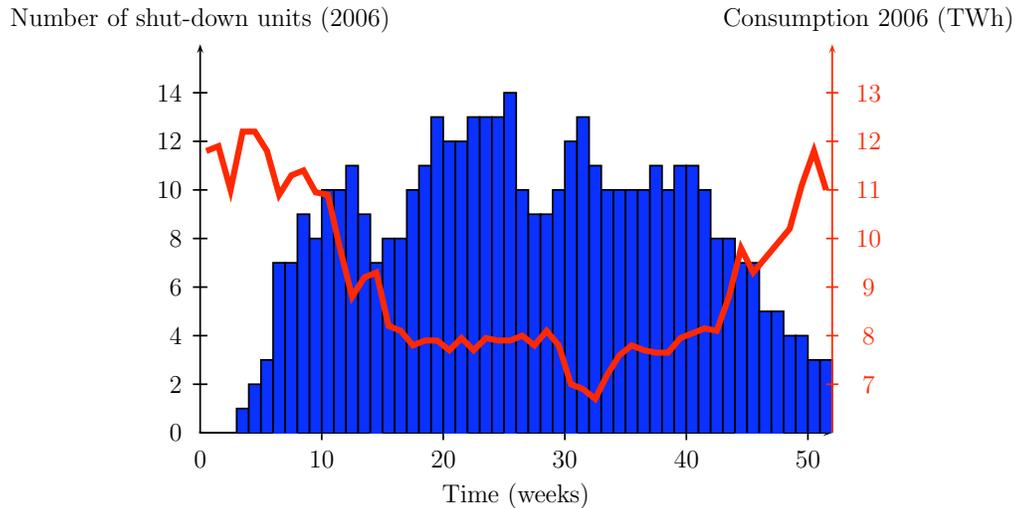


Figure 3: Availability of nuclear units, Source: EDF (2006)

ter) and concentrating them as much as possible in low demand (between May and September) (see Figure<sup>15</sup> 3). Thus, the producer takes into account the level of demand when it chooses when to reload the heart of the reactor. A fundamental point of the optimization of the French nuclear set is therefore the allocation of the shutdowns. Their timing and frequency determine the length of the campaigns for nuclear units.

To conclude, we note that a focus on the medium-term horizon of operation of flexible market based nuclear plants leads us to look at the seasonal flexibility of nuclear generation. In the medium-term, a flexible nuclear set is operated to follow a part of the seasonal variation of the demand level between seasons of high demand and seasons of low demand. In this context, nuclear fuel stock can be analyzed as a “reservoir” given that the nuclear units stop periodically (every 12 or 18 months) to reload their fuel. The operation of the reservoir allows different profiles of nuclear fuel uses during the different seasons of the campaign. A producer would then like to determine the temporal profile of generation to respond to the variation of demand and of price during a campaign in order to maximize the value of its production. To analyze the management of the nuclear fuel reservoir, we rely on an analogy with an hydro reservoir and we look at the literature related to the optimal scheduling of hydroelectric resources. Nevertheless, our analysis takes into consideration the inter-temporal management of the nuclear fuel stock during a period of production that consists of several campaigns as well as the constraints imposed by the flexible operation of the nuclear plants and the balancing of supply and demand due to the very high reliance in nuclear energy. Moreover, there exist differences between these two generation technologies (nuclear and hydro) regarding the characteristic of the “reservoir”. These differences mainly concern the timing and the frequency of reloading as well as the fact that production is interrupted during the period of reloading in the case of nuclear. However, a common point is the seasonality of reloading which has to be considered by the nuclear manager in both cases.

<sup>15</sup>Each blue bar shows the number of shut-down units during a week and the red line shows the evolution of the consumption over time. The different levels of consumption are measured on the right axis while the number of shut-down units is reflected on the left axis.

### 3 Model: Perfect competitive case

In this section, we describe our general deterministic model of a perfectly competitive electricity market where there exist two types of generation: nuclear and thermal non-nuclear. We assume a perfect competition according to which firms treat price as a parameter and not as a choice variable. Price taking firms guarantee that when firms maximize their profits (by choosing the quantity they wish to produce and the technology of generation to produce it with) the market price will be equal to marginal cost. This general framework is also characterized by perfect equilibrium between supply and demand and perfect information among producers. First, our modelling aims at determining the optimal short-term management of a flexible nuclear generation set in that competitive regime. We want to look out to the medium-term horizon which is characterized by the seasonal variation of demand between winter and summer. Second, there are production constraints imposed by the flexible operation of nuclear units, generation capacity and fuel storage that play a central role in determining the equilibrium outcomes in this wholesale electricity market.

We do not take into consideration the electricity importations/exportations within our model for simplicity reasons and in the absence of access to detailed data. However, in the theoretical case that electricity importations/exportations were part of our modelling, they could be considered either exogenous or endogenous to our model. If they were exogenous then the demand would be translated by the production output which is imported/exported that would not change our modelling. On the contrary, in the case that they were endogenous, the complexity of the modelling would augment since a number of new parameters has to be considered in our model e.g. technical constraints imposed by the transmission power lines, the price elasticity of foreign demand, etc.

#### 3.1 Modelling the demand

The demand, being exogenous, is considered perfectly inelastic which is clearly a simplification. However, it can be driven by some arguments. In the short-term to medium-term the demand is less responsive to price variations because it is already determined by preceding investments in electrical devices and lifestyles whose evolutions require time. Electricity is sold to consumers<sup>16</sup> by retailing companies. There is no bilateral contracting regime between retailers and producers. The wholesale spot prices are paid by the retailers directly to the producers.

Moreover, the level of the monthly demand is translated by the monthly hydro production resulting from the run-of-river hydro units. The run-of-river hydro units have little or no capacity for energy storage, therefore they can not co-ordinate the output of electricity generation to meet consumer demand. Consequently, they are used as base load power units. Since the hydro technology with no reservoir (run-of-river) is a base load generation technology which is presumably never marginal, it is necessary to choose nuclear to satisfy the various levels of demand. We do this translation in order to withdraw the part of the base load demand served by the run-of-river hydro units and hence to get a fully vision of the demand which will be served by the nuclear and non-nuclear thermal units. The seasonal variations of hydro production due to precipitation and snow melting are not taken into account. As a result of that we assume that the monthly hydro-production is constant through the entire time horizon of our model.

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<sup>16</sup>In fact in the French case, most of the consumers and an important part of the firms have a fixed price contract, being a regulated price contract set by the government and precisely by the French energy regulator (CRE) and the Ministry in charge of economy and energy. A French “market based” contract follows the evolution of the wholesale market price while a French fixed price contract does every twelve months.

We do not take into consideration the capacity derived from hydro units with possibility of storage (peaking<sup>17</sup> power plants) because of the supplementary capacity and storage constraints that would add to the complexity of the model.

### 3.2 Modelling the time horizon

The time horizon of the model is  $T = 36$  months<sup>18</sup> beginning by the month of January. We select a time horizon of 36 months for our modelling because we need a sufficiently extended time horizon to follow up the evolution of the optimal production output and storage fuel levels as well as the variations of the price and of the profit. We suppose that the value of profit is not discounted during the period  $T$  so that our model remains simple. In order to be consistent with this absence of taking into account the discount rate, we do not choose a longer time horizon. We can unquestionably take into consideration longer-lasting periods but the model will be less pertinent.

We now carry on with the modelling of the time horizon of the campaign of production. The period of a campaign is given by the maximum number of days during which a nuclear unit produces until exhaustion of its fuel of reloading. A French nuclear producer has two principal options with regard to the scheduling of fuel reloading: (i) 1/3 of fuel reservoir that corresponds to 18 months of campaign and 396 days equivalent to full capacity for a unit of 1300 MW, (ii) 1/4 of fuel reservoir that corresponds to 12 months of campaign and 258 days equivalent to full capacity for a unit of 1500 MW (Source: EDF and CEA (2008)). Note that both options come from the operational schema of EDF (Electricité de France). The scheduling of fuel reloading is entirely exogenous within our model because the regular length of a campaign depends on many factors (technical specificities of the reactor, size, age, management decision to reload the reactor's heart per a third or quarter of its full capacity, type of nuclear fuel put into the fuel bars, forecasted average rate of use of the reactor, regulatory constraints issued by safety inspectors...) which are hard to manage in order to endogenously determine the duration of the campaign within our model. The capacity of nuclear units as well as the capacity of non-nuclear thermal units are also exogenous in our model. Additionally, the quantity of the fuel reloaded in the reservoir is exogenous and is decided exclusively by EDF because of the exogenous scheduling of the fuel reloading (per third, per quarter of fuel reservoir) and the exogenous nuclear capacity. In order to get a tractable model, we need a cyclic model for the modelling of the campaign. We do not include the case of having both a campaign of 12 and of 18 months to avoid make complicated our model. We do not maintain the first modelling, thus a campaign of 18 months since it is not compatible with the "good" seasonal allocation of shutdowns of the nuclear units. As expected, if a nuclear operator reloads fuel in summer when the demand is low the date of the next reloading will be then in winter when the demand is high. Therefore, we retain a modelling close to the second modelling, so a duration of campaign equal to 12 months to obtain a cyclic model with a periodicity of one year. The one year period can be then broken down into 11 months being the period of production and 1 month corresponding to the month of reloading of the fuel.

We do not treat the question of the optimal allocation of the shutdowns in this paper for several reasons: (i) absence of operational data for confidentiality reasons, (ii) the length of the campaign is determined in advance in order to get a general scheduling of reloading for operational reasons and because of the intervention of many qualified persons external to the

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<sup>17</sup>Peaking power plants are power plants that generally run only when there is a high demand, known as peak demand, for electricity.

<sup>18</sup>The time horizon of the model is a multiplicative of twelve, being expressed in months. Therefore it could be modified.

nuclear producer for a reactor's reloading, (iii) lack of information with respect to the periodical inspections of nuclear reactors and the inspections imposed by the Nuclear Safety Authority. For these reasons, we assume that the refueling dates are exogenous within our model; they are decided through a programming realized by EDF (model ORION) which determines the optimal allocation of the shutdowns of nuclear units for reloading.

### 3.3 Modelling the generating units

We assume the existence of 12 types of nuclear generating units while we have only one type of non-nuclear thermal generating units. A producer operates with non-nuclear thermal generating units and with one or more types of nuclear generating units. Each type of nuclear units has its own single cost function. Moreover, we assume that these types of units differ by the available nuclear capacity that each of them holds as by the month of their fuel reloading (see Table 1, Section 8). We can then define twelve different "types" of nuclear units. Each type indexed by  $j = 1, \dots, 12$  corresponds to a different month of reloading of the nuclear unit. Then, a unit which belongs to the type of unit  $j = 1$  (respectively  $j = 2, \dots, j = 12$ ) shuts down in the month of January (respectively February,  $\dots$ , December).

Later, we will prove that, under some assumptions, an equilibrium of the original economy with  $N \geq 2$  producers is "equivalent" to an equilibrium of the alternative economy with a single producer. From a purely logical (mathematical) perspective, we use this proposition to reduce the number of optimization variables of our model and thus the degree of difficulty in determining an equilibrium of the original economy by working in an economy with one aggregate producer from now on. The aggregate producer holds the capacity of all the types of nuclear units as well as the total non-nuclear thermal capacity. In particular, this will avoid (most of the time) the notation  $q_{njt}^{nuc}$  that represents the level of the nuclear production during the month  $t = 1, \dots, T$  for the unit  $j$  of producer  $n = 1, \dots, N$  and the notation  $q_{nt}^{th}$  which symbolizes the level of the non-nuclear thermal production during the month  $t = 1, \dots, T$  of producer  $n = 1, \dots, N$  (and similarly for the stock).

The level of the nuclear production during the month  $t = 1, \dots, T$  for the unit  $j$  will be denoted by  $q_{jt}^{nuc}$ . Furthermore, the maximum nuclear production that can be realized by the unit  $j$  during a month is given by the parameter  $Q_{max}^{j,nuc}$ , while the minimum nuclear production is equal to  $Q_{min}^{j,nuc}$  (see Table 2).

Symmetrically, the level of the non-nuclear thermal production during the month  $t = 1, \dots, T$  will be denoted by  $q_t^{th}$ . Furthermore the maximum non-nuclear thermal production during a month is given by the parameter  $Q_{max}^{th}$  and corresponds to the nominal non-nuclear thermal capacity, while there is no minimum for non-nuclear thermal production  $Q_{min}^{th} = 0$  (see Table 2).

### 3.4 Modelling the production costs

The nuclear cost function is made of a fixed part determined by the cost of investment, the fixed cost of exploitation and taxes and a variable part which corresponds to the variable cost of exploitation and the fuel cost (see Table 3). We assume that the cost function<sup>19</sup>  $C_j^{nuc}(\cdot)$  of the nuclear production is linear and defined as

$$C_j^{nuc}(q_{jt}^{nuc}) = a_{nuc}^j + b_{nuc} q_{jt}^{nuc}.$$

<sup>19</sup>The coefficient  $a_{nuc}^j$  is proportional to the capacity  $Q_{max}^{j,nuc}$  since it corresponds to the fixed part of the nuclear cost function.

The non-nuclear thermal cost function is also made of a fixed part which corresponds to the cost of investment, the fixed cost of exploitation and taxes and a variable part covering the variable cost of exploitation, the fuel cost, the cost of CO<sub>2</sub> as well as the taxes on the gas fuel (see Table 3). We assume that the non-nuclear production has a quadratic cost function  $C^{th}(\cdot)$  which is the following

$$C^{th}(q_t^{th}) = a_{th} + b_{th}q_t^{th} + c_{th}q_t^{th^2}.$$

The nuclear and non-nuclear cost functions are monotone increasing and convex functions of  $q_{jt}^{nuc}$  and  $q_t^{th}$  respectively. We choose a quadratic cost function in the case of non-nuclear thermal because of the increasing marginal cost of the non-nuclear production since it results from different fossil fuel generation technologies (e.g. coal, gas -combined cycle or not-, fuel oil). Furthermore, the non-nuclear production needed a non constant marginal cost function in order to recover its fixed costs. Indeed, if we assume a constant marginal cost function for non-nuclear thermal then the value of the non-nuclear thermal production when non-nuclear is the marginal technology does not permit to recover its fixed costs. So, we assume that the marginal cost of nuclear  $mc^{nuc}(q_{jt}^{nuc})$  is a constant function of  $q_{jt}^{nuc}$  while that of the non-nuclear thermal  $mc^{th}(q_t^{th})$  is an increasing function of  $q_t^{th}$ .

### 3.5 Modelling the nuclear fuel stock

Let us denote  $S_{reload}^j$  the nuclear fuel stock of reloading<sup>20</sup> available to the unit  $j$ . The variable  $S_t^j \geq 0$ , which represents the quantity of fuel stored in the nuclear reservoir and available to the unit  $j$  at the beginning of the month  $t$ , is the potential energy that can be produced with this stock. The evolution of the nuclear fuel stock is then determined by the following rules:

$$S_1^j \text{ given, } S_{t+1}^j = \begin{cases} S_t^j - q_{jt}^{nuc}, & \text{if no reload during month } t \text{ for unit } j \\ S_{reload}^j, & \text{if unit } j \text{ reloads during month } t \end{cases} \quad (1)$$

The relationship 1 traces the evolution of the stock given the flow of the nuclear production. In the case that  $t$  is the month during which a producer reloads the fuel of the reactor, the stock at the beginning of the following month (beginning of the campaign) is equal to  $S_{reload}^j$ .

Moreover, we impose

$$S_{T+1}^j \geq S_1^j \quad (2)$$

The constraint (2) implies that a producer must keep its nuclear units at the end of the game at the same storage level as the initial one. A producer has to finish the period  $T$  at least with the same quantity of nuclear fuel as the initial one. In this way the producer has to “spare” its nuclear fuel during the production period. The absence of this constraint could lead to an “over-consumption” of the nuclear fuel stock in order to reach the maximum nuclear production level; this could generate some negative effects (e.g. insufficient level of stock to reach at least the minimum nuclear production level during some months (excluding the month of reloading)). In addition to this, the constraint (2) guarantees that the producer will start a new cycle of this game with a quantity of stock equal to  $S_1^j$  at the beginning of the game. Such a constraint is implicit if the end of period  $T$  coincides with the end of the campaign.

Let us notice that a producer spends all its nuclear fuel stock of reloading  $S_{reload}^j$  during a campaign (11 months), thus it disposes of a quantity of nuclear fuel stock equal to zero at

<sup>20</sup>The nuclear fuel stock of reloading ( $S_{reload}^j$ ) is given by the number of days (being expressed in hours) equivalent to full capacity during a campaign multiplied by the capacity of the unit  $j$ . We recall that this number corresponds to 258 days in our modelling (see subsection 3.2, Table 2).



$$\left\{ \begin{array}{l} \sum_{t=1}^{11} q_{12t}^{nuc} = S_{reload}^{12}, \quad \text{so that unit 12 uses stock of reloading from month 1} \\ \sum_{t=13}^{23} q_{12t}^{nuc} = S_{reload}^{12}, \quad \text{so that unit 12 uses stock reloaded during month 12} \\ \sum_{t=25}^{T-1} q_{12t}^{nuc} = S_{reload}^{12}, \quad \text{so that unit 12 uses stock reloaded during month 24} \end{array} \right. \quad (6)$$

### Proof

This proposition is an obvious corollary of Proposition 2.2, which is proved in subsection 2.5 of our paper (Lykidi, Glachant, Gourdel (2011)), since it is a particular case when  $N = 1$ .  $\square$

Subsequently, the length of a campaign will also correspond to the maximum number of days that a nuclear unit produces until the “available to the unit” nuclear fuel stock is exhausted.

## 4 Equilibrium and approaches of calculation

In this section, we introduce the notion of a merit order equilibrium and we extend it to the case of several producers. Under some assumptions, we show the “equivalence” of merit order equilibrium between an economy with  $N \geq 2$  producers and an economy with one aggregate producer. Then we present our different approaches to calculate a merit order equilibrium. In our first approach, we study the supply behaviour with respect to the merit order price. We show that supply being a correspondence and not a function of the merit order price can not be computed since a set of different feasible production levels (and not a single feasible production level) are associated with this price. Therefore, the determination of the equilibrium merit order price<sup>22</sup> by calculating a feasible supply level and by looking at the equality between supply and demand at each month is not possible. Consequently, we conclude that this first approach leads to a failure to calculate a merit order equilibrium. Then, we proceed with our second approach where a producer maximizes its profit in the month  $t$  by taking into consideration generation capacity constraints over the entire period  $T$  and the inter-temporal nuclear fuel storage constraints for all nuclear units  $j$  as well as the supply-demand equilibrium constraint during the month  $t$ . We show empirically that this approach does not permit the calculation of a merit order equilibrium because the nuclear manager is “short-sighted” with regards to future demand and does not succeed to equilibrate supply and demand in future periods without violating the production constraints. For this reason, we proceed with our third and last attempt to calculate a merit order equilibrium. In this approach, future demand is sufficiently taken into consideration in the optimal short-term management of the nuclear fuel reservoir so that the equality between supply and demand is respected over the entire time horizon of the model.

### 4.1 The notion of merit order equilibrium

Let us introduce the definition of a merit order equilibrium with respect to a system of prices  $p \in \mathbb{R}^T$ .

<sup>22</sup>We recall that the merit order price is determined by the marginal cost of the marginal technology which is the “last technology” of the merit order used to equilibrate supply and demand.

**Definition 4.1** The production vector  $\bar{q} = ((\bar{q}_{jt}^{nuc})_{j=1}^J, \bar{q}_t^{th})_{t=1}^T$  is a merit order equilibrium with respect to a system of prices  $p \in \mathbb{R}^T$  if:

(i)  $\bar{q}$  is a feasible production vector: (a) it respects the nuclear fuel constraints, for all  $j$  and (b) it respects the minimum/maximum production constraints, for all  $j, t$ .

(ii) the price at each month  $t$  is determined by the marginal cost of the marginal technology. It is called the merit order price associated with the production vector  $\bar{q}$ .

(iii) at each date  $t$ , it respects the equality between supply and demand

$$\sum_{j=1}^J \bar{q}_{jt}^{nuc} + \bar{q}_t^{th} = D_t - Q_t^{hyd}. \quad (7)$$

We recall from subsection 3.1 that the monthly demand which is considered in this model results from the difference between the level of demand  $D_t$  observed in month  $t$  and the hydro production  $Q_t^{hyd}$  provided during the month  $t$ .

Furthermore, in our model, the merit order price  $p$  associated with a feasible production vector  $q$ ,  $p = (p_t)_{t=1}^T = (\Phi_t(q_t))_{t=1}^T = (\Phi_t((q_{jt}^{nuc})_{j=1}^J, q_t^{th}))_{t=1}^T = \Phi(q)$  is calculated in month  $t$  as follows:

$$p_t = \Phi_t(q_t) = \begin{cases} mc^{th}(q_t^{th}), & \text{if } q_t^{th} > 0 \\ mc^{nuc}(q_{jt}^{nuc}), & \text{if } q_t^{th} = 0 \end{cases} = \begin{cases} b_{th} + 2c_{th}q_t^{th}, & \text{if } q_t^{th} > 0 \\ b_{nuc}, & \text{if } q_t^{th} = 0 \end{cases} \quad (8)$$

If nuclear is the “last technology” of the merit order which is called to equilibrate supply and demand during the month  $t$  ( $q_t^{th} = 0$ ), then the price is determined by the marginal cost of nuclear  $mc^{nuc}(q_{jt}^{nuc})$ . In the case that the non-nuclear thermal production is additionally used to cover the monthly levels of demand ( $q_t^{th} > 0$ ), the price is given by the non-nuclear thermal marginal cost  $mc^{th}(q_t^{th})$ . We deduce that the merit order price  $p_t$  is discontinuous on production vectors whose non-nuclear thermal component  $q_t^{th}$  is equal to zero (see Figure 4).

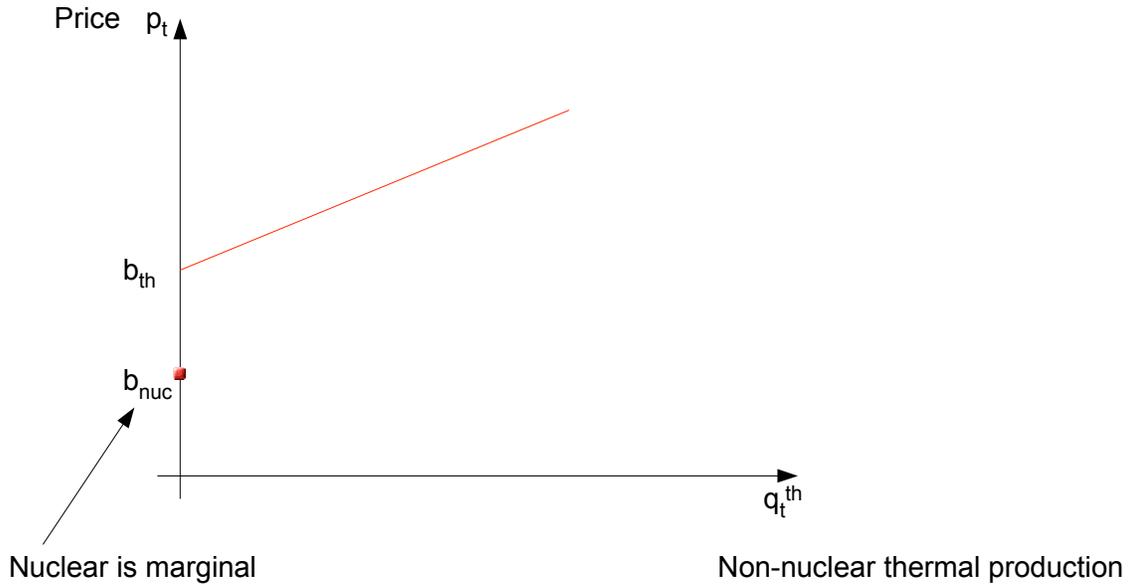


Figure 4: Price discontinuity

To end, the minimum/maximum nuclear and non-nuclear thermal production constraints have the following form

$$\begin{cases} Q_{min}^{j,nuc} \leq q_{jt}^{nuc} \leq Q_{max}^{j,nuc}, & \text{if no reload during month } t \text{ for unit } j \\ q_{jt}^{nuc} = 0, & \text{if unit } j \text{ reloads during month } t \end{cases} \quad (9)$$

$$0 \leq q_t^{th} \leq Q_{max}^{th} \quad (10)$$

The constraint (9) shows that the nuclear production of each month is bound by the minimum/maximum quantity of nuclear production which can be obtained during a month. If the unit  $j$  shuts down in month  $t$  for reloading, then its production during this month is equal to zero. The non-nuclear thermal production is a non-negative quantity which is bound by the maximum non-nuclear thermal production (constraint (10)). A producer may use the non-nuclear thermal resources to produce electricity until it reaches the level of demand of the corresponding month respecting at the same time the constraint (10). The nuclear fuel constraints for the unit  $j$  are given by Proposition 3.1.

## 4.2 An extension of merit order equilibrium in the case of $N$ producers

In view of the concept of merit order equilibrium, one can extend this model to the case of several producers operating with the same level of non-nuclear thermal capacity<sup>23</sup>. In addition, each producer holds a certain level of nuclear capacity of the unit  $j$ . Under the assumption that the non-nuclear thermal capacity is the same for all producers, we show that the merit order equilibrium in an economy with  $N \geq 2$  producers is “equivalent” to the merit order equilibrium in an economy with one aggregate producer.

### 4.2.1 The notion of merit order equilibrium in an economy with $N$ producers

In the case of a competitive electricity market with  $N \geq 2$  producers, the merit order equilibrium is defined as follows:

**Definition 4.2** *The production vector  $(\bar{q}_n)_{n=1}^N = ((\bar{q}_{1jt}^{nuc})_{j=1}^J, \bar{q}_{1t}^{th})_{t=1}^T, \dots, ((\bar{q}_{Njt}^{nuc})_{j=1}^J, \bar{q}_{Nt}^{th})_{t=1}^T$  is a merit order equilibrium with respect to a system of prices  $p \in \mathbb{R}^T$  if:*

(i) *for all  $n$ ,  $\bar{q}_n$  is a feasible production vector: (a) it respects the nuclear fuel constraints, for all  $j$  and (b) it respects the minimum/maximum production constraints, for all  $j, t$ .*

(ii) *the price, at each month  $t$ , is determined by the marginal cost of the marginal technology.*

*It is called the merit order price associated with the production vector  $(\bar{q}_n)_{n=1}^N$ .*

(iii) *at each date  $t$ , it respects the equality between supply and demand*

$$\sum_{n=1}^N \left( \sum_{j=1}^J \bar{q}_{njt}^{nuc} + \bar{q}_{nt}^{th} \right) = D_t - Q_t^{hyd}. \quad (11)$$

Note that in the case of  $N$  producers, the nuclear and non-nuclear thermal production costs are determined in subsection 2.4 of our paper (Lykidi, Glachant, Gourdel (2011)). According to Proposition 2.1 which appears in that subsection, the merit order price  $p$  associated with

<sup>23</sup>We can proceed with definition 4.2 without this assumption. However, in the case of asymmetry of the non-nuclear thermal individual capacities, the calculation of the merit order price (see relationship (12)) is no longer the same. More precisely, if the non-nuclear thermal capacity is not symmetric among producers, the formula (12) becomes  $p_t = \Phi_{nt}(q_{nt})$  for all  $n$  for which the non-nuclear thermal capacities are not saturated and  $p_t \geq \Phi_{nt}(q_{nt})$  in the case that the non-nuclear thermal individual capacities are saturated.

a feasible production vector  $(q_n)_{n=1}^N$ ,  $p = (p_t)_{t=1}^T = (\Phi_{nt}(q_{nt}))_{t=1}^T = (\Phi_{nt}((q_{njt}^{nuc})_{j=1}^J, q_{nt}^{th}))_{t=1}^T = \Phi_n(q_n)$  is calculated in month  $t$  as follows:

$$p_t = \Phi_{nt}(q_{nt}) = \begin{cases} mc_n^{th}(q_{nt}^{th}), & \text{if } q_{nt}^{th} > 0 \\ mc_n^{nuc}(q_{njt}^{nuc}), & \text{if } q_{nt}^{th} = 0 \end{cases} = \begin{cases} b_{th} + 2c_{th}^n q_{nt}^{th}, & \text{if } q_{nt}^{th} > 0 \\ b_{nuc}, & \text{if } q_{nt}^{th} = 0 \end{cases} \quad (12)$$

and is independent of  $n$ . Therefore, we can state the following remark:

**Remark 4.1** *If the non-nuclear thermal capacity is symmetric among producers then the production costs for non-nuclear thermal are the same for all producers and the non-nuclear thermal production is symmetric.*

Finally, the minimum/maximum nuclear and non-nuclear thermal production constraints take the form

$$\begin{cases} Q_{min}^{n,j,nuc} \leq q_{njt}^{nuc} \leq Q_{max}^{n,j,nuc}, & \text{if no reload during month } t \text{ for unit } j \\ q_{njt}^{nuc} = 0, & \text{if unit } j \text{ reloads during month } t \end{cases} \quad (13)$$

$$0 \leq q_{nt}^{th} \leq Q_{max}^{n,th} \quad (14)$$

where  $Q_{max}^{n,j,nuc}$  is the maximum nuclear production that can be realized by the unit  $j$  of producer  $n$  during a month and the minimum nuclear production is given by  $Q_{min}^{n,j,nuc}$ . The parameter  $Q_{max}^{n,th}$  represents the maximum non-nuclear thermal production during a month for the producer  $n$  while the minimum non-nuclear thermal production is equal to  $Q_{min}^{n,th} = 0$ . Moreover, the nuclear fuel constraints for the unit  $j$  are provided by Proposition 2.2 which appears in subsection 2.5 of our paper (Lykidi, Glachant, Gourdel (2011)) and constitutes a generalization of Proposition 3.1 in the case of  $N$  producers.

#### 4.2.2 Equivalence of merit order equilibrium between an economy with $N$ producers and an economy with one aggregate producer

Let us now proceed with the following proposition which shows, under the assumption that the non-nuclear thermal individual capacities are symmetric, the equivalence between the merit order equilibrium in an economy with  $N \geq 2$  producers and the merit order equilibrium in an economy with one aggregate producer.

**Proposition 4.1** *Let us consider an economy  $E$  with several producers and let  $\tilde{E}$  be the alternative economy with a unique producer obtained by the aggregation of the  $N$  producers of  $E$ . We assume that the non-nuclear thermal capacities are symmetric among producers. Let  $q$  be a merit order equilibrium of  $\tilde{E}$  then it can be decentralized as a merit order equilibrium  $(q_n)_{n=1}^N$  of  $E$ . Conversely, if  $(q_n)_{n=1}^N$  is a merit order equilibrium of  $E$  then its aggregation defined by  $q = \sum_{n=1}^N q_n$  is a merit order equilibrium of  $\tilde{E}$ .*

##### Proof

Initially, we show that if the production vector  $q = ((q_{jt}^{nuc})_{j=1}^J, q_t^{th})_{t=1}^T$  is a merit order equilibrium in an economy with one aggregate producer then the production defined later  $(q_n)_{n=1}^N = (((q_{njt}^{nuc})_{j=1}^J, q_{nt}^{th}))_{t=1}^T$  is a merit order equilibrium in an economy with  $N$  producers. We define the nuclear production  $q_{njt}^{nuc}$  of producer  $n$  as follows:

$$q_{njt}^{nuc} = \frac{Capacity^{j,nuc,n}}{Capacity^{j,nuc}} \cdot q_{jt}^{nuc}$$

where  $\text{Capacity}^{j,nuc} = \sum_{n=1}^N \text{Capacity}^{j,nuc,n}$ . The first term of this product is the share of the total nuclear capacity  $\text{Capacity}^{j,nuc}$  of unit  $j$  obtained by the producer  $n$  whose capacity is  $\text{Capacity}^{j,nuc,n}$ . As a consequence, the nuclear production  $q_{njt}^{nuc}$  of producer  $n$  is the part of the aggregate nuclear production  $q_{jt}^{nuc}$  that this producer can realize according to the capacity  $\text{Capacity}^{j,nuc,n}$  that it holds. In view of the Remark 4.1, the non-nuclear thermal production  $q_{nt}^{th}$  of producer  $n$  is defined by  $q_{nt}^{th} = \frac{1}{N} \cdot q_t^{th}$ .

Firstly, we will prove that  $(q_n)_{n=1}^N$  is a feasible production vector in the economy with  $N$  producers. Since  $q$  is a feasible production vector, we have that

$$Q_{min}^{j,nuc} \leq q_{jt}^{nuc} \leq Q_{max}^{j,nuc}, \quad \text{for all } j, t$$

and

$$0 \leq q_t^{th} \leq Q_{max}^{th}, \quad \text{for all } t$$

By multiplying the first relationship with the share of the total nuclear capacity of producer  $n$ , one obtains

$$\frac{\text{Capacity}^{j,nuc,n}}{\text{Capacity}^{j,nuc}} \cdot Q_{min}^{j,nuc} \leq \frac{\text{Capacity}^{j,nuc,n}}{\text{Capacity}^{j,nuc}} \cdot q_{jt}^{nuc} \leq \frac{\text{Capacity}^{j,nuc,n}}{\text{Capacity}^{j,nuc}} \cdot Q_{max}^{j,nuc}$$

which can be expressed as follows:

$$Q_{min}^{n,j,nuc} \leq q_{njt}^{nuc} \leq Q_{max}^{n,j,nuc}, \quad \text{for all } j, t$$

where  $n \in \{1, \dots, N\}$ ,  $Q_{min}^{j,nuc} = \sum_{n=1}^N Q_{min}^{n,j,nuc}$  and  $Q_{max}^{j,nuc} = \sum_{n=1}^N Q_{max}^{n,j,nuc}$ . In addition, in view of the Remark 4.1, by multiplying the second relationship with  $\frac{1}{N}$ , one has

$$\frac{1}{N} \cdot 0 \leq \frac{1}{N} \cdot q_t^{th} \leq \frac{1}{N} \cdot Q_{max}^{th}$$

or equivalently

$$0 \leq q_{nt}^{th} \leq Q_{max}^{n,th}, \quad \text{for all } t$$

where  $n \in \{1, \dots, N\}$  and  $Q_{max}^{th} = \sum_{n=1}^N Q_{max}^{n,th} = N \cdot Q_{max}^{n,th}$ . Moreover, the production vector  $q$  respects the nuclear fuel constraints (4), (5), (6). Similarly, by multiplying all the nuclear fuel constraints with the share of the total nuclear capacity of producer  $n$ , we deduce that the production vector  $(q_n)_{n=1}^N$  satisfies the nuclear fuel constraints in the case of  $N$  producers, where  $S_{reload}^j = \sum_{n=1}^N S_{reload}^{n,j}$ . Consequently, we conclude that  $(q_n)_{n=1}^N$  is a feasible production vector in an economy with  $N$  producers.

Secondly, we prove that the merit order price associated with the production  $(q_n)_{n=1}^N$  and the merit order price associated with the production  $q$  of one aggregate producer coincide. According to the determination of the merit order price in the case of one aggregate producer (8) and in the case of  $N$  producers (12) the marginal cost of nuclear is the same in both cases. Moreover, in view of Remark 4.1, the marginal cost of the non-nuclear thermal production in an economy with one aggregate producer coincides with the corresponding marginal cost in the case of an economy with  $N$  producers. Therefore, the merit order price is determined identically in both economies.

Finally, we show that at each date  $t$ ,  $(q_n)_{n=1}^N$  respects the equality between supply and demand in the economy with  $N$  producers. More precisely,

$$\sum_{n=1}^N \left( \sum_{j=1}^J q_{njt}^{nuc} + q_{nt}^{th} \right) = \sum_{n=1}^N \left( \sum_{j=1}^J \frac{\text{Capacity}^{j,nuc,n}}{\text{Capacity}^{j,nuc}} \cdot q_{jt}^{nuc} + q_{nt}^{th} \right) =$$

$$\sum_{j=1}^J \left( \frac{\sum_{n=1}^N Capacity^{j,nuc,n}}{Capacity^{j,nuc}} \cdot q_{jt}^{nuc} \right) + \sum_{n=1}^N q_{nt}^{th} = \sum_{j=1}^J q_{jt}^{nuc} + q_t^{th} = D_t - Q_t^{hyd}$$

Inversely, we show that if the production vector  $(q_n)_{n=1}^N = (((q_{njt}^{nuc})_{j=1}^J, q_{nt}^{th}))_{t=1}^T)_{n=1}^N$  is a merit order equilibrium in an economy with  $N$  producers then the aggregate production vector  $q = ((q_{jt}^{nuc})_{j=1}^J, q_t^{th})_{t=1}^T$  is a merit order equilibrium in an economy with one producer.

The production  $(q_n)_{n=1}^N$  is a merit order equilibrium thus, it is a feasible production vector which means that

$$Q_{min}^{n,j,nuc} \leq q_{njt}^{nuc} \leq Q_{max}^{n,j,nuc}, \text{ for all } j, t$$

and

$$0 \leq q_{nt}^{th} \leq Q_{max}^{n,th}, \text{ for all } t$$

By taking the sum of the nuclear component with respect to  $n$ , one can easily deduce that

$$\sum_{n=1}^N Q_{min}^{n,j,nuc} \leq \sum_{n=1}^N q_{njt}^{nuc} \leq \sum_{n=1}^N Q_{max}^{n,j,nuc}$$

which can be formulated as follows:

$$Q_{min}^{j,nuc} \leq q_{jt}^{nuc} \leq Q_{max}^{j,nuc}, \text{ for all } j, t$$

Similarly, by taking the sum of the non-nuclear component with respect to  $n$ , we deduce that

$$\sum_{n=1}^N 0 \leq \sum_{n=1}^N q_{nt}^{th} \leq \sum_{n=1}^N Q_{max}^{n,th}$$

or equivalently

$$0 \leq q_t^{th} \leq Q_{max}^{th}, \text{ for all } t$$

In addition, the production vector  $(q_n)_{n=1}^N$  respects the nuclear fuel constraints in an economy of  $N$  producers. If we take the sum of each of these constraints with respect to  $n$ , we conclude that  $q$  is a production that respects the nuclear fuel constraints (4), (5), (6) in the case of an aggregate producer. Hence,  $q$  is a feasible production vector in an economy with one aggregate producer.

Furthermore, the merit order price associated with the production vector  $q$  is the same as the merit order price associated with the production vector  $(q_n)_{n=1}^N$  as we proved previously.

To end, at each date  $t$ , the production vector  $(q_n)_{n=1}^N$  respects the equality between supply and demand

$$\begin{aligned} \sum_{n=1}^N \left( \sum_{j=1}^J q_{njt}^{nuc} + q_{nt}^{th} \right) &= D_t - Q_t^{hyd} \Leftrightarrow \sum_{j=1}^J \left( \sum_{n=1}^N q_{njt}^{nuc} \right) + \sum_{n=1}^N q_{nt}^{th} = D_t - Q_t^{hyd} \\ &\Leftrightarrow \sum_{j=1}^J q_{jt}^{nuc} + q_t^{th} = D_t - Q_t^{hyd} \end{aligned}$$

Thus, at each date  $t$ , the production vector  $q$  satisfies the supply-demand equilibrium constraint.

Consequently, we conclude that the production vector  $q$  is a merit order equilibrium in an economy with one aggregate producer and the proposition is proven.  $\square$

As a result, we may say that the merit order equilibrium  $q$  of the economy  $\tilde{E}$  is equivalent to the merit order equilibrium  $(q_n)_{n=1}^N$  of the decentralized economy  $E$ .

### 4.3 Supply behaviour with respect to the merit order price

In this section, we describe the supply behaviour with respect to the merit order price in order to emphasize the impossibility of computing a feasible production level during a month that satisfies the equality between supply and demand by looking at the merit order price during this month to maximize profit. Indeed, from a computational point of view, supply  $q = (q_t)_{t=1}^T = ((q_{jt}^{nuc})_{j=1}^J, q_t^{th})_{t=1}^T = (\Phi_t^{-1}(p_t))_{t=1}^T = \Phi^{-1}(p)$ , where  $q$  is a feasible production vector, does not behave well with respect to the merit order price within our model because  $\Phi^{-1}(\cdot)$  is not a function but a correspondence of the merit order price.

In figure 4, we can see that the non-nuclear thermal production in the month  $t$  is a continuous function of the form

$$q_t^{th}(p_t) = \begin{cases} \frac{p_t - b_{th}}{2c_{th}}, & \text{if } p_t \geq b_{th} \\ 0, & \text{if } p_t \leq b_{th} \end{cases} \quad (15)$$

However, we can not calculate the supply of nuclear  $((q_{jt}^{nuc}(p_t))_{j=1}^J)_{t=1}^T$  with respect to the price  $p_t$  in the month  $t$  because it is not a function of the price but a correspondence when nuclear is the marginal technology i.e.  $p_t = b_{nuc}$ , which means that the nuclear supply can take any value for this value of price. In addition to this, the nuclear component of the correspondence  $\Phi^{-1}(\cdot)$  is an empty set in the case that  $b_{nuc} < p_t < b_{th}$ .

Consequently, we conclude that a producer can not compute a feasible production vector at each month by looking at the merit order price during this month. For this reason, the determination of the price at the merit order equilibrium by calculating the supply  $q(p) = (q_t(p_t))_{t=1}^T = ((q_{jt}^{nuc}(p_t))_{j=1}^J, q_t^{th}(p_t))_{t=1}^T$  and by looking at the equality between supply and demand at each date  $t$

$$\sum_{j=1}^{12} q_{jt}^{nuc}(p_t) + q_t^{th}(p_t) = D_t - Q_t^{hyd}$$

is impossible numerically. Therefore, we deduce that it is not possible to calculate a merit order equilibrium in this first approach.

In the case that the nuclear production  $((q_{jt}^{nuc}(p_t))_{j=1}^J)_{t=1}^T$  was a function of the merit order price and not a correspondence, the problem would be different. More precisely, in view of the inter-temporal management of the nuclear fuel stock (see Proposition 3.1), a producer has to look at the equality between supply and demand over the entire time horizon  $T$  of the model in order to determine the price within the merit order equilibrium. In this typical case of  $T = 36$  months, we have to deal with a large non linear system

$$\sum_{j=1}^{12} q_{jt}^{nuc}(p_t) + q_t^{th}(p_t) = D_t - Q_t^{hyd}, \quad \text{for all } t$$

of 36 equations involving 36 unknowns  $((p_t)_{t=1}^T)$  which is difficult to solve numerically.

Note that this system is based on the auxiliary variables  $q_{jt}^{nuc}, q_t^{th}$  whose number is  $J \cdot T + T = 12 \cdot 36 + 36 = 468$ .

### 4.4 A second approach to calculate a merit order equilibrium

In this approach, a producer operating all the nuclear units  $j$  and the non-nuclear thermal units maximizes its profit in the month  $t$  given the optimal production levels realized in the previous months. It takes into account the minimum and maximum production constraints for both nuclear and non-nuclear thermal generation technologies for all months of period  $T$  and the

nuclear fuel constraints for each unit  $j$ . In addition, the supply-demand equilibrium constraint has to be satisfied during the month  $t$ .

At time  $t$ , an aggregate producer could try to solve the following optimal production problem:

$$\max_{(q_{j\tau}^{nuc}, q_{\tau}^{th})_{\tau=1}^T \in G^t} p_t \cdot \left( \sum_{j=1}^J q_{jt}^{nuc} + q_t^{th} \right) - \sum_{j=1}^J C_j^{nuc}(q_{jt}^{nuc}) - C^{th}(q_t^{th}) \quad (16)$$

where  $p_t$  is a given parameter and  $G^t$  is the set of feasible solutions of the optimization problem (16) defined as

$$G^t = \left\{ \begin{array}{ll} ((q_{j\tau}^{nuc})_{j=1}^J, q_{\tau}^{th})_{\tau=1}^T \in K \text{ s.t.} & \begin{array}{ll} q_{j\tau}^{nuc} = \tilde{q}_{j\tau}^{nuc}, & \text{for all } j \text{ and for all } \tau < t \\ q_{\tau}^{th} = \tilde{q}_{\tau}^{th}, & \text{for all } \tau < t \\ Q_{min}^{j,nuc} \leq q_{j\tau}^{nuc} \leq Q_{max}^{j,nuc}, & \text{for all } j \text{ and for all } \tau \\ 0 \leq q_{\tau}^{th} \leq Q_{max}^{th}, & \text{for all } \tau \end{array} \end{array} \right\}$$

The notation  $G^t$  is the reduced form of the notation  $G^t(((\tilde{q}_{j\tau}^{nuc})_{j=1}^J, \tilde{q}_{\tau}^{th})_{\tau=1}^{t-1})$ , where  $((\tilde{q}_{j\tau}^{nuc})_{j=1}^J, \tilde{q}_{\tau}^{th})_{\tau=1}^{t-1}$  is the optimal production vector of the months preceding the month  $t$ . The set  $K$  is defined by all the production vectors of the form  $q = ((q_{j1}^{nuc})_{j=1}^J, \dots, (q_{jT}^{nuc})_{j=1}^J, q_1^{th}, \dots, q_T^{th})$  that respect the nuclear fuel constraints (4), (5), (6) as well as the supply-demand equilibrium constraint

$$\sum_{j=1}^{12} q_{jt}^{nuc} + q_t^{th} = D_t - Q_t^{hyd}, \quad \text{in month } t.$$

The producer determines its optimal level of supply  $((q_{jt}^{nuc})_{j=1}^J, q_t^{th})$  during the month  $t$ , given the optimal production  $((\tilde{q}_{j\tau}^{nuc})_{j=1}^J, \tilde{q}_{\tau}^{th})_{\tau=1}^{t-1}$  of the previous months, by solving the optimal short-term production problem (16). Then, the price  $p_t$  is determined by the equality between supply and demand in month  $t$ . The solution of this problem determines the new level of stock  $S_{t+1}^j$ . A production vector  $((\bar{q}_{j\tau}^{nuc})_{j=1}^J, \bar{q}_{\tau}^{th})_{\tau=1}^T$  is an equilibrium of the optimal short-term optimization problem (16) if it is a merit order equilibrium and in addition to this it maximizes the profit of the producer during the month  $t$  on the set of feasible solutions  $G^t$ , for all  $t$ .

However, this approach could be qualified as “short sighted” since the equality between supply and demand in future periods is not taken into consideration in the optimization problem (16). This may lead to a failure of the system to equilibrate supply and demand in future periods while at the same time respecting the nuclear fuel and the generation capacity constraints. In fact, in our numerical example, we find that there exists a month  $t \in \{1, \dots, T\}$  such that the set of feasible solutions  $G^t$  is empty. More precisely, we verify (through a numerical test) the nonexistence of feasible solutions within the set  $G^{16}$  in the month of April ( $t = 16$ ) of the second year of period  $T$ . Therefore, the producer being “short sighted” with respect to future demand is not able to respect the equality between supply and demand during the entire time horizon of the model.

Consequently, we conclude that the calculation of a merit order equilibrium is not possible within this scenario. We intentionally present here a mistaken approach in order to show that too high a level of “short sightedness” with regards to future demand is not bearable. The equality between supply and demand has to be seen with a minimum anticipation in order to manage the current use of the nuclear fuel reservoir. For this reason, we proceed with the next and final approach to calculate a merit order equilibrium.

## 4.5 Final approach to calculate a merit order equilibrium

In view of the second approach to calculate a merit order equilibrium, the system operator can not find a feasible production vector which respects the equilibrium between supply and

demand during the entire time horizon of the model. This is because the producer does not sufficiently take into account the supply-demand equilibrium in future periods within the set of feasible solutions  $G_t$  of the optimal short-term production problem (16). This leads to a failure to calculate a merit order equilibrium of this optimization problem since the set  $G^t$  becomes empty ( $G^t = \emptyset$ , for  $t = 16$ ) in our numerical example. Therefore, the nuclear set has to be managed so that the equality between supply and demand is respected over the whole period  $T$ . For this reason, we provide a third and final scenario in which the supply-demand equilibrium constraint is taken into account in future periods within the set of feasible solutions of the optimization problem (16).

More precisely, at time  $t$ , an aggregate producer could try to solve the following optimal production problem

$$\max_{(q_{j\tau}^{nuc}, q_{\tau}^{th})_{\tau=1}^T \in H^t} p_t \cdot \left( \sum_{j=1}^J q_{jt}^{nuc} + q_t^{th} \right) - \sum_{j=1}^J C_j^{nuc}(q_{jt}^{nuc}) - C^{th}(q_t^{th}) \quad (17)$$

where  $H^t$  is the set of feasible solutions of the optimization problem (17) defined as follows:

$$H^t = \left\{ \left( (q_{j\tau}^{nuc})_{j=1}^J, q_{\tau}^{th} \right)_{\tau=1}^T \in M \text{ s.t. } \begin{array}{ll} q_{j\tau}^{nuc} = \tilde{q}_{j\tau}^{nuc}, & \text{for all } j \text{ and for all } \tau < t \\ q_{\tau}^{th} = \tilde{q}_{\tau}^{th}, & \text{for all } \tau < t \\ Q_{min}^{j,nuc} \leq q_{j\tau}^{nuc} \leq Q_{max}^{j,nuc}, & \text{for all } j \text{ and for all } \tau \\ 0 \leq q_{\tau}^{th} \leq Q_{max}^{th}, & \text{for all } \tau \end{array} \right\}$$

The notation  $H^t$  is used for  $H^t((\tilde{q}_{j\tau}^{nuc})_{j=1}^J, \tilde{q}_{\tau}^{th})_{\tau=1}^{t-1}$ , where  $((\tilde{q}_{j\tau}^{nuc})_{j=1}^J, \tilde{q}_{\tau}^{th})_{\tau=1}^{t-1}$  is the optimal production realized in the months preceding the month  $t$ . The set  $M$  is defined by all the production vectors of the form  $q = ((q_{j1}^{nuc})_{j=1}^J, \dots, (q_{jT}^{nuc})_{j=1}^J, q_1^{th}, \dots, q_T^{th})$  that respect the nuclear fuel constraints ((4), (5), (6)) as well as the supply-demand equilibrium constraint

$$\sum_{j=1}^{12} q_{jt}^{nuc} + q_t^{th} = D_t - Q_t^{hyd}, \quad \text{for all } t.$$

The optimal short-term production problem (17) determines the supply of the producer  $((q_{jt}^{nuc})_{j=1}^J, q_t^{th})$  during the month  $t$ , given the optimal nuclear and non-nuclear thermal production  $((\tilde{q}_{j\tau}^{nuc})_{j=1}^J, \tilde{q}_{\tau}^{th})_{\tau=1}^{t-1}$  realized in the previous months. Then, the price  $p_t$  is determined by the equality between supply and demand in the month  $t$ . The level of stock  $S_{t+1}^j$  of the next month is determined by the nuclear supply  $(q_{jt}^{nuc})_{j=1}^J$  of the current month. A production vector  $((\bar{q}_{j\tau}^{nuc})_{j=1}^J, \bar{q}_{\tau}^{th})_{\tau=1}^T$  is an equilibrium of the optimal short-term production problem (17) if it is a merit order equilibrium and it maximizes the profit of the producer during the month  $t$  on the set of feasible solutions  $H^t$ , for all  $t$ .

However, under some assumptions, we show that the absence of non-nuclear thermal production during the month  $t$  induces a decrease of the profit during this month which results from a decrease of the price.

Let us focus on the set<sup>24</sup>  $H_{th}^t$  defined as

$$H_{th}^t = \left\{ \left( (q_{j\tau}^{nuc})_{j=1}^J, q_{\tau}^{th} \right)_{\tau=1}^T \in M \text{ s.t. } \begin{array}{ll} q_{j\tau}^{nuc} = \tilde{q}_{j\tau}^{nuc}, & \text{for all } j \text{ and for all } \tau < t \\ q_{\tau}^{th} = \tilde{q}_{\tau}^{th}, & \text{for all } \tau < t \\ Q_{min}^{j,nuc} \leq q_{j\tau}^{nuc} \leq Q_{max}^{j,nuc}, & \text{for all } j \text{ and for all } \tau \\ 0 < q_t^{th} \leq Q_{max}^{th} & \end{array} \right\}$$

<sup>24</sup>The notation  $H_{th}^t$  is the reduced form of the notation  $H_{th}^t((\tilde{q}_{j\tau}^{nuc})_{j=1}^J, \tilde{q}_{\tau}^{th})_{\tau=1}^{t-1}$ , where  $((\tilde{q}_{j\tau}^{nuc})_{j=1}^J, \tilde{q}_{\tau}^{th})_{\tau=1}^{t-1}$  is the optimal production effectuated in the months preceding the month  $t$ .

**Remark 4.2** For all  $t \in \{1, \dots, T\}$ ,  $H_{th}^t$  is contained in  $H^t$  and  $H^t$  is contained in  $M$  ( $H_{th}^t \subset H^t \subset M$ ).

Since the marginal technology is the non-nuclear thermal on  $H_{th}^t$ , the price is determined by the non-nuclear thermal production during the month  $t$ . We now proceed with Proposition 4.2. We will make use of this proposition in order to prove the decrease of the profit at production vectors with zero levels of non-nuclear thermal production at date  $t$ .

**Proposition 4.2** For all  $t \in \{1, \dots, T\}$ , if  $H_{th}^t$  is a non-empty set, then  $\overline{H_{th}^t} = H^t$ .

**Proof**

First, we show that  $\overline{H_{th}^t} \subset H^t$ . Since  $H_{th}^t$  is contained in  $H^t$  (see Remark 4.2) and  $H^t$  is a compact set, we have that

$$H_{th}^t \subset H^t \Rightarrow \overline{H_{th}^t} \subset \overline{H^t} = H^t.$$

Secondly, we prove that  $\overline{H_{th}^t} \supset H^t$ . Let  $q \in H^t$  and  $q' \in H_{th}^t$ . For all  $m \in \mathbb{N}$ , there exists a sequence  $z_m = (1 - \frac{1}{m+1})q + \frac{1}{m+1}q'$  belonging to  $H_{th}^t$  such that  $\lim_{m \rightarrow \infty} z_m = \lim_{m \rightarrow \infty} (1 - \frac{1}{m+1})q + \frac{1}{m+1}q' = q$ . Hence,  $q \in \overline{H_{th}^t}$  and the inclusion is proven.

From the first and the second part of the proof, we conclude that  $\overline{H_{th}^t} = H^t$ . □

Let us notice that the non-emptiness of the set  $H_{th}^t$  obviously depends on the values of the exogenous variables  $(Q_{max}^{j,nuc}, Q_{min}^{j,nuc}, Q_{max}^{th}, S_{reload}^j, S_1^j, D_t, Q_t^{hyd})$  of the optimization problem (17).

From a geometrical point of view, it results from Proposition 4.2 that all the points of the set  $H^t$  and as a consequence those which belong to  $H^t \setminus H_{th}^t$  and therefore characterized by zero levels of non-nuclear thermal production in the month  $t$  can be approached by points that belong to  $H_{th}^t$ . This result plays a central role in order to prove the discontinuity and more precisely the decrease of profit at these particular points due to a decrease of the price in the month  $t$  (Bich and Laraki (2011)). In the next proposition, we show the decrease of the profit during the month  $t$  in the absence of non-nuclear thermal production.

**Proposition 4.3** For all  $t \in \{1, \dots, T\}$ , if  $H_{th}^t$  is a non-empty set,  $b_{nuc} < b_{th}$  and  $\bar{q} \in H^t \setminus H_{th}^t$ , then there exists a sequence  $(q_r)_{r \in \mathbb{N}} \in H_{th}^t$  with  $\lim_{r \rightarrow \infty} q_r = \bar{q}$  such that  $\lim_{r \rightarrow \infty} \pi_t(q_r) > \pi_t(\bar{q})$ .

**Proof**

According to the assumptions,  $\bar{q}$  is a production vector which belongs to  $H^t \setminus H_{th}^t \subset H^t$ , for some  $t \in \{1, \dots, T\}$ . Let us denote  $\bar{q} = ((\bar{q}_{j1}^{nuc})_{j=1}^J, \dots, (\bar{q}_{jT}^{nuc})_{j=1}^J, \bar{q}_1^{th}, \dots, \bar{q}_T^{th})$  while  $\bar{q}_t = ((\bar{q}_{jt}^{nuc})_{j=1}^J, \bar{q}_t^{th})$  is the production realized in the month  $t$ .

Profit's function  $\pi_t : H^t \rightarrow \mathbb{R}$  is defined as

$$p_t \cdot \left( \sum_{j=1}^J q_{jt}^{nuc} + q_t^{th} \right) - \sum_{j=1}^J C_j^{nuc}(q_{jt}^{nuc}) - C^{th}(q_t^{th})$$

In view of Proposition 4.2, the production vector  $\bar{q} \in \overline{H_{th}^t}$ . It follows that there exists a sequence  $(q_r)_{r \in \mathbb{N}}$  such that  $(q_r)_{r \in \mathbb{N}} \in H_{th}^t$  and  $\lim_{r \rightarrow \infty} q_r = \bar{q}$ . Let us denote  $q_r = ((q_{j1r}^{nuc}, \dots, q_{jTr}^{nuc})_{j=1}^J, q_{1r}^{th}, \dots, q_{Tr}^{th})$  and  $q_{tr} = ((q_{jtr}^{nuc})_{j=1}^J, q_{tr}^{th})$  the component of the sequence  $q_r$  which corresponds to the month  $t$ . For all  $r$ , we can compute the associated with the production  $q_{tr}$  merit order price  $p_{tr} = mc^{th}(q_{tr}^{th})$ . The price  $\bar{p}_t$  represents the merit order price associated with the production vector  $\bar{q}_t$ . Since at the limit, the value of  $q_{tr}$  is equal to  $\bar{q}_t$ , we deduce that the nuclear is the marginal technology during the month  $t$ . Thus, the price  $\bar{p}_t$  in month  $t$  is determined by the nuclear marginal cost  $b_{nuc}$ . It follows that

$$\lim_{r \rightarrow \infty} p_{tr} = \lim_{r \rightarrow \infty} mc^{th}(q_{tr}^{th}) = mc^{th}(\bar{q}_t^{th}) = mc^{th}(0) = b_{th} > b_{nuc} = \bar{p}_t. \quad (18)$$

At the limit, we obtain

$$\begin{aligned} & \lim_{r \rightarrow \infty} \pi_t(q_r) - \pi_t(\bar{q}) = \\ & \lim_{r \rightarrow \infty} (p_{tr}(\sum_{j=1}^J q_{jtr}^{nuc} + q_{tr}^{th}) - \bar{p}_t(\sum_{j=1}^J \bar{q}_{jt}^{nuc} + \bar{q}_t^{th})) - \\ & \lim_{r \rightarrow \infty} ((\sum_{j=1}^J C_j^{nuc}(q_{jtr}^{nuc}) + C^{th}(q_{tr}^{th})) - (\sum_{j=1}^J C_j^{nuc}(\bar{q}_{jt}^{nuc}) + C^{th}(\bar{q}_t^{th}))) \end{aligned}$$

Since  $\lim_{r \rightarrow \infty} q_{tr} = \bar{q}_t$  and in view of the relationship (18) and of the strictly positive nuclear production ( $\sum_{j=1}^J \bar{q}_{jt}^{nuc} \geq \sum_{j=1}^J Q_{min}^{j,nuc} > 0$ ) the first term is strictly positive. The second term converges to zero because of the continuity of the production cost functions.

Consequently, we deduce that  $\lim_{r \rightarrow \infty} \pi_t(q_{tr}) - \pi_t(\bar{q}_t) > 0 \Leftrightarrow \lim_{r \rightarrow \infty} \pi_t(q_{tr}) > \pi_t(\bar{q}_t)$  and the proposition is proven.  $\square$

The inequality  $b_{nuc} < b_{th}$  holds within our data; thus according to Proposition 4.3, the profit of a producer at date  $t$  decreases for all the production vectors whose non-nuclear thermal component at this date is equal to zero. Consequently, it is not profitable for a producer who wants to maximize its profit during the month  $t$  to run only its nuclear units and be remunerated at a price  $p_t = b_{nuc}$ . Therefore, the producer will search for a solution that maximizes its profit in the month  $t$  among the production vectors of the set  $H_{th}^t$ . The following corollary establishes the relation between the optimal short-term production problem on  $H^t$  and the optimal short-term production problem on  $H_{th}^t$ .

**Corollary 4.1** *The current monthly profit maximization problem determined on  $H^t$  is equivalent to the current monthly profit maximization problem determined on  $H_{th}^t$  (same set of solutions and same value<sup>25</sup>), for all  $t$ .*

### Proof

This corollary is an obvious consequence of Proposition 4.3.  $\square$

Let us mention that if the current monthly profit maximization problem is determined on  $H^t$  which is a compact set, the objective function is not continuous according to Proposition 4.3. If the current monthly profit maximization problem is determined on  $H_{th}^t$ , the objective function is continuous in view of Proposition 4.3 while the set  $H_{th}^t$  is not compact. Hence, it is not possible to conclude on the existence of solutions of this problem. However, if a solution of this optimization problem exists then it constitutes an equilibrium since all the conditions in order to be a merit order equilibrium are satisfied and it maximizes the producer's profit in the month  $t$ . In the next subsection, we provide a numerical illustration of the optimal short-term production problem (17).

Finally, we proceed with the following lemma which shows the concavity of the profit function of the optimal short-term production problem determined on  $H_{th}^t$ .

<sup>25</sup>The value of an optimization problem is defined as the upper bound of the set  $\{f(x)|x \in C\}$ , where  $f$  is the objective function and  $C$  is the set of feasible solutions. The value always exists (in the extended real line) even if the set of solutions is empty.

**Lemma 4.1** *For all  $t \in \{1, \dots, T\}$ , the profit function of the current monthly profit maximization problem on  $H_{th}^t$  is concave.*

**Proof of Lemma 4.1**

The profit  $\pi_t$  of producer at each month  $t$  is

$$p_t \cdot \left( \sum_{j=1}^J q_{jt}^{nuc} + q_t^{th} \right) - \sum_{j=1}^J C_j^{nuc}(q_{jt}^{nuc}) - C^{th}(q_t^{th})$$

We remind that the price  $p_t$  is given by the marginal cost of the non-nuclear thermal production on the set  $H_{th}^t$ . Following some calculations and using the supply-demand equilibrium constraint in the month  $t$ , we deduce that producer's profit  $\pi_t$  is a quadratic function of the non-nuclear thermal production  $q_t^{th}$

$$-c_{th}(q_t^{th})^2 + (2c_{th}(D_t - Q_t^{hyd}) + b_{nuc} - b_{th})q_t^{th} + c \quad (19)$$

where  $c_t = ((b_{th} - b_{nuc})(D_t - Q_t^{hyd}) - a_{th} - \sum_{j=1}^J a_{nuc}^j)$  is the constant part of the profit function.

We notice that this is a quadratic function of the form  $f_t(u_t) = au_t^2 + b_tu_t + c_t$ , where  $a = -c_{th}$  and  $b_t = (2c_{th}(D_t - Q_t^{hyd}) + b_{nuc} - b_{th})$ . Since  $a < 0$ , the function  $f_t(u_t)$  is concave, for all  $t$ . It is also a strictly concave function. Thus, taking into account the other variables, we conclude that the profit function (19) is concave, for all  $t$ . □

In particular, the strict concavity of the profit function  $\pi_t$  with respect to the non-nuclear thermal production (see Proof of Lemma 4.1) implies the unicity of solutions with respect to the non-nuclear thermal component. However, taking into account the other variables and according to Lemma 4.1 the profit function is concave and therefore the entire solution is not unique.

## 5 Numerical Illustration

In this section, we study the nuclear and non-nuclear thermal production decisions as well as the storage decisions analyzed in subsection 4.5 of the previous section, within a simple numerical model solved with Scilab.

### 5.1 Data

The data used in our numerical dynamic model is French. It is of several years due to the difficulty of collection: \* level of French demand during the year 2006 – 2007, \* fixed and variable costs of nuclear, coal and gas generation, \* generation capacity of hydro (run-of-river), nuclear and non-nuclear thermal and \* nuclear fuel stock of reloading. Consumption data comes from the French Transmission & System Operator (named RTE). It gives the daily consumption in MWh for the month of December 2006 and the entire year 2007 with which we determine the monthly consumption. RTE also provides the annual capacity of nuclear as well as the annual capacity of gas and coal for the year 2009. In addition, the nuclear fuel stock of reloading as well as the annual capacity and production of hydro have been provided by Electricité de France.

The costs of production come from the official report “Reference Costs of Electricity Production” issued by the ministry of industry (General Direction for Energy and Raw Materials (DGEMP) & Directorate for Demand and Energy Markets (DIDEME)) in 2003. This report

gives the technical characteristics, the costs and a sensitivity analysis for different types of generation technologies (nuclear, coal, gas, fuel). It also gives the life duration, the availability of the generating units as well as the typical management of the fuel for nuclear. This data is calculated for the year 2007 and 2015. The report also gives the cost of investment, the variable / fixed cost of operation, the cost of fuel as well as the external costs (e.g. cost of CO<sub>2</sub>, cost of a major nuclear accident, etc.). It also provides the total cost of production for a base load (8760h) and semi-base (3000h) operation. It gives the total cost for each technology as follows: cost of investment, variable and fixed cost of exploitation, fuel cost, taxes, R&D costs for the nuclear and cost of CO<sub>2</sub> per ton in the case of coal and gas for the same levels of operation that were previously mentioned. These costs are estimated for the year 2007 and 2015 with different discount rates (3%, 5%, 8%, 11%) taking into account the influence of exchange rate on the production cost. Finally, a sensitivity analysis links production costs to the main parameters for each technology (e.g. investment cost, availability, life duration, etc.).

The role of the discount rate is important in the determination of the nuclear costs for several reasons. We know that the level of the fixed costs depends crucially on the discount rate. For example, the investment cost for nuclear is set at 6,4 Euros/MWh with a discount rate of 3% and at 16,3 Euros/MWh for a discount rate of 8%. In addition to this, the nuclear fuel (being “enriched uranium”) has its own distinctive fuel cycle extremely different than other resources such as coal, oil and natural gas. Nuclear fuel is processed through several steps such as mining and milling, conversion, enrichment, and fuel manufacturing. A last main characteristic of nuclear is the storage of waste and the long process of the dismantling of plants (Source: Brite/Euram III: Projects). The resulting duration of the nuclear fuel cycle is significantly high. To end, the life duration retained for the non-nuclear thermal units is 30 years, while the new reactor EPR (European Pressurized Reactor) is conceived to operate for 60 years. Therefore, the discount rate is a very critical factor which can significantly affect the nuclear costs.

Moreover, the cost of non-nuclear thermal production is itself highly volatile for different reasons. One is the price volatility of CO<sub>2</sub>. The CO<sub>2</sub> future prices for 2007 delivery ranged between almost 0 and 30 euros per ton of CO<sub>2</sub> during just the twelve month period May 2006 - May 2007 in Phase I of the EU ETS (Source: The Brattle Group, Cambridge). The other considerable volatility factor has been the sharp move of oil prices (and gas prices) in these last few years (see Figure 5, Source: UNCTADstat, CNUCED).

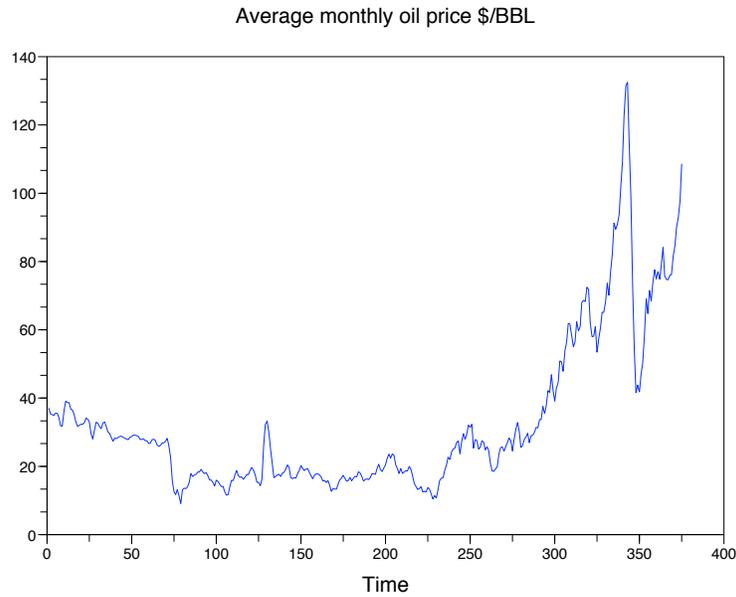


Figure 5: Average monthly oil price from January 1980 through March 2011 (\$/BBL).

U.S. Electricity Production Costs (1995–2009, In 2009 cents per kilowatt-hour)

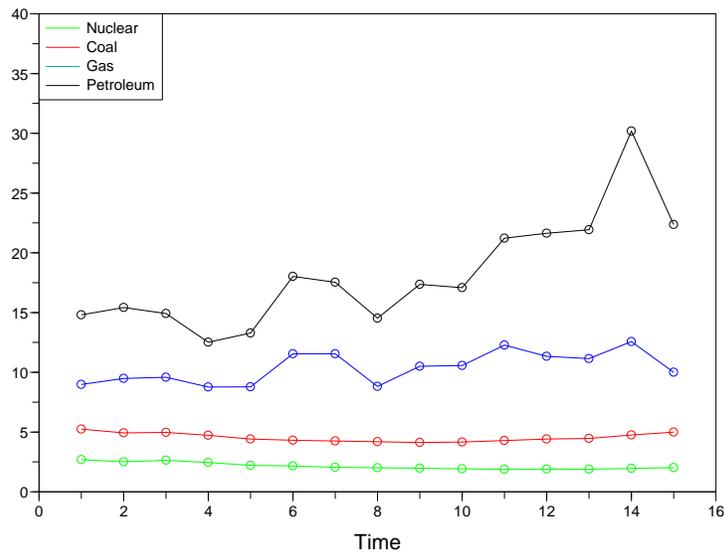


Figure 6: Evolution of U.S. electricity production costs (1995-2009, in 2009 cents/KWh).

On the contrary, the cost of nuclear fuel and the operation costs<sup>26</sup> are less volatile during that period than the costs of non-nuclear thermal production (see Figure 6, Source: Ventyx Velocity Suite, Nuclear Energy Institute). This is mainly due to two reasons; firstly the small level of uranium cost as a component of the final nuclear fuel cost, and secondly a stabilization factor resulting from the delay between the extraction of natural uranium and the final manufacturing

<sup>26</sup>The above data refers to fuel plus operation and maintenance costs only. It excludes capital costs since these vary greatly among utilities and states, as well as with the age of the plant.

of nuclear fuel.

Our modelling is based on a scenario in which one dollar is equal to one euro, the discount rate is 8%, the cost of CO<sub>2</sub> per ton reaches 20 euros, the price of coal is 30 dollars per ton and the price of gas is 3.3 dollars per MBtu (1 MBtu = 0.2931 KWh). The choice of this particular scenario is mainly based on the scenario considered by DGEMP & DIDEME for the estimation of costs of the different types of generation technologies. The value of the coefficient  $a_{th}$  involved in the non-nuclear thermal cost function corresponds to the fixed cost provided by the data (investment cost, fixed exploitation cost, taxes), while the other coefficients have been determined by interpolation in order to meet the variable cost of coal and gas provided by our data base (fuel cost, variable exploitation cost, CO<sub>2</sub> cost, taxes on the gas fuel). The consideration of the fixed costs in the production cost of both technologies (nuclear, non-nuclear thermal) permits us to obtain a more realistic vision of the value of profit within our medium-term horizon. The capacity of each nuclear unit has been simulated<sup>27</sup> in order to approximate the graph of figure 3, which shows the availability of nuclear units per week. For example, the capacity of the nuclear unit  $j = 1$  (respectively  $j = 2, \dots, j = 12$ ) corresponds to the sum of capacities of shut-downed units in December (respectively January,  $\dots$ , November). Moreover, the initial value of the nuclear fuel stock has been set by simulating the nuclear fuel stock of each unit available at the beginning of the time horizon of the model. We also take into account the electricity losses of the network, as estimated by RTE. Finally, an EPR reactor can maneuver between 25% of nominal capacity and 100% of nominal capacity in order to follow-up load. We take into account these levels of maneuverability within our numerical model to determine the minimum and maximum nuclear production constraints (see Table 2), Section 8).

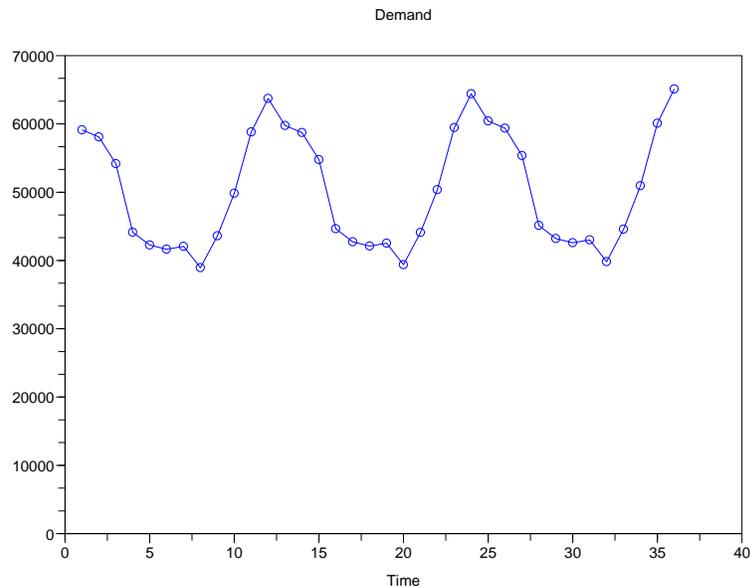


Figure 7: Simulated demand (in MW)

The levels of monthly demand<sup>28</sup> obtained for the period January 2007 – December 2009) are presented in figure 7 (we suppose an augmentation of the demand by a rate of 1% per

<sup>27</sup>Access to detailed nuclear capacity data for each short period of time is not possible due to the confidentiality of such data.

<sup>28</sup>Note that we did a rescaling on this data to take into account the diversity on the length of the months.

year). One can see the seasonal variation of the demand level between winter (high demand) and summer (low demand). We observe a high level of demand during November, December, January and February with demand peaks in December. The demand falls during spring as well as during summer (May – August). However, we do not observe any demand peaks during the summer period which implies that there were no significant extremes in temperature.

## 5.2 Simulation results

The hypothesis of Proposition 4.3 holds within our data, thus the discontinuity (more precisely the decrease) of the price at production vectors characterized by zero non-nuclear thermal production during a month induces a decrease of the value of profit during this month. A point that we need to stress is the nonexistence of an algorithm that maximizes a discontinuous function. We propose an alternative model, where the price is given by the non-nuclear thermal marginal cost ( $mc^{th}(0) = b_{th}$ ) instead of the nuclear marginal cost ( $b_{nuc}$ ) when nuclear is the marginal technology. This means that a producer receives at least  $b_{th}$  (Euros per MWh) when it runs only its nuclear units to cover the monthly demand. In view of this “regularization” of the merit order price, the current monthly profit being now a continuous function is maximized on the entire set of feasible solutions  $H^t$  within our numerical model. However, the “regularized” problem (continuous problem) and the economical problem described in subsection 4.5 (discontinuous problem) differ with respect to the objective function. More precisely, the profit of the economical problem is smaller than the profit of the “regularized” problem since the value of  $b_{nuc}$  (5.01 Euro/MWh) is less important than the value of  $b_{th}$  (26.24 Euro/MWh). Nevertheless, the value of the “regularized” problem and the value of the economical problem are identical, meaning that the value of the profit at the optimum is the same for both optimization problems (see Annex, Proposition 7.1). Consequently, the “regularized” problem is a “good” approximation of our economical problem (Boyd and Vandenberghe (2004)). It is also proven that the intersection of the set of solutions of the “regularized” problem and the set  $H_{th}^t$  whose production vectors are characterized by strictly positive non-nuclear thermal production levels, provides the set of solutions of the economical problem (see Annex, Proposition 7.2). The solution of the “regularized” problem whose results are analyzed in this section does not belong to the set  $H_{th}^t$ . Therefore, the set of solutions of the economical problem is empty (see Annex, Proposition 7.3) and this numerical solution constitutes only an “approximate” solution of our economical problem.

### 5.2.1 General results

The non-nuclear thermal is the marginal technology during most of the months of period  $T$  in order to equilibrate supply and demand while the nuclear technology is marginal only at the end of period  $T$  (see Figure 8, Figure 10). Nuclear stays marginal during almost the entire period of spring, summer and autumn of 2009. In addition, nuclear follows the seasonal variations of the demand by decreasing during summer and increasing during winter (see Figure 9). In the same way, non-nuclear thermal which is called to cover the residual demand every month follows-up the seasonal variations of the demand during the whole period  $T$ .

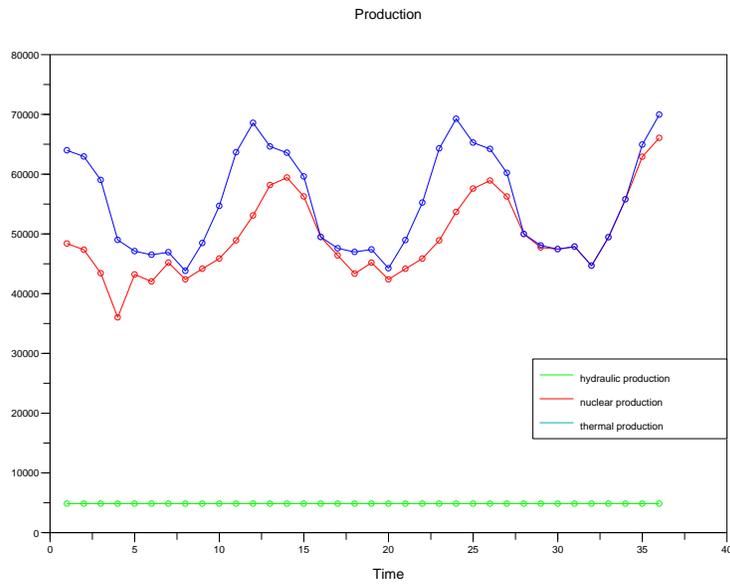


Figure 8: Simulated hydro/nuclear/non-nuclear thermal production (in MW)

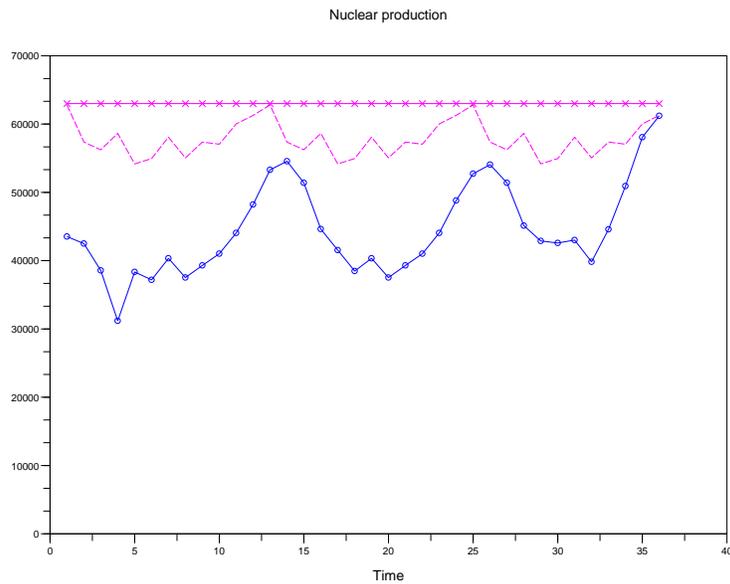


Figure 9: Simulated nuclear production (in MW)

Furthermore, one observes that the monthly nuclear production almost never reaches its maximum value<sup>29</sup> (see Figure 9) while the monthly non-nuclear thermal production reaches its maximum value<sup>30</sup> at the beginning of period T and then during the month of December of 2008

<sup>29</sup>The maximum nuclear production during the month  $t$  given that some unit is inactive during this month (month of reloading) is represented by the purple dotted line. This quantity is obviously below the nominal capacity of the French nuclear set represented by the crossed purple line.

<sup>30</sup>The maximum non-nuclear thermal production during a month is represented by the white blue dotted line and corresponds to the nominal non-nuclear thermal capacity (including coal, gas, fuel, etc.) of the French set.

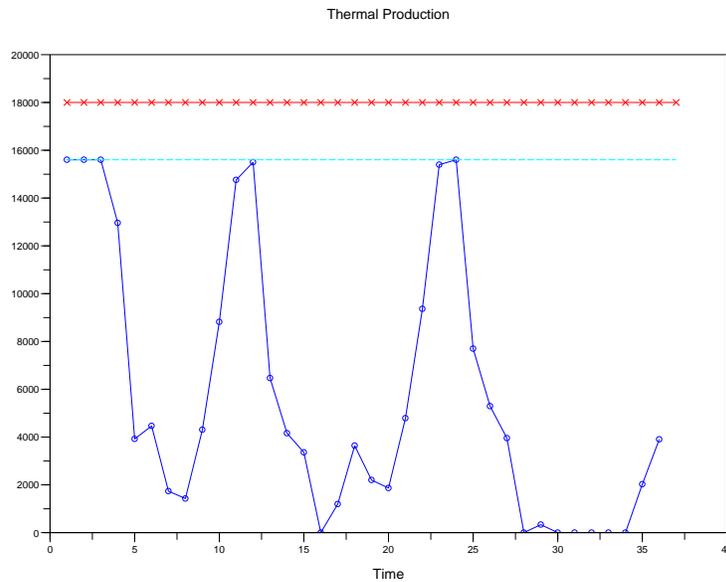


Figure 10: Simulated non-nuclear thermal production (in MW)

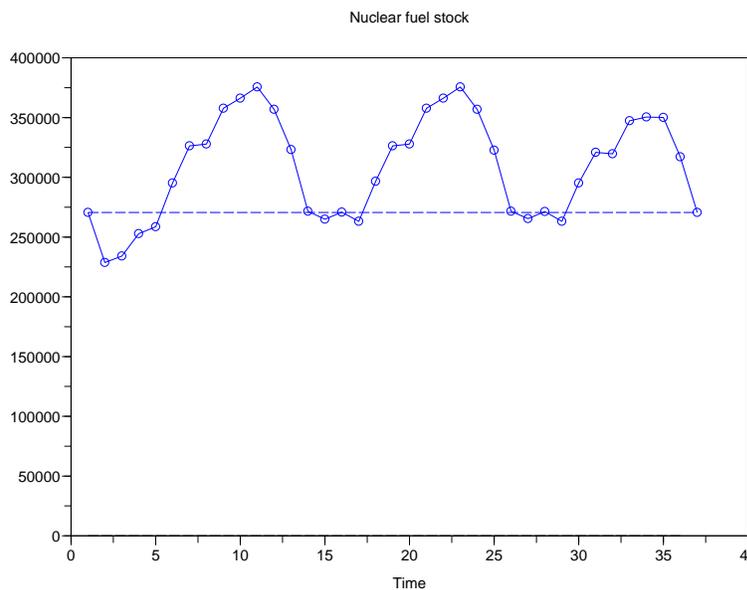


Figure 11: Simulated nuclear fuel stock (in MW)

and of 2009. However the reader should not focus on the precise amount of profit since its level depends on too many of the approximations we did (euro/dollar, oil prices, CO<sub>2</sub> cost, discount rate, no mark-up rate, absence of profits coming from the hydro technology (run-of-river), etc.) and because our modelling does not take into account the electricity importations/exportations or the production coming from renewable (see Figure 12, (see Table 4, Section 8)).

We separate period  $T$  into three sub-periods according to the evolution of both nuclear and non-nuclear thermal productions. According to Figure 8, we distinguish first a sub-period

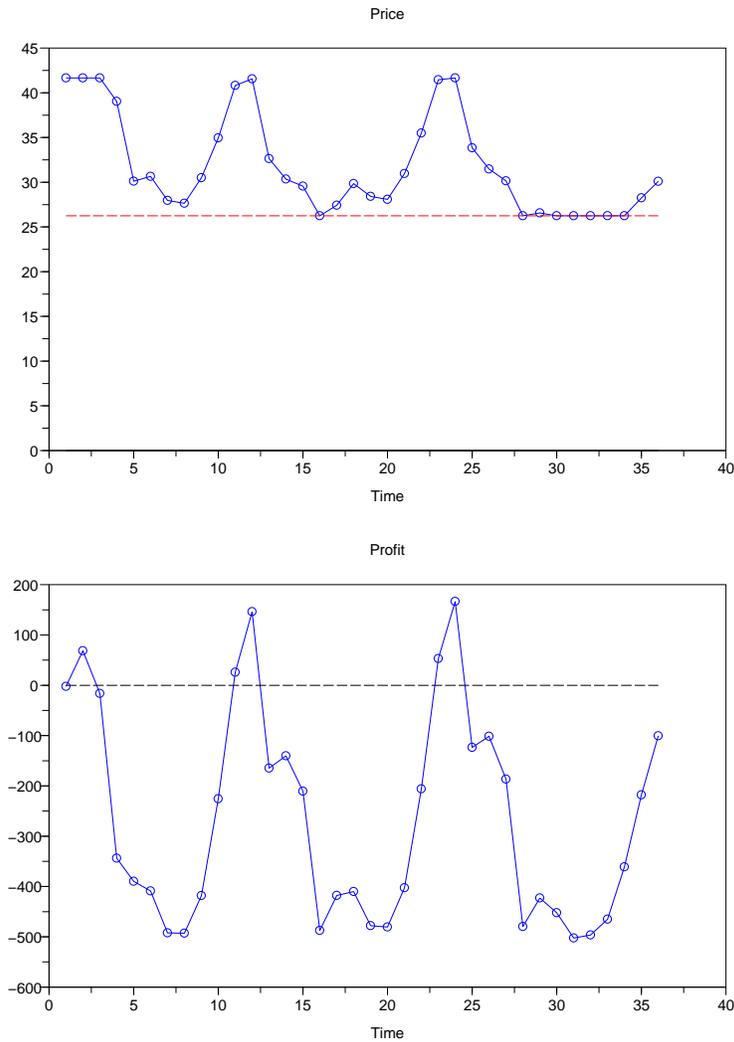


Figure 12: Simulated “regularized” price (in Euro/MWh)/Simulated “regularized” profit (in Euro (million))

during which non-nuclear is the marginal technology, a medium sub-period with a periodical evolution of the nuclear and non-nuclear thermal production and finally, a third sub-period during which nuclear is mainly the marginal technology.

### 5.2.2 First sub-period

The total nuclear production is significantly low during the first months of the simulation (January 2007 – April 2007) corresponding to an “underconsumption” of the nuclear fuel stock (see Figure 8, Figure 9, Figure 11) at the beginning of the game. But on the contrary, the non-nuclear thermal production increases significantly during these months in order to cover the increased levels of demand and the important decrease of the nuclear production (see Figure 8, Figure 10). In particular, the non-nuclear thermal production reaches its maximum value during the first three months of the model’s time horizon (January 2007 – March 2007). The price<sup>31</sup> and the profit during this period are higher than the price and the profit during the same months of the following years because of the importance of non-nuclear thermal production at

<sup>31</sup>The red dotted line indicates the “regularized” price level when nuclear is the marginal technology.

the beginning of period  $T$ . These significant levels of non-nuclear thermal production lead to a less important nuclear production compared to that realized in January – March in the following years (see Figure 12).

Note that if the nuclear fuel stock is “underused” at the beginning of the time horizon of the model (January 2007 – April 2007), significant gains<sup>32</sup> are generated if we compare them with the gains obtained during the same months of the years that follow (see Figure 12).

### 5.2.3 Medium sub-period

The nuclear production follows the seasonal variations of the demand (high production during winter, low production during summer) which means “high” levels of nuclear fuel stock during summer and “low” levels of nuclear fuel stock during winter (see Figure 9, Figure 11). Therefore, the periodical evolution of the nuclear production implies a periodical evolution for the nuclear fuel stock too. Note that the trend of the stock appears significantly above the “stock of reference”<sup>33</sup> suggesting that the nuclear fuel stock is not “overconsumed” during this period. However, during periods when nuclear production is important, the nuclear fuel stock decreases reaching its “reference” value.

Moreover, one can see that the non-nuclear thermal production is high during winter (respectively low during summer) because of the high (respectively low) level of demand. In particular, the non-nuclear thermal production is increasing during winter (beginning from the month of September) until it reaches its peak value during the month of November and of December. Afterwards, the non-nuclear thermal production decreases since summer is a low demand season. However, non-nuclear stays marginal during summer because of the very low levels of the nuclear production (see Figure 10). Consequently, price is high during the months of winter by taking its highest value during the period November – December and relatively low during summer. The optimal profit obtained by the producer is high during winter and at the beginning of spring while lower profits are realized during summer. Moreover, we can see that its value can be decomposed in a cyclical component and a linear trend which is increasing (see Figure 12).

### 5.2.4 Last sub-period

We observe in Figure 9 that the total nuclear production increases significantly during the last semester of 2009 and approaches its maximum level during the last months of the same year (November – December). The nuclear units “overproduce” during this period in order to consume the remaining from the first sub-period nuclear stock so that the producer gets its nuclear units at the end of the game with the same stock as the initial one (see Figure 11). On the contrary, the non-nuclear thermal production is equal to zero during this period (with only exception the month of November, December) since the demand is covered entirely by the nuclear production (see Figure 8, Figure 10). For this reason, the price and the profit reach their lowest levels during this sub-period (see Figure 12).

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<sup>32</sup>Recall that the mark-up rate is taken equal to zero.

<sup>33</sup>The “stock of reference” is represented by the blue dotted line which shows the value of stock at the beginning, being also the value of stock at the end.

### 5.2.5 General remarks

It should be noted that, within this time scale, we do not meet Spector's conclusion<sup>34</sup> about the insufficient size of the French nuclear set which makes the owner of that set (the French state) recipient of a scarcity rent (Spector (2007), Vassilopoulos (2007), Glachant and Finon (2007)). In our exercise, it does not seem to be significantly below the "optimal size" since the profit realized during the entire time horizon of the model is not very important (see Figure 12).

We also remark that if we modify the length of the time horizon  $T$  of the model (i.e.  $T \geq 36$  months), the behaviour of the producer does not change. The evolution of the nuclear and non-nuclear thermal production during the first and the last sub-period as well as the periodical evolution of the production during the medium sub-period are the same (e.g. for  $T = 72$ , see Figure 13).

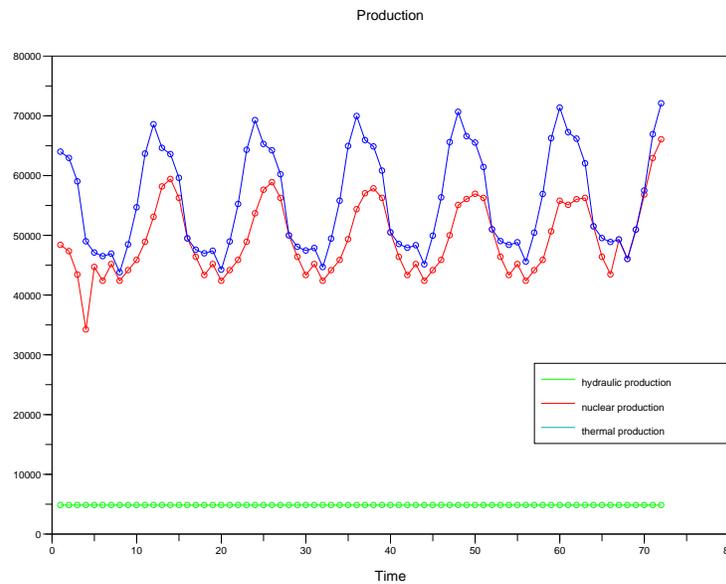


Figure 13: Simulated hydro/nuclear/non-nuclear thermal production (in MW) ( $T=72$ )

<sup>34</sup>In France, given the high participation of nuclear in the domestic electricity generation, studies like the one of Spector (2007) and Vassilopoulos (2007) pay special attention to the "optimal size" of the French nuclear set in a competitive electricity market. Nuclear plants are preferentially used to cover only the baseload demand and the main reason is the amortization of their high fixed costs. In a competitive market, if the marginal technology is nuclear all the year, the nuclear producers cannot cover their fixed costs. In such a market, the fixed costs and the variable costs will be covered, say, on a yearly basis only if the nuclear set has its "optimal size" within the whole generation set. In order to better understand this notion, we proceed with a numerical example given by Spector. Following this numerical example the optimal nuclear set for France corresponds to a nuclear marginality of 40%. This means that nuclear plants can cover all their fixed-costs through existing continental Europe market based prices during the 60% of marginality of the other thermal generation technologies (assuming that wind is not taken into account: basically coal, gas and fuel oil) on the basis of marginal costs of the latter. However, the nuclear set could also be "sub-optimal" that is to say, lower than its optimal level. In this case, even in presence of perfect competition, it would be remunerated above its marginal costs more than 60% of time. Consequently, its holders would profit from a scarcity rent, whatever the intensity of competition would be on the wholesale market. Spector and Vassilopoulos estimate that vis-à-vis the current size of the continental European market the French nuclear set is "sub-dimensioned", which makes the owner of that set (the French state) recipient of a scarcity rent (Glachant and Finon (2007)).

## 6 Conclusion

In view of the introduction of competition in the electricity markets one can ask what the optimal management of the nuclear generation set is in a competitive framework. In this paper, we studied the optimal short-term management of a flexible nuclear generation set (like the French set) which partly responds to the daily and seasonal variations of demand as a result of its significant size in a perfectly competitive regime. We focused on the seasonal variation of the demand between high demand (winter) and low demand (summer). The novelty of our model exists in the fact that the nuclear fuel functions like a “reservoir”, which allows different allocations of the nuclear production during the different seasons of a campaign of production. We showed that the nuclear fuel can operate as a “reservoir”. We built a deterministic multi-period model to study the perfect competitive case in a market where producers use both nuclear and non-nuclear units (section 3). We introduced the notion of a merit order equilibrium based on the merit order price rule and then, we presented our different attempts to calculate a merit order equilibrium of the optimal short-term production problem (section 4). To end, a numerical simulation was given by taking into account the actual size of a given nuclear set (the French) vis-à-vis the non-nuclear generation set (section 5).

From a mathematical perspective, we kept our model simple by assuming the existence of a unique producer that possesses the total capacity (nuclear and non-nuclear thermal). This permitted the reduction in the number of the optimization variables of the optimal short-term production problem and therefore a simplification of its resolution. In view of the concept of the merit order equilibrium, our model can be extended in the case of  $N \geq 2$  producers. However, considering Proposition 4.1, we were able to focus on the alternative economy with one aggregate producer since there is an equivalence of equilibrium between the original economy with  $N \geq 2$  producers and the alternative economy with one aggregate producer ( $N = 1$ ).

Three different approaches in order to determine a merit order equilibrium of the optimal short-term production problem were distinguished. The nuclear is traditionally used to cover the base load demand by functioning always at its maximum capacity. This is typically not the French case where nuclear is used to meet both the base load and the semi-base. In our first approach, we attempted to calculate a merit order equilibrium by computing a feasible production vector during the month  $t$  according to the observed merit order price level during this month and by taking into account the equality between supply and demand. However, it is not possible to calculate a merit order equilibrium within this approach as a result of the supply behaviour with respect to merit order price. In the second approach, a producer determines its production level at each month  $t$  by maximizing its profit in the month  $t$  subject to the production constraints imposed by generation capacity and nuclear fuel storage. In addition to this, the equality between supply and demand is taken into account for the month  $t$  but it is not considered in future months. We observed (via a numerical simulation) that there exists a month during which the production constraints and the supply-demand equilibrium constraint can not be respected simultaneously. Therefore, we deduced that a high level of “short-sightedness” with respect to future demand is intolerable and it leads to a failure to compute a merit order equilibrium of the optimal short-term production problem. In our third approach, we provided a last scenario in which the producer takes into consideration the supply-demand equilibrium constraint imposed by the operator of the network for the whole time period  $T$ . In this last approach, the equality between supply and demand is sufficiently anticipated in future periods in order to manage the current use of the nuclear fuel reservoir and a merit order equilibrium of the optimal short-term production problem is calculated within a numerical model.

The marginal cost of nuclear production being (importantly) lower than the marginal cost

of non-nuclear thermal production induces a discontinuity of the merit order price from a mathematical point view. We proved that the discontinuity and more precisely the decrease of price when nuclear changes into marginal technology implies a decrease of profit (Proposition 4.3).

In our next section, we proceeded with the analysis of the production decisions resulting from the optimal short-term management of flexible market based nuclear plants within a simple numerical model. The problem of discontinuity mentioned above was treated with the “regularization” of our economical problem (presented in subsection 4.5) which is achieved through the “regularization” of the merit order price. This “regularization” resulted in an approximate problem (continuous problem) which differs from the economical problem (discontinuous problem) by its objective function. However, we proved that the value of these optimization problems (the “regularized” problem and the economical problem) is the same, which implies that the “regularized” problem is a “good” approximation of our economical problem (see Annex, Proposition 7.1). Furthermore, we characterized the set of solutions of the economical problem as the intersection of the set of solutions of the “regularized” problem and the set  $H_{th}^t$  (see Annex, Proposition 7.2). Within our numerical example, we concluded that the set of solutions of the economical problem is empty because the solution obtained by the “regularized” problem does not belong to the set  $H_{th}^t$  (see Annex, Proposition 7.3). This numerical solution is only an “approximate” solution of our economical problem.

In this late frame, we presented the results coming from our numerical problem (“regularized” problem). We did find high levels of nuclear production during the months of high demand (winter) and low levels during the months of low demand (summer), which confirms that nuclear can theoretically be a load-following generation set. As expected, the evolution of the nuclear fuel stock is the opposite of the evolution of the nuclear production (low levels of stock during winter – high levels of stock during summer). In addition, we noticed that the different values of the nuclear fuel stock obtained during period  $T$  remain higher than the “reference” value of this fuel stock. However, the value of stock is very close to its “reference” value when nuclear production is significant. Furthermore, the producers increase their non-nuclear thermal production during winter and they decrease it during summer, according to the corresponding demand level and to the level of the nuclear production. Nevertheless, the non-nuclear thermal technology remains marginal during both seasons in the first and medium sub-period of time horizon  $T$ . The only exception is the last sub-period since the nuclear units “overproduce” so that the nuclear fuel constraint according to which the game ends with the same quantity of stock as the initial one is respected. Hence, in order to ensure the equilibrium between supply and demand, the management of the nuclear set has to take into account the thermal non-nuclear generation capacity during almost the entire time horizon  $T$ . Finally, in view of the “regularization” of the merit order price rule, the price is determined by the non-nuclear thermal marginal cost during the entire period  $T$ . In particular, we observed that price peaks during winter and reaches its lowest point during summer. Accordingly, producers obtain higher profits during winter and lower profits during summer.

We modelled the optimal production behaviour in a perfectly competitive electricity market as an optimal short-term production problem and we numerically determined a merit order equilibrium of this problem. This is a problem that consists of the maximization of the production value during the month  $t$  subject to the nuclear fuel constraints for each nuclear unit  $j$  and the minimum/maximum production constraints in the month  $t$ . Each month, the merit order equilibrium production levels satisfy the equality between supply and demand and the price is given by the marginal cost of the “last technology” of the merit order used to equilibrate supply and demand. This mode of operation is not based on the direct optimization of the production over the entire period  $T$ . It could, however, correspond to the prudent behaviour of a nuclear

set quitting the monopoly era and discovering step by step how competitive wholesale markets work. The discovery of a full inter-temporal optimization could take time as it would result from the maximization of the value of generation during the whole time horizon of the model. No doubt that market based management of flexible nuclear plants would then like to look at determining the global optimum of the optimal production problem. This further analysis is to be our future work.

## 7 Annex

Let us introduce some notations used in the annex. We call  $P_1^t$  the economical problem (presented in subsection 4.5)

$$\max_{q_t \in H^t} \pi_t(q_t)$$

and  $P_2^t$  the “regularized” problem (presented in subsection 5.2)

$$\max_{q_t \in H^t} \psi_t(q_t)$$

where  $\pi_t$  and  $\psi_t$  represent the current monthly profit functions of problem  $P_1^t$  and  $P_2^t$  respectively.

We will recall the properties proven in the paper in order to allow a self-content annex to the maximum extent. Functions  $\pi_t$  and  $\psi_t$  are such that  $\pi_t \leq \psi_t$  on  $H^t$ . This is because  $\pi_t = \psi_t$  on  $H_{th}^t$  which is a subset of  $H^t$  since the price is determined by the non-nuclear thermal marginal cost in both problems and  $\pi_t < \psi_t$  on  $H^t \setminus H_{th}^t$  because when nuclear is the marginal technology, the nuclear production is paid at price  $b_{th}$  within the “regularized” problem while it is paid  $b_{nuc}$  inside the economical problem and  $b_{nuc} < b_{th}$  (see Proposition 4.3). Let us also mention that  $\psi_t$  is a continuous function which, in addition, is strictly concave with respect to the non-nuclear thermal production  $q_t^{th}$  (see proof of Lemma 4.1). Moreover, the set  $H_{th}^t$  is dense in  $H^t$  (see Proposition 4.2).

We are now ready to state the following propositions:

**Proposition 7.1** *For all  $t \in \{1, \dots, T\}$ , the value of the economical problem ( $val(P_1^t)$ ) and the value of the “regularized” problem ( $val(P_2^t)$ ) are the same ( i.e.  $val(P_1^t) = val(P_2^t)$ ).*

**Proposition 7.2** *For all  $t \in \{1, \dots, T\}$ , the set of solutions of the economical problem ( $Sol(P_1^t)$ ) and the set of solutions of the “regularized” problem ( $Sol(P_2^t)$ ) are such that  $Sol(P_1^t) = Sol(P_2^t) \cap H_{th}^t$ .*

**Proposition 7.3** *For all  $t \in \{1, \dots, T\}$ , if  $q_t$  is a production vector that does not belong to  $H_{th}^t$  and  $q_t \in Sol(P_2^t)$  (it constitutes a solution of the “regularized” problem), then  $Sol(P_1^t) = \emptyset$ .*

We do not provide a proof for each of the above propositions since they are simple adaptations of the corresponding Propositions 8.1, 8.6 and 8.7 presented and proven in the annex of our paper (Lykidi, Glachant, Gourdel (2011)).

## 8 Tables

In this section, we present the values of the exogenous variables of our numerical model. We also provide the value of the total “regularized” profit obtained by the optimal short-term production problem. Moreover, we present the value of the total “regularized” nuclear and non-nuclear thermal profit.

Nuclear capacity of unit $j$ (in MW)	$j = 1$	$j = 2$	$j = 3$	$j = 4$	$j = 5$	$j = 6$	
Capacity $^{j,nuc}$	201	5641	6758	4359	8838	8045	
	$j = 7$	$j = 8$	$j = 9$	$j = 10$	$j = 11$	$j = 12$	Total
	4923	7956	5634	5950	2972	1723	63000

Table 1

The level of nuclear capacity that the unit  $j$  disposes is denoted by  $Capacity^{j,nuc}$ .

Capacity (in MW)	Nuclear	Non-nuclear thermal	Hydro
$Q_{min}^{j,nuc}$	$0.25 \times Capacity^{j,nuc}$	—	—
$Q_{max}^{j,nuc}$	$1 \times Capacity^{j,nuc}$	—	—
$Q_{min}^{th}$	—	0	—
$Q_{max}^{th}$	—	15600	—
$Q_t^{hyd}$	—	—	4851.6
Stock of reloading of unit $j$ (in MWh)			
$S_{reload}^j$	$1 \times Capacity^{j,nuc} \times 24 \times 258$	—	—

Table 2

where 258 corresponds to the number of days during which a nuclear unit can operate at full capacity.

Parameters	Nuclear	Non-nuclear thermal
$a_{nuc}^j$ (in Euro)	$22.79 \times Capacity^{j,nuc} \times (8760/11)$	—
$b_{nuc}$ (in Euro/MWh)	5.01	—
$a_{th}$ (in Euro)	—	$11.5 \times 10^7$
$b_{th}$ (in Euro/MWh)	—	26.24
$c_{th}$ (in Euro/MWh <sup>2</sup> )	—	$6.76 \times 10^{-7}$

Table 3

where 22.79 represents the total fixed cost of nuclear in Euro/MWh.

	Optimal short-term production problem
Total “regularized” profit (in Euro)	$-9.636 \times 10^9$
Total “regularized” nuclear profit (in Euro)	$-6.332 \times 10^9$
Total “regularized” non-nuclear thermal profit (in Euro)	$-3.304 \times 10^9$

Table 4

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