Investing in the environment : essays on energy efficiency and on the substitution of resources

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Investing in the Environment

Essays on energy efficiency and on the substitution of resources

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Dedicado a la memoria de mi padre:
José Luis Mosiño Irazaba
1946-2009

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I

Introduction (French version)

Introduction

Les gaz à effet de serre (GES) sont en partie responsables du changement climatique. L'humanité est donc confrontée à un choix : soit réduire les émissions des gaz qui sont la cause du problème, soit prendre des mesures pour permettre aux populations de surmonter les conséquences de ces changements. Dans cette thèse, nous nous concentrons sur la première solution sous la prémisse selon laquelle une grande partie de l'effet de serre provient des activités humaines. Plus précisément, nous proposons quelques essais sur la modélisation des déterminants des investissements ayant pour objet la réduction des GES, en particulier des investissements dans l'amélioration de l'efficacité énergétique, et des investissements dans la substitution de ressources (fossiles) non-renouvelables par des ressources renouvelables. Tout d'abord nous essayons et expliquons la lente diffusion de certains investissements dans l'efficacité énergétique dans un cadre d'équilibre général. Ensuite, nous étudions les déterminants de la substitution des ressources non-renouvelables par des ressources renouvelables lorsque celles-ci sont des substituts parfaits. Enfin, nous tenons compte de la nécessité permanente de ressources sales, même si des technologies plus propres sont disponibles. Toutes ces questions sont basées sur des modèles qui ne peuvent être entièrement résolus analytiquement. Par conséquent, nous proposons dans cette thèse une méthodologie basée sur les propriétés des polynômes de Chebyshev pour calculer les solutions.

Le contexte

La plupart des scientifiques climatologues s'accordent sur la principale cause du réchauffement climatique actuel : l'augmentation de l'effet de serre provenant des activités humaines. Cet effet de serre *anthropogénique* accroît l'effet de serre *naturel* de la Terre par l'ajout d'émissions de GES provenant de la combustion des combustibles fossiles (pétrole, charbon et gaz naturel), principalement dû à la production d'énergie. Le dioxyde de carbone ($C0_2$) est le gaz qui a le forçage radiatif le plus élevé, ce qui signifie que c'est le gaz qui a le potentiel de réchauffement global le plus élevé. Le charbon, en particulier le charbon brun, est la source d'énergie qui émet le plus d'émissions de GES par unité d'énergie. La combustion du charbon génère 70 pourcent de CO_2 de plus que le gaz naturel pour chaque unité d'énergie. En même temps, le charbon ne coûte pas cher et est le combustible fossile le plus largement disponible. En ce qui concerne le mazout, les taux moyens d'émissions de CO_2 aux États-Unis provenant de la production de fioul est de 1672lbs / Mwh.

Au niveau mondial, les émissions de CO_2 liées à l'énergie ont été mutlipliées par 145 depuis 1850 — passant de 200 millions de tonnes par an à 29 milliards de tonnes par an — et devraient augmenter encore de 54 pourcent d'ici 2030. La majeure partie des émissions mondiales proviennent d'un nombre relativement restreint de pays. Les 25 plus grands émetteurs, représentant 75 pourcent de la population mondiale et 90 pourcent du produit intérieur brut (PIB) mondial, totalisent environ 85 pourcent des émissions de GES mondiales.[1] Le Quatrième Rapport d'Évaluation datant de 2007 établi par le IPCC (AR4) a noté que « les changements dans les concentrations atmosphériques des gaz à effet de serre et des aérosols, de la couverture terrestre et du rayonnement solaire modifiaient l'équilibre énergétique du système climatique », et a conclu que « l'augmentation des concentrations de gaz à effet de serre d'origine anthropogénique est très susceptible d'avoir en grande partie causé l'augmentation des températures moyennes mondiales depuis le milieu du 20ème siècle ». Selon ce rapport, la température a augmenté d'environ 0,13 degrés Celsius par décennie au cours des 50 dernières années — près de deux fois l'augmentation de la température moyenne pour

[1] Climate Change Mitigation, UNEP, disponible en ligne à : `http://www.unep.org/climatechange/mitigation/Home/tabid/104335/Default.aspx`. Extrait du 20 juillet 2012.

les 100 dernières années, et onze fois durant les douze dernières années (1995-2006) se classant parmi les 12 années les plus chaudes (depuis 1850). Suite à cette évolution, on peut s'attendre à une élévation de la température allant de 1,8 degrés à 4 degrés Celsius [IPCC, 2007, Stern, 2007]. Ce réchauffement de la planète causera plusieurs préjudices en ce qui concerne l'activité économique, la vie humaine et l'environnement, souvent à cause de l'eau. La fonte des glaciers et l'élévation conséquente du niveau des mers se traduira par une réduction de l'approvisionnement en eau et de la migration de millions de personnes vivant dans des zones inondées. Des événements climatiques extrêmes seront davantage susceptibles de survenir, et le risque d'un rapide changement climatique et des répercussions majeures irréversibles (par exemple, la fonte de la calotte glaciaire du Groenland) sera plus importante. En outre, les rendements vont sérieusement baisser au niveau de la production alimentaire, les écosystèmes vulnérables seront incapables de maintenir leur forme actuelle et de nombreuses espèces devront faire face à une possible extinction.[2]

Compte tenu des graves impacts — négatifs — du réchauffement climatique, l'un des défis majeurs auxquels doivent faire face actuellement les décideurs du monde est la réduction des émissions de GES. Selon la version finale de la Déclaration de la 64ème Conférence Annuelle DPI / ONG [UNDPI, 2011] les gouvernements devraient avoir atteints des itinéraires clairs vers une durabilité climatique qui régule la hausse de la température mondiale en dessous de 1,5 degrés Celsius. Les émissions de GES devraient être réduits de 25 pourcent par rapport aux niveaux de 1990 d'ici 2020, 40 pourcent d'ici 2030, 60 pourcent d'ici 2040 et 80 pourcent d'ici 2050. Cet objectif est conforme à celui suggéré par des études antérieures, telles que le rapport Stern sur le changement climatique [Stern, 2007], qui préconise des interventions importantes et immédiates afin de réduire les émissions mondiales de 60 pourcent à 80 pourcent en 2050 ; Nordhaus [2007], qui propose un modeste contrôle à court terme, suivie par des réductions d'émissions nettes à moyen et à long terme, ou l'IPCC [2007], qui appelle à stabiliser les émissions à GES entre 445 et 490 ppm —- entre 50 à 85 pourcent de réductions par rapport aux niveaux de 2000 — pour garder des températures mondiales de 2 de-

[2]Voir, par exemple : The Consequences of global Warming, NRDC. Disponible en ligne à : `http://www.nrdc.org/globalwarming/fcons.asp`. Extrait du 20 Juillet 2012.

grés à 2,4 degrés Celsius au-dessus de la moyenne pré-industrielle.[3] Pour atteindre cet objectif, le récent Green Economy report » [UNEP, 2011] propose un objectif de 1,3 milliards de dollars (2 pourcent du PIB mondial) concernant des investissements verts (public et privé). Près de trois cinquièmes de cette somme serait investi dans l'efficacité énergétique — en particulier dans les bâtiments, l'industrie et les transports — et les énergies renouvelables. Des exemples de politiques publiques actuelles en matière d'efficacité énergétique sont celles du Département de l'Énergie des États-Unis, tel que le Programme de Renforcement des Technologies, dont le but stratégique est de créer des technologies et des approches conceptuelles qui permettent de mettre sur le marché des bâtiments qui ne consomment pas d'énergie, d'ici 2020 pour les maisons, et d'ici 2025 pour les bâtiments commerciaux ; ou l'initiative « Sunshot », dont le but est de rendre compétitive l'industrie de l'énergie solaire ainsi que celle d'autres formes d'énergies d'ici la fin de la décennie.[4]

Compte tenu de ce contexte, il est intéressant de modéliser les déterminants de l'investissement qui permet la réduction des émissions de GES, à la fois par l'amélioration de l'efficacité énergétique et par la substitution de ressources (fossiles) non-renouvelables (sales) par des ressources renouvelables (propres) pour l'approvisionnement d'énergie. Nous allons étudier les déterminants de ces investissements dans le but de fournir des recommandations de politique publique ; ceci est l'objectif principal de cette thèse. Nous abordons le problème de l'investissement dans l'efficacité énergétique en tenant compte du cas spécifique du secteur résidentiel (c'est-à-dire l'investissement dans l'efficacité énergétique pour les fenêtres, les portes, etc.). Le problème de la substitution des ressources est analysé selon deux scénarios différents. Dans le premier cas, nous supposons que les combustibles fossiles et les ressources renouvelables sont des substituts parfaits dans la production d'énergie. Puis, quand les entreprises décident de passer à des ressources renouvelables, elles continueront de produire avec cette nouvelle technologie par la suite. Dans le second cas, nous supposons que, bien que les entreprises peuvent passer à une technologie plus propre en utilisant des énergies renouvelables, l'économie n'est pas tout à fait « sans pollution »puisque les combustibles fossiles sont encore nécessaires dans l'économie.

[3] Il y a également Greenpeace, qui propose d'arrêter toute croissance économique dans le but de sauver la planète [Acemoglu et al., 2012].

[4] Le département américain de l'énergie : `http://www.eere.energy.gov/`. Extrait du 20 juillet 2012

Une particularité des modèles que nous présentons ici est que toutes les décisions sont prises dans un environnement stochastique. Dans le même temps, nous introduirons certaines irréversibilités inhérentes à presque toute politique environnementale. Nous tenons également compte du fait que, dans la plupart des cas, il est possible de retarder l'action et d'attendre pour obtenir des nouvelles informations. Nous obtenons que ces incertitudes, irréversibilités, et la possibilité de retarder l'action peuvent affecter de manière significative le moment optimal pour l'adoption de politiques. Par exemple, on obtient que l'incertitude sur les rendements financiers des ménages favorise l'adoption des technologies qui améliorent l'efficacité énergétique. L'incertitude affecte également l'efficacité de la politique économique qui vise à accélérer la substitution des ressources renouvelables pour des ressources non renouvelables dans la production d'énergie. Enfin, les gouvernements sont tentés d'attendre plus longtemps, le temps que des technologies plus propres soient suffisamment développées pour les adopter dans les secteurs de la production.

Dans les lignes qui suivent, nous justifions la nécessité de tenir compte de l'irréversibilité et l'incertitude dans l'économie environnementale. Nous allons ensuite présenter les quatre chapîtres qui composent ce mémoire.

Irreversibilité et incertitude

L'irréversibilité de l'investissement signifie que l'engagement pour des projets d'investissements se traduit par un coût irrécupérable — le coût initial — que l'on appelle « coûts irréversibles ». L'incertitude sur les bénéfices futurs et les coûts d'investissements sont également importants. En fait, ce sont l'effet combiné de l'irréversibilité et de l'incertitude qui a conduit Weisbrod [1964] à introduire le concept de la *valeur d'option.* Il fait valoir que si une décision a des conséquences irréversibles, alors la flexibilité — « l'option »— qui permet de choisir le moment le plus opportun pour prendre cette décision a une valeur qui doit être incluse dans une analyse coûts-bénéfices. Le concept de valeur d'option a deux interprétations différentes. Dans la première la valeur d'option est vue comme une prime de risque payée par les consommateurs de risques afin de réduire l'impact de l'incertitude dans la demande d'un bien environnemental [Cicchetti and Freeman, 1971, Schmalensee, 1972]. La seconde interprétation insiste sur l'aspect inter-temporel de l'irréversibilité et l'arrivée de nouvelles informations au fil du temps [Arrow and Fisher, 1974, Henry, 1974]. Cette seconde interprétation est celle qui a été le plus largement utilisée dans la littérature sur les *options réelles* [Myers, 1977]. Des exemples sont l'entrée, la sortie et les redémarrages de la production et de l'investissement ; les implications du moment de construction — et de l'option de renoncer à la construction — pour la valeur d'un projet ; et les déterminants du choix d'une entreprise quant à sa capacité [McDonald and Siegel, 1986, Brennan and Schwartz, 1985, Pindyck, 1988].

Pour avoir plus d'intuition à propos de l'irréversibilité, l'incertitude, et sur la signification de la valeur d'option, considérons le modèle proposé par Olsen and Stensland [1988]. Leur modèle se compose d'une industrie basée sur des ressources non renouvelables (par exemple l'extraction d'hydrocarbure extra-côtier) devant décider si elle doit ou non cesser son activité. Tout d 'abord, la décision de la cessation peut être considérée comme irréversible. Dans le modèle d'Olsen et Stensland l'irréversibilité signifie que les entreprises ne peuvent pas reprendre les opérations après la cessation (probablement en raison de coûts prohibitifs). De plus, Olsen et Stensland supposent que les conditions économiques — en particuliers les prix et les quantités de production — sont incertaines. L'incertitude conjointement à l'irréversibilité implique que la décision prise par l'entreprise implique l'exercice, ou l'abandon de l'option de fermer de manière optimale à tout moment dans l'avenir. Dans un tel cadre, il y a une valeur due à l'attente — la valeur d'option — puisque la firme a toujours la possibilité de reporter la cessation afin d'en apprendre plus sur les gains actuels et futurs. En conséquence, la règle habituelle de la cessation de l'activité lorsque les coûts marginaux de production enregistrés sont égaux aux gains marginaux attendus n'a plus lieu d'être valide. Pour pouvoir cesser l'activité, les coûts marginaux de production enregistrés doivent être supérieurs au revenu marginal attendu d'un montant égal à la valeur de maintien de l'option [Pindyck, 1988, Dixit and Pindyck, 1994]. Notons que la possibilité de cessation (ou la possibilité d'investir, ou toute autre opportunité en situation d'incertitude et d'irréversibilité) est analogue à une option sur une action ordinaire. Dans ce cas, nous avons une option (put) qui nous donne le droit (le prix d'exercice de l'option), mais pas l'obligation (en raison de la possibilité d'attendre) pour cesser (vendre) les activités (le sous-jacent), la valeur fluctuant de manière stochastique [Pindyck, 1991]. Bien sûr, dans le cas d'irréversibilité complète la valeur résiduelle de l'entreprise est égale à zéro [Dixit and Pindyck, 1994, Dangl and Wirl, 2004].

L'incertitude et l'irréversibilité peuvent également être envisagées dans certains autres problèmes environnementaux. Par exemple, Brennan and Schwartz [1985] analysent l'exploitation d'une mine, qui peut être fermée temporairement ; Paddock et al. [1988] proposent un modèle d'évaluation d'hydrocarbure extra-côtier loué en fonction du prix du marché du pétrole, et Clarke and Reed [1990] étudient la préservation des réserves sauvages naturelles. Le résultat standard est que, en présence d'irréversibilité de l'environnement, une analyse standard coûts-bénéfices est biaisée à l'encontre de la conservation [Arrow and Fisher, 1974, Henry, 1974]. Freixas and Laffont [1984] généralisent ce résultat, et Conrad [1980] lie la valeur d'option à la valeur attendue de l'information.

Dans certains documents plus récents, Pindyck [2000, 2002, 2007] explique le rôle de l'incertitude et de l'irréversibilité de la politique environnementale. Dans le cas de politiques axées sur le réchauffement global par exemple, l'incertitude provient du fait que nous ne savons pas de combien de degré la température moyenne s'élèvera avec ou sans réduction d'émissions de CO_2 ; nous ne connaissons pas non plus l'impact économique de la hausse des températures. Ensuite, l'irréversibilité apparaît, par exemple, dans les politiques visant à réduire la dégradation de l'environnement. Il y a des coûts irrécupérables qui prennent la forme d'investissements discrets (par exemple l'industrie qui utilise la combustion du charbon utilitaires pourrait être obligée d'installer des épurateurs ou des entreprises pourraient avoir à abandonner des machines existantes et à investir dans des machines plus économes en carburant), ou ils peuvent prendre la forme de flux de dépenses. Ces coûts irrécupérables créent un coût d'opportunité quant à l'adoption d'une politique maintenant, plutôt que d'attendre d'obtenir plus d'informations sur les impacts écologiques et leurs conséquences économiques. Il y a aussi une certaine irréversibilité des dommages environnementaux. Par exemple, l'accumulation de GES dans l'atmosphère est de longue durée, même si nous devions réduire drastiquement les émissions de GES, cela demandera de nombreuses années avant que les niveaux de concentration atmosphérique diminuent. Cela signifie qu'adopter une politique maintenant plutôt que plus tard a un bénéfice irrécupérable, qui est un coût d'opportunité négatif. Celas signifie aussi que l'analyse des coûts-bénéfices traditionnelle sera un parti pris contre l'adoption de politiques.

Qu'en est-il des cas particuliers de l'efficacité énergétique et de la substitution de ressources qui nous concerne ? Prenons le cas de la substitution des ressources. Imaginons une entreprise qui tente de décider si oui ou non il faut cesser d'utiliser du non renouvelable (combustibles fossiles) en faveur de la production renouvelable en ce qui concerne la production d'énergie. L'incertitude peut apparaître dans plusieurs cas, mais il semble particulièrement pertinent de se concentrer sur l'incertitude quant à la disponibilité des ressources [Pindyck, 1980, Epaulard and Pommeret, 2003, Smith and Son, 2005]. Plus précisément, nous pouvons avoir une idée de leur stock actuel, mais nous ne savons pas trop ce qu'il en sera quant à leur disponibilité future. Et même si nous étions en mesure de savoir quelle serait la disponibilité des ressources dans l'avenir, nous ne pouvons pas savoir l'effet résultant des décisions prises par les entreprises. Par exemple, plus les ressources renouvelables sont disponibles dans le futur, plus les entreprises sont tentées de les adopter au plus tôt pour produire de l'énergie. En outre, l'adoption d'énergies renouvelables impose des coûts irrécouvrables pour la société. C'est le cas en particulier si une plate-forme pétrolière extra-côtière se transforme en un parc éolien extra-côtier. Une fois cette décision prise, la quasi-totalité du capital déjà installé doit être démonté, et très peu de pièces de l'ancienne installation peuvent être réutilisées. Ce processus est évidemment très coûteux en termes de temps et d'argent. Comme précédemment, il y a une valeur d'option en ce sens que les investisseurs peuvent attendre d'obtenir de plus amples informations avant de se débarrasser des plates-formes pétrolières extra-côtières.

Considérons maintenant le cas spécifique d'un propriétaire investissant dans des nouveaux équipements afin de réduire sa facture énergétique. Nous pouvons supposer que cet investissement est irréversible, car la désinstallation du nouvel équipement peut être déraisonnablement coûteuse ou parce que l'ancien équipement a été mis au rebut. Une certaine incertitude peut également être envisagée. Par exemple, l'évolution de la richesse du ménage n'est pas entièrement connue, en particulier si l'on suppose que la richesse est fondée sur des actifs risqués. Sous ces conditions, la décision des ménages pour investir peut être considérée comme comportant une valeur d'option, car ils peuvent toujours différer l'investissement, afin de savoir si leur future richesse augmentera ou non. S'il n'y a pas d'arbitrage entre la consommation des ménages et la décision d'investir — c'est-à-dire dans le cadre d'un équilibre partiel, en vue d'investir dans cette technologie d'efficacité énergétique, la valeur marginale du nouvel équipement doit être supérieur au coût d'achat et d'installation, d'un montant égal à la valeur de maintien pour garder l'option d'investissement. Dans le cas d'un arbitrage entre la consommation et la décision d'investir — c'est-à-dire dans un cadre d'équilibre général, le moment optimal de l'adoption est non seulement sensible à l'incertitude, mais aussi sensible au degré d'aversion quant au risque des agents (Hugonnier et al. [2008] ; consulter aussi Pommeret and Schubert [2009] pour un exemple en matière d'économie de l'environnement).

Sur la résolution numérique des équations de Bellman

Dans cette thèse, nous sommes nous-même confrontés à des équations différentielles qui sont complexes et ne peuvent être entièrement résolues en utilisant des techniques analytiques standards. En particulier, les solutions analytiques pour les modèles dans les chapitres 3 et 5 ne sont valables que dans le cas d'un taux d'actualisation égal à zéro. Il y a un important débat dans l'économie de l'environnement en ce qui concerne la façon dont les utilités futures doivent être actualisées, surtout lorsque les décisions actuelles ont d'importantes conséquences à long terme (voir, par exemple, Portney and Weyant [1999] pour une enquête). Comme nous ne sommes pas particulièrement enclins en faveur d'un taux d'actualisation nul ou positif, une méthodologie pour résoudre tous les cas devient nécessaire. En outre, le modèle du chapitre 4 ne peut pas être entièrement résolu de façon analytique. Au moins pour les modèles que nous présentons ici, le problème est la forte non-linéarité des fonctions de valeur et le fait qu'elles doivent satisfaire certaines conditions limites. Ensuite, nous avons besoin de s'appuyer sur des méthodes numériques.

Comme suggéré par Judd [1992, 1998], nous utilisons les propriétés d'approximation des polynômes de Chebyshev pour calculer des solutions stables non divergentes des équations de Hamilton-Jacoby-Bellman. Plus précisément, nous transformons les fonctions de valeur et ces conditions pour obtenir des expressions avec les coefficients de Chebychev inconnus. En utilisant cette représentation, notre problème initial pour la résolution des équations à dérivées partielles se réduit à un problème où il faut résoudre des systèmes simples d'équations non-linéaires. Le modèle du chapitre 3 est résolu en utilisant un algorithme basé sur la méthode de Newton [Miranda and Fackler, 2004, Dangl and Wirl, 2004], tandis que les modèles des chapitres 4 et 5 sont résolus en utilisant la méthodologie présentée dans le chapitre 2.[5] En particulier, nous transformons la fonction de valeur et les conditions associées pour obtenir des équations matricielles avec des coefficients inconnus de Chebychev. En faisant de la sorte, notre problème se réduit à la résolution d'un système simple d'équations algébriques. Cette méthodologie est un sous-produit secondaire de cette thèse, mais elle constitue en elle-même une contribution originale à la littérature des méthodes numériques.

Efficacité énergétique

Les rénovations résidentielles sont généralement considérées comme étant pour les ménages un moyen très efficace pour réduire leurs dépenses d'énergie — et indirectement les GES — en améliorant l'efficacité, et elles deviennent donc une cible clé pour les politiques environnementales. En effet, les analyses coûts-bénéfices indiquent la viabilité économique de ces systèmes, même si le confort des co-bénéfices tels que l'amélioration de la qualité de l'air intérieur et de la protection contre le bruit ne sont pas pris en compte [Jakob, 2006, Ott et al., 2006]. Cependant, l'investissement réel dans ces systèmes est encore relativement rare [Banfi et al., 2008]. Dans le chapitre 3, nous visons à expliquer dans un modèle théorique pourquoi les ménages décident d'investir dans les rénovations résidentielles. Plus précisément, nous essayons d'expliquer la lente diffusion des investissements économiseurs d'énergie —- le soi-disant *paradoxe énergétique* ou *l'écart de rendement énergétique* [Jaffe and Stavins, 1994a].

La littérature existante qui explique le paradoxe énergétique (Hassett and Metcalf [1995], par exemple) considère des paramètres d'équilibre partiel, et ignore donc l'interaction entre la consommation optimale et l'adoption optimale, ainsi que les notions d'aversion au risque, ou les substitutions inter-temporelles. Dans de telles situations les

[5]La méthodologie du chapitre 2 peut également être adaptée pour résoudre le modèle du chapitre 3. En fait, les deux méthodes ont été utilisées a posteriori à des fins de comparaison.

consommateurs se comportent comme des entreprises au moment de décider d'investir dans des technologies qui permettent d'économiser l'énergie. Pour remettre en question ces résultats, nous reconsidérons l'effet conjoint de l'irréversibilité et l'incertitude concernant la décision d'un investissement économiseur d'énergie dans un cadre d'équilibre général [Hugonnier et al., 2008, Pommeret and Schubert, 2009]. En particulier, on peut se demander si l'explication du paradoxe de l'énergie basée sur l'existence d'une valeur d'option reste valable dans un modèle plus réaliste et plus général.

Pour résoudre ce problème, nous considérons le cas particulier d'un propriétaire qui peut investir dans une nouvelle isolation, ou dans du double vitrage en vue de réduire sa facture énergétique. Nous supposons que cet investissement est irréversible, car la désinstallation du nouvel équipement peut être excessivement coûteuse ou parce que l'ancien équipement a été mis au rebut. Nous supposons également que les avantages de ces technologies qui économisent l'énergie ainsi que les rendements financiers de l'épargne des ménages sont stochastiques. Nos résultats suggèrent que le seuil de déclenchement de la prise de décision dépend non seulement de paramètres techniques, mais aussi de paramètres de préférence. En outre, nous montrons que même si l'incertitude sur l'efficacité des technologies qui économisent l'énergie n'affecte que peu le moment de la prise de décision, l'incertitude sur les rendements financiers le favorise. Nous constatons également que l'existence d'une valeur d'option n'exclut pas le paradoxe énergétique.

Sur la substitution des ressources I

Le remplacement des combustibles fossiles par des ressources renouvelables offre des possibilités de réductions des émissions de GES permanentes. De nombreuses économies dans le monde sont donc incitées à s'orienter vers des énergies renouvelables. Dans le chapitre 4, nous considérons un modèle de *switch* technologique dans lequel l'énergie peut être produite à partir des deux facteurs de production disponibles : les ressources non renouvelables (combustibles fossiles) et les ressources renouvelables. Nous nous sommes donc particulièrement intéressés à la question suivante : Quand la société devrait-elle arrêter d'utiliser des ressources non renouvelables dans la production d'énergie et commencer à utiliser des ressources renouvelables ?

Nous proposons un modèle dans lequel les ressources disponibles (les énergies fossiles et les ressources renouvelables telles que l'eau, le vent, le solaire, la biomasse) sont des substituts parfaits dans la production d'énergie, et leurs stocks sont stochastiques. Nous supposons que les firmes commencent à produire de l'énergie en utilisant uniquement des combustibles fossiles, mais la possibilité de réaliser un investissement irréversible afin de switcher à un autre type de production est toujours ouverte. Grâce à nos hypothèses particulières, on obtient une fonction de valeur avant le changement qui est en forme de S. C'est tout à fait nouveau dans la littérature relative au switch technologique, où la fonction de valeur qui en résulte des programmes est souvent concave [Dixit and Pindyck, 1994]. Une autre originalité de notre modèle est que les firmes ne switch pas immédiatement dans le cas où le coût de switcher est égal à zéro (Pommeret and Schubert [2009], voir aussi le chapitre 3). Ceci résulte de l'augmentation des bénéfices obtenus par les firmes grâce aux profits plus élevés qu'elles reçoivent si elle utilisent des ressources non renouvelables, en particulier si elles sont abondantes.

Nous constatons que l'incertitude joue un rôle évident dans la décision de switcher. Plus il y a d'incertitude relative à la disponibilité de ressources non renouvelables, plus les firmes s'orientent tôt vers des ressources renouvelables ; et plus il y a d'incertitude sur la disponibilité des ressources renouvelables, plus le switch se fera tardivement. Le moment optimal pour switcher est également sensible à la demande d'énergie, aux coûts de production et la productivité relatifs des ressources.

Sur la substitution des ressources II

Nous savons déjà que l'une des politiques généralement menées par de nombreux pays pour réduire les émissions de GES consiste à substituer les sources d'énergie polluantes comme le charbon, le pétrole et le gaz, par une source d'énergie propre et renouvelable, comme l'énergie solaire et éolienne. Cependant, il semble que les combustibles fossiles continueront de représenter une partie importante de la combinaison dans le monde entier, même en 2050, année où une réduction d'environ 80 pourcent du total des émissions par rapport à celles de 1990 est attendue. Notre théorie est que tant que les énergies renouvelables ne sont pas très avancées et généralisées, (i) l'industrie aura encore besoin d'un pourcentage d'énergie qui provient de ressources sales, et (ii) la fourniture d'énergie propre aura besoin elle-même des ressources sales, au moins pour les matériaux nécessaires à la construction des usines (pensons à des panneaux solaires par exemple).

Dans le chapitre 5 nous modélisons une économie ayant accès à deux sources d'énergie différentes. La première provient d'une ressource naturelle polluante, tel que les combustibles fossiles. La seconde provient d'une ressource naturelle backstop, tel que le rayonnement solaire. En particulier, nous considérons le cas du rayonnement solaire étant converti en énergie au moyen de panneaux solaires. Il y a deux secteurs productifs de l'économie. Le premier est dédié à la production de ressources backstop. A tout moment, ce secteur nécessite des combustibles fossiles et l'énergie fournie par la backstop déjà disponible. Nous avons donc tenu compte de la nécessité des combustibles fossiles pour produire de l'énergie propre. Le deuxième secteur est consacré à la production de biens de consommation. Au départ, il utilise l'énergie provenant exclusivement des combustibles fossiles. Cependant, il y a toujours la possibilité de switcher pour une nouvelle technologie dont l'énergie provient de deux types de ressources. Dans ce cadre, nous cherchons à évaluer ce qui se passe quant la prise de décision concernant le type d'énergie à choisir si on tient compte de la nécessité des ressources sales, même dans une économie utilisant des énergies propres.

Dans la modélisation de cette décision de changement, nous tenons compte de l'incertitude sur les coûts et les bénéfices futurs. En particulier, nous supposons que l'accumulation de la backstop, et l'augmentation du stock de pollution — qui dans notre cas est égal à l'extraction de la ressource — sont stochastiques. Nous considérons également les irréversibilités associées à la politique de l'environnement. Plus précisément, l'adoption de la technologie propre impose des coûts irrécupérables sur le secteur de la consommation. Nos résultats suggèrent que, compte tenu de l'incertitude et de l'irréversibilité, les incitations à switcher pour une technologie plus propre dépendent de l'importance relative des combustibles fossiles dans la production de biens de consommation suite au switch. Nous constatons également que les améliorations technologiques dans le secteur des panneaux solaires sont d'une certaine importance afin de switcher pour des technologies plus propres. Si le changement technologique implique que la backstop peut être produite avec relativement moins de fossiles combustibles, le switch se produit plus tôt.

II

English version

1

Introduction

Greenhouse gases are responsible for some climate change. Humanity faces a choice: either reducing the emissions of these gases or adapt to climate change. In this dissertation we focus on the first solution under the premise that a large part of the greenhouse effect comes from human activities. More precisely, we propose some essays on modeling the determinants of investing in reducing greenhouse gases (GHG) through improving energy efficiency and substituting non-renewable resources (fossil fuels) by renewable resources. We first try and explain the slow diffusion of some energy efficient investments in a general equilibrium framework. We then study the determinants of switching from non-renewable resources to renewable resources when these are perfect substitutes. Finally, we account for the need of dirty resources even if cleaner technologies are available. All these issues are based on models that cannot be fully solved analytically, therefore we also propose in this dissertation a methodology based on the properties of Chebyshev polynomials to compute the solutions.

1.1 Context

Most climate scientists agree on the main cause of the current global warming: the expansion of the greenhouse effect coming from human activities. This *anthropogenic* greenhouse effect enhances the Earth's *natural* greenhouse effect by the addition of GHG emissions from the burning of fossil fuels (petroleum, coal, and natural gas), mainly to energy generation. Carbon dioxide ($C0_2$) is the gas with the higher radiative forcing, meaning that it is the gas with the higher global warming potential. Coal, particularly brown coal, is the energy source with the highest GHG emissions per energy unit. Burning coal generates 70 percent more CO_2 than natural gas for every unit of energy. At the same time, coal is cheap and is the most widely available fossil fuel. As far as burning oil is concerned, the average emissions rates of CO_2 in the United States from oil-fired generation is 1672 lbs/MWh.

Globally, energy related CO_2 emissions have risen 145-fold since 1850 —from 200 million tons to 29 billion tons a year— and are projected to rise another 54 percent by 2030. Most of the world's emissions come from a relatively small number of countries. The 25 largest emitters, with 75 percent of the world's population and 90 percent of the global gross domestic product (GDP), account for approximately 85 percent of global GHG emissions.[1] The 2007 Fourth Assessment Report compiled by the IPCC (AR4) noted that "changes in atmospheric concentrations of greenhouse gases and aerosols, land cover and solar radiation alter the energy balance of the climate system", and concluded that "increases in anthropogenic greenhouse gas concentrations is very likely to have caused most of the increases in global average temperatures since the mid-20th century". According to that report, temperature has increased about 0.13 degrees Celsius per decade over the last 50 years —nearly twice the average temperature rise for the last 100 years, and eleven of the last twelve years (1995–2006) rank among the 12 warmest years (since 1850). Following this trends, a temperature rise ranging from 1.8 degrees to 4 degrees Celsius can be expected [IPCC, 2007, Stern, 2007]. This global warming will cause several damages on economic activity, on human life and on the environment, often mediated through water. Melting glaciers and a consequent sea levels rise will result into a reduction of water supply and a migration of millions of people living in flooded areas. Extreme weather events will be more likely to appear, and the risk of a rapid climate change and of major irreversible impacts (for instance the melting of the Greenland ice sheet) will be higher. Also, crop yields will seriously decline affecting food production, vulnerable ecosystems will be unable to maintain current form and many species will face a possible extinction.[2]

[1] Climate Change Mitigation, UNEP. Available online at: `http://www.unep.org/climatechange/mitigation/Home/tabid/104335/Default.aspx`. Retrieved July 20, 2012.

[2] See, for instance: The Consequences of Global Warming, NRDC. Available online at: `http://www.nrdc.org/globalwarming/fcons.asp`. Retrieved July 20, 2012.

Given the severe —negative— impacts of global warming, one of the major challenges facing today world's policy makers is reducing GHG emissions. According to the final version of the 64th Annual DPI/NGO Conference Declaration [UNDPI, 2011] governments should have reached clear pathways towards climate sustainability that regulates the global temperature rise below 1.5 degrees Celsius. Emissions of GHG should be reduced to 25 percent of 1990 levels by 2020, 40 percent by 2030, 60 percent by 2040 and 80 percent by 2050. This target is consistent with that suggested by previous research, such as the Stern Review of Climate Change [Stern, 2007], who calls for extensive and immediate interventions in order to cut global emissions by 60 percent to 80 percent by 2050; Nordhaus [2007], who proposes a modest control in the short run followed by sharper emissions reductions in the medium and the long run; or the IPCC [2007], which calls to stabilize emissions at between 445 and 490ppm —50 to 85 percent reductions on 2000 levels— to keep global temperature 2 degrees to 2.4 degrees Celsius above the pre-industrial average.[3] To achieve this goal, the recent Green Economy report [UNEP, 2011] proposes a $1.3 trillion (2 percent of world GDP) target for green (public plus private) investments. Close to three-fifths of this sum would be invested in energy efficiency —particularly in buildings, industry and transport— and renewables. Examples of current public policies for energy efficiency are those of the U.S. Department of Energy, such as the Building Technologies Program, whose strategic goal is to create technologies and design approaches that lead to marketable net zero energy homes by 2020, and net zero energy commercial buildings by 2025; or the SunShot Initiative, whose goal is to make solar energy cost competitive with other forms of energy by the end of the decade.[4]

Under this context it is interesting to model the determinants of investment in reducing GHG, both through improving energy efficiency and by substituting non-renewable (dirty) resources (fossil fuels) by renewable (clean) resources for energy provision. Studying these investment determinants in order to provide recommendations on the best way public policy should deal with them is the main goal of this dissertation. We tackle the investment in energy efficiency problem by considering the specific case of home renovation (i.e. investment in energy-efficient windows, doors, or skylights). The

[3]There is also Greenpeace, which proposes to stop all economic growth in order to save the planet [Acemoglu et al., 2012].

[4]The U.S. Department of Energy: `http://www.eere.energy.gov/`. Retreived July 20, 2012.

problem of the substitution of resources is analyzed under two different scenarios. In the first one we assume that fossil fuels and renewable resources are perfect substitute inputs in energy production. Then, when firms decide to switch to renewable resources, they will keep producing with that technology thereafter. In the second one we assume that, although firms can switch to a cleaner technology using renewables, the economy is not completely "pollution-free" as fossil fuels are still required in the economy.

One particularity of the models we are presenting here is that all decisions are taken in a stochastic environment. At the same time we introduce some of the irreversibilities inherents to almost any environmental policy. We also account for the fact that, in most cases, it is feasible to delay action and wait for new information. We obtain that these uncertainties, irreversibilities, and the possibility of delay can significantly affect the optimal timing of policy adoption. For instance, we obtain that uncertainty on the financial returns of households fosters the adoption of energy savings technologies. Uncertainty also affects the effectiveness of economic policy in order to accelerate the substitution of renewable resources for non-renewable resources in energy production. Finally, governments are tempted to wait more in order for the cleaner technologies to be sufficiently developed before adopting them in the production sectors.

In following lines we justify the need to account for irreversibility and uncertainty in environmental economics. We next briefly present the four chapters comprising this dissertation.

1.2 Irreversibility and uncertainty

Irreversibility of investment amounts to saying that undertaking investment projects results in some unrecoverable initial cost —the so-called sunk cost. Uncertainty on future benefits and costs of investments is also important. In fact, it was the combined effect of irreversibility and uncertainty that led Weisbrod [1964] to introduce the concept of *option value*. He argues that if a decision has irreversible consequences, then the flexibility —the "option"— to choose the timing of that decision should be included in a cost-benefit analysis. The concept of option value has two different interpretations. The first views option value as a risk premium paid by risk averse consumers to reduce the impact of uncertainty in the supply of an environmental good [Cicchetti and Freeman, 1971, Schmalensee, 1972]. The second interpretation stresses the intertemporal aspect of irreversibility and the arrival of new information over time [Arrow and Fisher, 1974, Henry, 1974]. This second interpretation is the one that has been used more extensively in the literature on *real options* [Myers, 1977]. Examples are the entry, exit, and temporary shutdowns and re-startups on investment and output decisions; the implications of construction time —and the option to abandon construction— for the value of a project; and the determinants of a firm's choice of capacity [McDonald and Siegel, 1986, Brennan and Schwartz, 1985, Pindyck, 1988].

To gain more intuition about irreversibility, uncertainty, and about what the option value means, let us consider the model proposed by Olsen and Stensland [1988]. Their model consists of an industry based on non-renewable resources (e.g. off-shore oil extraction) having to decide whether or not shutting down its activity. First at all, the shut down decision can be thought to be irreversible. In Olsen and Stensland's model irreversibility means that firms cannot restart operations after shutting down (probably because of prohibitive high costs). Additionally, Olsen and Stensland assume that economic conditions —particularly prices and production quantities— are uncertain. Uncertainty jointly with irreversibility imply that firm's decision involves the exercising, or "killing" of an option —the option to optimally shut down at any time in the future. In such a framework, there is a value of waiting —the option value— since the firm has always the possibility to postpone the shut down in order to learn more about the present and future payoffs. As a result, the usual rule of "shutting down activity when the marginal production costs saved equal the marginal revenue foregone" is not longer valid. In order to shut down, the marginal production costs saved must exceed the marginal revenue foregone by an amount equal to the value of keeping the option alive [Pindyck, 1988, Dixit and Pindyck, 1994]. Notice that the opportunity of shutting down (or the opportunity to invest, or any other opportunity under uncertainty and irreversibility) is analogous to a option on a common stock. In this case we have a (put) option giving us the right (the exercise price of the option), but not the obligation (because of the opportunity to wait) to shut down (sell) activities (the underlying security), the value of which fluctuates stochastically [Pindyck, 1991]. Of course, in the case of complete irreversibility the salvage value of the firm is zero [Dixit and Pindyck, 1994, Dangl and Wirl, 2004].

Uncertainty and irreversibility has also being considered in some other environmental problems. For instance, Brennan and Schwartz [1985] analyze the operation of a mine, which can be temporarily closed; Paddock et al. [1988] propose a model to value offshore petroleum leases as a function of the market price of oil; and Clarke and Reed [1990] study the preservation of natural wilderness reserves. A standard result is that, in the presence of environmental irreversibility, a standard cost-benefit analysis is biased against conservation [Arrow and Fisher, 1974, Henry, 1974]. Freixas and Laffont [1984] generalize this result, and Conrad [1980] links option value to the expected value of information.

In some more recent papers, Pindyck [2000, 2002, 2007] explains the role of uncertainty and irreversibility in environmental policy. In the case of global warming-oriented policies for instance, uncertainty appears as we do not know how much average temperatures will rise with or without reduced CO_2 emissions, nor do we know the economic impact of higher temperatures. Second, irreversibility appears in, for instance, policies aimed at reducing environmental degradation. There are some sunk costs taking the form of discrete investments (e.g. coal-burning utilities might be forced to install scrubbers, or firms might have to scrap existing machines and invest in more fuel-efficient ones), or they can take the form of expenditure flows. These sunk costs create an opportunity cost of adopting a policy now, rather than waiting for more information about ecological impacts and their economic consequences. There is also some irreversibility in environmental damage. For example, atmospheric accumulations of GHG are long lasting; even if we were to drastically reduce GHG emissions, atmospheric concentration levels would take many years to fall. This means that adopting a policy now rather than waiting has a sunk benefit, that is a negative opportunity cost. Then traditional cost-benefit analysis will be biased against policy adoption.

What about our particular cases of energy efficiency and on the substitution of resources? Consider the case of resources substitution. Imagine a firm trying to decide whether or not stop using non-renewable (fossil fuels) in favor of renewable inputs in energy production. Uncertainty can appear in a lot of ways, but it seems particularly relevant to focus on the uncertainty over the availability of resources [Pindyck, 1980, Epaulard and Pommeret, 2003, Smith and Son, 2005]. Specifically, we can have an idea of their current stock, but we do not know too much about their future availability. And even if we knew how much of the resources are expected to be available in the future, we would not know the resulting effect on firm's decisions. For instance the more renewable resources are expected to be available in the future, the more the firms are tempted to adopt them sooner to produce energy. Moreover, the adoption of renewables imposes sunk costs on society. It is in particular the case if an offshore oil platform is being transformed into an offshore wind farm. Once this decision is made, almost all of the previously installed capital needs to be dismantled, and very few parts of the old facility can be re-utilized. This process is clearly very expensive in terms of time and money. As before, there is an option value in the sense that investors can wait for more information before getting rid of the offshore oil platforms.

Consider now the specific case of a homeowner investing in new "equipment" in order to reduce his energy bill. We can safely assume that this investment is irreversible, as uninstalling the new equipment can be unreasonably costly or because the old equipment has been scrapped. Some uncertainty can also be considered. For instance, the evolution of the household's wealth is not fully know, particularly if we assume that wealth depends on risky assets. Under this circumstances, household's decision to invest can be considered to have an option value, as she can always postpone the investment in order to learn whether future wealth is increasing or not. If there is no arbitrage between household's consumption and adoption —i.e. in a partial equilibrium setting, in order to invest in this energy saving technology, the marginal value of the new equipment must exceed the purchase and installation cost, by an amount equal to the value of keeping the investment option alive. In the case of an arbitrage between consumption and adoption —i.e. in a general equilibrium setting, the optimal adoption timing is not only sensitive to uncertainty, but also sensitive to the degree of agents' risk aversion (Hugonnier et al. [2008]; see also Pommeret and Schubert [2009] for an example in environmental economics).

1.3 On numerically solving Bellman equations

In this dissertation we face ourselves with functional equations that are complex and cannot be fully solved using standard analytical techniques. In particular, analytical solutions for the models in chapters 3 and 5 are only available in the case of a zero discount rate. There is a huge debate in environmental economics about the way future utilities should be discounted, especially when today's decisions have very long-term consequences (see, for instance, Portney and Weyant [1999] for a survey). As we are not particularly inclined in favour of a zero or a positive discount rate, a methodology to solve for all cases becomes necessary. Additionally, the model in chapter 4 can never be fully solved analytically. At least for the models we are presenting here, the problem is the high non-linearity of the value functions and the fact that they have to satisfy some boundary conditions. Then, we need to rely on numerical methods.

As suggested by Judd [1992, 1998], we use the approximation properties of Chebyshev polynomials to compute stable non-diverging solutions of the Hamilton–Jacoby–Bellman equations. Specifically, we transform the value functions and the given conditions into expressions with unknown Chebyshev coefficients. By using this representation, our original problem of solving partial differential equations reduces to a problem of solving simple systems of non-linear equations. The model of chapter 3 is solved by using an algorithm based on Newton's method [Miranda and Fackler, 2004, Dangl and Wirl, 2004]; while the models in chapters 4 and 5 are solved by using the methodology presented in chapter 2.[5] In particular, we transform the value function and the given conditions into matrix equations with unknown Chebyshev coefficients. By doing like this, our problem reduces to one of solving a simple system of algebraic equations. This methodology is a secondary by-product of this dissertation, but it constitutes by itself an original contribution to the numerical methods literature.

[5]The methodology of chapter 2 can also be adapted to solve the model of chapter 3. In fact, both methodologies were used *a posteriori* for comparison purposes.

1.4 Energy efficiency

Home renovations are generally asserted to be a highly effective means for households to lower expenditures on energy —and to indirectly reduce GHG— through improving efficiency, and they become therefore a key target for environmental policies. Indeed, cost-benefit analyses point to the economic viability of these systems even if the comfort co-benefits such as improvements in indoor air quality and protection against noise are not taken into account [Jakob, 2006, Ott et al., 2006]. However, actual investment in these systems is still relatively rare [Banfi et al., 2008]. In chapter 3 we aim to explain the home renovation decision of households in a theoretical model. Specifically, we try to explain the slow diffusion of energy efficient investments —the so-called *energy paradox* or *energy-efficiency gap* [Jaffe and Stavins, 1994a].

The existing literature that explains the energy paradox (Hassett and Metcalf [1995], for instance) considers partial equilibrium settings, and therefore ignores the interaction between optimal consumption and optimal adoption, as well as the notions of risk aversion, or intertemporal substitutions. In those settings consumers behave like firms when deciding energy-saving technology adoption. To challenge these results, we reconsider the joint effect of irreversibility and uncertainty on the energy-efficiency investment decision in a general equilibrium framework [Hugonnier et al., 2008, Pommeret and Schubert, 2009]. In particular, *we wonder whether the explanation of the energy paradox based on the existence of an option value still valid in a more realistic and general model.*

To tackle this issue we consider the specific case of a homeowner who may invest in new insulation, or double glazing in order to reduce her energy bill. We assume that this investment is irreversible, as uninstalling the new "equipment" can be unreasonably costly or because the old equipment has been scrapped. We also assume that the benefits of such energy-saving technologies as well as the financial returns on household's savings are assumed to be stochastic. Our results imply that the threshold triggering adoption depends not only on technological parameters but on preference parameters as well. Additionally, we show that while uncertainty on energy-saving technologies efficiency hardly affects adoption timing, uncertainty on financial returns fosters it. We also find that the existence of an option value does not rule out the energy paradox.

1.5 On the substitution of resources I

Substitution of fossil fuels provides permanent GHG emission reductions. Many economies in the world have therefore incentives to move towards renewable energy. In chapter 4 we consider a model of technology switching in which energy can be produced from two available inputs: non-renewable resources (fossil fuels) and renewable resources. We are particularly interested in tackling the following issue: *At what point should society stop using non-renewable resources to produce energy and start producing from renewables?*

We propose a model in which the available resources (fossil fuels, and renewable resources such as water, wind, solar, biomass) are perfect substitutes in energy production, and their stocks are stochastic. We assume that firms start producing energy using only fossil fuels, but the possibility to carry out an irreversible investment to switch to the other input is always open. Due to our particular assumptions, we obtain a value function before the switch that is S-shaped. This is completely new in the literature of technology switching, where the resulting value function is mostly concave [Dixit and Pindyck, 1994]. Another novelty of our model is that firms do not switch immediately in the case of switching cost being equal to zero (Pommeret and Schubert [2009]; see also chapter 3). This results from the higher profits the firms get from using nonrenewable resources, particularly if they are abundant.

We find that uncertainty plays a clear role in the decision to switch. The more the uncertainty about the availability of the non-renewable resources, the sooner the firms switch to the renewable resources; and the more the uncertainty about the availability of renewable resources, the later the switching time. The optimal switching time is also sensitive to energy demand, costs, and the relative productivity of resources parameters.

1.6 On the substitution of resources II

We already know that one of the policies commonly undertaken by many countries in order to reduce GHG emissions is to substitute dirty energy sources, such as coal, oil and gas, with a cleaner and renewable energy source, such as solar and wind energy. However, it seems that fossil fuels will continue to be an important part of the energy mix around the world even by 2050, a year in which a reduction of about 80 percent of total 1990-emissions is expected. Our theory is that as long as renewable energies are not very advanced and widespread, (i) industry will still need a percentage of energy that derives from dirty resources, and (ii) the provision of clean energy itself will require dirty resource at least as materials to build the plants (think of solar panels for instance).

In chapter 5 we model an economy having access to two different energy sources. The first one comes from a natural polluting resource, such as fossil fuels. The second one comes from a backstop natural resource, such as solar radiation. In particular, we consider the case of solar radiation being converted into energy by means of solar panels. There are two productive sectors in the economy. The first one is dedicated to manufacturing the backstop resources. At any time, this sector requires both fossil fuels and the energy provided by the backstop already available. We therefore account for the need of fossil fuels to provide clean energy. The second sector is devoted to production of the consumption good. Initially it uses energy coming exclusively from fossil fuels. However, it has always the possibility of switching to a new technology in which energy comes from both types of resources. Under this setting *we seek to appraise what happens for the energy adoption decision if we account for the need of dirty resources even in an economy using clean energy.*

In modeling this switching decision, we account for the uncertainty over the future costs and benefits. In particular, we assume that the accumulation of the backstop, and the increase in pollution stock —which in our case is equal to the resource extraction— are stochastic. We also introduce the irreversibilities associated with environmental policy. Specifically, adoption of the cleaner technology imposes sunk cost on the consumption sector. Our results imply that, under uncertainty and irreversibility, the incentives to switch to the cleaner technology depend on the relative importance of fossil fuels in the production of consumption goods after the switch. We also find that technological improvement in the solar panels sector is of some importance in order to switch to cleaner technologies. If the technological change implies that the backstop can be produced with relatively less of the fossil fuels, the adoption occurs sooner.

2

Using Chebyshev Polynomials to Approximate Partial Differential Equations*

Caporale and Cerrato [2009] propose a simple method based on Chebyshev approximation and Chebyshev nodes to approximate partial differential equations (PDEs). However, they suggest not to use Chebyshev nodes when dealing with optimal stopping problems. Here, we use the same optimal stopping example to show that Chebyshev polynomials and Chebyshev nodes can still be successfully used together if we solve the model in a matrix environment.

2.1 Introduction

Many problems in economics lead to functional equations that are complex and cannot be solved using standard analytical techniques. This is especially true for optimal stopping problems. Examples are the optimal sequential investment model in Majd and Pindyck [1985], and the incremental investment model in He and Pindyck [1992]. Both

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models are solved numerically by using finite difference methods. However, these techniques can sometimes fail to determine the value function accurately or are extremely slow (see, for instance, Dangl and Wirl [2004]).

Starting from Judd [1992], economists are increasingly interested in the use of projection methods as an alternative to finite difference methods. For example, Dangl and Wirl [2004] propose a modification of Judd's methodology to approximate the solution of a stopping problem.[1] In particular, they approximate the value function by using Chebyshev polynomials as a basis, and then find the projection coefficients by using Newton's algorithm. When one provides the Newton's algorithm with a good initial guess, the solution is stable and converges very fast to the real analytical solution. In this sense, Dangl and Wirl prove that collocation methods are superior to other standard numerical procedures, such as a modified shooting method -using a Runge-Kutta algorithm-,[2] and finite difference techniques.

In order to solve the same optimal stopping problem as Dangl and Wirl [2004], Caporale and Cerrato [2009] rely first on Chebyshev polynomials to approximate partial differential equations, but then they use Monte Carlo methods to solve the boundary conditions for the partial differential equation. Finally, they fit the functional at Chebyshev nodes to estimate the coefficients. Such a resolution has the advantage of being flexible and easy to program. However, in the particular case of the optimal shutdown time of a machine [Dixit and Pindyck, 1994], Caporale and Cerrato suggest not to use Chebyshev nodes, arguing it would result in a very poor fit.

Here, we are taking again the optimal shutdown time of a machine model to first introduce the Chebyshev matrix method. In short, we follow Judd [1992, 1998], Dangl and Wirl [2004], and Caporale and Cerrato [2009] in assuming that the value function can be well approximated by truncated Chebyshev series. However, in the spirit of Sezer and Kaynak [1996], we then express the approximation in matrix terms. This

[1] Dangl and Wirl [2004] consider the optimal shutdown time of a machine model first presented in Dixit and Pindyck [1994].

[2] The shooting method needs to be modified to solve free boundary problems. For standard shooting methods, see e.g. Deuflhard and Bornemann [2002].

expression is evaluated at Chebyshev nodes.[3] In contrast to Dangl and Wirl [2004], we do not use a Newton's algorithm to find the projection coefficients and only simple algebra is required. In this sense, we avoid the need of a first guess for the projection coefficients.

Our model also differs from Caporale and Cerrato [2009] in that we do not use Monte Carlo methods. Instead, we introduce the boundary conditions by a simple modification in our matrix expression. As a result, Chebyshev polynomials and Chebyshev nodes can still be successfully used together.

The rest of the chapter is as follows. In section 2.2, the model to derive the optimal shutdown time of a machine is described and analytically solved. In section 2.3 we present and adapt the Chebyshev matrix model, and it is shown how Chebyshev coefficients can be obtained by using linear algebra. In section 2.4 we show how our model can be extended to approximate non-linear problems. As we will see, the approximate numerical resolution in this case is too close to the one reported in Dangl and Wirl [2004]. Section 2.5 concludes.

2.2 The optimal stopping problem

The following framework -originally developed by Dixit and Pindyck [1994]- is useful first because it allows for an explicit analytical solution, so that we can compare it with the numerical approximation obtained in the following section. Second, the model is also used by Dangl and Wirl [2004], and Caporale and Cerrato [2009], so we can compare the performance of our approach with theirs. The model is described briefly, since it can be referred to the mentioned authors for further details.

[3] In their paper, Sezer and Kaynak [1996] find the approximate solution of a second order linear differential equation under some given boundary conditions. Our problem is a free-boundary problem. As we will see, our boundary conditions are given at some optimal stopping value which has to be determined.

2.2.1 Description of the model

Let $\pi(t)$ denote the profit flow in period t from using a particular equipment. Assume that $\pi(t)$ evolves according to:

$$\begin{aligned} d\pi(t) &= adt + \sigma dz, \\ \pi(0) &= \pi_0, \end{aligned} \tag{2.1}$$

where $a < 0$, for both economical obsolescence and physical and reasons. $\sigma > 0$ is the volatility of profit, and dz is a standard increment of a Wiener process. The stochastic part of equation (2.1), σdz, implies the existence of random shocks to the equipment's productivity due to the general business cycle, or because of idiosyncratic variations in the demand for the equipment's output. Also, note that there is no particular restriction on the value $\pi(t)$, i.e. the firm can accept some losses to keep the machine in operating condition.

Following Dixit and Pindyck [1994], Dangl and Wirl [2004], or Caporale and Cerrato [2009], suppose that $F(\pi, t)$ is a claim on the profit flow, $\pi(t)$; it is determined as:

$$F(\pi, t) = \max_{\{T\}} \mathbb{E} \int_0^T e^{-rt}\pi(t)dt, \tag{2.2}$$

where r is the subjective discount rate of the owner, and T is a random stopping time at which the machine is removed. Also, assume that equipment that is once removed cannot be reinstalled (or the re-installation cost is close to infinity), and that salvage value is zero.

The Bellman equation for this optimal stopping problem is:

$$F(\pi, t) = \max_{\{T\}} \left\{ 0, \pi dt + \frac{1}{1 + rdt}\mathbb{E}\left[F(\pi + d\pi, t + dt)\right] \right\}, \tag{2.3}$$

where the first argument applies if the machine is scrapped, and the second argument provides the value from continuation. The firm's optimal decision can be characterized by a threshold π^*, such that it is optimal to use the equipment as long as $\pi > \pi^*$, and to throw it away when the profit flow hits this threshold for the first time.

Using standard techniques, it is possible to show that, in the domain of continuation, the value function $F(\pi, t)$ has to satisfy:

$$rF(\pi, t) = \pi + aF_\pi + \frac{1}{2}\sigma^2 F_{\pi\pi}, \tag{2.4}$$

where $F_{(\cdot)}$ is the derivative with respect to the sub-script. Also, note that there is no explicit direct dependence on time t, then: $F(\pi, t) = F(\pi)$.

Three boundary conditions have to be met:

$$\begin{aligned} F(\pi^*(t)) &= 0 & (2.5)\\ F_\pi(\pi^*(t)) &= 0 & (2.6)\\ \lim_{\pi\to\infty}\left\{F(\pi) - \left(\frac{a}{r^2} + \frac{\pi}{r}\right)\right\} &= 0 & (2.7)\end{aligned}$$

Equation (2.5) is the value matching condition at the threshold value π^*: the unknown function $F(\pi)$ has to equal the known termination payoff function (zero in our example). Equation (2.6) is the smooth pasting condition. This equation matches the slope of $F(\pi)$ to that of the payoff function to ensure that the shutdown occurs at the optimal time. Finally, equation (2.7) implies that the opportunity to shut down has no value for $\pi \to \infty$. Then $F(\pi)$ reduces to the discounted sum of all future cash-flows.

2.2.2 Solving the model analytically

The general solution of the non-homogeneous linear differential equation (2.4) is:[4]

$$F(\pi) = \left[\frac{a}{r^2} + \frac{\pi}{r}\right] + c_1 \exp(\lambda_1\pi) + c_2 \exp(\lambda_2\pi). \tag{2.8}$$

As we can see, equation (2.8) encompasses the discounted sum of all future cash-flows, and some option values. $\lambda_1 > 0$ and $\lambda_2 < 0$ denote the roots of the characteristic polynomial of the corresponding homogeneous differential equation:

$$\lambda_{1,2} = \frac{-a \pm \sqrt{a^2 + 2\sigma^2 r}}{\sigma^2}. \tag{2.9}$$

[4]See Dangl and Wirl [2004].

Since $\lambda_1 > 0$, the boundary condition (2.7) can only be satisfied for:

$$c_1 = 0.$$

Hence, the closed form solution of the value function is:

$$F(\pi) = \begin{cases} \left[\frac{a}{r^2} + \frac{\pi}{r}\right] + c_2 \exp(\lambda_2 \pi) & \text{for } \pi \geq \pi^*, \\ 0 & \text{for } \pi \leq \pi^*. \end{cases} \tag{2.10}$$

Using equation (2.10), and solving the system of two equations (2.5) and (2.6) in the two unknowns π^* and c_2, we get:

$$\pi^* = \frac{1}{\lambda_2} - \frac{a}{r},$$

and

$$c_2 = -\frac{1}{r\lambda_2 \exp(\lambda_2 \pi^*)}.$$

Both Dixit and Pindyck [1994], and Dangl and Wirl [2004] provide some base case parameters: $a = -0.1$, $\sigma = 0.2$, $r = 0.1$. Using these we get:

$$\begin{aligned} \pi^* &= -0.17082039 \\ c_2 &= 10.1188. \end{aligned}$$

2.3 The matrix model

In this section we find a numerical approximation for equation (2.4) subject to equations (2.5) to (2.7). To do this, we adapt the procedure first presented in Sezer and Kaynak [1996] in the context of general linear differential equations. As we shall see, the solution of the free-boundary problem presented in the previous section is easy to set and only requires simple algebra.

2.3.1 A numerical approximation of the value function

Suppose that $\widehat{F}(\pi) \approx F(\pi)$ has a Chebyshev series solution of the form:[5]

$$\widehat{F}(\pi) = \frac{1}{2} a_0 T_0(\pi) + \sum_{i=1}^{N} a_i T_i(\pi), \tag{2.11}$$

[5] The quality of the approximation $\widehat{F}(\pi)$ is guaranteed by the Chebyshev approximation theorem. See Judd [1992] for deeper explanations.

for $\pi^* \leq \pi \leq \overline{\pi}$. In equation (2.11), $\overline{\pi}$ is an artificial (user defined) upper boundary for π, and $T_i(\pi)$, $i = 0, 1, ..., N$, is the general ith Chebyshev polynomial of the first kind. This can be obtained from the recurrence relation:

$$\begin{aligned} T_0(s(\pi)) &= 1, \\ T_1(s(\pi)) &= s, \text{ and} \\ T_{n+1}(s(\pi)) &= 2sT_n(s(\pi)) - T_{n-1}(s(\pi)), \end{aligned}$$

where:

$$s(\pi) = \frac{2\pi - (\pi^* + \overline{\pi})}{\overline{\pi} - \pi^*}. \tag{2.12}$$

In equation (2.11), a_i, $i = 0, 1, ..., N$, are the Chebyshev coefficients to be determined, and $N + 1$ is the degree of approximation. Also, assume that:

$$\widehat{F}^{(n)}(\pi) = \frac{1}{2} a_0^{(n)} T_0(\pi) + \sum_{i=1}^{N} a_i^{(n)} T_i(\pi), \tag{2.13}$$

where $\widehat{F}^{(n)}(\pi)$ is the nth derivative of $\widehat{F}(\pi)$ with respect to π, and $a_i^{(n)}$ are also Chebyshev coefficients. Obviously $a_i^{(0)} = a_i$ and $\widehat{F}^{(0)}(\pi) = \widehat{F}(\pi)$.

As $T_i(\pi)$ are generalized Chebyshev polynomials in equations (2.11) and (2.13), the recurrence relation between the Chebyshev coefficients $a_i^{(n)}$ and $a_i^{(n+1)}$ of $\widehat{F}^{(n)}(\pi)$ and $\widehat{F}^{(n+1)}(\pi)$ is given by:[6]

$$a_i^{(n+1)} = \frac{2}{\overline{\pi} - \pi^*} \left[2 \sum_{j=0}^{\infty} (i + 2j + 1) a_{i+2j+1}^{(n)} \right]. \tag{2.14}$$

If we take $i = 0, 1, ..., N$ and assume $a_i^{(n)} = 0$ for $i > N$, we can write equation (2.14) in the matrix form:

$$\mathbf{A}^{(n+1)} = 2\mathbf{M}^g \mathbf{A}^{(n)},\ n = 0, 1, 2, \tag{2.15}$$

where $\mathbf{A}^{(0)} = \mathbf{A}$, $\mathbf{M}^g = \frac{2}{\overline{\pi} - \pi^*}\mathbf{M}$, and $\mathbf{M}$ is as defined in Sezer and Kaynak [1996].

Equations (2.11) and (2.13) can also be expressed in matrix form:

$$\widehat{F}(\pi) = \mathbf{T}(\pi)\mathbf{A}, \tag{2.16}$$

$$\widehat{F}^{(n)}(\pi) = \mathbf{T}(\pi)\mathbf{A}^{(n)} \tag{2.17}$$

[6] See Sezer and Kaynak [1996] for computations in the case of usual Chebyshev polynomials: $T(x)$, $-1 \leq x \leq 1$.

so that:

$$\begin{aligned}
\mathbf{T}(\pi) &= \left[\begin{array}{cccc} T_0(\pi) & T_1(\pi) & \cdots & T_N(\pi) \end{array}\right], \\
\mathbf{A} &= \left[\begin{array}{cccc} \frac{1}{2}a_0 & a_1 & \cdots & a_N \end{array}\right]', \text{ and} \\
\mathbf{A}^{(n)} &= \left[\begin{array}{cccc} \frac{1}{2}a_0^{(n)} & a_1^{(n)} & \cdots & a_N^{(n)} \end{array}\right]'.
\end{aligned}$$

Using equation (2.15), equation (2.17) can be written as:

$$\begin{aligned}
\widehat{F}^{(n)}(\pi) &= \mathbf{T}(\pi)\mathbf{A}^{(n)} \\
&= 2\mathbf{T}(\pi)\mathbf{M^g}\mathbf{A^{(n-1)}} \\
&= 4\mathbf{T}(\pi)\left(\mathbf{M^g}\right)^2\mathbf{A^{(n-2)}} \\
&= ... \\
&= 2^n\mathbf{T}(\pi)\left(\mathbf{M^g}\right)^n\mathbf{A^{(0)}}. \qquad (2.18)
\end{aligned}$$

2.3.2 Collocating the collocation points

To obtain a Chebyshev solution of equation (2.4) in the form of (2.16), we first compute the Chebyshev coefficients by means of Chebyshev collocation points (nodes). These points are defined as:

$$s_i = \cos\left(\frac{i\mathrm{Pi}}{N}\right) \qquad (2.19)$$

where $i = 0, 1, ..., N$, and Pi refers to the standard mathematical constant. As a function of π is required, we use the definition of s in equation (2.12) to transform the collocation points as:

$$\pi_i = \frac{\overline{\pi} - \pi^*}{2}(s_i + 1) + \pi^*. \qquad (2.20)$$

Next, we write the numerical approximation to equation (2.4) as:

$$r\widehat{F}(\pi_i) - a\widehat{F}^{(1)}(\pi_i) - \frac{1}{2}\sigma^2\widehat{F}^{(2)}(\pi_i) = \pi_i. \qquad (2.21)$$

Consider now the following definitions:

$$\begin{aligned}
\phi_0 &: = r, \\
\phi_1 &: = -a, \\
\phi_2 &: = -\frac{1}{2}\sigma^2,
\end{aligned}$$

and:

$$\mathbf{\Phi}_j := \begin{bmatrix} \phi_j(\pi_0) & 0 & \cdots & 0 \\ 0 & \phi_j(\pi_1) & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \phi_j(\pi_N) \end{bmatrix}, \ j = 0, 1, 2. \tag{2.22}$$

If we use ϕ_0, ϕ_1, and ϕ_2 for this particular problem, equation (2.22) implies:

$$\mathbf{\Phi}_0 = r\mathbf{I}_{N+1}, \tag{2.23}$$

$$\mathbf{\Phi}_1 = -a\mathbf{I}_{N+1}, \tag{2.24}$$

$$\mathbf{\Phi}_2 = -\frac{1}{2}\sigma^2\mathbf{I}_{N+1}, \tag{2.25}$$

where $\mathbf{I}_{N+1}$ is an identity matrix of size $N+1$. Also, define:

$$\mathbf{F}^{(n)} := 2^n\mathbf{T}\left(\mathbf{M^g}\right)^n\mathbf{A}, \tag{2.26}$$

for:

$$\mathbf{T} = \left[\begin{array}{cccc} \mathbf{T}(\pi_0) & \mathbf{T}(\pi_1) & \cdots & \mathbf{T}(\pi_N) \end{array} \right]'.$$

And finally:

$$\mathbf{\Psi} := \left[\begin{array}{cccc} \pi_0 & \pi_1 & \cdots & \pi_N \end{array} \right]'. \tag{2.27}$$

Using equations (2.23) to (2.27), we can write equation (2.21) as:

$$\sum_{k=0}^{2} \mathbf{\Phi}_k\mathbf{F}^{(k)} = \mathbf{\Psi},$$

or, using equation (2.26):

$$\mathbf{\Gamma}\mathbf{A} = \mathbf{\Psi}, \tag{2.28}$$

which corresponds to a system of $N+1$ algebraic equations with the unknown Chebyshev coefficients so that:

$$\mathbf{\Gamma} = \sum_{k=0}^{2} 2^k\mathbf{\Phi}_k\mathbf{T}\left(\mathbf{M^g}\right)^k.$$

2.3.3 Value matching condition and Chebyshev coefficients

As one condition is needed to approximate π^*, we keep the smooth pasting condition apart for the time being and focus only on the value matching condition. Using equation (2.16), we may write equation (2.5) as:

$$\widehat{F}(\pi^*) = \mathbf{T}(\pi^*)\mathbf{A} = \mathbf{0}. \tag{2.29}$$

In order for this condition to be always satisfied, we replace the last row of matrix $\mathbf{\Gamma A}$ in equation (2.28) by the vector $\mathbf{T}(\pi^*)\mathbf{A}$, and the last row of vector $\mathbf{\Psi}$ by 0. Doing this we get the system:

$$\widetilde{\mathbf{\Gamma}}\mathbf{A} = \widetilde{\mathbf{\Psi}} \tag{2.30}$$

Hence, Chebyshev coefficients can be simply computed from equation (2.30):

$$\mathbf{A}(\pi^*) = \widetilde{\mathbf{\Gamma}}^{-1}\widetilde{\mathbf{\Psi}}. \tag{2.31}$$

2.3.4 Smooth pasting condition and the optimal stopping time

Equation (2.31) jointly with equation (2.16) gives us an approximate numerical solution for equation (2.4). This solution is a function of both π and π^*:

$$\widehat{F}(\pi, \pi^*) = \mathbf{T}(\pi)\mathbf{A}(\pi^*).$$

The smooth pasting condition means that:

$$\widehat{F}^{(1)}(\pi^*, \pi^*) = \mathbf{T}(\pi^*)\mathbf{A}(\pi^*) = 0, \tag{2.32}$$

which is an equation that can easily be solved for π^* by using Newton's method, Broyden's method, or any other methodology to solve nonlinear equations.

2.3.5 Results

Results for $N = 5$, 7, 10, and 15 are:[7]

$$\begin{aligned}
\pi^*_{N=5} &\approx -0.327379614433541, \\
\pi^*_{N=7} &\approx -0.186102170735499, \\
\pi^*_{N=10} &\approx -0.171001350671151, \\
\pi^*_{N=15} &\approx -0.170820408465840.
\end{aligned}$$

As expected, the approximation is better as N becomes larger. Moreover, the numerical solution with $N = 15$ is really close to the analytical solution. This confirms that the Chebyshev matrix model is doing a very good job when used jointly with Chebyshev nodes.

2.4 Introduction to non-linear optimal stopping problems

We want to show that our methodology is also useful to solve more complicated problems. For example, let us consider the optimal maintenance and shutdown model in Dangl and Wirl [2004]. According to this model, allowing for maintenance yields the following dynamic, stochastic problem:

$$F(\pi, t) = \max_{\{u(t), t\in[0,T], T\}} \mathbb{E} \int_0^T e^{-rt} \left(\pi(t) - \frac{1}{2} c u^2(t) \right) dt, \tag{2.33}$$

$$d\pi(t) = (a + u)dt + \sigma dz, \tag{2.34}$$

where $u > 0$ represents maintenance, $C(u) = \frac{1}{2}cu^2$ is the costs function, and c is just a constant.

The Bellman equation for this problem can be found to be:

$$rF = \pi + \left(a + \frac{1}{2c} F_\pi \right) F_\pi + \frac{1}{2} \sigma^2 F_{\pi\pi}, \tag{2.35}$$

[7] Refer to the appendix for step-by-step computations in the case of $N = 2$.

where we are using the usual techniques and the optimal maintenance value:

$$u^* = \frac{F_\pi}{c}. \tag{2.36}$$

Equation (2.35) is a non-linear, non-homogeneous, second order differential equation that has to be solved subject to the two boundary conditions (2.5) and (2.6), and the transversality condition:[8]

$$\lim_{\pi \to \infty} \left\{ F(\pi) - \left(\frac{a}{r^2} + \frac{1}{2c^3} + \frac{\pi}{r} \right) \right\} = 0. \tag{2.37}$$

However, this problem seems to have no explicit analytical solution.

2.4.1 Linearizing the non-linear equation

Following section 2.3.1, we assume that $\widehat{F}(\pi) \approx F(\pi)$ and its derivatives can be approximated as:

$$\widehat{F}^{(n)}(\pi) = 2^n \mathbf{T}(\pi) \left(\mathbf{M^g}\right)^n \mathbf{A^{(0)}}.$$

However, to apply the procedure described in section 2.3.2, we must first linearize equation (2.35) and the value matching condition (2.5).[9] For example, if we use the linear iterative scheme we can write:

$$r\widehat{F}_{k+1}(\pi_i) - \left(a + \frac{1}{2c}\widehat{F}_k^{(1)}(\pi_i) \right) \widehat{F}_{k+1}^{(1)}(\pi_i) - \frac{1}{2}\sigma^2 \widehat{F}_{k+1}^{(2)}(\pi_i) = \pi_i, \tag{2.38}$$

$$\widehat{F}_{k+1}(\widehat{\pi}^*) = \mathbf{T}(\widehat{\pi}^*)\mathbf{A} = \mathbf{0}, \tag{2.39}$$

for $k = 0, 1, 2, \ldots$, and $\widehat{\pi}^*$ being an initial guess for π^*.

To start iterating, we take the following initial guess:

$$\widehat{F}_0(\pi) = 0, \tag{2.40}$$

which satisfies the value matching condition as long as $\pi = \widehat{\pi}^*$. Inserting equation (2.40) into equation (2.38) we get:

$$r\widehat{F}_1(\pi_i) - \left(a + \frac{1}{2c}\widehat{F}_0^{(1)}(\pi_i) \right) \widehat{F}_1^{(1)}(\pi_i) - \frac{1}{2}\sigma^2 \widehat{F}_1^{(2)}(\pi_i) = \pi_i, \tag{2.41}$$

[8] Refer to Dangl and Wirl [2004] for further explanations.

[9] As we did in section 2.3.3, we keep the smooth pasting condition apart for a while.

$$\widehat{F}_1(\widehat{\pi}^*) = \mathbf{T}(\widehat{\pi}^*)\mathbf{A} = \mathbf{0}. \tag{2.42}$$

The linear differential problem of equations (2.41) and (2.42) can be easily solved by using the Chebyshev matrix method.[10] The resulting iterative approximation $\widehat{F}_1$ is then used to solve:

$$r\widehat{F}_2(\pi_i) - \left(a + \frac{1}{2c}\widehat{F}_1^{(1)}(\pi_i)\right)\widehat{F}_2^{(1)}(\pi_i) - \frac{1}{2}\sigma^2\widehat{F}_2^{(2)}(\pi_i) = \pi_i,$$

$$\widehat{F}_2(\widehat{\pi}^*) = \mathbf{T}(\widehat{\pi}^*)\mathbf{A} = \mathbf{0},$$

and son on. In general, the result of the kth iteration is used to activate the $(k+1)$th iteration. If the process is convergent, a fixed point will be reached after several iterations. The process is ended when the maximum absolute value of the difference between two consecutive estimates is less than a tolerance error ε, i.e.:

$$\widetilde{E}_{k+1} = \max_{\widehat{\pi}^* \leq \pi \leq \overline{\pi}} \left|\widehat{F}_{k+1}(\pi) - \widehat{F}_k(\pi)\right| \leq \varepsilon.$$

2.4.2 Using the smooth pasting condition

Assume that $\widehat{F}_k$ has reached a fixed point, and hence:

$$\widehat{F}_k(\widehat{\pi}^*) = \widehat{F}(\widehat{\pi}^*).$$

The last step is to evaluate our resulting expression by using the smooth pasting condition:

$$\widehat{F}^{(1)}(\widehat{\pi}^*) = 0. \tag{2.43}$$

[10]Define:

$$\begin{aligned}
\phi_{0,k+1} \quad &: \quad = r, \\
\phi_{1,k+1} \quad &: \quad = -\left(a + \frac{1}{2}F_0^{(1)}(\pi_i)\right), \\
\phi_{2,k+1} \quad &: \quad = -\frac{1}{2}\sigma^2,
\end{aligned}$$

and then redefine equations (2.22), and (2.26) to (2.31) in terms of the $(k+1)$th iteration.

If equation (2.43) is satisfied, it is obvious that $\widehat{\pi}^* = \pi^*$ is the optimal stopping value. On the contrary, if equation (2.43) is not satisfied, we have to guess another value for $\widehat{\pi}^*$ and start the whole process again.[11]

2.4.3 Results

We use the same parameters as in section 2.3.5 and $c = 200$. Results for $N = 5$, 7, 10, and 15 are:

$$\begin{aligned}
\pi^*_{N=5} &\approx -0.303894042968750, \\
\pi^*_{N=7} &\approx -0.163452148437500, \\
\pi^*_{N=10} &\approx -0.177474975585938, \\
\pi^*_{N=15} &\approx -0.179458618164063.
\end{aligned}$$

The numerical solution for $N = 15$ is really close to the one reported by Dangl and Wirl [2004]. So, the Chebyshev matrix model also does a very good job when used to approximate non-linear optimal stopping problems.

2.5 Conclusion

Many differential equations in economic models are usually difficult (or even impossible) to solve analytically. In many cases approximate solutions are required. Here, we follow the suggestion of Judd [1992, 1998], Dangl and Wirl [2004], and Caporale and Cerrato [2009] to use Chebyshev polynomials and collocation methods to approximate value functions. However, we transform the value function and the given conditions into matrix equations with unknown Chebyshev coefficients. By using this representation, our original problem of solving a partial differential equation reduces to a problem of solving a simple system of algebraic equations.

Contrary to one of the findings in Caporale and Cerrato [2009], our methodology suggests that Chebyshev series and Chebyshev nodes can still be sucessfully used together, even if the problem is non-linear. Also, by using the matrix method we avoid

[11] We can find the optimal stopping value by using a simple search algorithm.

the need of using a Newton's algorithm to find the projection coefficients as in Dangl and Wirl [2004].

2.A An illustrative example with *N=2*

In this section we follow the procedure described in the main text to solve for $\widehat{F}(\pi)$ and π^* step by step. In order to provide a very illustrative example we fix $N = 2$. This will lead to a very bad numerical approximation of equation (2.4), but will simplify our exposition.

For $N = 2$, the Chebyshev collocations points as defined by equation (2.19) are:

$$s_i = \{1, 0, -1\}.$$

Hence, equation (2.20) implies that:

$$\pi_i = \left\{ \overline{\pi}, \frac{1}{2}\left(\overline{\pi} + \pi^*\right), \pi^* \right\}.$$

For example, if $\overline{\pi} = 10$:

$$\pi_i = \left\{ 10, 5 + \frac{1}{2}\pi^*, \pi^* \right\}.$$

Using $a = -0.1$, $\sigma = 0.2$, $r = 0.1$ we can compute the matrices in equations (2.23) to (2.25):

$$\begin{aligned}
\mathbf{\Phi}_0 &= \begin{pmatrix} 0.1 & 0 & 0 \\ 0 & 0.1 & 0 \\ 0 & 0 & 0.1 \end{pmatrix}, \\
\mathbf{\Phi}_1 &= \begin{pmatrix} 0.1 & 0 & 0 \\ 0 & 0.1 & 0 \\ 0 & 0 & 0.1 \end{pmatrix}, \\
\mathbf{\Phi}_2 &= \begin{pmatrix} -0.02 & 0 & 0 \\ 0 & -0.02 & 0 \\ 0 & 0 & -0.02 \end{pmatrix},
\end{aligned}$$

the matrices in equation (2.26):

$$\mathbf{F}^{(0)} = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 0 & -1 \\ 1 & -1 & 1 \end{pmatrix} \mathbf{A},$$

$$\mathbf{F}^{(1)} = \begin{pmatrix} 0 & -\frac{2}{\pi^*-10} & -\frac{8}{\pi^*-10} \\ 0 & -\frac{2}{\pi^*-10} & 0 \\ 0 & -\frac{2}{\pi^*-10} & \frac{8}{\pi^*-10} \end{pmatrix} \mathbf{A},$$

$$\mathbf{F}^{(1)} = \begin{pmatrix} 0 & 0 & \frac{16}{(\pi^*-10)^2} \\ 0 & 0 & \frac{16}{(\pi^*-10)^2} \\ 0 & 0 & \frac{16}{(\pi^*-10)^2} \end{pmatrix} \mathbf{A},$$

and the matrix in equation (2.27):

$$\mathbf{\Psi} = \begin{pmatrix} 10 \\ 5 + \frac{1}{2}\pi^* \\ \pi^* \end{pmatrix}.$$

Hence, equation (2.28) implies:

$$\begin{pmatrix} 0.1 & \frac{1}{10}\frac{\pi^*-12}{\pi^*-10} & \frac{1}{50}\frac{5(\pi^*)^2-140\pi^*+884}{(\pi^*-10)^2} \\ 0.1 & -\frac{1}{5}\frac{1}{\pi^*-10} & -\frac{1}{50}\frac{5(\pi^*)^2-100\pi^*+516}{(\pi^*-10)^2} \\ 0.1 & -\frac{1}{10}\frac{\pi^*-8}{\pi^*-10} & \frac{1}{50}\frac{5(\pi^*)^2-60\pi^*+84}{(\pi^*-10)^2} \end{pmatrix} \mathbf{A} = \begin{pmatrix} 10 \\ 5 + \frac{1}{2}\pi^* \\ \pi^* \end{pmatrix}.$$

Before solving for $\mathbf{A}$, first we take the value matching condition into account. In this little example, equation (2.29) is:

$$\begin{pmatrix} 1 & -1 & 1 \end{pmatrix} \mathbf{A} = 0.$$

Replace the last row of matrix $\mathbf{\Gamma A}$ by this result, and the last row of vector $\mathbf{\Psi}$ by 0 to get:

$$\begin{pmatrix} 0.1 & \frac{1}{10}\frac{\pi^*-12}{\pi^*-10} & \frac{1}{50}\frac{5(\pi^*)^2-140\pi^*+884}{(\pi^*-10)^2} \\ 0.1 & -\frac{1}{5}\frac{1}{\pi^*-10} & -\frac{1}{50}\frac{5(\pi^*)^2-100\pi^*+516}{(\pi^*-10)^2} \\ 1 & -1 & 1 \end{pmatrix} \mathbf{A} = \begin{pmatrix} 10 \\ 5 + \frac{1}{2}\pi^* \\ 0 \end{pmatrix}.$$

Now, solve for $\mathbf{A}$ as indicated by equation (2.31):

$$
\begin{aligned}
\mathbf{A}(\pi^*) &= \begin{pmatrix} 0.1 & \frac{1}{10}\frac{\pi^*-12}{\pi^*-10} & \frac{1}{50}\frac{5(\pi^*)^2-140\pi^*+884}{(\pi^*-10)^2} \\ 0.1 & -\frac{1}{5}\frac{1}{\pi^*-10} & -\frac{1}{50}\frac{5(\pi^*)^2-100\pi^*+516}{(\pi^*-10)^2} \\ 1 & -1 & 1 \end{pmatrix}^{-1} \begin{pmatrix} 10 \\ 5+\frac{1}{2}\pi^* \\ 0 \end{pmatrix} \\
&= \begin{pmatrix} \frac{5}{2}\frac{5(\pi^*)^3-25(\pi^*)^2-1408\pi^*+11580}{5(\pi^*)^2-115\pi^*+674} \\ 20\frac{10(\pi^*)^2-251\pi^*+1510}{5(\pi^*)^2-115\pi^*+674} \\ -\frac{25}{2}\frac{\left(10+(\pi^*)^2-11\pi^*\right)(\pi^*-10)}{5(\pi^*)^2-115\pi^*+674} \end{pmatrix}.
\end{aligned}
$$

Hence, we have our approximated function $\widehat{F}(\pi,\pi^*) = \mathbf{T}(\pi)\mathbf{A}(\pi^*)$:

$$
\widehat{F}(\pi,\pi^*) = -20\frac{30\left(\pi^*\right)^2 - 5\pi\left(\pi^*\right)^2 - 25\pi\pi^* + 252 + 5\pi^2\pi^* - 252\pi - 5\pi^2}{5\left(\pi^*\right)^2 - 115\pi^* + 674}.
$$

Finally, we use the smooth pasting condition to find π^*. First, differentiate $\widehat{F}(\pi,\pi^*)$ with respect to π:

$$
\widehat{F}_\pi(\pi,\pi^*) = -20\frac{-5\left(\pi^*\right)^2 - 25\pi^* + 10\pi\pi^* - 252 - 10\pi}{5\left(\pi^*\right)^2 - 115\pi^* + 674},
$$

and then evaluate in $\pi = \pi^*$ as indicated by equation (2.32):

$$
\widehat{F}_\pi(\pi^*) = -20\frac{5\left(\pi^*\right)^2 - 35\pi^* - 252}{5\left(\pi^*\right)^2 - 115\pi^* + 674} = 0.
$$

This non linear equation can be easily solved for π^*:

$$
\pi^*_{N=2} = -4.415175298121956
$$

which is, of course, very far from the analytical solution.

3

Energy-saving Technology Adoption under Uncertainty in the Residential Sector*

Home renovation is generally asserted to be a highly effective means for households to lower expenditures on energy. In this sense, home renovation can also be thought as a means to reduce GHG emissions. In this chapter we consider a homeowner who makes an irreversible energy-saving investment in an uncertain environment. In a general equilibrium framework, we solve the program of a representative consumer who uses his wealth to invest in the energy-saving technology, to save or to consume energy goods and non-energy goods. Resolution is analytical in a zero discounting case and numerical for the general case, based on collocation and Chebyshev polynomials. In particular, we show that the usual explanation of the energy paradox based on the existence of an option value in partial equilibrium is no longer valid when the analysis is extended to a general equilibrium framework.

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3.1 Introduction

Reducing GHG emissions is nowadays becoming one of the priorities of policy makers in many developed countries. For instance, the French government wishes to reduce emissions by four by 2050. Nevertheless, such a concern appears in a context of growing energy demand. This phenomenon is in part due to the importance of residential energy: in France, buildings account for 23% of CO_2 emissions, of which 70% are generated by the residential sector, and for 46% of final energy consumption [ANAH, 2008]. Energy demand in the residential sector depends mainly on the intensity of use of energy appliances and on their efficiency [Hausman, 1979]. Indeed, home renovations are generally asserted to be a highly effective means for households to lower expenditures on energy through increased efficiency and they become therefore a key target for environmental policies. For instance, enhanced insulation and energy-efficient ventilation of residential buildings are new technologies that can considerably reduce the energy consumption for indoor heating and cooling [Farsi, 2010]. Cost-benefit analyses point to the economic viability of these systems even if the comfort co-benefits, such as improvements in indoor air quality and protection against noise, are not taken into account [Jakob, 2006, Ott et al., 2006]. However, actual investment in these systems is still relatively rare [Banfi et al., 2008]. This chapter aims at carefully explaining the home renovation decision of households in a theoretical model. In particular, we explicitly take into account that such a decision takes place in an uncertain environment, in which there exist arbitrages between consumption, savings, and investment in home renovation.

The literature has already tried to explain the slow diffusion of energy efficient investments -the so-called ”energy paradox” or ”energy-efficiency gap” [Jaffe and Stavins, 1994a]. Everything happens as if agents were discounting with unusually high rates to appraise energy-efficiency investment, ranging from 25% to 30% (see Brown [2001] and Sanstad et al. [1995]). The usual suspect is the option value generated by the irreversibility of the investment decision in a stochastic environment that drives a wedge between the investment valuation and the Net Present Value. Hassett and Metcalf [1995] consider models in which households minimize the cost of energy expenditures subject to a given level of comfort (moreover, accommodations are heterogeneous in

Hassett and Metcalf [1993]. Investment in renewable energy or in insulation is irreversible and future benefits are uncertain. Based on simulations of this model and energy price data, they obtain that the discount rate should be four times higher for energy-saving investments than for other kinds of investments. In Ansar and Sparks [2009], the potential investor may delay adoption not only because of the joint effect of irreversibility and uncertainty, but also to cash in on future experience-curve effects: with the passage of time, firms gain practical knowledge in producing and installing the energy-saving technology, enabling them to reduce the technology cost per unit of energy saved. Simulations for photovoltaic systems highlight the experience-curve effect as a fundamental reason for which households and firms delay making energy-saving investments until internal rates of return exceed values of 50%, consistent with observations in the economics literature.

Conversely, for Jaffe and Stavins [1994a,b] delaying energy-saving investment is costly. An example is the difference between incorporation of energy saving technology in a new home as opposed to an existing home. In the case of the new home, forgoing the technology at the time of construction typically means that the cost of installation later (if it is undertaken) will be higher. These investment decisions do not satisfy the assumptions of the option value model, but further development of the option value approach could overcome such shortcomings. Their conclusion is that there may simply be no way, using observations of purchase decisions alone and assuming optimizing behaviour, of disentangling the effects of consumer discounting, energy price expectations, and principal-agent problems, each of which could account for high implicit discount rates. Finally, some literature now turns to explanations such as behavioral and organisational barriers, leading to some bounded rationality [Sanstad and Howarth, 1994, Boulanger, 2007, Diaz-Rainey and Ashton, 2009].

In this chapter we go back to the standard assumptions of irreversible energy-saving technology adoption and of uncertain payoffs. Instead of relying on bounded rationality we focus on the characteristics of consumers that take the adoption decision in the residential sector. The existing literature that explains the energy paradox (Hassett and Metcalf [1995]for instance) considers partial equilibrium settings, and therefore ignores the interaction between optimal consumption and optimal adoption as well as the notions of risk aversion[1] or intertemporal substitutions: everything happens as if consumers behave like firms when deciding about energy-saving technology adoption. To challenge these results, we reconsider the joint effect of irreversibility and uncertainty on the energy-efficiency investment decision in a general equilibrium framework: is the explanation of the energy paradox based on the existence of an option value still valid in a more realistic and general model? Of course, such a framework may not be specific to the problem of energy-saving technology adoption in the residential sector. But we argue that it is better suited to analyse a decision made by consumers and may be viewed as a generalization of the existing specific models.

Most of the literature on adoption under uncertainty (including that on energy saving technology adoption) remains in partial equilibrium. Adoption reduces profits but there is no arbitrage between consumption and adoption. In such a framework, Abel and Eberly [2002, 2005]study the optimal adoption of the stochastic latest technology. By contrast, in Roche [2003], it may be optimal for an upgrading firm to keep some distance with the frontier technology. In Grenadier and Weiss [1997], adopting an innovation provides the firm with an option value to learn. Pavlova [2001] introduces the leaning-by-doing of Parente [1994] into the firm's choice of under uncertainty. Finally, Alvarez and Stenbacka [2001] study the optimal timing to adopt a technology that may be updated in the future. Theoretical analyses of technology adoption in general equilibrium and a stochastic environment are very recent and very limited in number. Hugonnier et al. [2008] study the optimal adoption of a new technology that increases the productivity of capital.[2] There exists then an optimal adoption timing, and this timing is highly sensitive to the size of uncertainty as well as to the degree of agents' risk aversion. Moreover,Pommeret and Schubert [2009] tackle the specific problem of abatement technology adoption under uncertainty in a general equilibrium. The authors first determine the socially optimal adoption timing that is affected by the existence of pollution. Second, they derive the tax scheme such that in a decentralized economy

[1] See Farsi [2010] for the role of risk aversion in the energy efficient investment decision based on an empirical approach.

[2] The latter paper, together with Hugonnier et al. [2005], provides the resolution for the optimal threshold that triggers an irreversible decision in a general equilibrium framework for the first time in the literature.

firms adopt the abatement technology for the same level of economic development as in the centralized economy.

Tackling the specific problem of a homeowner who may invest in new insulation, or double glazing in order to reduce his energy bill is a bit different. The determination of the optimal investment timing is affected by the consumption of goods and services other than residential energy services (called "non-energy goods" in the rest of the chapter). Therefore, we extend the general equilibrium model with a real option proposed by Hugonnier et al. [2008]. We solve the program corresponding to the optimal adoption of an energy-saving technology adoption by a representative consumer who uses his wealth to save or to consume energy goods and non-energy goods. We assume that the benefits of such energy-saving technologies are uncertain due to the lack of information about them.[3] The financial return on savings is assumed to be stochastic as well. Because of uncertainty, we obtain that it may be optimal to reduce both consumptions in order to foster adoption. As usual (see Hugonnier et al. [2008], or Pommeret and Schubert [2009]) the model can only be solved analytically if the utility discount factor is zero. Nevertheless, we confirm our results in the more general case with non-zero discounting using a numerical procedure based on collocation and Chebyshev polynomials. We show that the threshold triggering adoption depends not only on technological parameters but on preference parameters as well. In particular, the higher the risk aversion parameter, the smaller the level of wealth which is required for adoption. Finally, we also show that while uncertainty on energy-saving technologies efficiency hardly affects adoption timing, uncertainty on financial returns fosters it. The latter result is strikingly different from what is obtained in the existing literature that remains in partial equilibrium and manages to explain the energy paradox with the existence of option values (see for instance Sanstad et al. [1995]). In this chapter, we consider the proper framework to address the issue of energy-saving technology adoption in the residential sector, namely the arbitrage between consumption and adoption is taken into account. By better describing the decision problem, we manage to challenge the existing result of previous literature: the existence of an option value does not rule out the energy paradox.

[3] Another interpretation for this uncertainty (that do not perfectly fit our modelling) would come from the great fluctuations in energy prices.

The remainder of the chapter is as follows. Section 2 presents the general model, and explains the assumptions that will be valid for the whole chapter. Because this model is to be solved backwards, we start in section 3 by developing the general equilibrium framework once the energy-efficient technology has been adopted. In Section 4 we analytically solve the model before the technology adoption and derive the optimal adoption timing in the special case of zero discounting. We provide also some sensitivity analysis based on numerical resolutions. We confirm these results in section 5 with the more general case of a strictly positive discount rate. Section 6 concludes.

3.2 The Model

We assume that the homeowner holds risky assets. His income return encompasses a deterministic part, r, and a stochastic one, $\sigma_2 dz_2$. He consumes energy goods C_2 and non-energy goods C_1. The function of wealth accumulation consists of two components. The deterministic one is $rA_t - C_{1t} - xC_{2t}$,with A_t the level of wealth and x, the relative price of energy. The stochastic component comes from the stochastic financial returns. The function of wealth accumulation writes therefore:

$$dA = (rA - C_1 - xC_2)dt + \sigma_2 A dz_{2t} \text{ for } t < \tau \tag{3.1}$$

At any time τ, the household can lower the cost x of the energy service by switching to a new technology y. The initial cost of the new technology is β. This cost is unrecoverable. Moreover, there exists an uncertainty σ_1 that is linked to the cost of consumption in energy service after the adoption of the new technology. Indeed, we assume that the benefits of such energy-saving technologies are uncertain due to the lack of information about them. The amount of uncertainty grows with the time horizon. Thus, we learn about the efficiency of the new technology as time passes, but the efficiency in the future will always be unknown.[4] Notice that y must be less than x

[4]It could be more realistic to assume that after adoption some uncertainty is solved. However, the modelling of uncertainty we have chosen can account for shocks on climate (for instance thinking of solar panels) or on maintenance costs (again for solar panels, see Slade [2009]). And it is anyway well suited to account for the fact that prior to adoption, uncertainty about its efficiency grows with the time horizon.

because the cost of the energy service is otherwise higher with the new technology and the homeowner never adopts. We define the difference between x and y as the savings in energy efficiency. The function of wealth accumulation after the adoption of the new technology is:

$$dA = (rA - C_1 - yC_2)dt - \sigma_1 yC_2 dz_{1t} + \sigma_2 A dz_{2t} \text{ for } t \geq \tau \tag{3.2}$$

The homeowner preferences over consumption plans are represented by the lifetime expected utility functional:

$$E_0 \left[\int_0^\infty e^{-\rho t} U(C_{1t}, C_{2t}) dt \right] = E_0 \left[\int_0^\infty e^{-\rho t} \frac{(C_1^a C_2^b)^{1-\gamma}}{1-\gamma} dt \right] \tag{3.3}$$

To facilitate the presentation, let denote Θ the set of admissible plans, that is, the set of consumption plans and dates of adoption (C, τ) such that:

$$E_0 \left[\int_0^\infty e^{-\rho t} U(C_{1t}, C_{2t}) dt \right] < \infty \tag{3.4}$$

where ρ is the consumer subjective discount factor, γ is the constant relative risk aversion of the household with $\gamma \neq 1$ and $\gamma > 0$. The elasticities a and b are positives. We define the effective coefficient of risk aversion (see Smith and Son [2005]):

$$R = 1 - (a + b)(1 - \gamma) \tag{3.5}$$

The optimal switching time should maximize the intertemporal utility subject to the function of wealth accumulation, the non-negativity constraint and the initial condition A_0. The value function of the homeowner is :

$$V(A_0) = \sup_{(C,\tau)} E_0 \left[\int_0^\tau e^{-\rho t} \frac{(C_{1t}^a C_{2t}^b)^{1-\gamma}}{1-\gamma} dt + e^{-\rho \tau} W(A_\tau - \beta) \right] \tag{3.6}$$

where W is the value after having adopted the new technology and τ is the optimal adoption time.

This program can be solved in two stages. First, we solve for the optimal consumption plans of the representative agent after the adoption of the new technology. Then, there is no longer an adoption option in the value of the program. We find the expression of the value function which provides the boundary condition to compute the equilibrium of our general model with a technology adoption opportunity. Notice that it also provides a benchmark for the economy with the technology adoption option. Second, we formulate the choice of an optimal consumption plan and an optimal investment time prior to technology adoption.

3.3 The Optimal Path after Adoption

We assume in this section that the new technology has been adopted. The set of admissible plans collapses to the set of consumption plans such that :

$$E_\tau \left[\int_\tau^\infty e^{-\rho(t-\tau)} \left| U(C_{1t}, C_{2t}) \right| dt \right] < \infty \tag{3.7}$$

The value function of the household is :

$$W(A_\tau) = \sup_C E_\tau \left[\int_\tau^\infty e^{-\rho(t-\tau)} \frac{(C_{1t}^a C_{2t}^b)^{1-\gamma}}{1-\gamma} dt \right] \tag{3.8}$$

The Bellman equation may be written :

$$W(A_t) = \max_{C_{1t}, C_{2t}} \left\{ \frac{(C_{1t}^a C_{2t}^b)^{1-\gamma}}{1-\gamma} dt + e^{-\rho dt} E_t(W(A_{t+dt})) \right\} \text{ with } t \geq \tau \tag{3.9}$$

The first order conditions yield the optimal consumption of energy and non-energy goods:[5]

Proposition 3.1 *Under assumptions (3.2) and (3.3) the representative agent's consumptions and lifetime utility are:*

$$C_{1t}^* = \left[\frac{(a+b)}{a} M B^{-b(1-\gamma)} \right]^{\frac{1}{a(1-\gamma)-1}} A_t \tag{3.10}$$

$$C_{2t}^* = B A_t \tag{3.11}$$

$$W(A_t) = \frac{M A_t^{1-R}}{(1-\gamma)} \tag{3.12}$$

[5]See Appendix A.

with:

$$B = \frac{Ry \pm (1-\gamma)\sqrt{\Delta}}{R\sigma_1^2 y^2 \left[2R - b(1-\gamma)\right]}$$

$$\Delta = \left[y\left(a+b-\frac{1}{1-\gamma}\right)\right]^2 - 4\left[\frac{R}{2}\sigma_1^2 y^2\left(2a+b-\frac{2}{(1-\gamma)}\right)\right]\left[b\frac{R}{2}\sigma_2^2 - br + \frac{b\rho}{1-R}\right]$$

$$M = \frac{a}{a+b}B^{-R}\left[\frac{a}{b}\left(y + R\sigma_1^2 y^2 B\right)\right]^{[a(1-\gamma)-1]}$$

We notice that the optimal consumptions are both constant fractions of the wealth level. The feasibility condition imposes $B > 0$. Depending on the value of γ relative to unity we consider one or the other root of the second order equation.

The expected optimal wealth growth rate is:

$$\begin{aligned} \frac{E_t(dA_t/A_t)}{dt} &= r - E_t\left[\frac{C_1^*}{A_t}\right] - yE_t\left[\frac{C_2^*}{A_t}\right] \\ &= r - \left[\frac{M}{a}(a+b)B^{-b(1-\gamma)}\right]^{\frac{1}{a(1-\gamma)-1}} - By \end{aligned}$$

Notice that this is a more complex expression than usual (see for instance Smith and Son [2005]) since consumption expenditure is itself directly affected by uncertainty in the wealth accumulation equation. The following results can be obtained analytically:

$$\frac{\partial(C_{1t}/K_t)}{\partial\sigma_2^2} = \left[\frac{\partial(C_{1t}/K_t)}{\partial M}\frac{\partial M}{\partial B} + \left.\frac{\partial(C_{1t}/K_t)}{\partial B}\right|_{M=\overline{M}}\right]\frac{\partial B}{\partial \Delta}\frac{\partial \Delta}{\partial \sigma_2^2} \begin{array}{l} > 0 \text{ for } \gamma < 1 \\ \text{indeterminate for } \gamma > 1 \end{array}$$

$$\frac{\partial(C_{2t}/K_t)}{\partial\sigma_2^2} = \frac{\partial B}{\partial \Delta}\frac{\partial \Delta}{\partial \sigma_2^2} \begin{array}{l} > 0 \text{ for } \gamma < 1 \\ < 0 \text{ for } \gamma > 1 \end{array}$$

An increase in the uncertainty on the financial returns increases current consumption in both energy and non-energy consumption if the intertemporal elasticity of substitution $(1/\gamma)$ is greater than unity. Moreover, current consumption in energy goods decreases when uncertainty σ_2 rises if the intertemporal elasticity of substitution is less than unity. These results are consistent with the usual income and substitution effects: more uncertainty reduces the certainty equivalent of the financial returns which in turn generates an income effect (less current consumption) and a substitution effect (more current consumption). The substitution effect prevails if the intertemporal elasticity of substitution is large enough. Effects of σ_2^2 on C_{1t} for $\gamma > 1$, or of σ_1^2 on both C_{1t} and C_{2t} are analytically indeterminate. Figures providing a numerical computation of these effects can be found in 3.A.

It is easy to show that the condition on parameters such that the utility function is concave is $(a+b)(1-\gamma)-1 < 0$. The transversality condition requires the convergence of the value function, i.e.

$$\lim_{t\to\infty} E_0(W(A_t)) = 0$$

It is satisfied if the lifetime utility of wealth does not grow "too fast" in expectation (see Smith and Son [2005]). Applying Itô's lemma to $W(K_t)$, this requires that:

$$E(dA) = W_A E(dA) + \frac{1}{2} W_{AA} E(dA^2) < 0$$

$$\Leftrightarrow M \left[r - \left(\frac{M}{a}(a+b) B^{-b(1-\gamma)} \right)^{\frac{1}{a(1-\gamma)-1}} - yB - \frac{R}{2}\left(\sigma_1^2 y^2 B^2 + \sigma_2^2\right) \right] > 0.$$

We assume that this condition is fulfilled.

3.4 The Optimal Adoption Timing with no Discounting

Considering the analytical resolution helps understanding the mechanisms of the model. However, solving analytically is only possible in the special case in which the consumer's discount rate is equal to zero[6]. This is why we assume zero-discounting in this section. Note that the expressions of the optimal consumption path and of the value function after the switch that have been derived in the previous section remain valid but we now impose $\rho = 0$ in these expressions. Nevertheless, assuming that the consumer does not discount the future is not very realistic. Therefore, we will turn to numerical resolutions in the next section to show that introducing a discount factor does not change the nature of the results.

Recall that the homeowner has to choose both an optimal consumption plan and an optimal technology adoption timing. This choice is given by the maximization of the intertemporal utility function subject to the wealth accumulation equation. Once the

[6] See next section.

new technology has been adopted, the homeowner optimally follows the consumption plan described by equations (3.10) and (3.11). Therefore, the value function at the time of adoption is given by the following value matching and smooth pasting conditions :

$$V(A_\tau) = W(A_\tau - \beta) \tag{3.13}$$

$$V_A(A_\tau) = W_A(A_\tau - \beta) \tag{3.14}$$

where A_τ is the level of wealth for which is optimal to adopt. It implicitly determines the optimal switching time τ. The value matching condition (3.13) simply requires that, at the time of the switch (i.e for a level A_τ of wealth), the value before the switch is equal to the value after the switch once paid the initial costs. The smooth pasting condition (3.14) ensures the smoothness of the value function around the switch (V before the switch and W after the switch). It guarantees that adoption occurs for the optimal level of wealth.

Since it is always possible for the homeowner to indefinitely postpone the adoption of the new technology, another condition has to be satisfied, namely that for any level of wealth, the value with the adoption opportunity V cannot be smaller than W_0, the value without such an opportunity:

$$W_0(A_t) \leq V(A_t) \quad \forall t \tag{3.15}$$

The household's program is :

$$\begin{aligned} V(A_0) &= \sup_{C,\tau} E_0 \left[\int_0^\tau \frac{(C_{1t}^a C_{2t}^b)^{1-\gamma}}{1-\gamma} dt + W(A_\tau - \beta)_{\{\tau<\infty\}} \right] && (3.16) \\ s.t. \;\; dA &= (rA - C_1 - xC_2)dt + \sigma_2 A dz_{2t} && (3.17) \end{aligned}$$

To solve the program before the switch, we determine the marginal value of wealth which has to satisfy the smooth pasting condition. Integrating this value between zero and the level of wealth at the optimal switching time, we can use the value matching condition to get the optimal adoption date.

3.4.1 The Marginal Value of Wealth

The first order condition yields:[7]

$$C_{1t}^{*} = a^{\frac{1-b(1-\gamma)}{R}} \left(\frac{b}{x}\right)^{\frac{b(1-\gamma)}{R}} V_A^{-\frac{1}{R}} \tag{3.18}$$

$$C_{2t}^{*} = a^{\frac{a(1-\gamma)}{R}} \left(\frac{b}{x}\right)^{\frac{1-a(1-\gamma)}{R}} V_A^{-\frac{1}{R}} \tag{3.19}$$

Using a variable change, the Bellman equation leads to the following expression for the marginal value of wealth before the switch (see 3.B) :

$$\begin{aligned} V_A(A_t) &= \left[D_1 A_t^{-1} + \underbrace{D_2 A_t^{D_3}}_{G(A_t,A_\tau)} \right]^R && (3.20)\\ \text{with } D_1 &= \left[\frac{R}{\gamma-1} a^{\frac{a(1-\gamma)}{R}} \left(\frac{b}{x}\right)^{\frac{b(1-\gamma)}{R}} \right] \frac{1}{(r-\frac{1}{2}\sigma_2^2 R)} && (3.21)\\ \text{and } D_3 &= -\frac{2r}{R\sigma_2^2} && (3.22) \end{aligned}$$

Let $G(A_t, A_\tau) = D_2 A_t^{-\frac{r}{\frac{1}{2}R\sigma_2^2}}$ be the option value to switch.[8] D_2 is a constant which must be determined[9] using the smooth pasting condition (3.14). We obtain:

$$D_2 = \frac{[M(a+b)]^{\frac{1}{R}}}{(A_\tau - \beta)A_\tau^{D_3}} - \frac{D_1}{A_\tau^{D_3+1}} \tag{3.23}$$

If the homeowner does not have the opportunity to adopt a new technology, there is no option value to adopt the new technology, $G(A_t, A_\tau) = 0$, and the value function

[7] See 3.B.

[8] Recall that under uncertainty, it is possible to delay an irreversible investment. While the homeowner is waiting, he can take advantage of an opportunity to invest, similar to what happens with a financial option. Therefore, there exists an option value of the investment project that is killed at the time of investment (see Dixit and Pindyck [1994]). This option value represents an opportunity cost of investment that must be taken into account.

[9] See 3.B.

reduces to:[10]

$$W_0(A_t) \quad = \quad \left[\underbrace{\frac{\frac{R}{\gamma-1} a^{\frac{a(1-\gamma)}{R}} \left(\frac{b}{x}\right)^{\frac{b(1-\gamma)}{R}}}{\left(r - \frac{1}{2}\sigma_2^2 R\right)}}_{D_1}\right]^R \frac{A_t^{1-R}}{(a+b)(1-\gamma)} \tag{3.24}$$

$$\Leftrightarrow \quad W_0(A_t) = \frac{D_1^R}{(a+b)} \frac{A_t^{1-R}}{(1-\gamma)}$$

where $W_0(A_t)$ is the value function of the homeowner with no opportunity to switch. Moreover, the feasibility condition associated with the program in the absence of adoption opportunity writes:

$$D_1 > 0$$

Recall that $W_0(A_t)$ cannot be greater than the lifetime utility of the agent in an economy with the new technology; therefore we must have:

$$W_0(A_t) \leq W(A_t) \tag{3.25}$$

This condition ensures that there exists an optimal switching date, that is, in the absence of costs of switching to the new technology, the central planner would choose to immediately switch for any current level of wealth accumulation.

Using the expressions for W_{0A} (computed using equation (3.24)) and for W_A (computed using equation (3.12)), the marginal value $V_A(A_t)$ can be rewritten:

$$V_A(A_t) = \left[W_{0A}(A_t)^{\frac{1}{R}} + \underbrace{\left([W_A(A_\tau - \beta)]^{\frac{1}{R}} - W_{0A}(A_\tau)^{\frac{1}{R}}\right) \frac{A\tau}{A_t}^{\frac{2r}{R\sigma_2^2}}}_{=G(A_t,A_\tau),\text{ part due to the option to switch}}\right]^R \tag{3.26}$$

The marginal value of wealth differs significantly from the one that can be derived in the absence of technological change. This is due to the existence of an option to switch that generates an option value taken into account in the marginal value of wealth. In the absence of such an option, $G(A_t, A_\tau) = 0$ and the marginal value of wealth reduces

[10]See 3.B.

to $W_{0A}(A_t)$. This option value is the difference between the marginal value after the switch (having paid initial cost β) and the marginal value in the absence of opportunity to switch, discounted by the distance to the switch, that is related to the ratio between the current wealth A_t and $A\tau$.

We have two cases :

- If $\gamma < 1$, we have: $W_{0A}(A_t)^{\frac{1}{R}} < W_A(A_t)^{\frac{1}{R}} < W_A(A_t - \beta)^{\frac{1}{R}}$ that ensures that $G(A_t, A_\tau) > 0$. There is no problem of existence of V_A in this case. The marginal value of wealth in the economy with an opportunity to switch is greater than the marginal value of wealth in an economy without this opportunity. It means that consumption at each time is smaller in an economy with opportunity to adopt a new technology, compared to the consumption which prevails in an economy in which the opportunity does not exist.
- If $\gamma > 1$, we have $W_{0A}(A_t)^{\frac{1}{R}} > W_A(A_t)^{\frac{1}{R}}$. In this case, the sign of $G(A_t, A_\tau)$ is ambiguous.
  - $G(A_t, A_\tau) < 0$. It means that the part due to the option to adopt a new technology in the expression of the marginal value of wealth is negative. Therefore, consumption at each period is greater in an economy with an opportunity to switch compared to the consumption which prevails in an economy without such an opportunity. In this case, the homeowner does not like to substitute and the option to adopt a new technology is an incentive to rise his consumption today to smooth his consumption path that is expected to grow more once the technology is adopted. Thus, adoption is delayed. However such a consumption path cannot happen since for small values of A_t, namely for $A_t < (-D_1/D_2)^{1/(1-2r/R\sigma_2)}$, the expression of $V_A^{\frac{1}{R}}$ becomes negative and the program is no longer defined. Therefore, $G(A_t, A_\tau) < 0$ cannot be considered.
  - $G(A_t, A_\tau) > 0$. It implies that when integrating V_A, the following condition $W_0(A_t) \leq V(A_t) \quad \forall t$ can no longer be satisfied (since for $\gamma > 1$, feasibility condition implies $2r/(\sigma_2^2 R) > 1$). Therefore, $G(A_t, A_\tau) > 0$ cannot be considered.

- $G(A_t, A_\tau) = 0$. It ensures both that $V_A^{\frac{1}{R}}$ is positive (the program is defined) and that the condition $W_0(A_t) \leq V(A_t) \quad \forall t$ is satisfied. It is the sole solution that we can consider if $\gamma > 1$. It involves that the consumption is not affected by the existence of an option to adopt a new technology. We denote A^* the special value of A_t such that $G(A_t, A_\tau) = 0$.

Therefore, we obtain using equation (3.24) and (3.26):

$$A^* = \frac{\beta}{1 - \frac{[M(a+b)]^{\frac{1}{R}}}{D_1}} \tag{3.27}$$

A^* must be positive, which requires $M(a+b) < D_1^R$ that is ensured by condition (3.25). Obviously, this threshold raises with the cost to pay for adoption and decreases with the savings in energy efficiency (see numerical resolutions in 3.B). The better the new technology compared to the old one, the smaller the threshold.

3.4.2 Boundary Conditions

The level of wealth A_τ is such that, at the time to the adoption, the value with the initial technology is equal to the value with the new technology once the cost β is paid (this is the value matching condition):

$$V(A_\tau) = W(A_\tau - \beta) \tag{3.28}$$

It is straightforward to obtain Proposition 2 for $\gamma < 1$ using equations (3.18), (3.19), (3.26) with $G(A_t, A_\tau) > 0$ and (3.28):

Proposition 3.2 *Assume that the problem is undiscounted, that $\gamma < 1$ and that the marginal propensity to consume is positive. The optimal technology adoption threshold is then the unique solution to the non-linear equation:*

$$\int_0^{A_\tau} V_A(A, A_\tau)dA = W(A_\tau - \beta) \ \textit{since} V(0) = 0 \ \textit{for} \ \gamma < 1 \tag{3.29}$$

with

$$V_A(A_t, A_\tau) = \left[W_{0A}(A_t)^{\frac{1}{R}} + G(A_t, A_\tau)\right]^R \tag{3.30}$$

where

$$G(A_t, A_\tau) = \left([W_A(A_\tau - \beta)]^{\frac{1}{R}} - W_{0A}(A_\tau)^{\frac{1}{R}} \right) \frac{A\tau}{A_t}^{\frac{2r}{R\sigma_2^2}}$$

$G(A_t, A_\tau)$ *is the part of the marginal value which from the option to switch to a new technology. Moreover, the value function and optimal consumptions strategies before technology adoption are given by:*

$$V(A_t) = \int_0^{At} V_A(A)dA$$

$$C_{1t}^* = a^{\frac{1-b(1-\gamma)}{R}} \left(\frac{b}{x}\right)^{\frac{b(1-\gamma)}{R}} V_A^{-\frac{1}{R}} \tag{3.31}$$

$$C_{2t}^* = a^{\frac{a(1-\gamma)}{R}} \left(\frac{b}{x}\right)^{\frac{1-a(1-\gamma)}{R}} V_A^{-\frac{1}{R}} \tag{3.32}$$

We now illustrate this case. Equation (3.29) can be solved numerically. Simulations are driven using the following values for the parameters: $\sigma_1 = 0.013$; $b = 0.25$; $a = 0.7$; $\gamma = 0.5$; $x = 10$; $y = 0.25$; $r = 0.05$; $\sigma_2 = 0.5$; $\beta = 0.1$. The value of the effective coefficient of risk aversion is $R = 0.525$.

Figure 3.1 shows the three value functions: $V(A)$ before the switch, $W(A - \beta)$ after the switch, and $W_0(A)$ without the option to switch. The threshold that triggers the switch is $A_\tau = 0.82$. Notice that at the time of adoption, yC_2 (with C_2 computed after adoption) is smaller that xC_2 (with C_2 computed before adoption). It implies that, at least at the time of adoption, the rebound effect does not prevail on energy consumption.

It is straightforward to obtain Proposition 3 for $\gamma > 1$ using equations (3.18), (3.19), (3.26) with $G(A_t, A_\tau) = 0$ and (3.28):

Proposition 3.3 *Assume that the problem is undiscounted, that $\gamma > 1$, and that the marginal propensity to consume is positive. The optimal technology adoption threshold A_τ is then:*

$$A_\tau = A^* = \frac{\beta}{1 - \frac{[M(a+b)]^{\frac{1}{R}}}{D_1}} \tag{3.33}$$

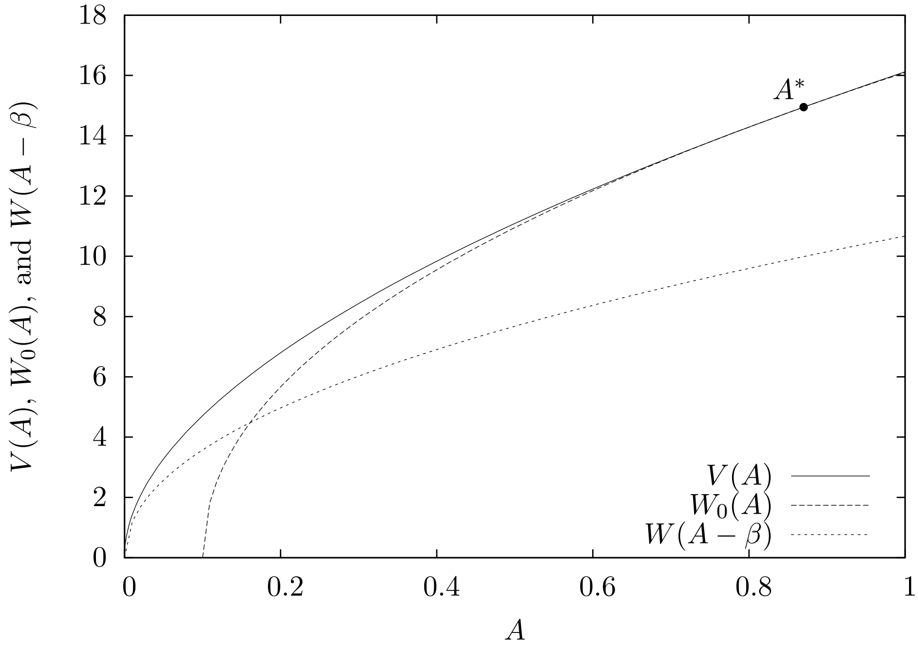

Figure 3.1: The Value Functions for $\gamma < 1$

Moreover, the value function and optimal consumptions strategies before technology adoption are given by:

$$V(A_t) = \frac{1}{1-R}\left[D_1^R A_t^{1-R} + [M(a+b)]\,(A_\tau - \beta)^{1-R} - D_1^R A_\tau^{1-R}\right] \tag{3.34}$$

$$C_{1t}^* = a^{\frac{1-b(1-\gamma)}{R}} \left(\frac{b}{x}\right)^{\frac{b(1-\gamma)}{R}} V_A^{-\frac{1}{R}} \tag{3.35}$$

$$C_{2t}^* = a^{\frac{a(1-\gamma)}{R}} \left(\frac{b}{x}\right)^{\frac{1-a(1-\gamma)}{R}} V_A^{-\frac{1}{R}} \tag{3.36}$$

We now illustrate this case. Note that both A^* and $V(A_t)$ are analytically defined with $\gamma > 1$. Simulations to draw the value functions are driven using the following values for the parameters : $\sigma_1 = 0.013$; $b = 0.25$; $a = 0.7$; $\gamma = 2$; $x = 10$; $y = 0.25$; $r = 0.05$; $\sigma_2 = 0.1$; $\beta = 0.1$. The value of the effective coefficient of risk aversion is $R = 1.95$.

Figure 3.2 shows the three value functions: $V(A)$ before the switch, $W(A-\beta)$ after the switch, and $W_0(A)$ without the option to switch. The threshold which triggers the switch is $A^* = 0.265$ (see equation (3.27)). Contrary to what happened for $\gamma < 1$, one may compute that the rebound effect prevails at the time of adoption.

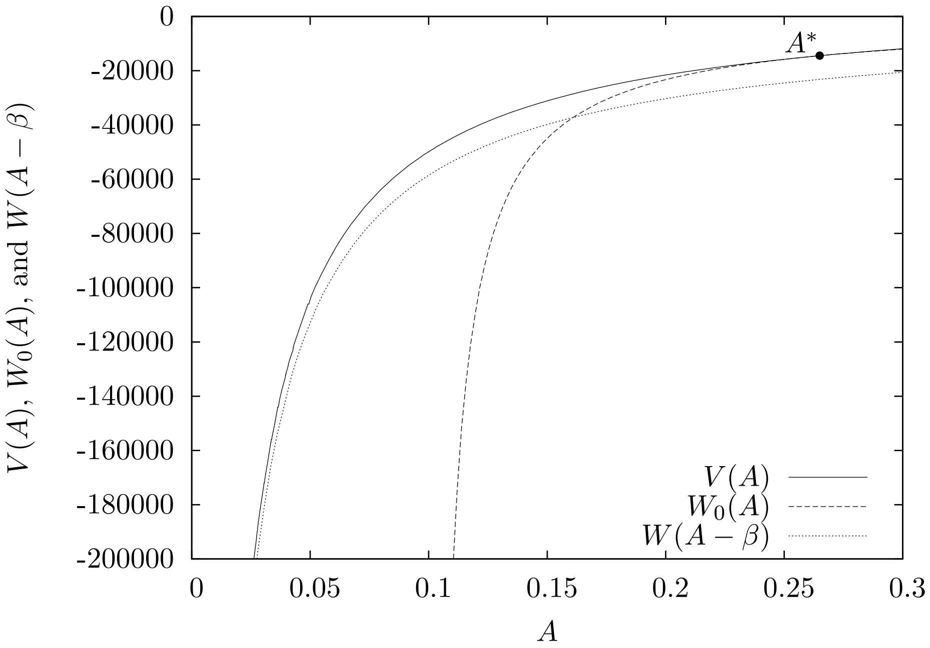

Figure 3.2: The Value Functions for $\gamma > 1$

3.4.3 Comparative Statics

We provide Table 3.1 in 3.C to show the effect of each parameters. Parameters' values used to draw the value functions previously are now considered as baseline parameters for the simulations.

The level of wealth for which it is optimal to switch is a decreasing function of x and an increasing function of y. It is quite intuitive that the larger the gain of adoption, the sooner the homeowner wishes to adopt and therefore the lower the level of wealth for which he wishes to adopt. Of course, we obtain that the higher the adoption cost, the higher the threshold wealth and the later the adoption.

Let us consider the effects of preference parameters. As far as parameters a (on non-energy goods C_1) and b (on energy goods C_2)[11] are concerned, we obtain the following results: the more sensitive the utility of the household to non-energy consumption (i.e. the higher a), the later the homeowner wants to devote resources to adoption and then, the higher the optimal adoption threshold, but the effect of b depends on the value of γ relative to unity. The relative risk aversion coefficient γ has a complex effect because it summarizes both the attitude with respect to risk and that with respect to intertemporal substitution. Simulations show that the higher γ, the smaller the level of wealth that triggers adoption.

Let us now turn to the effect of uncertainties. The level of wealth for which it is optimal to switch is not sensitive to the uncertainty (σ_1) related to the efficiency of consumption in energy service after the adoption of the new technology. It is an increasing function of the deterministic part of financial return (r) while it is a decreasing function of the uncertainty on the financial returns (σ_2). First, the larger the deterministic return on wealth, the more reluctant the homeowner is to devote part of his wealth to technology adoption. Second, σ_2 reduces the certainty equivalent of the wealth rate of return ($r - \frac{1}{2}\sigma_2^2 R$) and it is no surprise that it affects adoption in the opposite way compared to r. These effects of r and σ_2 only appear for $\gamma < 1$. For $\gamma > 1$ adoption becomes insensitive to these parameters.

This numerical example illustrates that contrary to what happens in partial equilibrium, uncertainty does not increase the adoption threshold. An explanation of the energy paradox based on the existence of option values is therefore no longer valid when one considers more carefully the consumer's decision problem.

[11]Note that we only present the results for values of b between 0.2 and 0.4 because b cannot be greater than a.

3.5 The Optimal Adoption Timing with Discounting

It is not possible to go on with the analytical resolution if the consumer discount rate is not equal to zero. Indeed, the Bellman equation before the switch is:

$$\rho V = \max_{C_{1t},C_{2t}} \left\{ \frac{(C_{1t}^a C_{2t}^b)^{1-\gamma}}{1-\gamma} + V_A(rA - C_1 - xC_2) + \frac{1}{2}\sigma_2^2 A^2 V_{AA} \right\} \tag{3.37}$$

Maximizing wit respect to both C_{1t} and C_{2t} leads to

$$\rho V.V_A^{\frac{1-R}{R}} = \frac{R}{1-\gamma} a^{\frac{a(1-\gamma)}{R}} \frac{b}{x}^{\frac{b(1-\gamma)}{R}} + rAV_A^{\frac{1}{R}} + \frac{1}{2}\sigma_2^2 A^2 V_A^{\frac{1-R}{R}} V_{AA} \tag{3.38}$$

Such an equation can no longer be solved using the variable change proposed in appendix B for $\rho = 0$. As this Bellman equation cannot be solved analytically, we turn to a numerical resolution. More precisely, we adapt Judd's methodology (see Judd [1992]) based on Chebyshev polynomials and projection methods as proposed in Dangl and Wirl [2004].

3.5.1 An Approximate Value Function Before Adoption

Using equation (3.38), let us define L:

$$L(V)(A) = \frac{R}{1-\gamma} a^{\frac{a(1-\gamma)}{R}} \frac{b}{x}^{\frac{b(1-\gamma)}{R}} + rAV_A^{\frac{1}{R}} + \frac{1}{2}\sigma_2^2 A^2 V_A^{\frac{1-R}{R}} V_{AA} - \rho V_A^{\frac{1-r}{R}} V.$$

L is an operator, or a function that maps functions to functions, and $A \in [0, A_\tau]$. As noted in Judd [1992, 1998], the domain of L includes all the C^1 functions, and its range is C^0. The differential equation (3.38), combined with the value matching and smooth pasting conditions, equations (3.13) and (3.14) respectively, can be viewed as the problem of finding a C^1 function V such that:

$$L(V)(A) = 0 \tag{3.39}$$

$$V(A_\tau) = W(A_\tau - \beta) \tag{3.40}$$

$$V'(A_\tau) = W'(A_\tau - \beta). \tag{3.41}$$

The projection method simplifies the original problem (3.39) by approximating the function $V(A)$ by means of polynomials.[12] As we decided to use Chebyshev polynomials, our approximation can be written as:

$$\widehat{V}(A, \mathbf{c}) = \frac{1}{2}c_0 T_0(A) + \sum_{i=1}^{N} c_i T_i(A), \tag{3.42}$$

where $A \in [0, A_\tau]$, and $T_i(A)$ is the general ith Chebyshev polynomial of the first kind that is defined by the following recurrence relationship:

$$\begin{aligned} T_0(s(A)) &= 1, \\ T_1(s(A)) &= s(A), \text{ and} \\ T_{n+1}(s(A)) &= 2sT_n(s(A)) - T_{n-1}(s(A)), \end{aligned}$$

or by the trigonometric identity:

$$T_n(s(A)) = \cos(n \arccos s(A)),$$

where $s(A)$ is a linear transformation such that $-1 \leq s(A) \leq 1$. Written in this way, the quality of our approximation is guaranteed by the Chebyshev approximation theorem (see Judd [1992]).

3.5.2 Choosing the Coefficients

We need to choose $c = \{c_0, c_1, c_2, ..., c_N\}$ so that $\widehat{V}(A, c)$ nearly solves the differential equation (3.38). To do this, we first ignore the conditions (3.40) and (3.41), and define the residual function:

$$RF(A, \mathbf{c}) \equiv L(\widehat{V})(A). \tag{3.43}$$

Equation (3.43) is the deviation of $L(\widehat{V})(A)$ with respect to the zero target value. The projection method adjusts the set of coefficients until a set c is found that makes $RF(A, c)$ sufficiently close to the zero function. Equation (3.42) has then to be inserted

[12] By Weierstrass theorem, we know that any C^1 function can be properly approximated by a large sums of polynomial terms. So, as N becomes larger in our equation (3.42), we are sure that $\widehat{V}(A, c)$ is converging to $V(A)$.

into equation (3.43). Note that we have $N+1$ coefficients c_i to be found. Therefore, we choose $N+1$ collocation values $A_i \in [0, \widehat{A}_\tau]$, where $\widehat{A}_\tau$ is an arbitrarily chosen value of A_τ. For example, the Chebyshev-Gauss-Lobatto collocation points can be used. They are defined as follows:

$$s_i = \cos\left(\frac{i\pi}{N}\right).$$

Applying such a collocation method, the initial problem is reduced to that of solving a set of $N+1$ non linear equations:

$$RF_i(A_i, \mathbf{c}) = 0,\ i = 0, 1, ..., N. \tag{3.44}$$

Boundary conditions, i.e. the value matching and smooth pasting conditions, need then to be considered. For instance, let us start by introducing the value matching condition. Choosing an initial value $\widehat{A}_\tau$ for A, equation (3.42) and equation (3.40) imply that:

$$\widehat{V}(\widehat{A}_\tau) - W(\widehat{A}_\tau - \beta) = 0. \tag{3.45}$$

To impose that our solution satisfies the value matching, one condition of the set (3.44) is then replaced by equation (3.45). The resulting system can be solved iteratively starting with a guess $c^0 = (c_i^0)$. Specifically, we use Newton's method: $c^{k+1} = c^k - (J_{\mathbf{c}^k})^{-1} P(c^k)$, where $J_{\mathbf{c}^k}$ is the Jacobian of $RF(A, c)$ evaluated at the respective point c^k. Finally, the optimal switching time A_τ is found using a search algorithm in order satisfy the smooth pasting condition. We solve for our non-linear system until a value $\widehat{A}_\tau = A_\tau$ is found that solves:

$$\widehat{V}'(\widehat{A}_\tau) = W'(\widehat{A}_\tau - \beta)$$

3.5.3 Results

In our computations we are using $N = 10$ and the baseline parameters' values described in the previous section (with no discounting) except that $\rho = 0.0001$. Such a small value allows to compare the results with those obtained under the assumption of no-discounting.[13] Simulations are driven using Matlab software. Figures 3.3 and 3.4 4 show the value functions before and after the switch and the optimal switching level of wealth A_τ:

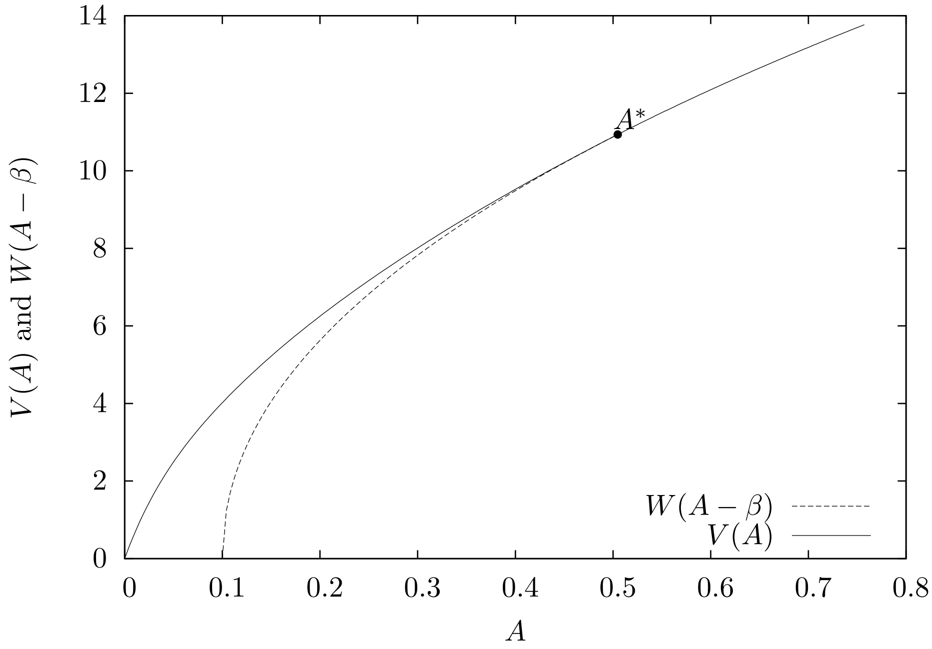

Figure 3.3: Value functions $\rho \neq 0$, $\gamma < 1$. $A_\tau = 0.5049$

Table 3.2 in 3.C show the sensitivity of the optimal adoption timing to the model parameters, starting with the usual baseline.

First, this analysis proves to be fully consistent with that driven under the no-discounting assumption. In particular we obtain again that one cannot rely on uncertainty to explain the energy paradox. Second, the table shows that the more concerned about the present the household is (larger ρ), the earlier he adopts the new technology, in order to get the benefits sooner.

3.6 Conclusion

In this chapter we consider a homeowner who makes an irreversible energy-saving investment under uncertainty. Both financial returns and the energy-saving technology efficiency are stochastic. In a general equilibrium framework, we solve the program

[13] A more realistic value for ρ can be found in the sensitivity analysis.

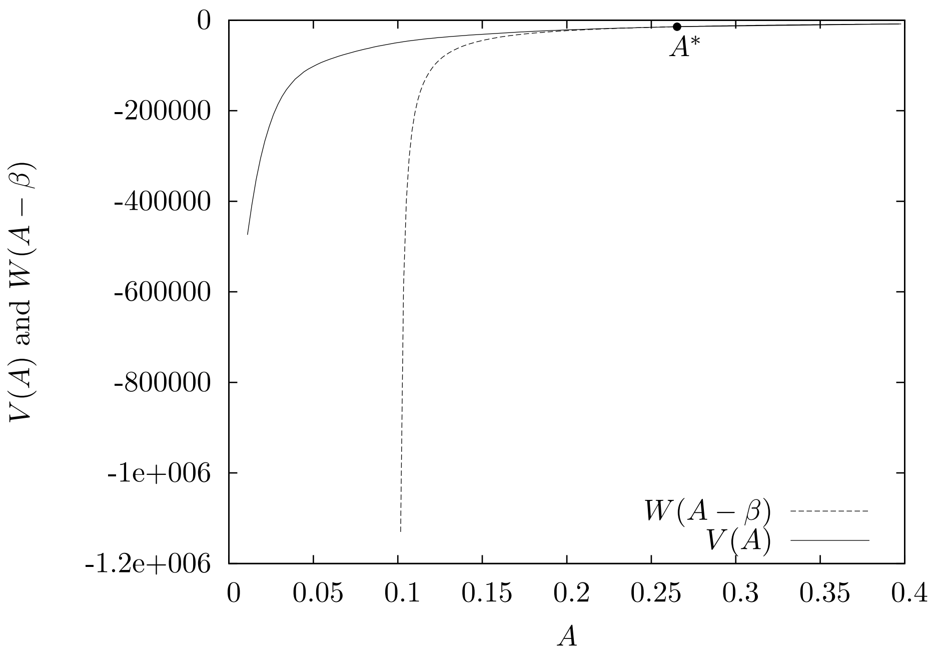

Figure 3.4: Value functions $\rho \neq 0$, $\gamma > 1$. $A_\tau = 0.2653$

corresponding to the optimal adoption of an energy-efficiency technology adoption by a representative consumer who uses his wealth to save or to consume energy goods and non-energy goods. We therefore explicitly take the arbitrage between consumption and adoption into account. This is not the case in the existing literature that explains the energy paradox. The model can only be solved analytically if the utility discount factor is zero. We confirm the results in the more general case with non-zero discounting using a numerical procedure. We show that the threshold triggering adoption depends not only on technological parameters but on preference parameters as well. In particular, the higher the risk aversion parameter, the smaller the level of wealth which is required for adoption. Finally, we also show that while uncertainty on energy-saving technologies efficiency does not affect adoption timing, uncertainty on financial returns fosters it. The latter result is strikingly different from what is obtained in partial equilibrium: we show that the usual explanation of the energy paradox based on the existence of an option value is no longer valid when the analysis is extended to a general equilibrium framework.

3.A Solving the Optimal Program after Adoption

The Bellman equation is written as:

$$W(A_t) = \max_{C_{1t},C_{2t}} \left\{ \frac{(C_{1t}^a C_{2t}^b)^{1-\gamma}}{1-\gamma} dt + E_t(W(A_{t+dt})) \right\} \text{ with } t \geq \tau$$

Using Itô's lemma, this equation becomes:

$$\max_{C_{1t},C_{2t}} \left\{ \frac{(C_{1t}^a C_{2t}^b)^{1-\gamma}}{1-\gamma} dt + W_A(rA - C_1 - yC_2)dt + \left[\frac{\sigma_1^2}{2} y^2 C_2^2 + \frac{\sigma_2^2}{2} r^2 A^2 \right] W_{AA} dt \right\} = 0 \tag{3.46}$$

The first order conditions yields:

$$C_{1t}^* = \left[W_A \frac{C_{2t}^{-b(1-\gamma)}}{a} \right]^{\frac{1}{a(1-\gamma)-1}} \quad \text{and} \quad C_{2t}^* = \frac{yW_A}{bC_{2t}^{b(1-\gamma)-2} C_{1t}^{a(1-\gamma)} + \sigma_1^2 y^2 W_{AA}}$$

We guess that $C_{2t}^* = B.A$ and $W(A_t) = W(A_t) = MA^{1-R}/(1-\gamma)$, with B and M being constant to de determined and $R = 1-(a+b)(1-\gamma)$ being the effective coefficient of risk aversion (equation (3.5) in the text). Then:

$$W_A = M(a+b)A^{(a+b)(1-\gamma)-1}$$

$$W_{AA} = M(a+b)\left[(a+b)(1-\gamma)-1\right] A^{(a+b)(1-\gamma)-2}$$

By replacing the consumptions by their optimal expressions into the bellman equation we get:

$$\begin{aligned} 0 \;=\; & \frac{1}{1-\gamma} \left[\left(\frac{W_A (BA)^{-b(1-\gamma)}}{a} \right)^{\frac{a(1-\gamma)}{a(1-\gamma)-1}} (BA)^{b(1-\gamma)} \right] \\ & + W_A \left[rA - \left(\frac{W_A (BA)^{-b(1-\gamma)}}{a} \right)^{\frac{1}{a(1-\gamma)-1}} - y(BA) \right] + W_{AA} \left[\frac{\sigma_1^2}{2} y^2 (BA)^2 + \frac{\sigma_2^2}{2} A^2 \right] \end{aligned}$$

$$\Leftrightarrow M^{\frac{1}{a(1-\gamma)-1}} B^{\frac{-b(1-\gamma)}{a(1-\gamma)-1}} = \left(\frac{a}{a+b}\right)^{\frac{1}{1-a(1-\gamma)}} \left(\frac{a(1-\gamma)}{1-a(1-\gamma)}\right) \left[\frac{R}{2}\left(\sigma_1^2 y^2 B^2 + \sigma_2^2\right) - r + yB\right]$$

Moreover, using the expression of C_{2t}^* we can get:

$$M^{\frac{1}{a(1-\gamma)-1}} B^{\frac{-b(1-\gamma)}{a(1-\gamma)-1}} = \frac{B}{b} a^{\frac{a(1-\gamma)}{a(1-\gamma)-1}} (a+b)^{\frac{1}{1-a(1-\gamma)}} \left(y + R\sigma_1^2 y^2 B\right)$$

Hence:

$$\begin{aligned}
&\left(\frac{1-\gamma}{1-a(1-\gamma)}\right)\left[\left(\sigma_1^2 y^2 B^2 + \sigma_2^2\right)\frac{R}{2} - r + yB\right] = \frac{B}{b}\left(y + R\sigma_1^2 y^2 B\right) \\
&\Leftrightarrow B^2\left[\frac{R}{2}\sigma_1^2 y^2\left(\frac{1-\gamma}{[1-a(1-\gamma)]} - \frac{2}{b}\right)\right] + B\left[y\left(\frac{(1-\gamma)}{1-a(1-\gamma)} - \frac{1}{b}\right)\right] \\
&\qquad + \left[\frac{\rho}{1-R} - r + \frac{R}{2}\sigma_2^2\right]\left(\frac{1-\gamma}{1-a(1-\gamma)}\right) = 0
\end{aligned}$$

$$\Leftrightarrow B^2\left[\frac{R}{2}\sigma_1^2 y^2\left(b - 2\frac{[1-a(1-\gamma)]}{1-\gamma}\right)\right] + B\left[y\left(b - \frac{[1-a(1-\gamma)]}{1-\gamma}\right)\right] + \left[b\frac{R}{2}\sigma_2^2 - br + \frac{b\rho}{1-R}\right] = 0$$

$$\Leftrightarrow B^2\left[\frac{R}{2}\sigma_1^2 y^2\left(2a + b - \frac{2}{1-\gamma}\right)\right] + B\left[y\left(a + b - \frac{1}{1-\gamma}\right)\right] + \left[b\frac{R}{2}\sigma_2^2 - br + \frac{b\rho}{1-R}\right] = 0$$

$$\Delta = \left[y\left(a + b - \frac{1}{1-\gamma}\right)\right]^2 - 4\left[\frac{R}{2}\sigma_1^2 y^2\left(2a + b - \frac{2}{(1-\gamma)}\right)\right]\left[b\frac{R}{2}\sigma_2^2 - br + \frac{b\rho}{1-R}\right]$$

Therefore:

$$\begin{aligned}
B &= \frac{-y\left((a+b)(1-\gamma) - 1\right) \pm (1-\gamma)\sqrt{\Delta}}{R\sigma_1^2 y^2\left[(2a+b)(1-\gamma) - 2\right]} \\
&= \frac{-y\left((a+b)(1-\gamma) - 1\right) \pm (1-\gamma)\sqrt{\Delta}}{R\sigma_1^2 y^2\left[2(a+b)(1-\gamma) - 2 - b(1-\gamma)\right]} \\
&= \frac{Ry \pm (1-\gamma)\sqrt{\Delta}}{R\sigma_1^2 y^2\left[2R - b(1-\gamma)\right]}
\end{aligned}$$

$$W(A_t) = B^{b(1-\gamma)}\left[\frac{B}{b} a^{\frac{a(1-\gamma)}{a(1-\gamma)-1}} (a+b)^{\frac{1}{1-a(1-\gamma)}} \left(y + R\sigma_1^2 y^2 B\right)\right]^{[a(1-\gamma)-1]} \frac{A^{(a+b)(1-\gamma)}}{(1-\gamma)}$$

We insert the expression of the effective coefficient of risk aversion in the precedent equation and we obtain (equation 3.12 of the text):

$$W(A_t) = B^{b(1-\gamma)} \left[\frac{B}{b} a^{\frac{a(1-\gamma)}{a(1-\gamma)-1}} (a+b)^{\frac{1}{1-a(1-\gamma)}} \left(y + R\sigma_1^2 y^2 B \right) \right]^{[a(1-\gamma)-1]} \frac{A^{1-R}}{(1-\gamma)}$$

Moreover, consumption expenditure can be affected by uncertainty. Effects of σ_2^2 on C_{1t} if $\gamma > 1$ or of σ_1^2 on both C_{1t} and C_{2t} are analytically indeterminate. Figures 3.5, 3.6, and 3.7 provide a numerical computation of these effects based on the set of parameters' values described in section 4.2.

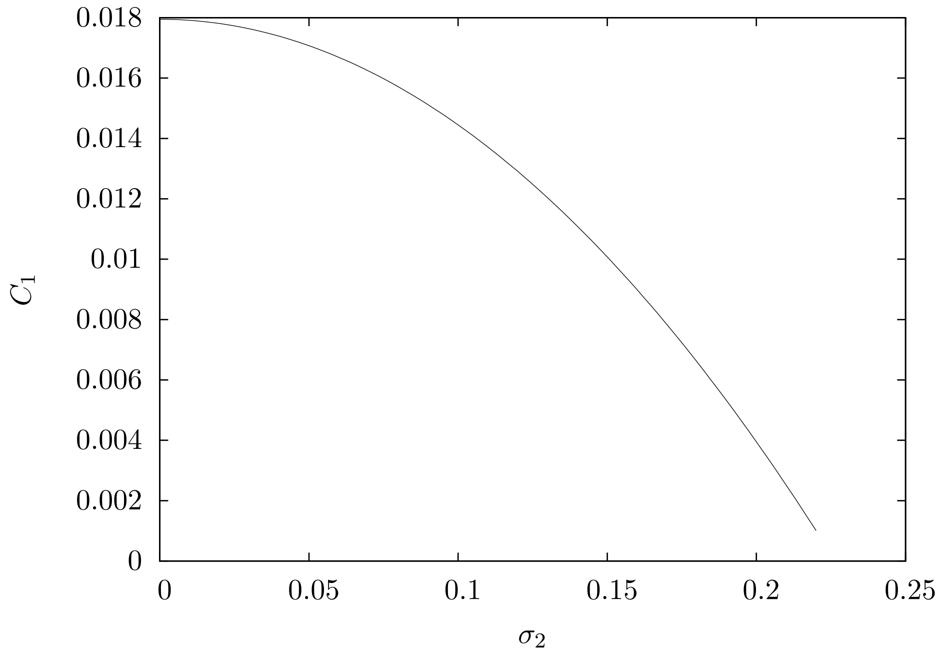

Figure 3.5: Effect of σ_2^2 on C_{1t} when $\gamma > 1$.

3.B Solving the Optimal Program Before Adoption

Using Itô's lemma, the value function before adoption $V(A\tau)_{\{t<\tau\}}$ has to satisfy:

$$\max_{C_{1t}, C_{2t}} \left\{ \frac{(C_{1t}^a C_{2t}^b)^{1-\gamma}}{1-\gamma} dt + V_A (rA - C_1 - xC_2) dt + \frac{1}{2} \sigma_2^2 A^2 V_{AA} dt \right\} = 0 \tag{3.47}$$

The first order conditions yield:

$$\begin{aligned} aC_{1t}^{a(1-\gamma)-1}C_{2t}^{b(1-\gamma)} &= V_A \\ C_{1t}^{a(1-\gamma)}bC_{2t}^{b(1-\gamma)-1} &= xV_A \end{aligned}$$

Therefore:

$$C_{1t}^* = \left[a^{1-b(1-\gamma)}(\frac{b}{x})^{b(1-\gamma)}\right]^{\frac{1}{R}} V_A^{\frac{-1}{R}}$$

$$C_{2t}^* = a^{\frac{a(1-\gamma)}{R}}(\frac{b}{x})^{\frac{1-a(1-\gamma)}{R}} V_A^{\frac{-1}{R}}$$

Replacing consumption by its optimal expression in the Bellman equation and multiplying by $V_A^{\frac{1-R}{R}}$ yields:

$$\frac{R}{1-\gamma}a^{\frac{a(1-\gamma)}{R}}\left(\frac{b}{x}\right)^{\frac{b(1-\gamma)}{R}} + AV_A^{\frac{1}{R}}r + \frac{1}{2}\sigma_2^2A^2V_A^{\frac{1-R}{R}}V_{AA} = 0$$

We make the following variable change: $f(A_t) = V_A^{\frac{1}{R}}$ and $f'(A_t) = \frac{1}{R}V_A^{\frac{1-R}{R}}V_{AA}$. Hence, the preceding equation may be written as:

$$\frac{R}{1-\gamma}a^{\frac{a(1-\gamma)}{R}}\left(\frac{b}{x}\right)^{\frac{b(1-\gamma)}{R}} + f(A_t)A_tr + \frac{1}{2}\sigma_2^2A_t^2Rf'(A_t) = 0$$

We guess that $f(A_t)$ can be written as follows:

$$f(A_t) = \frac{D_1}{A_t} + D_2A_t^{D_3} \;\Rightarrow\; f'(A_t) = -\frac{D_1}{A_t^2} + D_2D_3A_t^{D_3-1}$$

where D_1, D_2 and D_3 are constants to be determined. Then:

$$\frac{R}{1-\gamma}a^{\frac{a(1-\gamma)}{R}}\left(\frac{b}{x}\right)^{\frac{b(1-\gamma)}{R}} + \left(\frac{D_1}{A_t} + D_2A_t^{D_3}\right)A_tr + \frac{1}{2}\sigma_2^2RA_t^2\left(-\frac{D_1}{A_t^2} + D_2D_3A_t^{D_3-1}\right) = 0$$

This equation is of the form: $g(A_t) + \upsilon = 0$. In order for this condition to be satisfied whatever A_t, one must have: $g(A_t) = 0$ and $\upsilon = 0$. Therefore:

$$\frac{R}{1-\gamma} a^{\frac{a(1-\gamma)}{R}} \left(\frac{b}{x}\right)^{\frac{b(1-\gamma)}{R}} + D_1 \left(r - \frac{1}{2}\sigma_2^2 R\right) = 0$$

and:

$$r + \frac{1}{2}\sigma_2^2 R D_3 = 0$$

Thus, we obtain D_1 and D_3:

$$D_1 = \left[\frac{R}{\gamma - 1} a^{\frac{a(1-\gamma)}{R}} \left(\frac{b}{x}\right)^{\frac{b(1-\gamma)}{R}}\right] \frac{1}{\left(r - \frac{1}{2}\sigma_2^2 R\right)}$$

$$D_3 = -\frac{r}{\frac{1}{2}R\sigma_2^2}$$

We show that the marginal value of wealth before the switch is:

$$V_A(A_t) = \left[\underbrace{\frac{\frac{R}{\gamma-1} a^{\frac{a(1-\gamma)}{R}} \left(\frac{b}{x}\right)^{\frac{b(1-\gamma)}{R}}}{\left(r - \frac{1}{2}\sigma_2^2 R\right)}}_{D_1} A_t^{-1} + D_2 A_t^{-\frac{r}{\frac{1}{2}R\sigma_2^2}}\right]^R$$

D_2 is a constant which must be determined using the smooth pasting condition (see equation (3.14) of the text). From the last equation we obtain:

$$\begin{aligned} V_A^{\frac{1}{R}} &= f(A) = \frac{D_1}{A} + D_2 A^{D_3} \\ &\Rightarrow \; V_A = \left[\frac{D_1}{A} + D_2 A^{D_3}\right]^R \end{aligned}$$

and using the smooth pasting condition (equation (3.14) in the text):

$$V_A = W_A(A_\tau - \beta) = M(a+b)(A_\tau - \beta)^{-R}.$$

Hence:

$$\begin{aligned}
\left[\frac{D_1}{A_\tau} + D_2 A_\tau^{D_3}\right]^R &= M(a+b)(A_\tau - \beta)^{-R} \\
&\Leftrightarrow \frac{D_1}{A_\tau} + D_2^{D_3} A_\tau = \frac{[M(a+b)]^{\frac{1}{R}}}{A_\tau - \beta} \\
&\Leftrightarrow D_2 A_\tau^{D_3} = \frac{[M(a+b)]^{\frac{1}{R}}}{A_\tau - \beta} - \frac{D_1}{A_\tau} \\
&\Leftrightarrow D_2 = \frac{[M(a+b)]^{\frac{1}{R}}}{(A_\tau - \beta)A_\tau^{D_3}} - \frac{D_1}{A_\tau^{D_3+1}}.
\end{aligned}$$

This is the equation (3.23) in the text.

$$\Longrightarrow \quad V_A = \left[\frac{D_1}{A_t} + \left(\frac{[M(a+b)]^{\frac{1}{R}}}{(A_\tau - \beta)A_\tau^{D_3}} - \frac{D_1}{A_\tau^{D_3+1}}\right) A_t^{D_3}\right]^R$$

$$= \left[\underbrace{\frac{D_1}{A_t}}_{W_{0A}(A_t)^{1/R}} + \left(\underbrace{\frac{[M(a+b)]^{\frac{1}{R}}}{(A_\tau - \beta)}}_{W_A(A_\tau-\beta)^{1/R}} - \underbrace{\frac{D_1}{A_\tau}}_{W_{0A}(A_\tau)^{1/R}}\right)\left(\frac{A_t}{A_\tau}\right)^{D_3}\right]^R$$

The expression of the marginal value of wealth can be written as follows:

$$V_A(A_t) = \left[\left[\frac{\frac{R}{\gamma-1} a^{\frac{a(1-\gamma)}{R}} \left(\frac{b}{x}\right)^{\frac{b(1-\gamma)}{R}}}{\left(r - \frac{1}{2}\sigma_2^2 R\right)}\right] A_t^{-1} + G(A_t, A_\tau)\right]^R$$

where $G(A_t, A_\tau)$ is the option value. It is the equation (3.20) in the text.

Notice that $W_0(A_t)$, the value function of the homeowner in an economy with no technological change, has to satisfy:

$$\begin{aligned}
0 &= \frac{R}{\gamma-1} a^{\frac{a(1-\gamma)}{R}} \left(\frac{b}{x}\right)^{\frac{b(1-\gamma)}{R}} + A W_{0A} r + \frac{1}{2}\sigma_2^2 A^2 W_{0AA} \\
&\Leftrightarrow W_0(A_t) = \left[\frac{\frac{R}{\gamma-1} a^{\frac{a(1-\gamma)}{R}} \left(\frac{b}{x}\right)^{\frac{b(1-\gamma)}{R}}}{\left(r - \frac{1}{2}\sigma_2^2 R\right)}\right]^R \frac{A^{1-R}}{(a+b)(1-\gamma)} \\
&\Leftrightarrow W_0(A_t) = D_1^R \frac{A^{1-R}}{(a+b)(1-\gamma)}
\end{aligned}$$

Finally, notice that in the special case in which $\sigma_1 = 0$ and $x = y$, we have $M = D_1^R/(a+b)$.

3.C Comparative Statics

	$\gamma < 1$			$\gamma > 1$		
x	10	20	30	2	7	15
A_τ	0.82	0.75	0.7	0.42	0.29	0.24
y	2	4	8	0.2	1	1.5
A_τ	1.5	2.5	9	0.25	0.39	0.46
β	0.2	0.4	0.8	0.2	0.5	0.8
A_τ	1.6	3.4	6.6	0.53	1.33	2.12
a	0.1	0.4	0.7	0.1	0.4	0.7
A_τ	0.22	0.39	0.82	0.20	0.23	0.26
b	0.25	0.30	0.40	0.08	0.2	0.4
A_τ	0.82	0.90	1.5	0.65	0.31	0.20
γ	0.45	0.6	0.8	1.2	1.8	2.2
A_τ	1.19	0.56	0.41	0.67	0.29	0.25
σ_1	0.2	0.4	0.8	0.5	1	1.5
A_τ	0.82	0.82	0.82	0.265	0.265	0.265
σ_2	0.6	1	1.4	0.02	0.06	0.1
A_τ	0.76	0.58	0.38	0.265	0.265	0.265
r	0.01	0.02	0.04	0.025	0.035	0.045
A_τ	0.36	0.39	0.57	0.265	0.265	0.265

Table 3.1: $\rho = 0$

	$\gamma < 1$			$\gamma > 1$		
x	5	10	15	5	10	15
A_τ	0.530	0.505	0.493	0.314	0.265	0.245
y	0.25	1	1.5	0.25	1	1.5
A_τ	0.505	0.564	0.589	0.265	0.392	0.466
β	0.1	0.15	0.2	0.1	0.15	0.2
A_τ	0.505	0.757	1.009	0.265	0.399	0.533
a	0.1	0.4	0.7	0.1	0.4	0.7
A_τ	0.149	0.249	0.505	0.201	0.233	0.265
b	0.1	0.25	0.5	0.1	0.25	0.5
A_τ	0.410	0.505	0.828	0.542	0.265	0.176
γ	0.5	0.6	0.7	1.8	2	2.2
A_τ	0.505	0.328	0.229	0.292	0.265	0.248
σ_1	0.013	1	2	0.013	1	2
A_τ	0.505	0.505	0.505	0.265	0.265	0.266
σ_2	0.5	0.75	1	0.01	0.1	0.2
A_τ	0.505	0.318	0.291	0.263	0.265	0.244
r	0.02	0.035	0.05	0.02	0.035	0.05
A_τ	0.311	0.374	0.505	0.260	0.265	0.265
ρ	0.0001	0.02	0.04	0.0001	0.02	0.04
A_τ	0.505	0.349	0.297	0.265	0.221	0.200

Table 3.2: $\rho \neq 0$

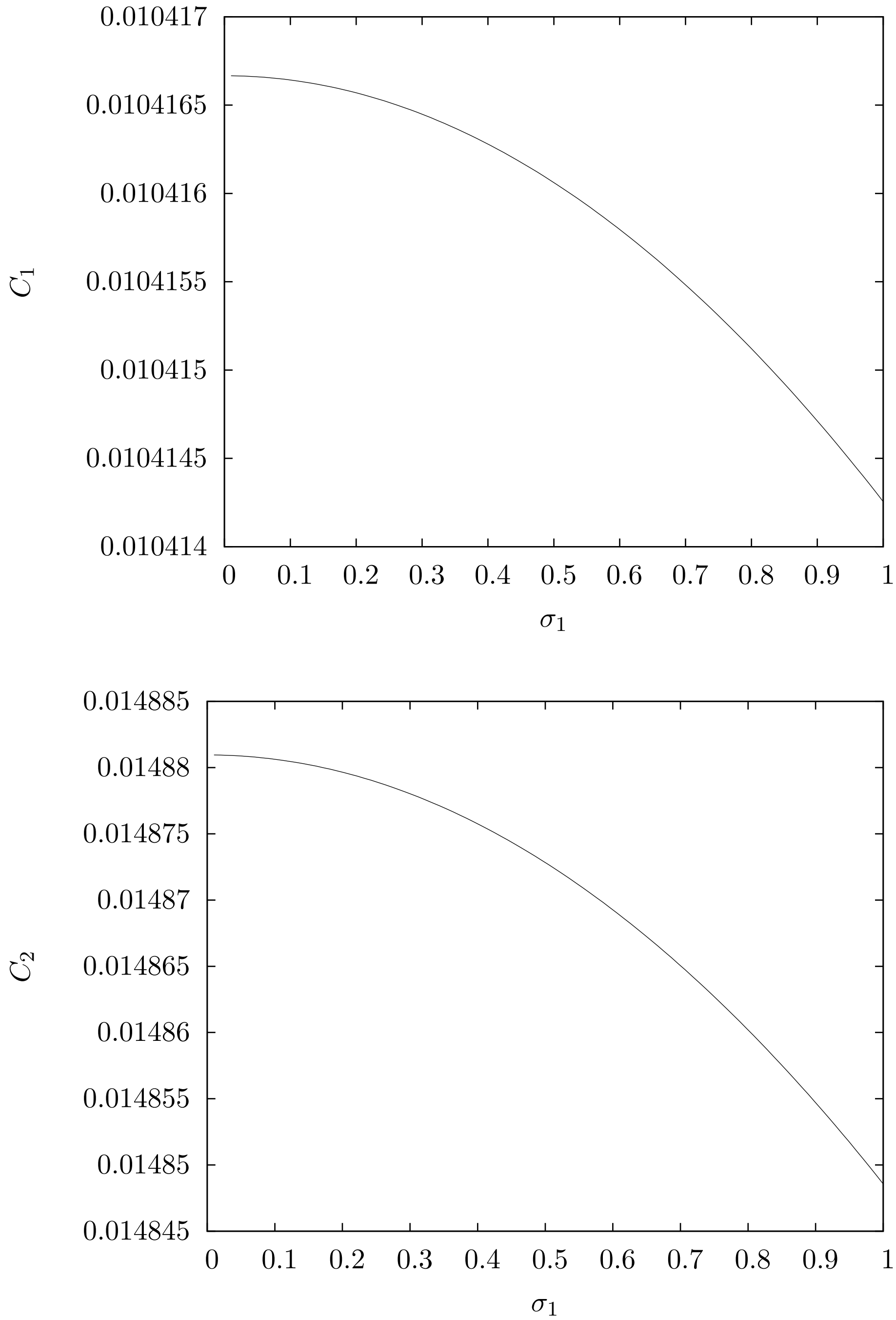

Figure 3.6: Effects of σ_1^2 on both C_{1t} and C_{2t} when $\gamma < 1$

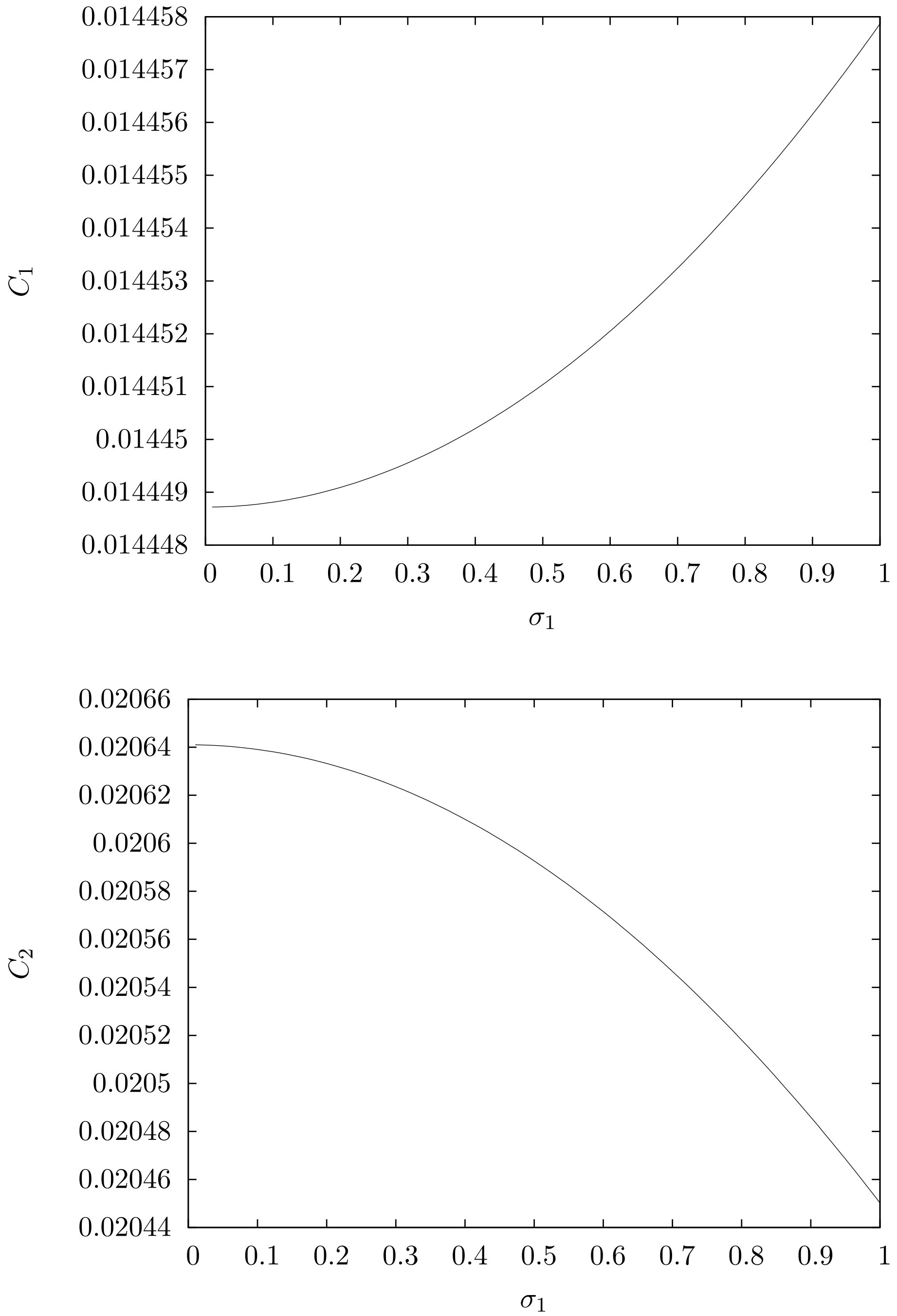

Figure 3.7: Effects of σ_1^2 on both C_{1t} and C_{2t} when $\gamma > 1$

4

Producing Energy in a Stochastic Environment: Switching from Non-Renewable to Renewable Resources*

In this chapter, we study the determinants of switching from non-renewable natural resource inputs to renewable resource inputs in energy production. We assume that the stocks of both natural resources are stochastic, and that the adoption of renewable resources is costly and irreversible. Our formulation gives raise to an optimal stopping/switching problem that cannot be solved analytically, then we turn to numerical simulations. Our results suggest that the optimal switching time depends not only on the uncertainty parameters, but also on energy demand, costs, and the relative productivity of the resources.

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4.1 Introduction

Nowadays the substitution of renewable for non-renewable resources in the energy sector is among the most important environmental and economic issues. On the one hand, this is due to the recognition of the excessive emission of carbon dioxide generated by burning of fossil fuels as a major cause of global warming. On the other hand, the substitution of renewable for non-renewable resources is also recognized as a key strategy to prevent the depletion of the stock of natural capital. Despite this, most analyses in environmental economics have been focused on only renewable resources, or only non-renewable resources, and very few exceptions can be found in the literature.

Among the studies on both renewable and non-renewable resources, two types of models can be identified. The first type corresponds to aggregate capital-resource growth models in which a backstop technology is a perfect substitute input for a non-renewable resource. For instance, in the model of Krautkraemer [1986], and the more recent model of Tahvonen and Salo [2001] the substitution of resources depends crucially on the structure of the extraction costs and the cost of using the backstop. Given their relative costs, the economy can use either renewables only, the backstop only, or both resources at the same time. One extension to Krautkraemer and Tahvonen and Salo's type models is of particular interest. André and Cerdá [2005] include in their model a natural growth function to account for the fact that the stock of a renewable resource can regenerate and grow by natural means. As a result, the optimal solution has a particular shape with three stages corresponding to three different extraction regimes. These stages depend on the characteristics of the natural growth function.

The second type of studies on the substitution of resources is that of technical innovation. In Chakravorty et al. [1997] for instance, innovation results in a decrease in the cost of renewable energy, in such a way that it can be substituted more easily for the other non-renewable (traditional) forms of energy. Another more recent study by Acemoglu et al. [2012] considers the interaction of renewables and non-renewables in the form of clean and dirty inputs. In their model, if the elasticity of substitution between dirty and clean inputs is sufficiently high, innovation in the long run will be directed towards the clean sector only. Similar conclusions are found in Grimaud and

Rougé [2008], in which the (non-polluting) renewable input enters in the form of a labour resource. In both Acemoglu et al., and Grimaud and Rouge's models however, the non-renewable resource is essential in production and then it will be exhausted at some time.

Despite the very interesting results of the literature we have mentioned, we believe that a complete theory on the substitution of resources is far from being complete. In this chapter, we direct our attention towards three important and yet somewhat neglected characteristics that must be taken into account to evaluate the adoption of any environmental policy [Pindyck, 2000, 2002]. First, the uncertainty over the future costs and benefits. Second, the irreversibilities associated with environmental policy. Third, the fact that policy adoption is rarely a now or never proposition, such that, in most cases, it is feasible to delay action and wait for new information. By including these characteristics the analysis becomes more accurate, as these uncertainties, irreversibilities, and the possibility of delay can significantly affect the optimal timing of policy adoption. Specifically, at what point should society stop using non-renewable resources to produce energy and start producing from renewables? First, although we have an idea of their current stock, we do not know too much about the future availability of the resources. But, even if we knew how much of the resources are expected to be available in the future, we would not know the resulting effect on firms decisions. Second, adoption of renewables imposes sunk costs on society. In addition, economic constraints (particularly those associated with the production process) make the adoption of renewables difficult to reverse, so that these sunk costs are incurred over a long period of time, even if the original rationale for the switching disappears. These kinds of sunk costs create an opportunity cost of adopting renewables now, rather than waiting for more information.[1]

As in the previous literature, we propose a model in which energy can be produced from two available inputs which are perfect substitutes: non-renewables (fossil fuels), and renewables (water, wind, solar, biomass). In our model, however, there is some

[1] A second source of irreversibility that is not considered here is that of environmental damage [Pindyck, 2000, Pommeret and Prieur, 2009]. For instance, increases in greenhouse gas (GHG) concentrations are long lasting. Then, even if radical policies were adopted to drastically reduce GHG emissions, these concentrations would take many years to fall. This creates an opportunity cost of adopting renewables later.

uncertainty about the availability of both resources. Following the literature on investment under uncertainty, we assume that the current stock of both resources is known with certainty, but the instantaneous change in their stocks is random.[2] As in André and Cerdá [2005] we also assume that renewables regenerate and grow by natural means. But contrary to Krautkraemer [1986] and Tahvonen and Salo [2001], our assumptions imply that both types of resources are never used simultaneously. This result may be a loss of generality, but it allows us to be more focused on the optimal timing of the switching decision, and on the factors influencing the decision to switch.

We assume that firms start producing energy using only fossil fuels, but the possibility to carry out an irreversible investment to switch to the other input is always open. We simplify our model by assuming that once the irreversible investment is undertaken, it is not possible to switch back to the use of the non-renewable resources.[3] This simplification implies that complete exhaustion of the non-renewable resource can be avoided. It also implies that our model can be viewed as an option value problem in the sense of Dixit and Pindyck [1994].

The optimal switching time is found in three steps. In the first step we assume that firms use only non-renewable resources, and they do never switch. In the second step, we assume that firms in the market have switched to the renewable input. These two steps can be solved analytically in some particular cases [Pindyck, 1984], and provide the boundary conditions that help us find the optimal switching time in the third step. However, due to the particular structure of our model (and specifically because of the boundaries found in the first and second steps) our problem cannot be fully solved analytically. Then, the optimal switching time (in the form of an optimal switching level of the non-renewable stock) can only be found by numerical approximations.

As we shall see, due to the particular assumptions in our analysis, and especially the boundary conditions (both, the transversality condition and the condition at the

[2] See Dixit and Pindyck [1994] for a general treatment of these kind of models, and Pindyck [2000, 2002] for examples of optimal timing problems in environmental economics. The seminal paper of Dasgupta and Heal [1974], and Mason [2001] also present models of non-renewable extraction under uncertainty.

[3] As the structure of firms producing energy from non-renewable resources is usually very different from that of firms producing energy from renewables, this assumption seems to be appropriate in our context.

switching time), and the fact that the stock of the non-renewable resource is decreasing, the resulting value function before the switch is S-shaped. This is completely new in the literature of technology switching, where the resulting value function is mostly concave [Dixit and Pindyck, 1994]. Another novelty of our model is that firms do not switch immediately in the case of switching cost being equal to zero [Pommeret and Schubert, 2009, Charlier et al., 2011]. This results from the higher profits the firms get from using non-renewable resources, particularly if they are abundant.

The rest of the chapter is as follows. In section 4.2 we describe the assumptions and equations governing our economy. In section 4.3 we carefully describe the decisions of the firms before and after the switch; in that section we also describe the economic conditions at the optimal switching time. In section 4.4 we describe one of the fundamental determinants of the optimal switching time, namely the renewable resource's self regeneration function; we also describe the conditions under which the equilibrium in the natural resource market converges to the steady state. In section 4.5 we solve the model and perform a sensitivity analysis. We conclude in section 4.6.

4.2 The model

We consider a competitive market in which a large number of identical firms are engaged in energy production. In the aggregate, firms are assumed to use $q_1(t)$ units of a non-renewable natural resource to produce $E(t)$ units of energy at time t according to the following production function:

$$E(t) = a_1 q_1(t), \tag{4.1}$$

for $a_1 = 1/\mu_1$, and μ_1 being the fabrication coefficient, i.e. how many units of q_1 are needed to produce one unit of energy.[4] We assume that the stock of the non-renewable resource is stochastic and evolves according to:

$$dS_1 = -q_1(t)dt + \sigma(S_1)dz_1, \tag{4.2}$$

with $\sigma'(\cdot) > 0$, $\sigma(0) = 0$ to ensure that the resource stock is always non-negative, and dz_1 being the standard increment of a Wiener process. In particular, we assume that

[4] See, for instance, Førsund [2007].

$\sigma(S_1) = \sigma_1 S_1$. Then, equation (4.2) implies that the current stock of the resource is known with certainty, but that percentage changes in the stock, dS_1/S_1, have a random component that is normally distributed. Examples of random negative shocks in our context are earthquakes, hurricanes, floods, and droughts, while a random positive shock can be the discovery of an unexpected field.[5] $q_1(t) > 0$ is the aggregate rate of extraction or "harvest".

When using the non-renewable natural resource as an input, firms are assumed to pay total (extraction) costs given by:

$$C(q_1, S_1) = c(S_1)q_1(t),$$

where marginal cost $c(S_1)$ is decreasing and strictly convex, and $c(0) = \infty$. We also assume that $D(p)$ is the downward sloping market demand function for energy. Specifically, we consider iso-elastic demand, and iso-elastic marginal cost functions:

$$D(p) = bp^{-\eta} \tag{4.3}$$

$$c(S_1) = cS_1^{-\gamma}. \tag{4.4}$$

Consumer plus producer surplus (or more precisely, consumer surplus minus total extraction costs) can thus be computed as:[6]

$$\pi_1(q_1, S_1) = \int_0^{E^*} D^{-1}(E)dE - c(S_1)q_1(t). \tag{4.5}$$

[5] Processes as the one in equation (4.2) have been used to model the dynamics of fossil fuels, such as oil and coal (see, for instance, the stochastic "cake-eating"problems in Epaulard and Pommeret [2003], and Smith and Son [2005]. However, there is no complete agreement on whether the uncertainty on the stock is multiplicative or additive. One can imagine that the loss due to a negative shock is proportional to the known stock, while it is easier to think of positive shocks as being additive [Pindyck, 1980]. We prefer the shock to be multiplicative, as in this way it can also be interpreted as a random rate of depreciation [Beltratti, 1996].

[6] Alternatively, we could have assumed that each firm i ($i = 1, 2, \ldots n$) in the market is interested in maximizing its individual profits given by:

$$pE^i - C^i(q_1^i, S_1),$$

where the price of energy p is taken as exogenous. Both expositions are equivalent if the property rights in the market are clearly defined, and each firm own equal shares of the resource stock. Pindyck [1984] presents this latter case.

Suppose that firms in the market have the option to switch to a renewable (perfect substitute) input to produce energy. This switching is available at a fixed and irreversible cost $I > 0$. We assume that, once the irreversible investment is undertaken, it is not worthwhile to switch back to the non-renewable resource, even if profits fall due to an excess supply of the resource. This assumption is not new in the literature, and seems to be highly justified in our context.[7] Consider for instance the differences between an offshore oil platform and an offshore wind farm. Once the oil platform decides to become a wind farm, almost all of the previously installed capital needs to be dismantled, and very few parts of the old facility can be re-utilized.[8] This process is very expensive in terms of time and money, and so is the reverse procedure, making it difficult to the firm to even consider the possibility of switching back.[9]

Notice also what our previous argument implies for production before and after the switching decision. In Tahvonen and Salo [2001] for instance, both resources are used simultaneously at least with resource stocks implying that the marginal costs of using the renewables and the marginal cost of extracting the non-renewables are equal. However, uncertainty, irreversibility, and the very nature of the decision in our model make using both resources at the same time to be highly improbable. In the particular case of oil and wind, no more energy can be produced from the former resource from the very beginning of the offshore oil platform's dismantling process, and even the parts being re-utilized need to be adapted before using them in the wind farm. Then, if firms decide to switch to the new input, energy will be produced according to the following production function:

$$E(t) = a_2 q_2(t), \tag{4.6}$$

where $a_2 = 1/\mu_2$, and μ_2 is the new fabrication coefficient. As for the non-renewable

[7] Classical readings are Brennan and Schwartz [1985], McDonald and Siegel [1986], and Pindyck [1988].

[8] Spar buoys, and deepwater cabling plus pipeline are good examples. See publications of the National Renewable Energy Laboratory at: `http://www.nrel.govforinstance.` (Retrieved on November 25, 2011).

[9] In a more general setting, firms may still be able to switch back to the use of non-renewables in the case of, for instance, a profit fall. This is particularly the case if the offshore oil platforms are not transformed into wind farms but simply abandoned, and the switching-back and maintenance costs are not prohibitively high. We do not consider this possibility here, but refer interested readers to Wirl [2006] for an example of optimal stopping and switching back decisions in environmental economics.

resource case, we assume that the stock of the renewable natural resource, S_2, is stochastic, but it now evolves according to:

$$dS_2 = [f(S_2) - q_2(t)]\, dt + \sigma(S_2) dz_2, \tag{4.7}$$

where $q_2(t) > 0$ is the new aggregate rate of extraction, and dz_2 is the standard increment of a Wiener process. For simplicity, we assume that z_1 and z_2 are uncorrelated. Additionally, we assume that $\sigma(S_2) = \sigma_2 S_2$.

In equation (4.7), $f(S_2)$ is the resource's self regeneration function. This function is assumed to be is strictly concave, with $f(0) = 0$. Specifically:

$$f(S_2) = \frac{\alpha}{\beta} S_2 \left(1 - \left(\frac{S_2}{K} \right)^{\beta} \right), \tag{4.8}$$

which is a modified version of the logistic equation proposed by P.F. Verhulst in 1838. We study this equation in more detail in section 4.4.1.

Processes as the one in equation (4.7) have been widely used in the population ecology literature,[10] and clearly are very appropriate in the case of biomass energy. In particular, many of the crops used to produce biofuels, such as biodiesel and bioethanol, can be assumed to display a logistic growth in yields. For instance, Waggoner [1995] uses a logistic projection for maize yields, with carrying capacity depending on the technology available. Also, Harris and Kennedy [1999] show that there are strong indications in the data that the logistic model is superior to the exponential model. This is true for maize, but also true for other crops like wheat, and rice. Uncertainty in this context can result, for example, from random fluctuations in nutrient availability, or from stochastic changes in natural conditions that affect reproduction.[11] Other sources of energy can be considered, like the total amount of water into a reservoir [Thompson et al., 2004, Zhao and Davison, 2009], or the local levels of incoming solar radiation Hamlen et al. [1978].

We also make the following assumptions in order to capture two of the main differences between the use of non-renewable and renewable natural resources in energy

[10] See Pindyck [1984] for a survey.

[11] Justification of equation (4.7) in the case of modelling biofuels can also be found in some recent literature on engineering. See for instance [Benavides and Diwekar, 2011, Ulas and Diwekar, 2004, Rico-Ramirez et al., 2003].

production. First, we assume that extracting renewables is cheaper than extracting non-renewables. This is usually the case, compare for instance the cost of extracting oil with the cost of using wind or water. While the marginal cost ot extracting oil is clearly positive, wind or water are freely available. To simplify, we assume that marginal costs are zero in the case of renewable energy production. Then, the total surplus function becomes:

$$\pi_2(q_2, S_2) = \int_0^{E^*} D^{-1}(E)dE. \tag{4.9}$$

Second, we assume that non-renewables are more productive than renewables. This is again the case. For instance, 1MWH of electricity can be produced from about 25 gallons of oil, or from about 800 gallons of water.[12] Then, we assume that $a_2 = \lambda a_1$, $\lambda < 1$.

Reasons to switch are evident at this point. If firms decide not to switch, their profits increase as the non-renewable resources are more productive; but at the same time, their profits decrease as marginal costs increase. Thus, the final effect on profits depends on which effect is the strongest. However, firms know that their profits will eventually fall to zero, as the only inputs available are depletable. Switching allows the firms to use a resource that is less productive but that can be tentatively used forever. Uncertainty can affect the decision to switch in several ways. The most obvious is related to the availability of the resources. As we shall see, if firms are afraid about the future availability of the non-renewable resources, they can be tempted to switch sooner. On the other hand, if firms believe that the availability of the renewable resource is very uncertain (which can be the case of solar energy for instance), they can be tempted to switch later.[13]

[12] See, for instance, the Energy Conversion Factors avaliable at: `http://www.bioenergy.ornl.gov/papers/misc/energy_conv.html` (Retrieved on October 25, 2011)

[13] There are other important reasons to switch that our model does not account for. For instance, renewable resources are non-pollutant. Thus, firms would like to switch if they particularly interested in the environment. There can also be some other sources of uncertainty. For example, current and future climate change policies are expected to influence the cost of the fossil fuels. Additionally, R&D can also affect the cost of resource extraction, energy production, and the cost of the switch.

4.3 Three steps to solve for the optimal switching time

To solve for the optimal switching time T, we proceed as follows. First, we develop the program to be solved by the firms when using the non-renewable resource as an input, and they do never switch. Second, we develop and solve the program assuming that all the firms in the market have switched to the renewable input. The results of the first and second steps will provide the transversality and boundary conditions, respectively, to compute the equilibrium of the general model in which the switching opportunity is taken into account. Of course, this equilibrium is computed as a third step, in which the optimal switching time is implicitly found.

4.3.1 Step 1: Production with non-renewable inputs

As a starting point, we assume that the possibility of switching to a renewable input is not available. With a discount rate of ρ, the social value function W_0 can be computed as:

$$W_0 = W_0(S_1) = \max_{q_1} \mathbb{E}_0 \int_0^{\infty} e^{-\rho t} \pi_1(t) dt,$$

subject to equations (4.2) and (4.5), and $\mathbb{E}_0$ indicating the expectation at time 0. We also assume that $S_1(0) < \infty$ to guarantee that the problem is bounded. In this case, the fundamental equation of optimality is:

$$\rho W_0 dt = \max_{q_1} \left\{ \pi_1(t) dt + \mathbb{E}_0 dW_0 \right\}. \tag{4.10}$$

Notice that the instantaneous return on energy production $\rho W_0 dt$ has two components: the instantaneous total surplus flow $\pi_1(t)dt$, and the instantaneous expected capital gain $\mathbb{E}_t dW_0$. Also, on the margin any increase in the total surplus flow will just be offset by a decrease in expected capital gain. By using Ito's lemma, equation (4.10) can be rewritten as:

$$\rho W_0 = \max_{q_1} \left\{ \int_0^{E^*} D^{-1}(x) dx - c(S_1) q_1(t) - q_1 W_{0,S_1} + \frac{1}{2} \sigma^2(S_1) W_{0,S_1 S_1} \right\}, \tag{4.11}$$

where subscripts denote partial derivatives. The first order condition for the optimal choice of q_1 is:

$$a_1 D^{-1}(E^*) - c(S_1) - W_{0,S_1} = 0, \tag{4.12}$$

where $E^* = a_1 q_1^*$. Equation (4.12) implies that the marginal value function W_{0,S_1} equals the profit that can be obtained by extracting, manufacturing and selling one unit of the natural resource as energy.

We restrict attention to the special case where $\gamma = 1/\eta$, which allows us to characterize explicitly the solution before the switch. Later in the chapter we relax this assumption, which requires solving the model by using numerical simulations. By using the methodology proposed in 4.A, we can show that fixing $\gamma = 1/\eta$ does not change any of the main results of our analysis, then we keep this equality from now on. This leads us to the following proposition.

Proposition 4.1 *If firms use the non-renewable resource forever, the welfare maximizing outcome can be found to be:*

$$W_0 = \phi_1 \frac{\eta}{\eta - 1} S_1^{\frac{\eta-1}{\eta}}, \ \forall\ \eta \neq 1, \tag{4.13}$$

where $\phi_1 > 0$ *solves:*

$$\theta_1 \phi_1^{\frac{1}{1-\eta}} - \phi_1 - c = 0,$$

and:

$$\theta_1 = a_1 \left(\frac{\rho}{b}\eta + \frac{\eta - 1}{\eta} \frac{\sigma_1^2}{2b} \right)^{\frac{1}{1-\eta}}.$$

As a result, the optimal rate of resource extraction is:

$$q_1^*(S_1) = \frac{a_1^{\eta-1} b}{(\phi_1 + c)^{\eta}} S_1. \tag{4.14}$$

Proof See 4.B. ∎

Notice in proposition 4.1 that we restrict the constant ϕ_1 to be strictly larger than zero. This guarantees the concavity of the social value function W_0.

4.3.2 Step 2: The optimal path after the switch

We now assume that the economy is at time T, which is defined as the optimal switching time. From that point on, the social value function W is computed as:

$$W = W(S_2) = \max_{q_2} \mathbb{E}_T \int_T^{\infty} e^{-\rho(t-T)} \pi_2(t) dt,$$

subject to equations (4.7) and (4.9), and $\mathbb{E}_T$ indicating expectation at time T. As in section 4.3.1, we assume that $S_2(T) < \infty$ to guarantee that the problem is bounded. Then, it is easy to show that our problem consists in finding the function W that solves:

$$\rho W = \int_0^{E^*} D^{-1}(x)dx + [f(S_2) - q_2^*(S_2)]\, W_{S_2} + \frac{1}{2}\sigma^2(S_2) W_{S_2 S_2}. \tag{4.15}$$

In general this problem must be solved numerically, but a special case that admits an analytical solution does exist. Specifically, we can characterize explicitly the solution after the switch in the special case where $\beta = 1/\eta - 1$. This assumption is not new in the literature. In particular, Pindyck [1984] shows that increasing the elasticity of demand and then skewing the biological growth function to the left reduces the effect of uncertainty on the physical scarcity of the resource.[14] We relax this assumption later in the chapter. By using the methodology proposed in 4.A, we can show that $\beta = 1/\eta - 1$ has no critical implications for our analysis, and then it can be safely be held throughout the chapter. This leads us to the following proposition.

Proposition 4.2 *If firms use the renewable resource forever, the welfare maximizing outcome can be found to be:*

$$W = \phi_2 \frac{\eta}{\eta - 1} \left(S_2^{\frac{\eta-1}{\eta}} \frac{\alpha}{\rho} K^{\frac{\eta-1}{\eta}} \right), \tag{4.16}$$

where $\phi_2 > 0$ solves:

$$\theta_2 \phi_2^{\frac{1}{1-\eta}} - \phi_2 = 0,$$

[14] In Pindick's paper, this conclusion depends on both the concavity of the biological growth function and the convexity of the costs function. In particular, resource rent increases due to the concavity of the biological growth function, but reduces due to the convexity of the costs function.

and:

$$\theta_2 = a_2 \left(\frac{\rho + \alpha}{b} \eta + \frac{\eta - 1}{\eta} \frac{\sigma_2^2}{2b} \right)^{\frac{1}{1-\eta}}.$$

As a result, the optimal rate of resource extraction is:

$$q_2^*(S_2) = \frac{a_2^{\eta-1} b}{\phi_2^{\eta}} S_2. \tag{4.17}$$

Proof See 4.B. ∎

In proposition 4.2, we again restrict attention to parameters implying that $\phi_2 > 0$. This guarantees the concavity of the social value function W.

Finally, notice that nothing in our analysis prevents S_2 from falling to zero. As we will see in section 4.4.2, some conditions on parameters need to be imposed in order to ensure that the stock S_2 fluctuates around some steady-state expected value, and that $q_2^* > 0$.

4.3.3 Step 3: The opportunity to switch and the optimal switching time

As in section 4.3.1, all firms are using a non-renewable input S_1 to produce energy. The stock of this resource is still evolving according to equation (4.2). However, in this case as an opportunity to switch does exist, society needs to choose both the optimal extraction plan, $q_1^*(S_1)$, and the optimal switching time, T. The value function before the switch is in this case:

$$V = V(S_1) = \max_{q_1, T} \mathbb{E}_0 \int_0^T e^{-\rho t} \pi_1(t) dt + e^{-\rho T} \left(W(S_{2,T}) - I \right), \tag{4.18}$$

where $\pi_1(t)$ is as defined in equation (4.5), and $\mathbb{E}_0$ is expectation at time 0. Notice in equation (4.18) that all firms switch to the new input at time T, and get aggregate discounted profits W from that point on. Also notice that the cost of switching is $I > 0$, as mentioned before this cost is irreversible. We also assume that $S_{2,T} = \vartheta$, which is some fixed initial stock of the renewable resource.[15]

[15] See section 4.4.1.

As usual, the firm's optimal decision can be characterized by a threshold $S^* = S_{1,T}$, such that it is optimal not to switch as long as $S_1 > S^*$, and to switch when the level of the natural resource hits this threshold for the first time.[16] In the domain of continuation, the fundamental equation of optimality can be written as:

$$\rho V = \max_{q_1} \left\{ \pi_1(t) + \frac{1}{dt} \mathbb{E}_T dV \right\}. \tag{4.19}$$

By going through the same steps as in sections 4.3.1 and 4.3.2, we can find the social optimality condition before the switch, V:

$$\rho V = \int_0^{E^*} D^{-1}(x)dx - c(S_1)q_1^*(S_1) - q_1^*(S_1)V_{S_1} + \frac{1}{2}\sigma^2(S_1)V_{S_1S_1}, \tag{4.20}$$

where $q_1^*(S_1)$ is again determined by equation (4.12).

Contrary to what happened in sections 4.3.1 and 4.3.2, no analytical solution is available in this case. This is because equation (4.20) has to satisfy the following boundary conditions:

$$V(S^*) = W(\vartheta) - I, \tag{4.21}$$

$$V_{S_1}(S^*) = 0. \tag{4.22}$$

Equation (4.21) is the value matching condition at the threshold value S^*: the unknown function $V(S_1)$ has to equal the known termination pay-off function, $W(\vartheta) - I$, which is constant in our case. Equation (4.22) is the smooth pasting condition. This equation matches the slope of $V(S_1)$ to that of the pay-off function, zero in our case, to ensure that the switching occurs at the optimal time. It also ensures the smoothness of the value function around the switch.

To ensure that the option to switch is of some value, we also require that:

$$V(S_1) \geq W_0(S_1),$$

and:

$$\lim_{S_1 \to \infty} [V(S_1) - W_0(S_1)] = 0, \tag{4.23}$$

[16] I.e., the random switching time T is determined by $T = \inf \{\tau > 0 | S_{1,\tau} \leq S^*\}$.

where W_0 is as defined in equation (4.13). Equation (4.23) is the transversality condition. It implies that if the stock of the non-renewable resource is infinite, firms do not care about having the renewable resource as an input substitute in production.[17]

4.4 The natural resource's renewal function and the steady state behaviour of the resource after the switch

Equation (4.8) plays an important role in the decision to switch. As we will show in section 4.4.1, the larger the stock of the renewable resource, the less the incentives to switch to the use of this resource. But additionally, function $f(S_2)$ also has interesting implications for the behaviour of the steady-state equilibrium.[18] As we shall see, depending on $f(S_2)$ (and also on $\sigma(S_2)$, and $D(p)$), the stock S_2 may eventually fluctuate around some steady-state expected value. Alternatively, the equilibrium $q_2^*(S_2)$ might yield a degenerate steady-state distribution for S_2, i.e. with probability 1 the stock will eventually fall to zero [Pindyck, 1984]. In section 4.4.2, we will derive the steady state probability distribution of S_2, and derive the conditions under which this distribution is not degenerate.

4.4.1 The natural resource's renewal function

As we mentioned above, equation (4.8) plays an important role in the decision to switch. In that equation, $\frac{\alpha}{\beta} > 0$ is the intrinsic growth rate, and the constant $K > 0$ is the environmental carrying capacity, saturation level, or natural equilibrium level of the resource.[19] In the presence of a positive harvest rate (e.g. $q_2^*(S_2) = q < max(f(S_2))$, equation (4.8) possesses two equilibria, $\underline{S}_2$ and $\overline{S}_2$. In this case, $f(S_2) > 0$ when S_2 lies between these two values, while $f(S_2) < 0$ elsewhere. It follows that $\overline{S}_2$ is a stable

[17]Although the switching time requires to be found numerically in any case, the procedure is simplified as we have analytical expressions for $W(\vartheta)$ and $W_0(S_1)$ in equations (4.21) and (4.23). Otherwise, the model needs to be entirely approximated. See 4.A.

[18]In a deterministic model, the steady-state equilibrium is found by setting $\dot{S}_2 = 0$. When the resource grows stochastically, that equilibrium can only be described in terms of probability distributions and moments [Pindyck, 1984].

[19]See Clark [1990] for a deeper analysis of this model.

equilibrium. This is shown in Figure 4.1, where we used the parameters of Table 4.1, and a hypothetical harvest rate of 5%.

Table 4.1: Base Case Parameters

a_1	1.0
b	1.0
c	1.0
I	1.0
K	2.0
α	0.21
ρ	0.05
λ	0.5
σ_1	0.01
σ_2	0.05
η	0.8

Notice in Figure 4.1 that there exists a maximum sustained yield MSY at $q_{MSY} = max\,(f(S_2))$, with the property that any larger harvest rate will lead to the depletion of the resource. Also, the stock level $S_2 = S_{MSY} = (\beta + 1)^{-1/\beta} K$ at which the productivity of the resource is maximized is different from the natural equilibrium level K.

If the renewable resource is not used in other sectors in the economy apart from the energy sector, we get from the properties of equation (4.8) that $\overline{S}_2 = K$, and hence $\vartheta = K$ in equation (4.21). An optimal switching time exists as long as $W_0(S_1)$ and $W(K) - I$ intersect each other. In Figure 4.2 for instance, firms are not interested in using renewables as long as $S_1 \to \infty$. This is because social benefits from using non-renewables, $W_0(S_1)$, are larger than the social benefits from start using the current stock of renewables, $W(K) - I$. However, as S_1 becomes smaller, and hence W_0 and $W(K) - I$ approach each other, it becomes more and more interesting for firms to switch. Obviously, if W_0 and $W(K) - I$ do not intersect, firms will be tempted to switch immediately. In general, we need the following condition to be satisfied:

$$W(K) - I < \lim_{S_1 \to \infty} W_0(S_1) = \begin{cases} \phi_1 \frac{\eta}{\eta - 1} & \text{if } \eta > 1 \\ 0 & \text{Otherwise} \end{cases} . \tag{4.24}$$

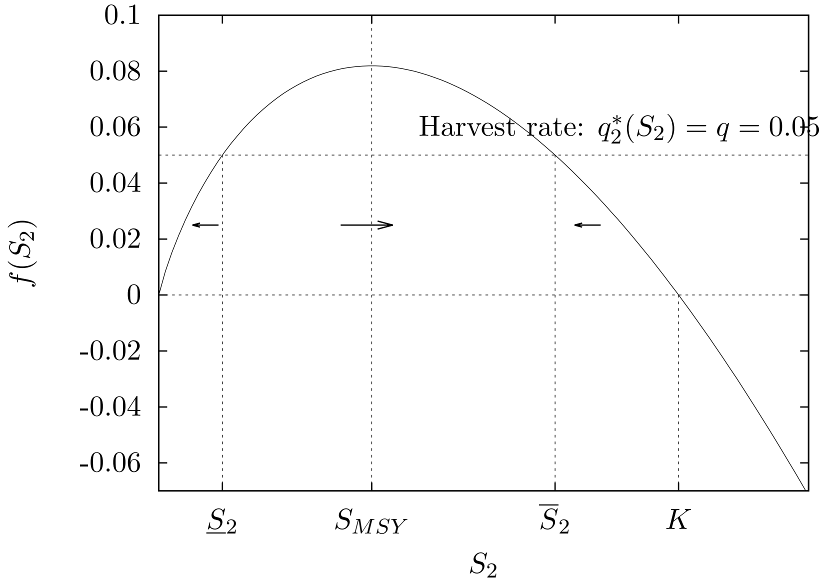

Figure 4.1: Renewable Natural Resource's Growth Function.

It is interesting to notice that condition (4.24) is always satisfied if $\eta < 1$. In this case, then, firms prefer to take advantage of the higher profits they get from using non-renewable resources before switching. This is true even if I is equal to zero. Notice also from equation (4.24) or from Figure 4.2, that if the initial stock of the renewable resources is sufficiently small — or I is sufficiently high, firms are tempted to wait before switching until the stock of the non-renewable resource is almost exhausted. These results are both confirmed in section 4.5.

4.4.2 The steady state behaviour of the resource after the switch

Production from a renewable resource stock needs not fall to zero. However, the equilibrium $q_2^*(t)$ in equation (4.7) may lead to a situation in which the stock S_2 falls to zero with probability 1 (e.g. Figure 4.1). Alternatively, the stock S_2 may fluctuate around some steady-state expected value. Substitution of the equilibrium extraction rate, equation (4.17), into equation (4.7) yields a stochastic differential equation that completely describes the evolution of S_2:

$$dS_2 = [f(S_2) - q_2^*(t)]\, dt + \sigma(S_2)dz_2.$$

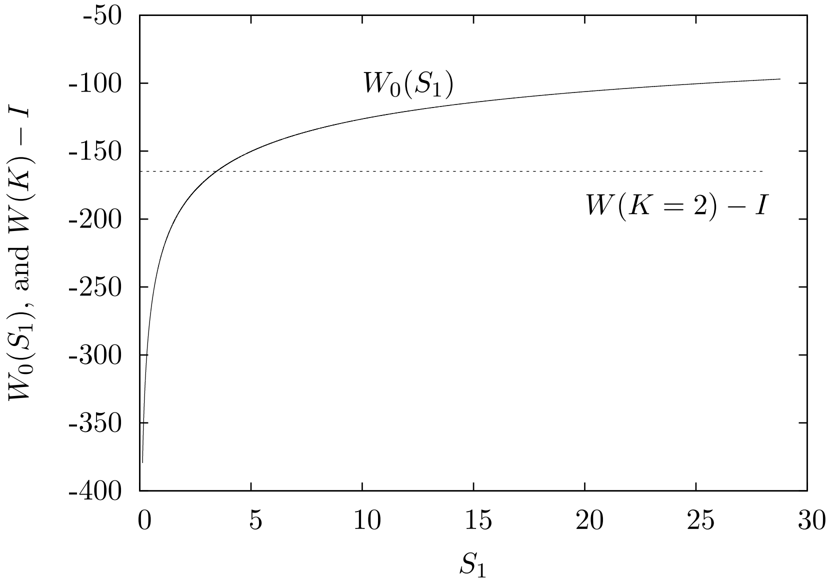

Figure 4.2: Value functions before and after the switch.

As shown in Merton [1975],[20] if the probability distribution of S_2 is not degenerate, i.e. if in the long run the stock S_2 fluctuates around some steady-state expected value, it is given by:

$$\pi_\infty(S_2) = \frac{m}{\sigma^2(S_2)} \exp\left[2\int^{S_2} \frac{[f(\nu) - q_2^*(\nu)]}{\sigma^2(\nu)} d\nu\right], \tag{4.25}$$

with m chosen so that $\pi_\infty(S_2)$ integrates to unity.[21]

Using equations (4.8) and (4.17), we can show that a non-degenerate steady-state distribution for S_2 exists if:

$$0 < \sigma_2^2 < 2\left(\alpha\frac{\eta}{1-\eta} - \frac{a_2^{\eta-1}b}{\phi_2^\eta}\right), \tag{4.26}$$

which in turn implies:

$$\alpha\frac{\eta}{1-\eta} > \frac{a_2^{\eta-1}b}{\phi_2^\eta}.$$

[20] See also Pindyck [1984].

[21] On the other hand, the probability distribution of S_2 is degenerate, i.e. the stock S_2 will eventually fall to zero, if $\int_0^\infty \pi_\infty(S_2)dS_2$ is unbounded.

If condition (4.26) is satisfied, then equation (4.25) can be shown to be:

$$\pi_\infty(S_2) = \frac{\beta\omega_2^{\omega_1/\beta}}{\Gamma(\omega_1/\beta)} S_2^{\omega_1-1} \exp\left(-\omega_2 S_2^\beta\right), \tag{4.27}$$

where:

$$\begin{aligned}
\omega_1 &= \frac{2}{\sigma_2^2}\left(\frac{\alpha}{\beta} - \frac{a_2^{\eta-1}b}{\phi_2^\eta}\right) - 1 \\
\omega_2 &= \frac{2}{\sigma_2^2\beta K^\beta}\frac{\alpha}{\beta},
\end{aligned}$$

and $\Gamma(\cdot)$ is the gamma function. From (4.27) we determine the steady-state expected stock:

$$\mathbb{E}_\infty(S_2) = \overline{S}_{2,\infty} = \frac{\Gamma\left(\frac{\omega_1+1}{\beta}\right)}{\Gamma(\omega_1/\beta)}\omega_2^{-1/\beta}, \tag{4.28}$$

and the steady-state expected extraction rate:

$$\overline{q}_{2,\infty}^* = \frac{a_2^{\eta-1}b}{\phi_2^\eta}\overline{x}_{2,\infty}. \tag{4.29}$$

Notice that equation (4.26) is not satisfied in the particular case of $\eta > 1$. Hence the renewable resource stock does not have a non-degenerate steady-state distribution in this case. This is because, for small S_2, the expected (absolute) change in the stock is of order $S_2^{1/\eta<1}$ but the standard deviation of that change is of order S_2. Then, S_2 will eventually be exhausted. Fortunately, it is precisely the case of $\eta < 1$ the one that seems to be empirically relevant, as shown in the literature on energy demand.[22] This is the case we will be focused on from now on.

4.5 Results

To approximate the value function before the switch, we can use the procedures developed by Dangl and Wirl [2004], Caporale and Cerrato [2009], Mosiño [2012], or Balikcioglu et al. [2011]. In our computations we use the base case parameters of Table 4.1. Figure 4.3 shows the value function before the switch, $V(S_1)$, and the value function after the switch at the initial stock of the renewable resource, $W(K) - I$. As

[22] See Bohi and Zimmerman [1984], and Maddala et al. [1997] for instance.

we can see from this figure, the optimal switching level of S_1 is $S^* = 0.1229$. We also illustrate in Figure 4.4 that the value function before the switch shows the asymptotic behaviour demanded by the transversality condition (4.23). Finally, although it is not evident due to the little value of S^*, notice that Figures 4.3 and 4.4 show that the resulting value function before the switch is S-shaped.

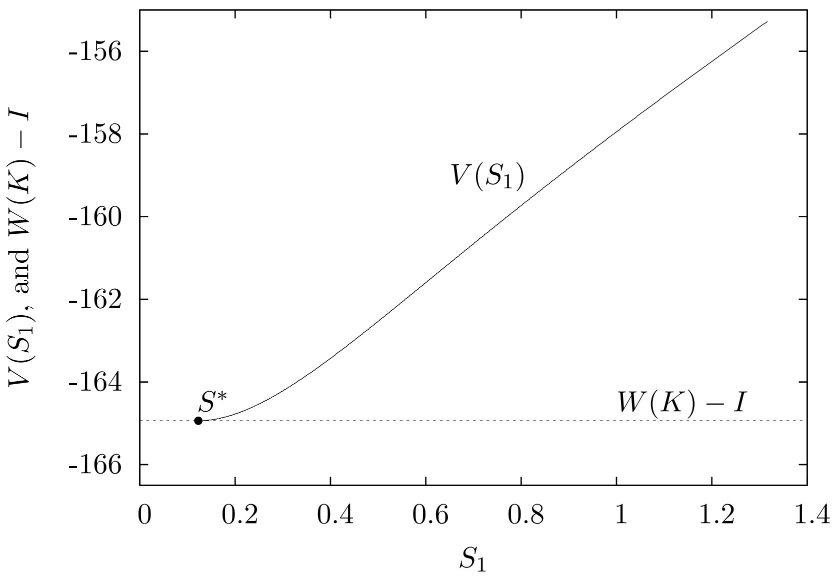

Figure 4.3: Value function before the switch and the optimal switching time.

This latter result is very interesting. From a technical point of view, the S-shaped value function before the switch it is the result of (1) the negative drift (extraction) function in the dynamics of the non-renewable resource; (2) the fact that the value function before the switch must converge to a constant as $S_1 \to S^*$; and (3) the fact that the value function before the switch must converge to a concave function as $S_1 \to \infty$. From an economical point of view, the S-shaped value function can be interpreted as follows. If the non-renewable resource is very abundant, the gain to society from having an additional unit (or alternatively, the loss of using an additional unit) is close to zero. As the resource is being exhausted, society starts caring more about using the resource, and so the social value of using an additional unit is increasing. However, as the resource approaches to its optimal switching level, firms become more interested in using renewables in the production process, and so the social value of having an additional unit of non-renewables goes back to zero.[23]

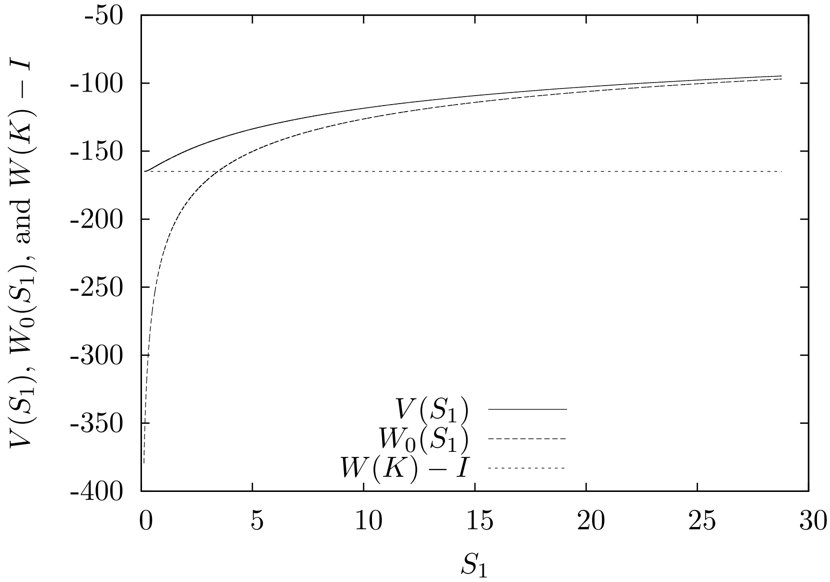

Figure 4.4: Value functions before the switch and the transversality condition.

Table 4.2 shows the effect of each parameter on the optimal switching level of S_1. Specifically, it shows the percentage change of S^* with respect of its base case value. As we can see, the optimal switching level of S_1 is a decreasing function of b and η, the demand parameters. This is probably due to the higher productivity of the non-renewable resources. As long as the demand for energy becomes larger, firms are able to satisfy the demand easier if they stick to non-renewable resources. Then, they are tempted to switch later.

S^* is also a decreasing function of I, the cost of switching, and an increasing function of c, the marginal cost of using non-renewables. This is quite intuitive. On the one hand, as long as the switching becomes more expensive, firms prefer waiting a bit more and continue producing with the non-renewable resources. On the other hand, firms have more incentives to switch as long as the extraction cost of the non-renewable resources becomes larger. Additionally, it is worth noting that firms will not switch

[23]Of course, in the absence of a switching option, the social value of using an additional unit of the non-renewable resource becomes infinite as the resource is completely exhausted. In this case, the social value before the switch is concave, and equal to $W_0(S_1)$.

Table 4.2: Sensitivity Analysis[a]

	−5%	+5%
b	5.9943%	−5.4584%
c	−4.7450%	4.6866%
I	0.1431%	−0.1429%
K	−5.8727%	5.8509%
α	−4.2395%	4.2294%
ρ	−1.5480%	1.5442%
λ	−5.8727%	5.8509%
σ_1	−0.0001%	0.0001%
σ_2	0.1279%	−0.1344%
η	15.3608%	−15.4567%

[a] Percentage change of S^* (with respect of its base case value) following a −5% or +5% change in the model's parameters.

immediately if $I = 0$. This results from the higher profits the firms get from using non-renewable resources, particularly if they are abundant.

The biological growth function's parameters also play an important role in the decision to switch. The optimal switching level of S_1 is an increasing function of both α and K. As the renewable resource reproduces faster, and its saturation level is larger, firms will switch sooner to take advantage of the more abundant (and cheaper) resource.

Of course, S^* is an increasing function of λ, the productivity of renewables relative to non-renewables. Firms will always take advantage of the most productive resource; if the renewable resource is becoming more productive, firms will want to use it sooner. In the same way, S^* appears to be an increasing function of ρ. The more concerned about the present the firms are, the sooner the switching time. This is particularly true if firms take into account the fact that, as the non-renewable resource gets depleted profits become smaller over time.

The role played by uncertainty is also clear. In particular, Table 4.2 shows that S^* is an increasing function of σ_1, but a decreasing function of σ_2. The former result implies that firms are tempted to switch sooner as long as the uncertainty about the availability of the non-renewable resources is larger. The later result implies that firms will be less interested in switching as long as the uncertainty on the availability of the renewable resources becomes larger.

We can run some simulations to evidence some other implications of uncertainty. In general, when global uncertainty increases (i.e. when both, σ_1 and σ_2 increase), S^* as a function of any of the parameters shifts down. This means that firms adopt later for any given value of b, c, I, K, α, ρ, λ, or η. We can also show that S^* is less sensitive to b and η as global uncertainty increases. This implies that economic policy focused on energy demand is less effective in the presence of uncertainty. The contrary is true for c, K, α, ρ, and λ: when global uncertainty increases economic policy focused on these parameters seems to be more effective. One exception is the switching cost variable I, whose effectiveness on the optimal switching time does not seem to change as uncertainty changes.

Table 4.3: Sensitivity Analysis: β and γ^a

	-5%	$+5\%$
β	0.5433%	-0.5401%
γ	-11.7305%	11.6807%

[a] Percentage change of S^* (with respect of its base case value) following a -5% or $+5\%$ change in the model's parameters.

Finally, we can know the effect of β and γ by dropping the assumptions of sections 4.3.1 and 4.3.2. In this case the optimal switching level can be found by entirely approximating the optimal switching time.[24] As we can see in Table 4.3, the optimal level of S_1 is a decreasing function of β. In other words, the more skewed to the right the resource's self regeneration function is — and the less the intrinsic growth rate α/β, the later the switch. We also see that the optimal level of S_1 is an increasing function of γ: the higher the elasticity of the marginal cost function before the switch, the sooner the adoption time.

[24] Refer to 4.A.

4.6 Conclusion

In this chapter we consider a model of technology switching in which energy can be produced from two available inputs: non-renewable resources (fossil fuels) and renewable resources. The stock of both resources is stochastic, and the decision to adopt renewables in production is costly and irreversible. Our model is simplified in two ways. First, we constrain the transition from one resource to the other one only into one direction. This can be the result of the dismantling of the infrastructure to extract fossil fuels and, of course, the fact that investment cannot be recovered. Second, we assume that both types of resources are never used simultaneously. These assumptions allow us to be more focused on the optimal timing of the switching decision, and on the factors influencing the decision to switch.

We find that uncertainty plays a clear role in the decision to switch. The more the uncertainty about the availability of the non-renewable resources, the sooner the firms switch to the renewable resources; and the more the uncertainty about the availability of renewable resources, the later the switching time. The optimal switching time is also sensitive to energy demand, costs, and the relative productivity of resources parameters. These later results have some implications for economic and environmental policy. For instance, the government can accelerate the substitution of renewable for non-renewable resources in energy production by increasing the marginal cost of using non-renewable resources, through a tax, or by decreasing the cost of switching through a subsidy. The government may also implement measures to reduce energy demand, or apply policies to increase the productivity of renewable resources with respect to non-renewable resources (e.g. through innovation). The effectiveness of any of these policies also depends on the degree of uncertainty surrounding the economy.

Our model can be extended to consider some other sources of uncertainty, and in particular those related to climate policies. For instance, switching can help firms to ward off mandatory future environmental regulations, so they may be tempted to switch sooner. At the same time, firms can decide to wait until the new regulations are announced, and thus they delay the switching time. Uncertainty can also appear in the productivity of the renewable resource, in the demand for electricity, or even in climate change (e.g. a future rise in temperature).

Some other possible extensions are the following. 1) Taxes and subsidies can be incorporated in an explicit way. 2) Relative productivity of resources is not necessarily known with certainty. 3) Learning or adjustment costs can be introduced in such a way that both resources can be used simultaneously. 4) Consumers decisions can be included to account for the negative effects of environmental degradation. 5) Finally, the possibility of switching back can also be considered. However, including all these possibilities at the same time can result in a model difficult to treat, so we left them as a future work.

4.A Numerical approximations and more on sensitivity analysis

In this section we provide a sketch of the way we can entirely approximate the optimal switching value S^* of section 4.3.3. Our problem is to find the function $V(S_1)$ that solves the second order non-linear differential equation (4.20), subject to the boundaries (4.21) and (4.22). An entirely numerical approximation of $V(S_1)$, and the optimal switching level of S_1, can be found as follows:

1. Approximate the function $W_0(S_1)$. This can be easily done by following, for instance, the procedure described in Miranda and Fackler [2004]. The procedure uses a combination of Chebyshev polynomials and Chebyshev nodes, and the resulting function is approximated by using Newton's method. This step is useful for at least two reasons. The first one is that the resulting equation provides the transversality condition as described in section 4.3. The second reason is that the resulting Chebyshev coefficients can be used as an initial guess for the Chebyshev coefficients in the following steps.

2. Approximate the function $W(S_2)$. As in the previous step, as no additional boundaries are required, the procedure in Miranda and Fackler [2004] can be applied straightforwardly.

3. From the previous step we recover the value $W(K)$. Then, we compute $W(K)-I$, which is one of the boundaries we are going to use in the following steps.

4. Approximate the function $V(S_1)$. To do this, we recommend the procedures described in Dangl and Wirl [2004] and Mosiño [2012]. Both procedures use a combination of Chebyshev polynomials and Chebyshev nodes, but the first one approximates the appropriate coefficients (and hence function $V(S_1)$) by using Newton's method, while the second one uses a more algebraic procedure (avoiding then the need for an initial guess for the Chebyshev coefficients). Both procedures allow the user to easily modify the algorithm in order to include the appropriate boundaries. In particular, any of the algorithms need to be modified in order to include the value matching condition.

5. Use the smooth pasting condition to solve for the optimal switching value of S_1, S^*.

One of the advantages of using the procedure we have just described, is that there is no need to set $\gamma = 1/\eta$, nor $\beta = 1/\eta - 1$. This procedure then allows us to perform a sensitivity analysis on these parameters. The results of this exercise are those shown in Table 4.3 in the main text.

4.B Solving the Hamilton-Jacobi-Bellman equation

In this section we solve a somewhat more general problem in order to prove propositions 4.1 and 4.2 simultaneously. We also show that, in the case of S being a regulated process, the value function cannot be found explicitly, and then numerical approximations are required.

4.B.1 The non-regulated process case

Assume that the social value function, J, can be computed as:

$$J = J(S) = \max_q \mathbb{E}_T \int_T^\infty e^{-\rho(t-T)}\pi(t)dt, \tag{4.30}$$

subject to:

$$\begin{aligned} dS &= \left[f(S) - q(t)\right] dt + \sigma(S) dz, && (4.31) \\ \pi(t) &= \int_0^{E^*} D^{-1}(x) dx - c(S) q(t), && (4.32) \\ S(T) &< \infty, && (4.33) \end{aligned}$$

where dz is the standard increment of a Wiener process. As usual, the fundamental equation of optimality is:

$$\rho J dt = \max_q \left\{ \pi(t) + \mathbb{E}_T dJ \right\}.$$

By using Ito's Lemma, it is easy to find that:

$$\rho J = \int_0^{E^*} D^{-1}(x) dx - c(S_1) q^*(t) + \left[f(S) - q^*(t)\right] J' + \frac{1}{2}\sigma^2(S) J'', \tag{4.34}$$

where:

$$q^* = \frac{1}{a} D\left(\frac{1}{a}\left[c(S) + J'\right]\right), \tag{4.35}$$

and $E^* = aq^*$.

Using:

$$\begin{aligned} D(p) &= bp^{-\eta}, \\ c(S) &= cS^{-\gamma}, \\ \sigma(S) &= \sigma S, \end{aligned}$$

and:

$$f(S) = \frac{\alpha}{\beta} S \left(1 - \left(\frac{S}{K}\right)^\beta\right),$$

equation (4.34) becomes:

$$\begin{aligned} \rho J = & \frac{b}{\eta - 1} a^{\eta - 1} \left(cS_1^{-\gamma} + J'\right)^{1-\eta} \\ & + \frac{\alpha}{\beta} S \left(1 - \left(\frac{S}{K}\right)^\beta\right) J' + \frac{1}{2}\sigma^2 S^2 J'', \end{aligned} \tag{4.36}$$

and equation (4.35):

$$q^* = \frac{a^{\eta-1}b}{cS^{-\gamma} + J'}. \tag{4.37}$$

If S is a non-regulated process, no additional restrictions are imposed to equation (4.36). Then, it admits a closed-form solution as long as:

$$\gamma = \frac{1}{\eta}, \text{ and } \beta = \frac{1}{\eta} - 1.$$

It is easy to verify (by substitution) that this solution is:

$$J(S) = \phi \frac{\eta}{\eta - 1} \left(S^{\frac{\eta-1}{\eta}} - \frac{\alpha}{\rho} K^{\frac{\eta-1}{\eta}} \right), \tag{4.38}$$

where ϕ satisfies:

$$\theta \phi^{\frac{1}{1-\eta}} - \phi - c = 0,$$

and:

$$\theta = a \left[\frac{\alpha + \rho}{b} \eta + \frac{\eta - 1}{\eta} \frac{1}{2b} \sigma^2 \right]^{\frac{1}{1-\eta}}.$$

Equations (4.13) and (4.16) in the main text are found using the appropriate processes (i.e. either S_1 or S_2, and parameters), and by fixing: 1) $\alpha = 0$, and $T = 0$, and 2) $c = 0$, respectively. Obviously, equations (4.14) and (4.17) are found by substituting the resulting value function in equation (4.37).

4.B.2 The regulated process case

Assume that the social value function is now:

$$J = J(S) = \max_q \mathbb{E}_T \int_0^T e^{-\rho t} \pi(t) dt + e^{-\rho T} G(\cdot),$$

where S and $\pi(t)$ satisfy equations (4.31), (4.32), and (4.33), and $G(\cdot)$ is a known function — or constant. In the continuation (waiting to switch) region, the value function can be rewritten as in equation (4.34), and then the optimal extraction value is as in equation (4.37). However, as S is now a regulated process, equation (4.34) needs to satisfy the following boundary conditions:

$$\begin{aligned} J(S^*) &= G(\cdot), \\ J'(S^*) &= G'(\cdot). \end{aligned}$$

Unfortunately, the specific form of this non linear differential equation prevents us from having a complete analytical solution. A particular solution of the full equation is obviously identical to equation (4.38). However, a complementary function (which is also the option value of waiting to switch, [Dixit, 1991, Dixit and Pindyck, 1994]) is imposible to find. Also notice that, as the optimal extraction value, q^*, depends on the value function $J(S)$, this is going to differ from the one we computed in the absence of regulated processes.[25]

[25]The fact that the Bellman equation does not allow for an analytical solution is not just a peculiar feature of our model. Similar second order non-linear differential equations with no analytical solutions can be found in Raman and Chatterjee [1995], and Dangl and Wirl [2004], for instance.

5

Switching to clean(er) technologies in a stochastic environment

In this chapter, we consider an economy having access to two different energy sources. The first one is coming from natural polluting resources; and second one is coming from a backstop natural resource. There are two productive sectors in the economy. The first one is dedicated to manufacturing the backstop resources; the second one is devoted to production of the consumption good. Both sectors are dirty in the sense that both use the polluting resource at any time. The social planner sector, however, has always the possibility of paying an irreversible fixed cost to switch the consumption sector towards the use of a cleaner technology. Additionally, we assume that the accumulation of the backstop, and the increase in pollution stock are stochastic. Our results imply that the incentives to switch to the cleaner technology depend on the relative importance of fossil fuels in the production of consumption goods after the switch. We also find that technological improvement in the solar panels sector is of some importance in order to switch to cleaner technologies.

5.1 Introduction

The Intergovernmental Panel on Climate Change [IPCC, 2007] and the International Energy Agency [IEA, 2008] estimate that in order to limit the rise of average global temperature to 2 degrees Celsius, the concentration of greenhouse gases (GHG) should not exceed 450 parts per million (ppm) CO_2. This translates to a peak of global emissions in 2015 and at least a 50 per cent cut in global emissions by 2050, compared with 2005 [UNEP, 2011]. This objective is consistent with that of many developed countries. For instance, France has been committed in 2003 to divide by four the 1990 national level of GHG, while the U.S. and Canada aim at reducing GHG emissions by more than 80 percent by 2050.

To achieve this goal, one of the policies commonly undertaken by many countries is to substitute dirty energy sources, such as coal, oil and gas, with a cleaner and renewable energy source, such as solar and wind energy. For instance, the Directive 2009/28/EC on renewable energy, implemented by Member States by December 2010, sets ambitious targets for all Member States, such that the European Union will reach a 20 percent share of energy from renewable sources by 2020. In spite of this, fossil fuels will continue to be an important part of the energy mix around the world even by 2050. In particular, as long as renewable energies are not very advanced and widespread, (i) industry will still need a percentage of energy that derives from dirty resources, and (ii) the provision of clean energy itself will require dirty resources at least as materials to build the plants (think of solar panels for instance). This is the main idea that we address here. We seek to account for the need of dirty resources even if a clean energy can be used.

In this chapter, we consider an economy having access to two different energy sources. The first one comes from a natural polluting resource, such as fossil fuels. The second one comes from a backstop natural resource, such as solar radiation. In particular, we consider the case of solar radiation being converted into energy by means of solar panels. There are two productive sectors in the economy. The first one is dedicated to manufacturing the backstop resources. At any time, this sector requires both fossil fuels and the energy provided by the backstop already available. We therefore account for the need of fossil fuels to provide clean energy. The second sector is devoted to production of the consumption good. Initially it uses energy coming exclusively from fossil fuels. However, it has always the possibility of switching towards a new technology in which energy comes from both types of resources. As the backstop is being accumulated such a switch becomes more attractive. In particular, it gets worth paying a fixed cost to use the existing stock of new solar panels and avoid —at least partially— the use of the polluting input. With this specification, the economy becomes cleaner after the switch although not completely clean, in the sense that the clean energy cannot fully replace fossil fuels to produce the consumption good. Therefore we account for the fact that even if the new technology is used, fossil fuels are still required in the industry. While taking into account these two levels of dependence with respect to the fossil fuels (namely (i) to run the economy, and (ii) to produce clean energy) after the switch, we pay particular attention to the optimal timing of the switching decision, and on the factors influencing the decision to switch.

In modelling this switching decision we include three important characteristics that must be taken into account to evaluate the adoption of any environmental policy [Pindyck, 2000, 2002]. First, we account for the uncertainty over the future costs and benefits. In particular, we assume that the accumulation of the backstop, and the increase in pollution stock —which in our case is equal to the resource extraction— are stochastic. Then, the future availability of the backstop, and the future levels of pollution —affecting the utility function— are not completely known. Second, we introduce the irreversibilities associated with environmental policy. Specifically, adoption of the cleaner technology imposes sunk cost on the consumption sector. Finally, we take into account the fact that technology adoption is rarely a now or never proposition, such that, in most cases, it is feasible to delay action and wait for new information. As the adoption of the new technology is difficult to reverse, the sunk costs are incurred over a long period of time, even if the original rationale for the switching disappears. These kind of sunk costs create an opportunity cost of adopting the new technology now, rather than waiting for more information. Our results imply that the incentives to switch to the cleaner technology depend on the relative importance of fossil fuels in the production of consumption goods after the switch. Specifically, if fossil fuels are relatively less important than solar panels to produce consumption, the central planner tends to wait more in order to switch to the new technology. This is because the solar panels sector needs to be sufficiently developed to prevent some consumption loss once the new technology is adopted. But, if fossil fuels are relatively more important than solar panels to produce consumption, switching to the new technology is easier —smoother—, and then the incentives for the central planner to wait vanishes. We also find that technological improvement in the solar panels sector is of some importance in the decision to switch to cleaner technologies. If the technological change implies that the backstop can be produced with relatively less of the fossil fuels, the adoption occurs sooner .

The rest of the chapter is as follows. In Section 5.2 we describe the assumptions and equations governing our economy. In Section 5.3 we develop the general equilibrium framework once the cleaner technology has been adopted by the consumption sector. In section 5.4 we solve the model before the technological switch by assuming that the discount rate is zero, and derive the socially optimal adoption timing. We additionally perform a comparative statics exercise in Section 5.5. In Section 5.6 we relax the assumption of a zero discount rate and then solve the model by using numerical methods. We conclude in section 5.7.

5.2 The model

We consider an economy with access to two different energy sources: one dirty, and another one clean. Dirty energy comes from a natural polluting resource, R_t, such as fossil fuels (e.g. oil). Clean energy comes from a backstop natural resource, such as solar radiation. Specifically, we consider the case of solar radiation being converted into energy by means of solar panels, S_t.

There are two productive sectors in this economy. The first one is devoted to production of the consumption good. Initially, it uses energy coming exclusively from dirty inputs to run a given constant stock of capital $\overline{K_1}$. At some point, however, the backstop becomes more developed in the economy, such that the consumption sector is more interested to switch to a new technology using both types of energy, i.e. electricity from solar panels and oil, to run the capital. Such a switch becomes more attractive as the backstop is being accumulated: it gets worth paying a fixed cost to use the existing stock of solar panels. In particular, we assume that:

$$C_t = B_1(\kappa_t R_t)\overline{K_1}, \tag{5.1}$$

for $t < T$, and:

$$C_t = A_1(\kappa_t R_t)^{\eta}(\lambda_t S_t)^{1-\eta}\overline{K_1}, \tag{5.2}$$

for $t \geq T$, where T is the time of the switching, i.e. when the backstop becomes more active —sufficiently developed— in the economy. In equations (5.1) and (5.2), B_1, $A_1 > 0$ are technological parameters, and η ($0 \leq \eta \leq 1$) is the share of the polluting resource in the consumption function. Notice that after the switch, the smaller the parameter η, the cleaner the consumption sector. However, even in the limit case of $\eta = 0$, the economy is not completely "pollution-free" due to the fact that solar panels still require fossil fuels to be produced by the other sector (see below equations (5.3) and (5.4)).

The second sector is dedicated to manufacturing the backstop resource. This sector requires both fossil fuels and the energy provided by the backstop already available. On can think of solar panels whose fabrication requires some given constant stock of capital $\overline{K}_2$, as well as solar panels (for electricity provision) and oil as a source of energy or of materials to be built. This particular assumption is in line with a physician view of environmental economics that stresses the need for oil in order to turn to a new energy (and some of them even doubting that current reserves are sufficient for this energy change). Moreover, efficiency in this sector is stochastic, since there is a lot of uncertainty surrounding the productivity of solar panels in energy provision and the maintenance costs of these panels. Uncertainty is assumed to be multiplicative, meaning that the larger the number of solar panels already built the larger future uncertainty on solar panel accumulation. We assume that the backstop is accumulated according to:

$$dS_t = B_2\left[\beta(1-\kappa_t)R_t + (1-\beta)S_t\right]\overline{K}_2 dt + \sigma_S \overline{K}_2 S_t dz_S, \tag{5.3}$$

for $t < T$, and:

$$dS_t = A_2\left[\alpha(1-\kappa_t)R_t + (1-\alpha)(1-\lambda_t)S_t\right]\overline{K}_2 dt + \sigma_S \overline{K}_2 S_t dz_S, \tag{5.4}$$

for $t \geq T$, with $S_t > 0$, and S_0 given. In equations (5.3), and (5.4) B_2, $A_2 > 0$ as well as α and β $(0 \leq \alpha, \beta \leq 1)$ are technological parameters, and dz_S is the standard increment of a Wiener process. The parameters κ_t and λ_t $(0 \leq \kappa_t, \lambda_t \leq 1)$ are endogenously chosen fractions of the polluting and backstop resources, respectively, used in the consumption sector. We assume that a 100 percent of the extracted polluting resources, and a 100 percent of the backstop already available are used in the economy. Hence, by choosing optimally κ_t and λ_t, the central planer is implicitly choosing $(1 - \kappa_t)$ and $(1 - \lambda_t)$ to be the fractions of the polluting and backstop resources, respectively, used in the backstop production sector. It is worth noting that if $\kappa_t = \lambda_t = 1$, there is no production of solar panels after the switch —though they are still stochastically evolving as a simple Brownian motion, then we go back to a one sector formulation. It is also important to notice that before T the backstop resource is not used in the consumption sector, though it is accumulated according to equation (5.3). Additionally, uncertainty affects solar panels accumulation in the same way irrespective of whether panels are used in the consumption sector or not.

Our formulation in equations (5.1) to (5.4) also implies that from period T the economy becomes cleaner in the sense that the clean resource can be used to provide the consumption good, but there is also a switch in technology since the accumulation process of solar panels changes. Particularly, technology in solar panel accumulation improves after the switch as long as $A_2 > B_2$. Additionally, the green effect of the switching is reinforced by assuming that $\alpha < \beta$, such that the backstop production sector is less polluting-resource dependent after the switch. We analyse this case, and the less general case in which there is no technological improvement ($A_2 = B_2$ and $\alpha = \beta$) in Section 5.5.

We assume that the increase in the pollution stock is equal to the resource extraction. It is also subject to some multiplicative uncertainty that for instance takes into account that Nature assimilation of CO_2 released after oil combustion is not well-known. This is described by the following equation:

$$dP = R_t dt + \sigma_P P_t dz_P, \tag{5.5}$$

with $P_t \geq 0$, and P_0 given. dz_P is another standard increment of a Wiener process. For simplicity, we assume that dz_S and dz_P are uncorrelated.

The social preferences derived from consumption and environmental quality can be represented by the lifetime expected utility:

$$\mathbb{E}_0\left[\int_0^\infty e^{-\rho\tau}U(C_\tau, P_\tau)d\tau\right] = \mathbb{E}_0\left[\int_0^\infty e^{-\rho\tau}\frac{\left(C_\tau P_\tau^\phi\right)^{1-\varepsilon}}{1-\varepsilon}d\tau\right], \tag{5.6}$$

where $\phi < -1$, and $\rho \geq 0$ is the rate of time preference. This specification satisfies some conditions that are now common in the literature, and takes into account the fact that the combustion of fossil fuels is responsible for an important part of CO_2 emissions and other pollutants, and provides a (negative) amenity to households. The cross derivative U_{cP} is negative which means that utility exhibits a "distaste effect", in the terminology of Michel and Rotillon [1995]: a decrease in pollution increases the marginal utility of consumption and implies that households have a higher desire to consume.

Since there are two arguments in the utility function, it is not immediately obvious what risk aversion or intertemporal substitution means (see Debreu [1976] and Kihlstrom and Mirman [1974] for the literature on multivariate risk aversion). Equation (5.6) can be rewritten as:

$$\mathbb{E}_0\left[\int_0^\infty e^{-\rho\tau}\frac{\left(C_\tau^{\frac{1}{1+\phi}}P_\tau^{\frac{\phi}{1+\phi}}\right)^{1-\Gamma}}{1-\varepsilon}d\tau\right]$$

Debreu [1976] calls the function in the braces the "least concave utility function". The exponents of this function may be interpreted as governing ordinal preferences between the two goods in the absence of risk. The transforming function $[\cdot]^{1-\Gamma}$ can then be interpreted as governing aversion to risk. A simple calculation then reveals that the appropriate measure of risk relative aversion is Γ. Then, following the terminology in Smith [1999] or Pommeret and Schubert [2009] we will call Γ the effective coefficient of relative risk aversion and $\mathcal{E}$ the inverse of the effective elasticity of intertemporal substitution Since Γ depends on ϕ, pollution changes risk aversion:

$$\begin{aligned}\Gamma &= 1-(1-\varepsilon)(1+\phi)\\ \mathcal{E} &= 1-(1-\varepsilon)=\varepsilon.\end{aligned}$$

From now on, we keep the notation ε for the inverse of the effective elasticity of intertemporal substitution. We let Ω denote the set of admissible plans, that is the set κ_t, λ_t, extraction rates and dates of adoption $(\kappa,\ \lambda, R, T)$, such that:

$$\mathbb{E}_0 \left[\int_0^\infty e^{-\rho\tau} \left| U(C_\tau, P_\tau) \right| d\tau \right] < \infty.$$

In this case, we can write the value function of the central planner as:

$$V(S_0, P_0) = \sup_{(\kappa_\tau, \lambda_\tau, R_\tau, T) \in \Omega} \mathbb{E}_0 \left\{ \int_0^T e^{-\rho\tau} \frac{\left(C_\tau P_\tau^\phi \right)^{1-\varepsilon}}{1-\varepsilon} d\tau + e^{-\rho T} W(S_T - IP_T^{-\phi}, P_T) \right\},$$

where $W(\cdot)$ is the value function after the switch and $IP_t^{-\phi}$ is the switching cost. Notice that this cost is increasing with the level of pollution, and can be expressed in terms of solar panels by means of some constant $I > 0$. In this sense, the cost of switching to a new production technology in the consumption sector is assimilated to a lost of some solar panels. This program can be solved in two stages. We first solve for the problem for the representative agent assuming that the backstop energy is used actively. We next determine the optimal time for adopting the backstop in the consumption sector.

5.3 The optimal path after the switch, and the case without an option to switch

5.3.1 The after the switch case

We assume that the backstop energy is used actively in the economy. The set of admissible plans collapses to the set $(\kappa,\ \lambda, R)$ such that:

$$\mathbb{E}_t \left[\int_t^\infty e^{-\rho(\tau - t)} \left| U(C_\tau, P_\tau) \right| d\tau \right] < \infty.$$

The value function of the central planner is:

$$W\left(S_t, P_t\right) = \sup_{(\kappa_\tau, \lambda_\tau, R_\tau) \in \Omega} \mathbb{E}_t \left\{ \int_t^\infty e^{-\rho\tau} \frac{\left(C_\tau P_\tau^\phi \right)^{1-\varepsilon}}{1-\varepsilon} d\tau \right\},\ t \geq T.$$

Then, the Hamilton-Jacobi-Bellman equation can be written as:

$$\rho W\left(S_t, P_t\right)=\max_{\kappa_t,\lambda_t,R_t}\left\{\frac{\left(C_t P_t^{\phi}\right)^{1-\varepsilon}}{1-\varepsilon}+\mathbb{E}_t\left[W\left(S_{t+dt}, P_{t+dt}\right)\right]\right\}, t\geq T. \tag{5.7}$$

After maximizing the right hand side of equation (5.7), we can rewrite the Hamilton-Jacobi-Bellman equation by defining the following *pollution-adjusted* version of the variables:

$$c_t \;:\; = C_t P_t^{\phi} \tag{5.8}$$

$$s_t \;:\; = S_t P_t^{\phi} \tag{5.9}$$

$$r_t \;:\; = R_t P_t^{\phi}. \tag{5.10}$$

Then our problem is simplified to one of solving the following second order differential equation in *one* variable:

$$\rho\omega\left(s_t\right)=\mathbf{a}_1\left[\omega'\left(s_t\right)\right]^{\frac{\varepsilon-1}{\varepsilon}}+\mathbf{a}_2\omega'\left(s_t\right)s_t+\mathbf{a}_3\omega''\left(s_t\right)s_t^2,$$

where:

$$\begin{aligned}
\mathbf{a}_1 &= \frac{\varepsilon}{1-\varepsilon}\left[\frac{(1-\eta)\Theta_A}{A_2(1-\alpha)\overline{K_2}}\right]^{\frac{1-\varepsilon}{\varepsilon}}\\
\mathbf{a}_2 &= A_2(1-\alpha)\overline{K_2}+\frac{1}{2}\phi(\phi-1)\sigma_P^2\\
\mathbf{a}_3 &= \frac{1}{2}\left(\sigma_S^2\overline{K_2}+\phi^2\sigma_P^2\right),\\
\Theta_A &= A_1\left(\frac{1-\alpha}{\alpha}\frac{\eta}{1-\eta}\right)^{\eta}\overline{K_1},
\end{aligned}$$

and:

$$W(S_t, P_t)\equiv\omega\left(s_t\right).$$

This formulation leads us to the following Proposition.

Proposition 5.1 *If the clean energy is used actively in the consumption sector, the value of the pollution-adjusted solar panels is:*

$$\omega(s_t)=\mathbf{A}\frac{1}{1-\varepsilon}s_t^{1-\varepsilon}, \tag{5.11}$$

where:

$$\mathbf{A} = \left[\frac{1}{\mathbf{a}_1}\left(\frac{\rho}{1-\varepsilon} - \mathbf{a}_2 + \varepsilon\mathbf{a}_3\right)\right]^{-\varepsilon}.$$

The optimal consumption, and the optimal amount of the dirty input used in the consumption sector are:

$$c_t^* = \Theta_A \lambda^* s_t \tag{5.12}$$

$$(\kappa_t r_t)^* = \frac{1-\alpha}{\alpha}\frac{\eta}{1-\eta}\lambda^* s_t, \tag{5.13}$$

where:

$$\lambda^* = \lambda_t^* = \left[\frac{(1-\eta)\Theta^{1-\varepsilon}}{A_2(1-\alpha)\mathbf{A}}\right]^{\frac{1}{\varepsilon}}$$

is a constant.

Proof See 5.A. ∎

Therefore, we obtain that the repartition of the stock of solar panel between the consumption sector and the backstop manufacturing sector is constant over time. Notice that it is not necessarily the case for κ_t that governs the repartition of fossil fuel extraction between the two sectors. Moreover, (pollution-adjusted or not) consumption is a constant fraction of (pollution-adjusted or not) solar panels. The latter result follows from the fact that $(\kappa_t r_t)^*$, i.e. the fossil fuel input in the consumption good process, is a constant fraction of s_t.

In Proposition 5.1 we require that $\mathbf{A} > 0$, so we impose:

$$\begin{array}{ll} \frac{\rho}{1-\varepsilon} - \mathbf{a}_2 + \varepsilon\mathbf{a}_3 > 0, & \text{if } \varepsilon < 1 \\ \frac{\rho}{1-\varepsilon} - \mathbf{a}_2 + \varepsilon\mathbf{a}_3 < 0, & \text{if } \varepsilon > 1. \end{array} \tag{5.14}$$

The transversality condition requires the convergence of the value function, i.e.:

$$\lim_{t\to\infty} \mathbb{E}_0\left[\omega(s_t)\right] = 0.$$

This condition is satisfied if $\omega(s_t)$ does not grow too fast in expectation. This requires that:

$$\mathbb{E}\left[d\omega(s_t)\right] = \omega_s(s_t)\mathbb{E}\left(ds_t\right) + \frac{1}{2}\omega_{ss}(s_t)\mathbb{E}\left(ds_t\right)^2 < 0.$$

Hence:

$$(\phi+1)\left\{\frac{1-\alpha}{\alpha}\frac{\eta}{1-\eta}\left(-\frac{\alpha A_2}{\phi}\right)\frac{\lambda^*}{\kappa^*}+\frac{1}{2}\sigma_P^2\left[\phi-\varepsilon\left(\phi+1\right)\right]\right\}<0. \tag{5.15}$$

As $\phi+1<0$, guaranteeing condition (5.15) to be satisfied requires the term inside the curly brackets to be strictly positive. We can show that sufficient conditions are:

$$\phi<\frac{\varepsilon}{1-\varepsilon}$$

in the case of $\varepsilon>1$, and:

$$\frac{1-\alpha}{\alpha}\frac{\eta}{1-\eta}\left(-\frac{\alpha A_2}{\phi}\right)\lambda^*>\left|\frac{1}{2}\sigma_P^2\left[\phi-\varepsilon\left(\phi+1\right)\right]\right|$$

otherwise.

5.3.2 The no option to switch case

Having solved the program once the clean energy has been adopted in the consumption sector, one can easily deduce the solution of the central planner's problem in an economy in which this kind of energy is never available to this sector. We will consider the fictive case in which —even though there is no possibility of switching— there is still a second sector which produces solar panels according to equation (5.3). The hypothetical results —although not intuitively relevant— will be theoretically useful for what follows. We let $W_0(S_t,P_t)$ be the value function of the central planner of the economy with no clean energy used in the consumption sector, with:

$$W_0\left(S_t,P_t\right)=\sup_{(\kappa_\tau,R_\tau)\in\Omega}\mathbb{E}_0\left\{\int_t^\infty e^{-\rho(\tau-t)}\frac{\left(C_\tau P_\tau^\phi\right)^{1-\varepsilon}}{1-\varepsilon}d\tau\right\}$$

Following the above steps and definitions, we can show that the value function in this case can be written as:

$$\rho\omega_0\left(s_t\right)=\mathbf{b}_1\left[\omega_0'\left(s_t\right)\right]^{\frac{\varepsilon-1}{\varepsilon}}+\mathbf{b}_2\omega_0'\left(s_t\right)s_t+\mathbf{b}_3\omega_0''\left(s_t\right)s_t^2,$$

with:

$$
\begin{aligned}
\mathbf{b}_1 &= \frac{\varepsilon}{1-\varepsilon}\left[\frac{B_1\overline{K_1}}{\beta B_2\overline{K_2}}\right]^{\frac{1-\varepsilon}{\varepsilon}} \\
\mathbf{b}_2 &\equiv \mathbf{a}_2 \\
\mathbf{b}_3 &\equiv \mathbf{a}_3.
\end{aligned}
$$

and:

$$W_0(S_t, P_t) \equiv \omega_0\left(s_t\right).$$

The solution is as in the following Proposition.

Proposition 5.2 *If the economy cannot use the clean energy in the consumption sector, the value of the pollution-adjusted panels is:*

$$\omega_0(s_t) = \mathbf{B}\frac{1}{1-\varepsilon}s_t^{1-\varepsilon}, \tag{5.16}$$

where:

$$\mathbf{B} = \left[\frac{1}{\mathbf{b}_1}\left(\frac{\rho}{1-\varepsilon} - \mathbf{b}_2 + \varepsilon\mathbf{b}_3\right)\right]^{-\varepsilon}.$$

The optimal consumption, and the optimal amount of the dirty input used in the consumption sector are:

$$
\begin{aligned}
c_t^* &= B_1\Theta_B s_t, \\
(\kappa r_t)^* &= \Theta_B s_t,
\end{aligned}
$$

where:

$$\Theta_B = \left[\frac{\left(B_1\overline{K_1}\right)^{1-\varepsilon}}{B_2\beta\mathbf{B}\overline{K_2}}\right]^{\frac{1}{\varepsilon}}.$$

Proof See 5.B. ▮

In Proposition 5.2 we require that $\mathbf{B} > 0$. We then impose:

$$
\begin{array}{ll}
\frac{\rho}{1-\varepsilon} - \mathbf{b}_2 + \varepsilon\mathbf{b}_3 > 0, & \text{if } \varepsilon < 1 \\
\frac{\rho}{1-\varepsilon} - \mathbf{b}_2 + \varepsilon\mathbf{b}_3 < 0, & \text{if } \varepsilon > 1.
\end{array} \tag{5.17}
$$

For any s_t the value function in equation (5.16) cannot be greater than the lifetime utility of the agent in an economy with the clean energy available in the consumption sector. Then, we must have:

$$\omega_0(s_t) \le \omega(s_t). \tag{5.18}$$

This condition ensures that there exists an optimal switching date; that is, in the absence of costs of switching to the cleaner energy, the central planner would choose to immediately switch for any current level of pollution-adjusted capital accumulation. A necessary and sufficient condition for equation (5.18) to be satisfied is:

$$\begin{aligned} \mathbf{A} &\geq \mathbf{B}, \text{ if } \varepsilon < 1 \\ \mathbf{A} &\leq \mathbf{B}, \text{ if } \varepsilon > 1, \end{aligned}$$

which we impose.

5.4 The optimal switching time, the undiscounted case

The choice of an optimal consumption plan (through the choice of the optimal extraction rate, and the variables κ_t and λ_t) and of an optimal adoption time, is given by the maximization of the intertemporal utility function subject to the laws of solar panels and pollution accumulations. Once the new energy has been adopted, the central planner optimally follows the consumption plan described by equation (5.12). Therefore, the value function at the time of the switch is given by the following value-matching and smooth pasting conditions:

$$V(S_T, P_T) = W(S_T - IP_T^{-\phi}, P_T), \tag{5.19}$$

$$V_S(S_T, P_T) = W_S(S_T - IP_T^{-\phi}, P_T), \tag{5.20}$$

where $V(S_T, P_T)$ is the value function before the switch —evaluated at T, and subscripts denote partial derivatives. The central planner's problem becomes then:

$$V(S_0, P_0) = \sup_{(\kappa_\tau, \lambda_\tau, R_\tau) \in \Omega} \mathbb{E}_0 \left\{ \int_0^T e^{-\rho\tau} \frac{\left(C_\tau P_\tau^\phi\right)^{1-\varepsilon}}{1-\varepsilon} d\tau + e^{-\rho T} W(S_T - IP_T^{-\phi}, P_T) \right\},$$

subject to equations (5.1), (5.4), (5.5), and conditions (5.19), and (5.20). Notice that the value function before the switch depends on the current stock of solar panels even though these panels are not used before T. This is because solar panels have some value due to the existence of an opportunity to switch in the future.

By using the notation in equations (5.8) to (5.10), we can show that after maximization, the problem collapses to one of solving the following differential equation:

$$\rho\upsilon\left(s_t\right)=\mathbf{b}_1\left[\upsilon'\left(s_t\right)\right]^{\frac{\varepsilon-1}{\varepsilon}}+\mathbf{b}_2\upsilon'\left(s_t\right)s_t+\mathbf{b}_3\upsilon''\left(s_t\right)s_t^2, \tag{5.21}$$

with:

$$V(S_t,P_t)\equiv\upsilon\left(s_t\right),$$

and the following boundary conditions:

$$\upsilon(s_T) = \omega(s_T-I), \tag{5.22}$$

$$\upsilon_s(s_T) = \omega_s(s_T-I), \tag{5.23}$$

which represent the pollution-adjusted version of the value matching and smooth pasting conditions. In the problem above, s_T is the level of the pollution-adjusted solar panels stock for which it is optimal to switch. This value implicitly determines the optimal switching time T. Additionally, given that the central planner can always choose not to switch to the technology using panels to produce consumption, another condition that must be satisfied is:

$$\omega_0(s_t)\leq\upsilon(s_t)\ \forall t. \tag{5.24}$$

The problem in equations (5.21) to (5.23) has an analytical solution only if the discount rate ρ is equal to zero. Let us assume that it is the case. Then, we can find an expression for the marginal value of the pollution-adjusted capital before the switch:

$$\upsilon_s(s_t)=\left(\frac{D_1}{s_t}+D_2s_t^{D_3}\right)^{\varepsilon}, \tag{5.25}$$

where:

$$\begin{aligned} D_1 &= \mathbf{B}^{\frac{1}{\varepsilon}},\\ D_3 &= -\frac{\mathbf{b}_2}{\varepsilon\mathbf{b}_3}, \end{aligned}$$

and D_2 is a constant that must be determined using the smooth pasting condition, equation (5.23). We can show that this is:

$$D_2=\frac{1}{s_T^{D_3}}\left(\frac{\mathbf{A}^{\frac{1}{\varepsilon}}}{s_T-I}-\frac{D_1}{s_T}\right).$$

We define:

$$G(s_t, s_T) := D_2 s_t^{D_3}$$

as the part of the value function due to the option to switch. Notice that in the absence of such an option, i.e. $G(s_t, s_T) \equiv 0$, the marginal value of the pollution-adjusted capital reduces to $\omega_{0s}(s_t)$. We deduce from the following Propositions that the value of $G(s_t, s_T)$, and hence the value function and the optimal switch time crucially depend on the value of ε.

Proposition 5.3 *If $\varepsilon < 1$, then $G(s_t, s_T) > 0$, and $\upsilon_s(s_t)$ is always defined. As a consequence the stock of the pollution-adjusted panels s_T can be found by solving:*

$$\int_0^{s_T} \upsilon_s(s_t)dt = \omega(s_T). \tag{5.26}$$

Proof See 5.C. ∎

Equation (5.26) can be solved numerically. Numerical resolution is driven using the parameters in Table 5.1. Figure 5.1 shows the three value functions: before the switch, $\upsilon(s_t)$, after the switch, $\omega(s_t - I)$, and without the option to switch, $\omega_0(s_t)$. The threshold that triggers the switch is $s_T = 0.4825$.

Table 5.1: Base Case Parameters, $\varepsilon < 1$

ϕ	-2.0
ε	0.7
σ_S	0.5
σ_P	0.05
A_1	1.0
A_2	0.1
α	0.5
η	0.5
B_1	1.0
B_2	0.1
β	0.7
I	0.1

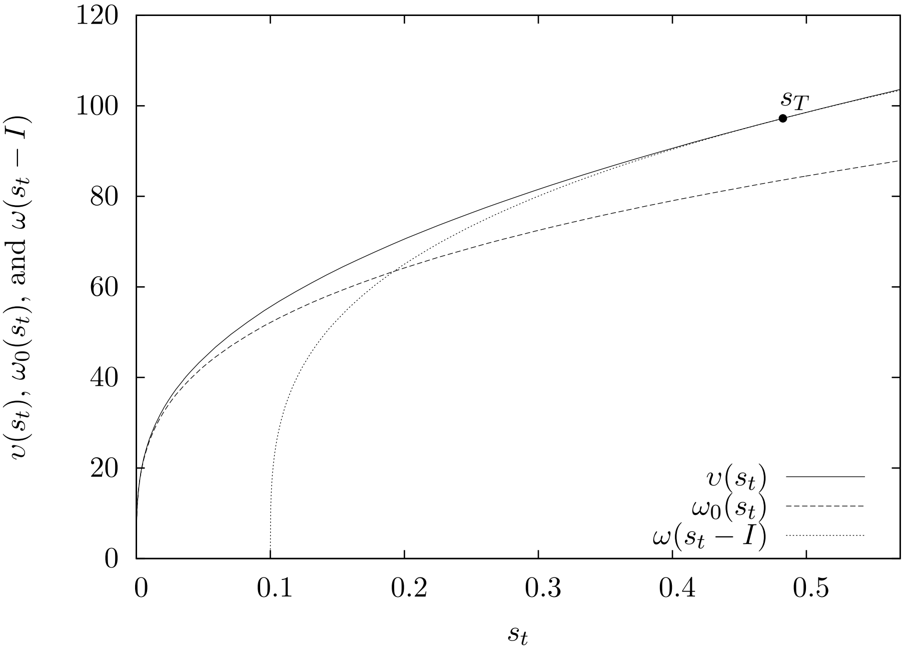

Figure 5.1: Optimal switching level of s_t, $\varepsilon < 1$.

Proposition 5.4 *If $\varepsilon > 1$ the option $G(s_t, s_T)$ collapses to zero. In this case, the value function before the switch can be found to be:*

$$v(s_t) = \mathbf{B}\frac{1}{1-\varepsilon}s_t^{1-\varepsilon} + \mathbf{B}_0, \tag{5.27}$$

for:

$$\mathbf{B}_0 = \frac{1}{1-\varepsilon}\left[\mathbf{A}\left(s_T - I\right)^{1-\varepsilon} - \mathbf{B}s_T^{1-\varepsilon}\right].$$

The optimal switching is then:

$$s_T = \frac{I}{1 - \left(\frac{\mathbf{b}_1}{\mathbf{a}_1}\right)^{\frac{1}{\varepsilon}}} := s^*. \tag{5.28}$$

The optimal consumption, and the optimal amount of the dirty input used in the consumption sector remain as in Proposition (5.2).

Proof See 5.C. ▮

As before, (pollution-adjusted or not) consumption is a constant fraction of (pollution-adjusted or not) solar panels. This is because the fossil fuel input in the consumption good process, $(\kappa_t r_t)^*$, is still a constant fraction of s_t.

As an example, we drive a numerical resolution by using the values in Table 5.2. Figure 5.2 shows the three value functions: before the switch, $\upsilon(s_t)$, after the switch, $\omega(s_t - I)$, and without the option to switch, $\omega_0(s_t)$. The threshold that triggers the switch is $s_T = 0.2514$.

Table 5.2: Base Case Parameters, $\varepsilon < 1$

ϕ	-2.0
ε	1.7
σ_S	0.2
σ_P	0.2
A_1	1.0
A_2	0.6
α	0.5
η	0.5
B_1	1.0
B_2	0.6
β	0.7
I	0.1

5.5 Comparative Statics

In this section we begin our analysis by considering the simplifying assumptions of $A_2 = B_2$, and $\alpha = \beta$. In this case, there is no technological improvement in the backstop production sector, nor a reinforcement of the "green effect" after the switch. By imposing these assumptions, the only advantage of the backstop production sector when switching is that solar panels are to be shared with the consumption sector. Next, we compare our results with those of the more general case of $A_2 > B_2$, and $\alpha < \beta$.

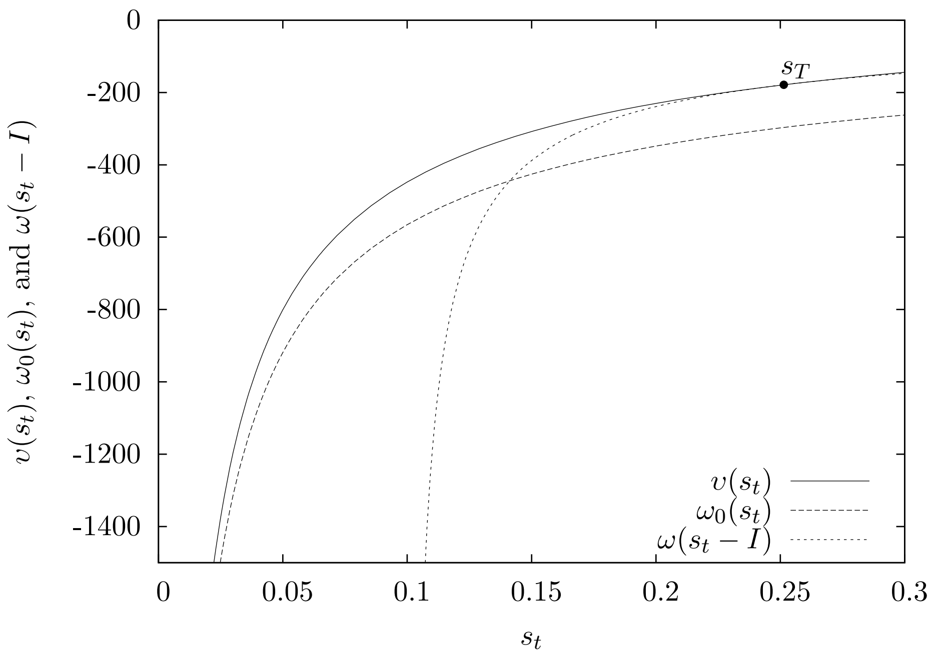

Figure 5.2: Optimal switching level of s_t, $\varepsilon > 1$.

5.5.1 No technological improvement in the solar panel process

As a base case, we use the same parameters as in Table 5.1, and Table 5.2, in which it is already assumed that $A_2 = B_2$, and we set $\alpha = \beta = 0.85$. Then we get that $s_T = 1.4378$ when $\varepsilon < 1$ (Figure 5.3), and $s_T = 1.4451$ when $\varepsilon > 1$ (Figure 5.4). We next consider the effect of each of the parameters on the optimal switching value of the pollution-adjusted panels.

We consider first the effect of ϕ on the level of the pollution-adjusted solar panels stock triggering their adoption by the consumption sector. As we can deduce from Table 5.3, s_T is a decreasing (and convex) function of ϕ: the larger (less negative) ϕ, the technology using solar panels is adopted by the consumption sector for a smaller pollution-adjusted panels stock. This is a priori counter-intuitive: more negative values of ϕ means that the central planner cares more about pollution affecting the utility of households and adoption should occur for a smaller solar panel stock. However, it has to be kept in mind that, from the definition of s_T, more negative values of ϕ, and

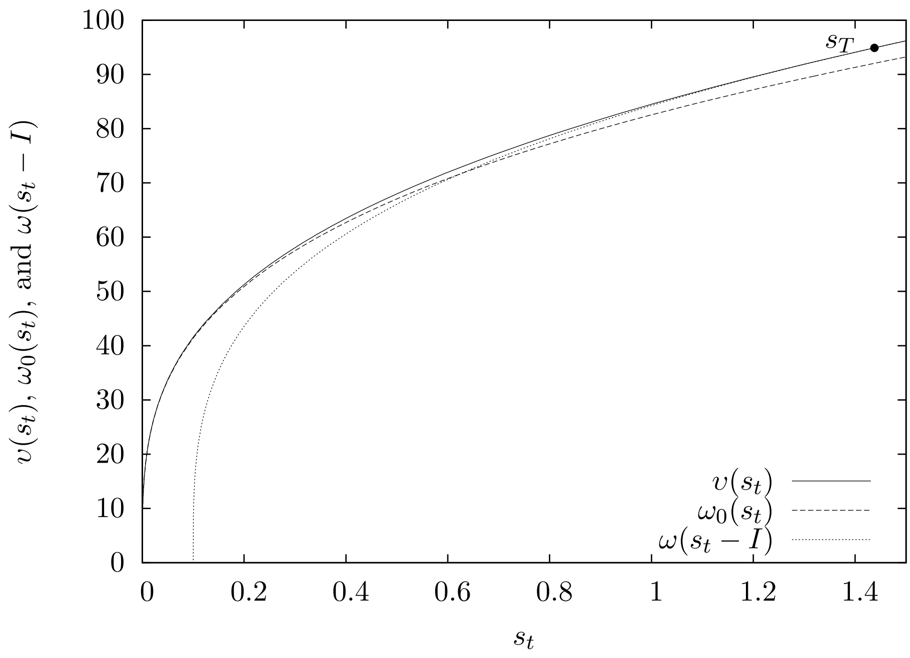

Figure 5.3: Optimal switching level of s_t without technological change, $\varepsilon < 1$.

then smaller values of s_T, may perfectly correspond to higher levels of non-pollution-adjusted panels, S_T, since pollution may be smaller. Moreover, there exists another effect of ϕ through the effective risk aversion $\Gamma = 1 - (1 - \varepsilon)(1 + \phi)$ (see also equation (5.11) in which the two effects of this parameter clearly appear through the constants $\mathbf{a}_2$ and $\mathbf{a}_3$). The larger ϕ, the smaller the risk aversion. This may explain that a smaller accumulated stock of pollution-adjusted solar panels is required to switch. In the case $\varepsilon > 1$, the switch is triggered by the equality between the marginal values before and after the switch that depend in the same way from ϕ; therefore this parameter does not affect s_T. Again, it does not mean that it does not affect S_T.

We now consider the effect of ε. As we know, this parameter is the inverse of the effective intertemporal elasticity of substitution. On the one hand, larger values of ε reduce the effective intertemporal elasticity of substitution. On the other hand, larger values of ε increase the effective coefficient of risk aversion $\Gamma = 1 - (1 - \varepsilon)(1 + \phi)$. We deduce from Table 5.3 that the optimal level of the pollution-adjusted solar panels is a decreasing function of ε: less taste for intertemporal substitution erodes the option

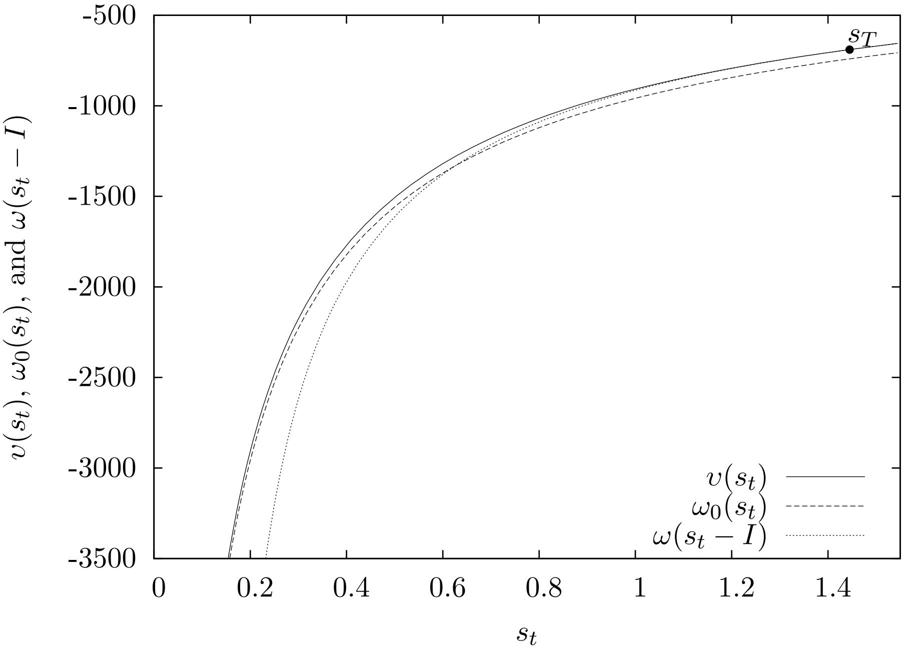

Figure 5.4: Optimal switching level of s_t without technological change, $\varepsilon < 1$.

value to wait and therefore induces an adoption for a smaller stock of pollution adjusted panels.

Uncertainty plays an interesting role in the decision to switch, particularly in the case of $\varepsilon < 1$. The level of the pollution-adjusted solar panels at which it is optimal to switch is a decreasing function of the uncertainty on the accumulation of solar panels. This result on "economic" uncertainty fully reverses that of the partial equilibrium literature (e.g. Pindyck [2000]), in which higher levels of uncertainty increase the incentives to wait rather than adopt the policy now. What happens here is that this uncertainty reduces the value before the switch more than the value after it, therefore reducing the level of pollution adjusted panels stock that triggers the switch. On the contrary, uncertainty on pollution accumulation is consistent with the usual partial equilibrium effect of uncertainty. The effect of both σ_S and σ_P disappears in the case of $\varepsilon > 1$ because uncertainties affect in the same way the marginal values before and after the switch and therefore do not affect s_T (it does not mean that it does not affect S_T) as can be seen in equations (5.11) and (5.27). This is in standard result in general equilibrium (see, for instance, Pommeret and Schubert [2009]).

We also have that the level of the pollution-adjusted solar panels at which it is optimal to switch is a decreasing function of the technological parameter A_1, and an increasing function of the technological parameter B_1: the larger the technology gain due to the switch, the smaller the pollution adjusted panels stock that triggers adoption. Again this is due to an increase of the value after the switch compared to that before the switch. On the other hand, s_T is an increasing function of A_2: the more the level of technology in the solar panels production sector, the later the adoption. As $A_2 = B_2$, the larger the level of technology in this sector the less the incentives to switch. Such an effect necessarily arises from the effect of solar panels technology on the option value to switch. This last effect, however, disappears in the case of $\varepsilon > 1$ because value functions before and after the switch are affected in the same way.

Our simulations show that s_T is an increasing (and convex) function of η: as the participation of the polluting resource in the production of the consumption good after the switch increases the central planner will choose to adopt for a larger s_T; the larger this parameter, the less the incentive to switch. On the opposite, the larger α, the share of the polluting resource required to accumulate solar panels (before and after the switch), the most important it is to use less of the fossil fuel in the production of the consumption good and therefore the smaller the s_T that triggers the switch. Finally, the central planner will decide to adopt for a higher s_T if the irreversible investment cost is higher.

5.5.2 Technological improvement in the solar panels sector

We now relax the assumptions of $\alpha = \beta$. and $A_2 = B_2$, and see how much the results change in the presence of technological improvement, i.e. $\alpha < \beta$ and $A_2 > B_2$, starting from the parameters of Table 5.1, and Table 5.2. Some effects are quite similar to those found in the previous section. For instance, the optimal level of the pollution-adjusted solar panels is still a decreasing (but now convex) function of ε, a decreasing function of A_1, and an increasing function of B_1. We also get that the central planner will decide

Table 5.3: Comparative statics without technological improvement

	$\varepsilon < 1$		$\varepsilon > 1$	
	$f'(\cdot)$	$f''(\cdot)$	$f'(\cdot)$	$f''(\cdot)$
ϕ	≤ 0	≥ 0	$= 0$	$= 0$
ε	≤ 0	≥ 0	≤ 0	≥ 0
σ_S	≤ 0	≥ 0	$= 0$	$= 0$
σ_P	≥ 0	≥ 0	$= 0$	$= 0$
A_1	≤ 0	≥ 0	≤ 0	≥ 0
A_2	≥ 0	≥ 0	$= 0$	$= 0$
α	≤ 0	≥ 0	≤ 0	≤ 0
η	≥ 0	≥ 0	≥ 0	≥ 0
B_1	≥ 0	≥ 0	≥ 0	≤ 0
I	≥ 0	$= 0$	≥ 0	$= 0$

to adopt for a higher s_T the higher the irreversible investment cost. However, most of the results are inverted. Let us consider each of them.

As before, we first consider the effect of ϕ on the level of the pollution-adjusted solar panels stock triggering their adoption by the consumption sector. As we can see in Table 5.4, s_T is an *increasing* function of ϕ: the larger (less negative) ϕ, the higher the value of the adjusted-solar panels in order the technology using solar panels to be adopted by the consumption sector. This result seems to be more intuitive than previously. In this particular case, i.e. if the switch allows using less polluting resource for both consumption and solar panels accumulation, it is the direct effect of ϕ on utility the one that matters the most: more negative values of ϕ mean that the central planer cares more about pollution affecting the utility of households and can increase the intertemporal utility thanks to the technology improvement.

We now consider the effect of uncertainty. The role played by uncertainty on the accumulation of solar panels still depends on the value of ε relative to unity, and they are now reversed for $\varepsilon < 1$. To explain these new results we can focus on the effect of the technological improvement after the switch. Whatever the effective intertemporal elasticity of substitution, the pollution-adjusted solar panels stock that triggers the switch is a decreasing function of the uncertainty on pollution accumulation. This comes from the fact that the central planner tries to mitigate the bad effect of an increasing pollution uncertainty by adopting the new technology sooner. Uncertainty on solar panels accumulations has a different effect. The role played by uncertainty on the accumulation of solar panels depends again on the value of ε relative to unity. In particular, for $\varepsilon < 1$, a larger σ_S now leads to a larger s_T and such a result is consistent with the existing literature on technology adoption under uncertainty in partial equilibrium. But the effect is reversed for $\varepsilon > 1$. What happens is that more uncertainty on solar panels accumulation unambiguously reduces the value after the switch, but may increase the value before the switch through the option part of the value. This should trigger adoption for a larger s_T; this is what occurs if the agent likes to substitute in time, but it is no longer the case for $\varepsilon > 1$ for which there is no option part in marginal value before the switch.

Moreover, the level of the pollution-adjusted solar panels at which it is optimal to switch is a decreasing function of A_2 and an increasing function of B_2 whatever ε: the more the gain in technology thanks to the switch the smaller the adoption threshold s_T. This results confirm that technological improvement in either sector is an important incentive (absent in the previous section) to switch. It is also clear that s_T is an increasing function of α, and a decreasing function of β. The more important is the polluting resource to produce solar panels after the switch, the later the adoption. The more important is the polluting resource to produce solar panels before the switch the sooner the adoption. Of course, the fact that β is larger than α and that $A_2 > B_2$ provide the central planner with an additional incentive to switch, as solar panels production process is more efficient after the switch and, in particular, it requires less of the polluting resource in their production process. In other words, this sector becomes "greener"".

Our simulations on η are as in Figure 5.5. Notice that s_T is an increasing (decreasing) function of η as long as $\eta \leq 0.5$ ($\eta > 0.5$). This is the result of the constant returns to scale in the production of the consumption good after the switch. If fossil fuels are relatively less important than solar panels to produce consumption, the central planner tends to wait for a larger value of s_T in order to switch to the new technology. This is because the solar panels sector needs to be sufficiently developed to not loosing consumption once the new technology is adopted. But, if fossil fuels are relatively more important than solar panels to produce consumption, switching to the new technology is easier (smoother), and then the incentives to wait for the central planner start vanishing.

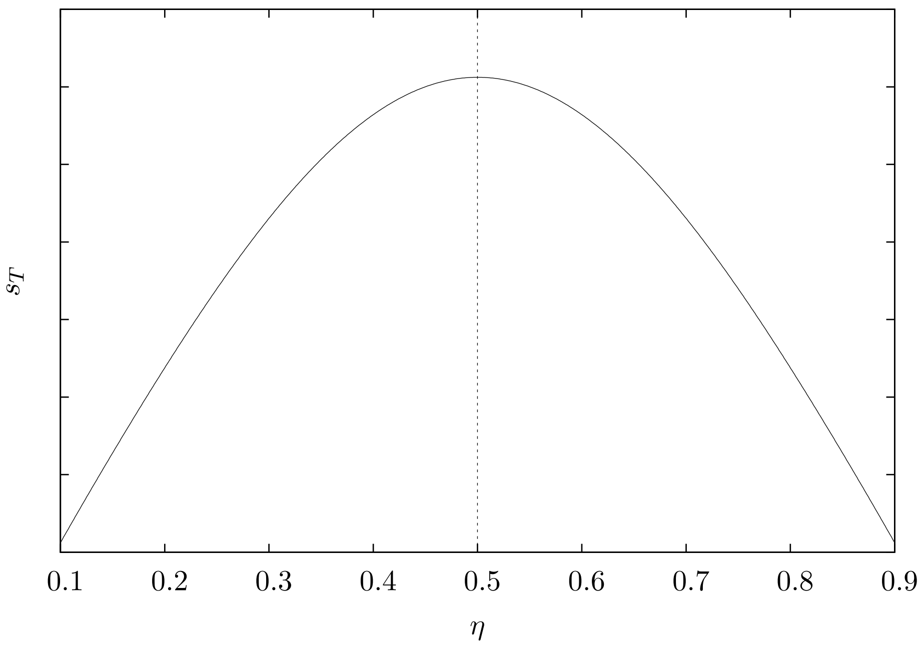

Figure 5.5: The effect of η on the optimal switching level of s_t.

The effect of all of the parameters on s_T is summarized in Table 5.4.

5.6 The optimal switching time, the discounted case

When $\rho \neq 0$ there is no analytical solution to the problem described by equations (5.21) to (5.24). Hence, numerical methods are necessary to calculate the value function in this case. As suggested by Judd [1992, 1998], we can use the approximation properties of Chebyshev polynomials to compute stable non-diverging solution of the Hamilton–Jacoby–Bellman equation (5.21). In this section, we follow Mosiño (2012) in transforming the value function and the given conditions into matrix equations with unknown Chebyshev coefficients. By using this representation, our original problem of solving a partial differential equation reduces to a problem of solving a simple system

Table 5.4: Comparative statics with technological improvement

	$\varepsilon < 1$		$\varepsilon > 1$	
	$f'(\cdot)$	$f''(\cdot)$	$f'(\cdot)$	$f''(\cdot)$
ϕ	≥ 0	$= 0$	≥ 0	≤ 0
ε	≤ 0	≤ 0	≤ 0	≤ 0
σ_S	≥ 0	≥ 0	≤ 0	≤ 0
σ_P	≤ 0	≤ 0	≤ 0	≤ 0
A_1	≤ 0	≥ 0	≤ 0	≥ 0
A_2	≤ 0	≥ 0	≤ 0	≥ 0
α	≥ 0	≥ 0	≥ 0	≥ 0
η	≤ 0	≤ 0	≤ 0	≤ 0
B_1	≥ 0	≥ 0	≥ 0	≥ 0
B_2	≥ 0	≥ 0	≥ 0	≥ 0
β	≤ 0	≥ 0	≤ 0	≥ 0
I	≥ 0	$= 0$	≥ 0	$= 0$

of algebraic equations. Interested readers can also follow Dangl and Wirl [2004], which propose an algorithm using Newton's method.

5.6.1 A numerical approximation of the value function

In the computations that follow we suppress time subscripts as they are not necessary for clarity. Suppose that $\widehat{v}(s) \approx v(s)$ has a Chebyshev series solution of the form:

$$\widehat{v}(s) = \frac{1}{2}\gamma_0 T_0(s) + \sum_{i=1}^{N} \gamma_i T_i(s), \tag{5.29}$$

for $\underline{s} \leq s \leq s_T$. In equation (5.29), $\underline{s}$ is an artificial lower bound for s, and $T_i(s)$, $i = 0, 1, \ldots, N$, is the general i-th Chebyshev polynomial of the first kind. This can be obtained from the recurrence relation:

$$\begin{aligned} T_0(h(s)) &= 1, \\ T_1(h(s)) &= s, \text{ and} \\ T_{n+1}(h(s)) &= 2hT_n(h(s)) - T_{n-1}(h(s)), \end{aligned}$$

where:

$$h(s) = \frac{2s - (\underline{s} + s_T)}{s_T - \underline{s}}. \tag{5.30}$$

In equation (5.29), γ_i, $i = 0, 1, \ldots, N$, are the Chebyshev coefficients to be determined, and $N + 1$ is the degree of approximation. We also assume that:

$$\widehat{\upsilon}^{(n)}(s) = \frac{1}{2}\gamma_0^{(n)} T_0(s) + \sum_{i=1}^{N} \gamma_i^{(n)} T_i(s), \tag{5.31}$$

where $\widehat{\upsilon}^{(n)}(s)$ is the n-th derivative of $\widehat{\upsilon}(s)$ with respect to s, and $\gamma_i^{(n)}$ are also Chebyshev coefficients. Obviously $\gamma_i^{(0)} = \gamma_i$, and $\widehat{\upsilon}^{(0)}(s) = \widehat{\upsilon}(s)$.

Equations (5.29) and (5.31) can also be expressed in matrix form:

$$\widehat{\upsilon}(s) = \mathbf{T}(s)\mathbf{\Gamma}, \tag{5.32}$$

$$\widehat{\upsilon}^{(n)}(s) = 2^n \mathbf{T}(s)(\mathbf{M}^g)^n \mathbf{\Gamma}, \tag{5.33}$$

where:

$$\begin{aligned} \mathbf{T}(s) &= [T_0(s)\ T_1(s)\ \cdots\ T_N(s)], \\ \mathbf{\Gamma} &= \left[\frac{1}{2}\gamma_0\ \gamma_1\ \cdots\ \gamma_N\right]', \end{aligned}$$

and $\mathbf{M}^g$ is as defined in Mosiño (2012).

To obtain a Chebyshev solution of equation (5.21) in the form of (5.32), we first linearise the non-linear equation (5.21):

$$\rho\widehat{\upsilon}_{k+1}(s_i) = \left[\mathbf{b}_1\left(\widehat{\upsilon}_k^{(1)}(s_i)\right)^{-\frac{1}{\varepsilon}} + \mathbf{b}_2 s_i\right]\widehat{\upsilon}_{k+1}^{(1)}(s_i) + \mathbf{b}_3\widehat{\upsilon}_{k+1}^{(2)}(s_i)\, s_i^2, \tag{5.34}$$

where $k = 0, 1, 2, \ldots$ refers to the k-th iteration on equation (5.34). Also:

$$s_i = \frac{\widehat{s_T - \underline{s}}}{2}(h_i + 1) + \underline{s},$$

for $\widehat{s_T}$ being an initial guess of s_T, and h_i being the i-th collocation point defined as:

$$h_i = \cos\left(\frac{i\pi}{N}\right),$$

where $i = 0, 1, \ldots, N$, and π is the standard mathematical constant. We also write the "iterative" version of equation (5.22):

$$\widehat{\upsilon}_{k+1}(\widehat{s_T}) = \mathbf{T}(\widehat{s_T})\mathbf{\Gamma} = \omega(\widehat{s_T} - I). \tag{5.35}$$

Notice that we are not taking the smooth pasting condition, equation (5.23), into account. This condition will be useful only at the end of the process.[1]

To start iterating, we take the following initial guess:

$$\widehat{v}_0(s) = \omega(s - I), \tag{5.36}$$

which satisfies equation (5.35) as long as $s = \widehat{s_T}$. Inserting equation (5.36) into equation (5.34) we get:

$$\begin{aligned} \rho\widehat{v}_1\left(s_i\right) &= \left[\mathbf{b}_1\left(\widehat{v}_0^{(1)}\left(s_i\right)\right)^{-\frac{1}{\varepsilon}} + \mathbf{b}_2 s_i\right]\widehat{v}_1^{(1)}\left(s_i\right) + \mathbf{b}_3\widehat{v}_1^{(2)}\left(s_i\right)s_i^2, &(5.37)\\ \widehat{v}_1(\widehat{s_T}) &= \omega(\widehat{s_T} - I). &(5.38)\end{aligned}$$

The linear differential problem of equations (5.37) and (5.38) can be easily solved by using the Chebyshev matrix method in Mosiño (2012). The resulting approximation $\widehat{v}_1$ is then used to solve:

$$\begin{aligned} \rho\widehat{v}_2\left(s_i\right) &= \left[\mathbf{b}_1\left(\widehat{v}_2^{(1)}\left(s_i\right)\right)^{-\frac{1}{\varepsilon}} + \mathbf{b}_2 s_i\right]\widehat{v}_2^{(1)}\left(s_i\right) + \mathbf{b}_3\widehat{v}_2^{(2)}\left(s_i\right)s_i^2,\\ \widehat{v}_2(\widehat{s_T}) &= \omega(\widehat{s_T} - I),\end{aligned}$$

and so on. In general, the result of the k-th iteration is used to activate the $(k+1)$-th iteration. If the process is convergent, a fixed point will be reached after several iterations. The process is ended when the maximum absolute value of the difference between two consecutive estimates is less than a tolerance error ϵ, i.e.:

$$\widetilde{E}_{k+1} = \max_{\underline{s} \leq s \leq s_T} |\widehat{v}_{k+1}(\widehat{s_T}) - \widehat{v}_k(\widehat{s_T})| \leq \epsilon.$$

Finally, assume that $\widehat{v}_{k+1}$ has reached a fixed point. Hence:

$$\widehat{v}_k(\widehat{s_T}) = \widehat{v}(\widehat{s_T}).$$

The last step is to evaluate our resulting expression by using the smooth pasting condition:

$$\widehat{v}^{(1)}(\widehat{s_T}) = \omega_s(\widehat{s_T} - I). \tag{5.39}$$

If equation (5.39) is satisfied, we conclude that $\widehat{s_T} = s_T$ is the optimal threshold value. Otherwise, we have to guess another value for $\widehat{s_T}$ and start the whole process again.[2]

[1] Also notice that the transversality condition does not play an important role in the computations. We can say that this condition is satisfied as long as the system is stable.

[2] If $\widehat{s_T} = s_T$ is not satisfied, we can find the optimal threshold value by using a simple search algorithm.

5.6.2 Results

Table 5.5: Optimal switching time - Discounted Case

ρ	0.005	0.01	0.015
$\varepsilon < 1$	0.6570	0.9336	1.3263
$\varepsilon > 1$	0.2530	0.2593	0.2656

In our computations we are using the base case parameters of Tables 5.1 and 5.2, and $N = 15$. Figures 5.6 and 5.7 illustrate the particular example of $\rho = 0.01$ in the more general case of technology improvement.

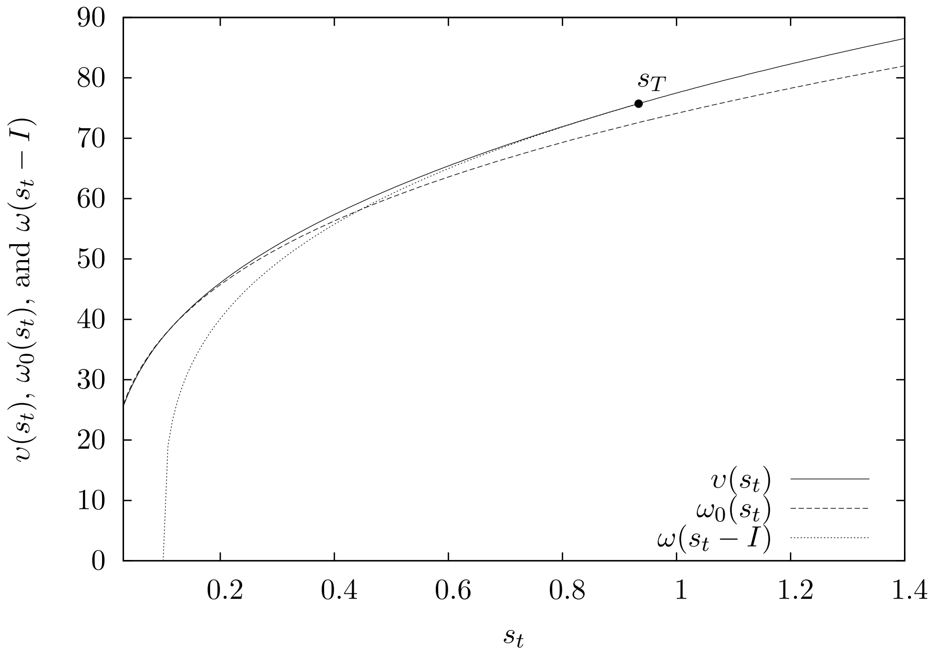

Figure 5.6: Optimal switching level of s_t, $\rho \neq 0$ $\varepsilon < 1$.

By running some simulations we can show that the qualitative results of Section 5.5 remain the same. Then, we focus on the comparative statics with respect to ρ, whose results are shown in Table 5.5. As we can see, the level of the pollution-adjusted solar panels at which it is optimal to switch is an increasing function of ρ: the higher the discount rate, the later the adoption. This result fully reverses the results of previous literature (e.g. Hugonnier et al. [2008], and Charlier et al. [2011]). Our intuition

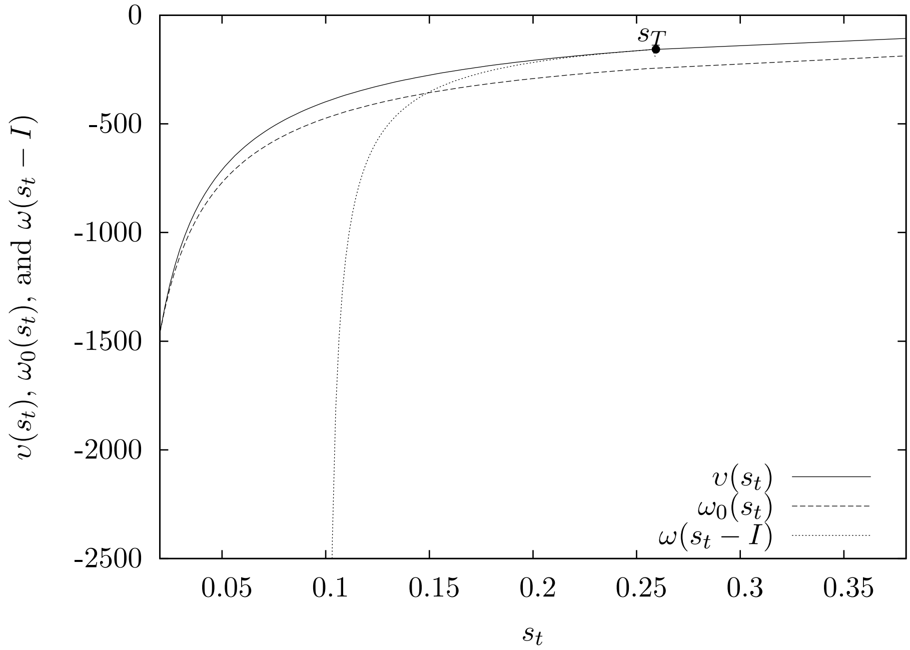

Figure 5.7: Optimal switching level of s_t, $\rho \neq 0$ $\varepsilon < 1$.

suggests that, as the social planner is becoming more concerned about the present, she prefers waiting the solar panels sector to be more developed before switching. This is because before any action, the consumption sector can take advantage of the higher productivity of the polluting resource.

5.7 Conclusions

In this chapter we consider a model in which two sectors interact to produce consumption. The first sector is dedicated to manufacturing a backstop resource —solar panels for instance. At any time, this sector requires both fossil fuels and the energy provided by the backstop already available. The second sector is the one that produces the consumption good. Initially it uses energy coming exclusively from fossil fuels. However, it has always the possibility of switching to a new technology in which energy comes from both types of resources. Using fossil fuels pollutes the economy. We assume that the accumulation of pollution, as well as the accumulation of the backstop, are stochastic.

We also assume that, as this backstop resource is being accumulated, it gets worth paying a fixed and irreversible cost to use the existing stock of new solar panels and avoid —at least partially— the use of the polluting input. With this specification, the economy becomes cleaner after the switch —although not completely clean. Particularly, we account for the fact that even if the new technology is used, fossil fuels are still required in the industry.

We find that the threshold triggering adoption crucially depends on technological parameters. In particular, the incentives to switch to the cleaner technology depend on the relative importance of fossil fuels in the production of consumption goods after the switch. Technological improvement in the solar panels sector is also important in order to switch to cleaner technologies. If the technological change implies that the backstop can be produced with relatively less of the fossil fuels, the adoption occurs sooner. These conclusions are important in terms of economic policy. They imply that policy is to be focused on (i) reducing the dependence of countries on fossil fuels — which is particularly important for oil-dependent developing countries, and on (ii) innovation.

We also find that the effect of uncertainty depends on the existence of technological improvement in the backstop production sector, and on the value of inverse of the effective intertemporal elasticity of substitution relative to unity. If the inverse of the effective intertemporal elasticity of substitution is less than unity, and in the absence of any technological improvement, the cleaner technology is adopted sooner as the uncertainty on the accumulation of solar panels increases. The cleaner technology is adopted later for higher levels of uncertainty on pollution. The former result fully reverses that of the partial equilibrium literature, while the latter is fully consistent with it. If the inverse of the effective intertemporal elasticity of substitution is larger than unity, however, both uncertainties affect the the marginal values before and after the switch in the same way, and then their effects disappear. This is a standard result in general equilibrium settings.

When there is technological improvement the effect of uncertainty is even more important. On the one hand, whatever the effective intertemporal elasticity of substitution, the pollution-adjusted solar panels stock that triggers the switch is a decreasing function of the uncertainty on pollution accumulation. This comes from the fact that the central planner tries to mitigate the bad effect of an increasing pollution uncertainty by adopting the new technology sooner. On the other hand, the uncertainty on solar panels accumulation unambiguously reduces the value after the switch, but may increase the value before the switch through the option part of the value.

An extension to this model seems to be particularly relevant. In this chapter we assume that the increase in the pollution stock is equal to the resource extraction. However, exhaustibility of the resource is not taken into account explicitly. To deal with this we can either (i) include another process for the resource stock, or (ii) bound the pollution process to take into account the fact that the resource stock cannot be negative. This issue is left for future work.

5.A Proof of Proposition 5.1

The Hamilton-Jacobi-Bellman equation after the switch is as in equation (5.7):

$$\rho W\left(S_t, P_t\right) = \max_{\kappa_t, \lambda_t, R_t} \left\{ \frac{\left(C_t P_t^{\phi}\right)^{1-\varepsilon}}{1-\varepsilon} + \mathbb{E}_t\left[W\left(S_{t+dt}, P_{t+dt}\right)\right] \right\} \tag{5.40}$$

Following the usual techniques (see Dixit and Pindyck [1994] for instance), and using equations (5.2), (5.4), and (5.5) we can rewrite equation (5.40) as:

$$\begin{aligned} \rho W &= \max_{\kappa,\lambda,R} \left\{ \frac{\left(CP^{\phi}\right)^{1-\varepsilon}}{1-\varepsilon} + W_S A_2 \left[\alpha(1-\kappa)R + (1-\alpha)(1-\lambda)S\right] \overline{K_2} dt + W_P R \right\} \\ &+ \frac{1}{2}\left[W_{SS}\sigma_S^2 \overline{K_2}^2 S^2 + W_{PP}\sigma_P^2 P^2\right], \end{aligned} \tag{5.41}$$

where time subscripts and arguments have been suppressed for ease of exposition. Subscripts in equation (5.41) represent partial derivatives. The first order conditions are:

$$\left(CP^{\phi}\right)^{1-\varepsilon} \frac{\eta}{R} + W_S A_2 \left[\alpha(1-\kappa)\right] \overline{K_2} + W_P = 0, \tag{5.42}$$

$$\left(CP^{\phi}\right)^{1-\varepsilon} \frac{(1-\eta)}{\lambda} + W_S A_2 \left[(1-\alpha)S\right] \overline{K_2} = 0, \tag{5.43}$$

$$\left(CP^{\phi}\right)^{1-\varepsilon} \frac{\eta}{\kappa} - W_S A_2 \left[\alpha R\right] \overline{K_2} = 0. \tag{5.44}$$

From equations (5.43) and (5.44) we get that:

$$\kappa R = \frac{1-\alpha}{\alpha}\frac{\eta}{1-\eta}\lambda S. \tag{5.45}$$

Combining this result with equation (5.2) we obtain:

$$C = \Theta_A \lambda S, \tag{5.46}$$

where Θ_A is as defined in the main text. From this result and equation (5.43) we also get:

$$\lambda S = \left[\frac{(1-\eta)\left(\Theta_A P^{\phi}\right)^{1-\varepsilon}}{W_S A_2 (1-\alpha)\overline{K_2}}\right]^{\frac{1}{\varepsilon}}. \tag{5.47}$$

Using equations (5.46) and (5.47) — and after a few simplifications , we are able to rewrite equation (5.41) as:

$$\begin{aligned} \rho W &= \mathbf{a}_1 \left(W_s P^{-\phi}\right)^{\frac{\varepsilon-1}{\varepsilon}} + A_2(1-\alpha)\overline{K_2} W_S S \\ &+ \frac{1}{2}\left[W_{SS}\sigma_S^2 \overline{K_2}^2 S^2 + W_{PP}\sigma_P^2 P^2\right]. \end{aligned} \tag{5.48}$$

We now consider the following transformation:

$$W(S,P) \equiv \omega(s);\ s := SP^{\phi}.$$

Then:

$$\begin{aligned} W_s &= \omega_s P^{\phi}, \\ W_P &= \frac{\phi}{P}\omega_s s, \end{aligned}$$

and:

$$\begin{aligned} W_{ss} &= \omega_{ss} P^{2\phi}, \\ W_{PP} &= \phi^2 \omega_{ss}\frac{S^2}{P^2} + \phi(\phi-1)\omega_s \frac{S}{P^2}. \end{aligned}$$

This last expressions allow us to rewrite equation (5.48) as in the main text:

$$\rho\omega = \mathbf{a}_1 \omega_s^{\frac{\varepsilon-1}{\varepsilon}} + \mathbf{a}_2 \omega_s s_t + \mathbf{a}_3 \omega_{ss} s_t^2. \tag{5.49}$$

Solution to equation (5.49) can be easily find to be:

$$\omega = \mathbf{A}\frac{1}{1-\varepsilon}s^{1-\varepsilon}, \tag{5.50}$$

which is equation (5.11) in the main text.

Finally, notice that equation (5.47) can be rewritten as:

$$\lambda S P^{\phi} = \lambda s = \left[\frac{(1-\eta)\Theta_A^{1-\varepsilon}}{\omega_S A_2(1-\alpha)\overline{K_2}}\right]^{\frac{1}{\varepsilon}}.$$

By combining this equation with equation (5.50) we find that:

$$\lambda = \lambda^* = \left[\frac{(1-\eta)\Theta^{1-\varepsilon}}{A_2(1-\alpha)\mathbf{A}}\right]^{\frac{1}{\varepsilon}}$$

is a constant. Direct application of this result on equations (5.45) and (5.46), gives us equations (5.12) and (5.13) in the main text.

5.B Proof of Proposition 5.2

We now use equations (5.1), (5.3), and (5.5) to write the Bellman equation before the switch as:

$$\begin{aligned}\rho V &= \max_{\kappa,R}\left\{\frac{\left(CP^{\phi}\right)^{1-\varepsilon}}{1-\varepsilon} + V_S B_2\left[\beta(1-\kappa)R + (1-\beta)S\right]\overline{K_2}dt + V_P R\right\} \\ &\quad + \frac{1}{2}\left[V_{SS}\sigma_S^2\overline{K_2}^2 S^2 + V_{PP}\sigma_P^2 P^2\right], \end{aligned}\tag{5.51}$$

where again time subscripts and arguments have been suppressed for ease of exposition. Subscripts in equation (5.51) represent partial derivatives. The first order conditions are:

$$\left(CP^{\phi}\right)^{1-\varepsilon}\frac{1}{R} + V_S B_2\left[\beta(1-\kappa)\right]\overline{K_2} + V_P = 0, \tag{5.52}$$

$$\left(CP^{\phi}\right)^{1-\varepsilon}\frac{1}{\kappa} - V_S B_2\left[\beta R\right]\overline{K_2} = 0. \tag{5.53}$$

From equations (5.53) and (5.1) we get that:

$$\kappa R = \left[\frac{\left(B_1\overline{K_1}P^{\phi}\right)^{1-\varepsilon}}{V_S\beta B_2\overline{K_2}}\right]^{\frac{1}{\varepsilon}}, \tag{5.54}$$

and of course:

$$C = B_1 \left[\frac{\left(B_1 \overline{K_1} P^{\phi}\right)^{1-\varepsilon}}{V_S \beta B_2 \overline{K_2}} \right]^{\frac{1}{\varepsilon}} \overline{K_1}. \tag{5.55}$$

These results allow us to rewrite equation (5.51) as:

$$\begin{aligned} \rho V &= \mathbf{b}_1 \left(V_s P^{-\phi} \right)^{\frac{\varepsilon-1}{\varepsilon}} + B_2(1-\beta)\overline{K_2} V_S S \\ &+ \frac{1}{2} \left[V_{SS} \sigma_S^2 \overline{K_2}^2 S^2 + V_{PP} \sigma_P^2 P^2 \right]. \end{aligned} \tag{5.56}$$

We now consider the following transformation:

$$V(S,P) \equiv \upsilon(s); \; s := SP^{\phi}.$$

Then:

$$\begin{aligned} V_s &= \upsilon_s P^{\phi}, \\ V_P &= \frac{\phi}{P} \upsilon_s s, \end{aligned}$$

and:

$$\begin{aligned} V_{ss} &= \upsilon_{ss} P^{2\phi}, \\ V_{PP} &= \phi^2 \upsilon_{ss} \frac{S^2}{P^2} + \phi(\phi-1) \upsilon_s \frac{S}{P^2}. \end{aligned}$$

This last expressions allow us to rewrite equation (5.56) as in the main text:

$$\rho \upsilon = \mathbf{b}_1 \upsilon_s^{\frac{\varepsilon-1}{\varepsilon}} + \mathbf{b}_2 \upsilon_s s_t + \mathbf{b}_3 \upsilon_{ss} s_t^2. \tag{5.57}$$

If an option to switch is not available, we redefine:

$$\upsilon(s) = \omega_0(s),$$

and equation (5.57) can be solved directly. Solution can be easily found to be:

$$\upsilon = \mathbf{B} \frac{1}{1-\varepsilon} s^{1-\varepsilon}, \tag{5.58}$$

which is equation (5.16) in the main text.

Finally, using equation (5.58) on equations (5.54) and (5.55) we get that:

$$\begin{aligned} c_t^* &= B_1 \Theta_B s_t, \\ (\kappa r_t)^* &= \Theta_B s_t, \end{aligned}$$

as in the main text.

5.C Proof of Propositions 5.3 and 5.4

The marginal value of the adjusted solar panels —equation (5.25) in the main text— is:

$$v_s(s_t) = \left(\frac{D_1}{s_t} + G(s_t, s_T)\right)^{\varepsilon}. \tag{5.59}$$

Equations (5.11) and (5.16), allow us to rewrite equation (5.59) as:

$$v_s(s_t) = \left(\left[\omega_0'(s_t)\right]^{\frac{1}{\varepsilon}} + \left(\left[\omega'(s_T - I)\right]^{\frac{1}{\varepsilon}} - \left[\omega_0'(s_T)\right]^{\frac{1}{\varepsilon}}\right)\left(\frac{s_t}{s_T}\right)^{D_3}\right)^{\varepsilon}, \tag{5.60}$$

where:

$$G(s_t, s_T) = \left(\left[\omega'(s_T - I)\right]^{\frac{1}{\varepsilon}} - \left[\omega_0'(s_T)\right]^{\frac{1}{\varepsilon}}\right)\left(\frac{s_t}{s_T}\right)^{D_3},$$

is the part of the value function due to the option to switch.

5.C.1 Proof of Proposition 5.3

If $\varepsilon < 1$, it can be easily deduced that:

$$\omega_0'(s_t) \leq \omega'(s_t) < \omega'(s_t - I).$$

This implies that $G(s_t, s_T) > 0$, and hence $v_s(s_t)$ (> 0) is always defined. Then the stock of the pollution-adjusted solar panels can be found by integrating equation (5.59) and using the value matching condition, equation (5.22) in the main text.

5.C.2 Proof of Proposition 5.4

If $\varepsilon > 1$, the sign of $G(s_t, s_T)$ is ambiguous:

- Assume first that $G(s_t, s_T) < 0$. In this case, there is a value s_{inf}:

  $$s_{\text{inf}} = \left(\frac{D_1}{D_2}\right)^{\frac{1}{D_3 - 1}},$$

  such that $v_s(s_t) < 0$, and hence the program is not longer defined —because a contradiction with the smooth pasting condition, equation (5.23) in the main text. Then, this case cannot be considered.

- Now, assume that $G(s_t, s_T) > 0$. In this case the marginal value function is positive as required. However, by integrating this we get that $v(s_t) > 0$, which is a contradiction with the value matching condition —equation (5.22) in the main text. Then this case cannot be considered either.
- $G(s_t, s_T) = 0$ ensures both that $v(s_t) > 0$ and that the value matching condition can be satisfied. Then, this constitutes the sole case we can consider. Equation (5.27) and s^* in the main text can be easily found by integrating $v_s(s_t)$ and using the value matching condition, equation (5.22).

6

Conclusion

This dissertation has been devoted to modeling the determinants of investing in reducing greenhouse gases (GHG) through improving energy efficiency and substituting non-renewable resources (fossil fuels) by renewable resources. We have first presented a general equilibrium framework in which we try and explain the slow diffusion of energy efficient investments. In particular, we consider the specific case of a homeowner who may invest in new insulation, or double glazing in order to reduce her energy bill. This investment is irreversible; there is also uncertainty over the benefits of such energy-saving technologies, and over the financial returns on savings. We show that the threshold triggering adoption depends not only on technological parameters but on preference parameters as well. In particular, the higher the risk aversion parameter, the smaller the level of wealth which is required for adoption. We also show that while uncertainty on energy-saving technologies efficiency hardly affects adoption timing, uncertainty on financial returns fosters it. By considering the arbitrage between consumption and adoption, we manage to challenge the existing result of previous literature that remains in partial equilibrium: the existence of an option value does not rule out the energy paradox.

We also presented two models on the substitution of resources. In the first model we consider two different resources —fossil fuels, and renewable resources— that are perfect substitutes for energy production. The stocks of both resources are stochastic. In this model, firms start producing energy using only fossil fuels, but the possibility to carry out an irreversible investment to switch to the other input is always open. We find that uncertainty plays a clear role in the decision to switch. The more the uncertainty on the availability of the non-renewable resources, the sooner the firms switch to the renewable resources; and the more the uncertainty on the availability of renewable resources, the later the switching time. The optimal switching time is also sensitive to energy demand, costs, and the relative productivity of resources parameters. These later results have some implications for economic and environmental policy. For instance, the government can accelerate the substitution of renewable for non-renewable resources in energy production by increasing the marginal cost of using non-renewable resources, through a tax, or by decreasing the cost of switching through a subsidy. The government may also implement measures to reduce energy demand, or apply policies to increase the productivity of renewable resources with respect to nonrenewable resources (e.g. through innovation). The effectiveness of any of these policies also depends on the degree of uncertainty surrounding the economy.

In the second model on the substitution of resources we consider a different scenario. Our theory is that as long as renewable energies are not very advanced and widespread, (i) industry will still need a percentage of energy that derives from dirty resources, and (ii) the provision of clean energy itself will require dirty resource at least as materials to build the plants. Particularly the economy has access to two different energy sources. The first one comes from a natural polluting resource, such as fossil fuels. The second comes from a backstop natural resource, such as solar radiation. There are two productive sectors in the economy. The first one is dedicated to manufacturing the backstop resources. At any time, this sector requires both fossil fuels and the energy provided by the backstop already available. We therefore account for the need of fossil fuels to provide clean energy. The second sector is devoted to production of the consumption good. Initially it uses energy coming exclusively from fossil fuels. However, it has always the possibility of switching to a new technology in which energy comes from both types of resources. The backstop accumulation and pollution are both stochastic, and the adoption of the cleaner technology imposes sunk costs on the consumption sector. We find that the effect of uncertainty depends on the existence of technological improvement in the backstop production sector, and on the value of inverse of the effective intertemporal elasticity of substitution relative to unity.

We also show that the threshold triggering adoption crucially depends on technological parameters. In particular, the incentives to switch to the cleaner technology depend on the relative importance of fossil fuels in the production of consumption goods after the switch. Technological improvement in the solar panels sector is also important in order to switch to cleaner technologies. If the technological change implies that the backstop can be produced with relatively less of the fossil fuels, the sooner the adoption. These conclusions are important in terms of economic policy. They imply that policy to speed up clean energy adoption is to be focused on (i) reducing the dependence of countries on fossil fuels —which is particularly important for oil-dependent developing countries, and on (ii) innovation.

As a result of the extensive use of numerical methods, another contribution of this dissertation is a methodology for solving non-linear Hamilton-Jacobi-Bellman equations. Particularly, we transform the value functions —partial differential equations— and the given conditions into expressions with unknown Chebyshev coefficients. In this case, we reduce our original problem of solving partial differential equations to one of solving simple systems of —algebraic— non-linear equations.

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Abstract

Greenhouse gases (GHG) are responsible for some climate change. Humanity faces a choice: either reducing the emissions of these gases or adapt to climate change. In this dissertation we focus on the first solution under the premise that a large part of the greenhouse effect comes from human activities. More precisely, we propose some essays on modeling the determinants of investing in reducing GHG through improving energy efficiency and substituting non-renewable resources (fossil fuels) by renewable resources. We first try and explain the slow diffusion of some energy efficient investments in a general equilibrium framework. We then study the determinants of switching from non-renewable resources to renewable resources when these are perfect substitutes. Finally, we account for the need of dirty resources even if cleaner technologies are available. All these issues are based on models that cannot be fully solved analytically, therefore we also propose in this dissertation a methodology based on the properties of Chebyshev polynomials to compute the solutions.

Keywords: Greenhouse gases; Irreversible investment; Uncertainty; Natural resources; Numerical solutions

Résumé

Les gaz à effet de serre (GES) sont en partie responsables du changement climatique. L'humanité est donc confrontée à un choix : soit de réduire les émissions des gaz qui sont la cause du problème, ou bien de prendre de mesures pour permettre aux populations de surmonter les conséquences de ces changements. Dans cette thèse, nous nous concentrons sur la première solution sous la prémisse selon laquelle une grande partie de l'effet de serre provient des activités humaines. Plus précisément, nous proposons quelques essais sur la modélisation des déterminants des investissements ayant pour objet la réduction des GES, en particulier des investissements dans l'amélioration de l'efficacité énergétique, et des investissements dans la substitution de ressources (fossiles) non-renouvelables par des ressources renouvelables. Tout d'abord nous essayons et expliquons la lente diffusion de certains investissements dans l'efficacité énergétique dans un cadre d'équilibre général. Ensuite, nous étudions les déterminants de la substitution des ressources non-renouvelables par des ressources renouvelables lorsque celles-ci sont des substituts parfaits. Enfin, nous tenons compte de la nécessité permanente de ressources sales, même si des technologies plus propres sont disponibles. Toutes ces questions sont basées sur des modèles qui ne peuvent être entièrement résolus analytiquement. Par conséquent, nous proposons dans cette thèse une méthodologie basée sur les propriétés des polynômes de Chebyshev pour calculer les solutions.

Mots-clés : Gaz à effet de serre; Investissement irréversible; Incertitude; Ressources Naturelles; Solutions numériques