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Gigahertz laser resonant ultrasound spectroscopy for the evaluation of transverse elastic properties of micrometric fibers

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Abstract

The cross section eigenmodes of micrometric cylinders were measured in the range of several tenth of MHz to about 0.5 GHz. The vibrations were excited using subnanosecond laser pulses. The cross section eigenmodes were simulated using finite element modeling in a 2D geometry. Using the method of resonant ultrasound spectroscopy, the vibration spectrum of an aluminum wire of diameter 33 μm served to determine the transverse Young modulus and Poisson ratio with a precision of 0.7% and 0.3%, respectively. The calculated and measured frequencies of cross section eigenmodes were fitted with a precision better than 0.5% in the 50-500 MHz range.

Keywords:

fiber, laser ultrasonics, resonant ultrasound spectroscopy (RUS), finite element modeling, laser interferometry, cylinder, eigenmodes, Rayleigh modes, wave gallery modes, design of experiment (DOE)

1. Introduction

Resonant ultrasound spectroscopy (RUS) has been extensively used during the past two decades to evaluate the elastic constants of various anisotropic solid materials with an ul-

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4 mate precision [1–4]. From the measurement of the mechanical resonance frequencies of a
5 sample, the elastic constants of the material can be determined.

6 The PZT-RUS method, *i.e.* the RUS technique using piezoelectric transducers for both
7 the excitation and the measurement of vibrations, is the most widely used technique. The
8 optimum size of the sample for PZT-RUS is about 1 cm, but the technique can be used
9 for small samples whose dimensions are of the order of 0.5 mm with specially designed
10 transducers [3]. For such small samples, the RUS technique may require the measurement
11 of resonance frequencies up to 50 MHz [5].

12 However, even in the case of samples of several millimeters in size, contact forces between
13 the ultrasonic transducers and the sample may lead to spurious frequency shifts for the
14 resonance frequencies in addition to the widening of resonances [6–8]. The problem of
15 contacts comes up more and more crucial as the size of samples decreases.

16 The suppression of contact forces by using lasers for both the excitation and the detection
17 of vibrations results in getting sharp resonances, which is beneficial to the accuracy of
18 measurement [7, 8]. The excitation of vibrations can be carried out by using a periodic
19 laser modulation, whose frequency is swept over the required range [9]. Alternatively, short
20 laser pulses can be used to excite the vibrations of the sample. The advantage of the pulsed
21 method lies in the possibility of exciting simultaneously many eigenmodes over a very large
22 frequency range. The broad band excitation provided by short optical pulses is particularly
23 appropriate to the study of very small samples. The vibrations are measured in the time
24 domain and the eigenfrequencies are obtained from the Fourier transform of the measured
25 signal. The pulsed Laser-RUS method is preferable to determine the elastic constants of
26 small samples [7, 8, 10, 11] or to detect flaws in a small composite structure [12–15]. In this
27 paper, we present a pulsed LRUS technique applied to the evaluation of elastic constants of
28 micrometric fibers, whose diameters are in the range 5–50 μm . For such small dimensions,
29 the eigenfrequencies must be measured in the 0.05–1 GHz range.

30 The LRUS allows one to evaluate the elastic constants on a single fiber, in contrast
31 with indirect methods of evaluation [16]. From the evaluation of the elastic constants of a
32 unidirectional fiber composite, the elastic constants of the fibers are calculated. This indirect

33 method is based on specific assumptions about the nature of the reinforcing effect of fibers
34 in the polymer matrix. These assumptions may not be valid for all kinds of reinforced
35 polymers, so the application of this method may lead to erroneous results. Therefore, it is
36 is much more reliable to evaluate the elastic constants directly on a single fiber.

37 A direct evaluation of elastic constants of carbon fibers by a non-contact technique was
38 first achieved using the technique of laser picosecond ultrasonics [17]. The transverse elastic
39 constant C_{11} of carbon fibers was evaluated through the measurement of the time of flight of
40 acoustic pulses that propagate back and forth across a fiber diameter, which determines the
41 longitudinal acoustic velocity and then the C_{11} elastic constant. However, the evaluation of
42 the C_{44} elastic constant, which requires the measurement of the transverse acoustic velocity,
43 could not be achieved because of the difficulty to generate and to detect picosecond transverse
44 acoustic waves.

45 Fibers are commonly used in reinforced polymers, such as glass or carbon fibers. The
46 interest of the method is great for anisotropic fibers (carbon and vegetal fibers) because
47 of the lack of reliable and accurate methods for the evaluation of the elastic constants of
48 fibers, in particular the elastic constants characterizing the transverse elastic properties.
49 The evaluation of the longitudinal Young modulus of fibers is generally carried out using
50 the tensile test method [18, 19]. But this method is impracticable to measure the transverse
51 elastic properties.

52 We illustrate the application of the LRUS method to the evaluation of the transverse
53 Young modulus and Poisson ratio of a micrometric aluminum wire. The evaluation of the
54 transverse elastic properties of fibers requires the excitation of the *cross section* eigenmodes.
55 In our approach, the propagation of guided acoustic waves along the fiber axis is neglected,
56 so the mechanical displacements are restricted to a plane perpendicular to the fiber axis.
57 The restriction in 2D of the cylinder vibrations results in a discrete vibration spectrum,
58 which is required for the application of the RUS method. In section 2, we will describe the
59 experimental method used to excite and detect cross-section modes of a fiber. In section 3,
60 we will present the method used to solve the inverse problem, which leads to the evaluation
61 of the transverse elastic properties of an aluminum fiber. We will also discuss the conditions

62 of validity RUS applied to a cylinder.

63 **2. Measurement of ultrasonic vibrations**

64 *2.1. Laser ultrasonic setup*

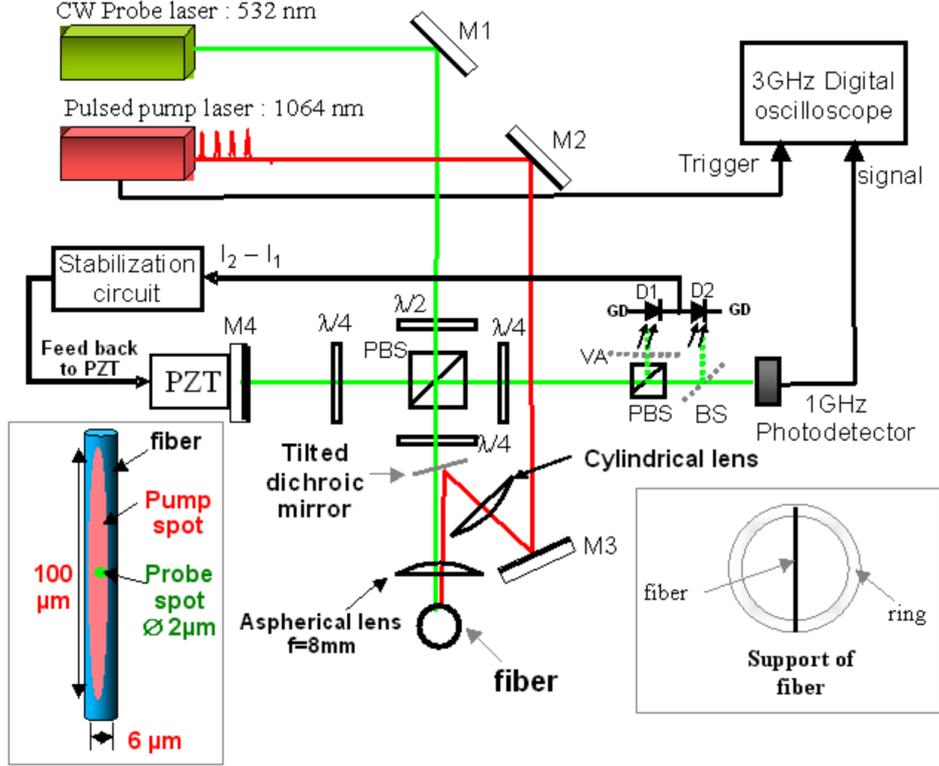
65 The irradiation of a cylinder with a laser pulse can excite many acoustic guided modes.
66 The propagation of guided modes along the cylinder axis is determined by the dispersion
67 relation $\omega(k)$, where k is the propagation wave vector along the cylinder axis and $\omega/(2\pi)$
68 is the frequency of the guided mode. The dispersion relation determines both the phase
69 velocity $v_\phi = \omega/k$ and the group velocity $v_g = d\omega/dk$ of the guided mode.

70 Both the geometry of the laser spot focused on the cylinder surface and the duration
71 of the laser pulse determine the spectral range of guided modes that can be excited. If
72 the pump beam is focused to form a linear spot parallel to one of the generating line of
73 the cylinder, then a linear acoustic source will be formed and a cylindrical acoustic wave
74 propagating perpendicularly to the cylinder axis will be generated. Consequently, a linear
75 acoustic source parallel to the cylinder axis, which is very long compared to the fiber diameter
76 will favor the excitation of guided modes which very small wave vectors $k \approx 0$, *i.e.* with
77 very large wavelengths $\lambda = 2\pi/k$ compared to the fiber diameter. Moreover, such a linear
78 acoustic source will force the displacements in the directions perpendicular to the cylinder
79 axis and the displacements will be almost independent of the z -coordinate around the center
80 of the acoustic source.

81 Figure 1 shows the LRUS setup, which includes two parts: the first part is devoted to the
82 excitation of vibrations and the second part, the interferometer, serves to measure ultrasonic
83 vibrations. The vibrations of fibers are excited using a Q-switched Nd:YAG microchip laser
84 at 1064 nm from *TEEM photonics* whose pulses have a maximum energy of 10 μ J and a
85 duration of 0.6 ns at a maximum repetition rate of 5 kHz. With a 0.6 ns pump pulse, it is
86 possible to generate vibrations up to the 1-2 GHz frequency range, which is sufficient to
87 excite many eigenmodes of a fiber whose diameter is in the 5-50 μ m range.

88 The fibers are mounted on a 1 cm diameter plastic ring (Inset of Fig. 1, at the bottom
89 right). In order to excite acoustic waves which propagate mainly perpendicularly to the

Figure 1: Experimental setup of laser ultrasonic resonance spectroscopy (L-RUS) and support of the fiber. BS: beam splitter, PBS : polarizing beam splitter, VA: variable attenuator, $\lambda/4$: quarter-wave plate, $\lambda/2$: half-wave plate, M1, M2, M3, M4: mirrors, PZT: piezoelectric actuator, GD: ground potential.



90 fiber axis, the length of the pump spot must be much larger than the fiber diameter. The
 91 pump laser is therefore shaped to form a narrow line, parallel to the fiber axis. By using a
 92 cylindrical lens placed in the optical path of the pump beam, we obtain an elliptical spot
 93 with dimensions $6 \mu\text{m} \times 100 \mu\text{m}$ at the fiber surface, with the main axis of the ellipse aligned
 94 with the fiber axis (see left inset of Fig. 1). By a proper adjustment of the pulse energy with
 95 an variable attenuator (not represented on Fig. 1), the vibrations of the fiber can be excited
 96 in the non destructive thermoelastic regime [20].

97 For the measurement of radial ultrasonic vibrations, a stabilized homodyne Michelson
 98 interferometer is used [20]. The probe beam at 532 nm is focused onto the fiber surface using
 99 an aspherical lens (x20) having 8 mm focal length and 0.5 numerical aperture. The resulting
 100 focused spot centered on the pump line has a diameter of about $2 \mu\text{m}$ (inset of Fig. 1).
 101 An incident probe power of 1-2 mW is sufficient to achieve a measurement sensitivity of

102 1 mV nm⁻¹ on a metallic surface.

103 The optical interferometric signal is received by a high-speed photodetector with a band-
104 pass of 1 GHz. Using a digital oscilloscope with a band-pass of 3 GHz, the signals are
105 recorded in synchronization with the pump pulses during about 2 μs after the pump laser
106 pulse. Thanks to the averaging of 10 000 acquisitions, which necessitates a only two seconds
107 at the pulse repetition rate of 5 kHz, we obtain a detection threshold of about 5 pm for the
108 measurement of out-of-plane displacements, which is sufficient to detect the vibrations with
109 a satisfactorily signal to noise ratio.

110 The high frequency ultrasonic vibrations are superimposed to a slow and huge displace-
111 ment of the fiber; a displacements of about 5-10 nm which is probably induced by the sudden
112 expansion of the air in contact with the heated fiber surface, thus inducing the recoil of the
113 fiber. This slow drift of the signal is filtered out in order to magnify the high frequency
114 vibrations corresponding to the fiber *cross-section eigenmodes*. The vibration spectrum is
115 obtained with 0.5 MHz resolution by calculating the Fast Fourier Transform (FFT) of the
116 digitized signal (Fig. 2).

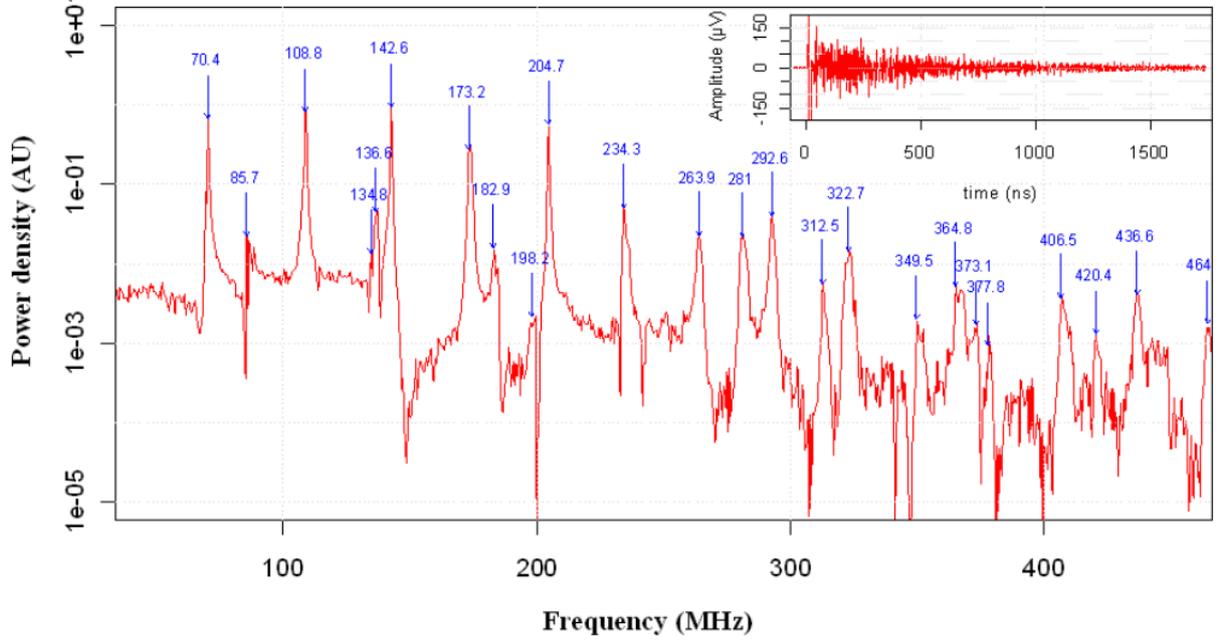
117 2.2. *Vibration spectrum of an aluminum wire*

118 The test sample is an aluminum bonding wire with 1% silicon, manufactured by *HER-*
119 *AEUS* [21]. With a scanning electron microscope (SEM), we measured the wire diameter
120 at several points of the wire. A mean diameter of (32.7 ± 0.1) μm was determined over a
121 length of 1 cm.

122 The ultrasonic measurements were made at several points each millimeter over a wire
123 length of 5 mm.

124 Figure 2 shows one of the measured vibration spectra of the aluminum wire. The inset
125 of Fig. 2 represents the high frequency ultrasonic component used to calculate the vibration
126 spectrum. Each eigenfrequency can be determined with an accuracy of about 0.5 MHz. For
127 each spectral line, the mean frequency and standard deviation were calculated from the
128 vibration spectra measured at different points of the wire. The standard deviations was
129 found of the order of the spectral resolution 0.5 MHz. The repeatability of the vibration

Figure 2: Eigenfrequency spectrum of an aluminum wire of diameter $32.7\ \mu\text{m}$ calculated from the signal shown in the inset, which shows the high frequency component of the recorded signal, corresponding to the vibrations of the wire cross-section.



130 spectra assesses the constancy of the wire diameter over a length of 5 mm.

131 The next step before determining the transverse Young modulus and Poisson ratio of the
 132 aluminum wire is the identification of eigenmodes.

133 3. Evaluation of the transverse elastic properties of an aluminum fiber

134 3.1. Two-dimensional finite element modeling

135 In the following, we will focus our attention on $k = 0$ guided modes, whose displacements
 136 are restricted within the xy -plane. From the experimental conditions depicted before, we
 137 expect that the excited eigenmodes are mainly the $k = 0$ guided modes. We will refer to such
 138 guided modes as *cross-section* modes. The eigenfrequencies of cross-section modes depend
 139 only on the transverse Young modulus and Poisson ratio of the cylinder. Consequently,
 140 from the measured eigenfrequencies, the transverse Young modulus and Poisson ratio can
 141 be determined.

142 A 2D mechanical model of the aluminum wire is carried out. It is assumed that the
 143 elastic properties are homogeneous and invariant by rotation around the fiber axis, *i.e.* the
 144 mechanical properties of the material are *transverse isotropic*. Thus, the eigenmodes are
 145 determined by: the cylinder diameter d , the Young modulus E , the Poisson ratio ν and
 146 density ρ . The vibration eigenmodes of an aluminum wire are determined with the software
 147 *COMSOL Multiphysics*. The mode shapes of the first 20 eigenmodes are illustrated in Fig.
 148 3.

149 3.1.1. Nomenclature of modes

150 The guided modes of a cylinder are named according to the nomenclature of Silk and
 151 Bainton [22]. The modes are labeled $X(m, n)$ (or simply Xm, n as in Fig. 4 or Table 1)
 152 where $X = L, T,$ or F and m, n are positive integers. The letters L, T or F are the initials of
 153 *longitudinal, torsional* and *flexural* modes, respectively. The number m of a $X(m, n)$ mode
 154 characterizes the rotational symmetry of the mode shape and n the ordering of the Xm
 155 mode in the frequency scale. In cylindrical coordinates (r, θ, z) , the u_r and u_θ components
 156 of the displacement field vary according to the functions $\cos m\theta$ or $\sin m\theta$, which means that
 157 the mode shape is invariant after a rotation of $2\pi/m$ rad around the cylinder z -axis. The
 158 longitudinal and torsional modes, which have both a rotational symmetry for an arbitrary
 159 rotation angle around the cylinder axis have $m = 0$. The *longitudinal* and the *torsional*
 160 modes are characterized by the displacements $u_\theta = 0$ and $u_r = 0$, respectively. The modes
 161 with $m > 0$ are *flexural modes*.

162 According to Victorov [23, 24], the cross-section modes belong to one of the two cat-
 163 egories: the *Rayleigh modes* (R) or the *Wave Gallery modes* (WG). The *Rayleigh modes*
 164 are flexural modes of the $n = 0$ series, which we denote more explicitly $R(m, 0)$ instead of
 165 $F(m, 0)$. The Rayleigh modes series starts with the $R(2, 0)$ mode, as the $R(1, 0)$ mode is
 166 the trivial Rayleigh mode corresponding to the translation of the cylinder along a radial
 167 direction, having thus the frequency zero. The *WG modes* are denoted $WG(m, n)$ instead of
 168 $F(m, n)$, with $n > 0$ and $m > 0$. The first *longitudinal mode*, denoted $L(0, 1)$, is often called
 169 the *breathing mode*. The first *torsional mode* is denoted $T(0, 1)$. The $L(0, 0)$ and $T(0, 0)$

170 modes are trivial modes with their eigenfrequency equal to zero.

171 The distinction between *Rayleigh* and some series of *Wave Gallery* modes will be useful
172 to consider in the perspective of mode identification, as all Rayleigh modes have significant
173 radial components u_r and weak orthoradial components u_θ at the surface of the cylinder.
174 On the contrary, the $n = 1$ series of Wave Gallery modes is characterized by weak radial
175 components compared to the orthoradial ones and the displacements are mainly concentrated
176 in the vicinity of the surface. The displacements for WG modes of the series $n > 1$ may
177 have very weak components at the surface. In contrast, for some particular WG modes (for
178 example see mode 12 in Fig. 3 and Table 1), the magnitude of the radial displacement
179 may be significant. Considering the correlation between the relative magnitude of radial
180 displacements predicted by the modeling and the amplitudes of the spectral lines may help
181 to identify the modes (cf. section 3.2).

182 The longitudinal modes $L(0, n)$ and the torsional modes $T(0, n)$ are not degenerate.
183 On the contrary, the $m > 0$ modes are two-fold degenerate. This property arises from the
184 existence of the two angular functions $\cos m\theta$ and $\sin m\theta$, expressing the angular dependence
185 of a mode shape. The two degenerate mode shapes can be deduced from each other by a
186 rotation of $\pi/(2m)$ rad. If a fiber deviates a little from the circular symmetry, a degeneracy
187 lifting may occur. If the spectral resolution is sufficient, it could be possible to appreciate an
188 elliptical shape of the fiber cross-section. In the spectrum of Fig. 2, the absence of doublets
189 in the spectrum demonstrates the almost perfect circular cross-section of the aluminum wire.

190 3.2. Modes identification

191 The RUS technique requires the correct mode identification of each measured eigenfre-
192 quency. This can be done by determining experimentally the mode shapes, which can be
193 then compared to the calculated ones [10, 25]. A raster-scanning of the probe laser on the
194 surface can image the mode shape.

195 However, in the case of a non-planar surface, like a fiber, it is not possible to scan
196 the probe laser and thus to determine experimentally the mode shapes. Nevertheless, the
197 digital holography technique could be an alternative to determine experimentally the mode

198 shapes of fibers [26, 27], but this technique is not yet implemented for small object as fibers.
 199 therefore, we will try to guess which eigenmode corresponds to each measured eigenfrequency.

200 The Rayleigh eigenmodes are characterized by strong radial components at the surface
 201 and would thus be more likely attributed to the strongest lines of the spectrum. Indeed,
 202 Raleigh modes can be observed in the spectrum up to the order $m = 15$ (eigenfrequency at
 203 464 MHz). Conversely, WG modes which have predominant orthoradial components at the
 204 surface would correspond to spectral lines of smaller amplitude. The torsional modes, with
 205 zero radial components, are missing modes.

206 Eigenfrequencies up to about 200 MHz are well separated in the spectrum and can be
 207 identified without ambiguity, except for the two close lines at 135 MHz and 137 MHz, which
 208 may be attributed either to the first longitudinal mode $L(0, 1)$ (the breathing mode) or to
 209 the $WG(2, 1)$ mode. The strongest line at 137 MHz would correspond more likely to the
 210 $L(0, 1)$ mode (mode 5), as this mode has exclusively radial components. The $WG(2, 1)$
 211 mode (mode 4 in Fig. 3) displays very weak radial components, so it would thus more likely
 212 corresponds to the weakest spectral line.

213 The mode identification can be carried out without ambiguity for 9 eigenfrequencies
 214 among the first 11 eigenfrequencies shown in Table 1. The values in Table 1 corresponds
 215 to the mean experimental frequencies measured on several spectra. Most of the frequencies
 216 displayed in Fig. 2 are close to the frequencies of Table 1 within 0.5 MHz.

217 3.3. Solving the inverse problem

218 In order to determine the mechanical parameters, we have to search the set of mechanical
 219 parameters which minimizes the root mean square of the distance between the experimental
 220 frequencies f_i^{exp} and the calculated ones f_i^{calc} , which is:

$$\sigma_{res} = \sqrt{(1/N) \sum_{i=1}^N (f_i^{exp} - f_i^{calc})^2} \quad (1)$$

221 A non linear regression algorithm (Gauss-Newton) where N is the number of the exper-
 222 imental frequencies that are considered for the fit. Only frequencies below 210 MHz are

223 considered, as these lines can be identified without ambiguity. Eigenfrequencies beyond
 224 210 MHz have been omitted, as the overlap of some unresolved spectral lines may lead to
 225 an ambiguous identification. Only 9 eigenfrequencies among the 11 below 210 MHz can be
 226 identified unambiguously. We do not consider the weak line around 135 MHz, probably the
 227 $WG(2, 1)$ mode, which is visible only a few times among all the measured spectra, and thus
 228 this line cannot be measured accurately. The torsional $T(0, 1)$ mode, which is missing in the
 229 spectra, is not considered.

230 Solving an inverse problem requires in general many iterations. For each iteration, the
 231 eigenfrequencies are calculated for a set of input parameters. The use of finite element cal-
 232 culation is not very practicable for iterative processes. For a cylindrical fiber, an analytic
 233 calculation of eigenfrequencies would be probably the best method to calculate the eigen-
 234 frequencies. However, we preferred to use a more general method which can be applied to a
 235 fiber of arbitrary cross-section shape. In order to calculate quickly the eigenfrequencies for
 236 a given set of the parameters E , ν and ρ , we determined first the coefficients of polynomial
 237 interpolation functions, which can be used to calculate the eigenfrequencies. For a prede-
 238 termined diameter d_0 , each interpolation function can be expressed in the form of a Taylor
 239 series of the three variables E , ν and ρ , which can be express in the form:

$$f_i(E, \nu, \rho) = \sum_{\alpha, \beta, \gamma} C_i^{\alpha \beta \gamma} (E - E_0)^\alpha (\nu - \nu_0)^\beta (\rho - \rho_0)^\gamma \quad (2)$$

240 where $E_0 = 70$ GPa and $\nu_0 = 0.33$ and $\rho_0 = 2700$ kg/m³.

241 A second-order Taylor series is sufficient to get an accurate prediction of the frequencies,
 242 so that only 10 coefficients are determined, those verifying the inequality $\alpha + \beta + \gamma \leq 2$. For
 243 each frequency i , the 10 coefficients $C_i^{\alpha \beta \gamma}$ were determined from a set of 15 finite element
 244 models with the parameters (E, ν, ρ) suitably chosen inside the 3-dimensional domain \mathbf{D}
 245 defined by the intervals: $E = (70 \pm 3)$ GPa, $\nu = 0.33 \pm 0.03$ and $\rho = (2700 \pm 75)$ kg/m³.
 246 The method of *design of experiment* (DOE) for quadratic models [28] was used for this
 247 purpose.

248 The interpolation functions of Eq. 2 are used to predict the eigenfrequencies with an

249 error inferior to 0.05 MHz in the **D**-domain, which is sufficient compared to the experimental
 250 uncertainties.

251 For a different diameter d of the cylinder, the eigenfrequencies were calculated by first
 252 using Eq. 2 and by multiplying the eigenfrequencies vector by the scale factor d_0/d .

253 3.4. Results

254 With the 9 interpolation functions, we searched the set of parameters (E, ν, ρ) which
 255 minimizes the standard residual σ_{res} . For the first run, we fixed the diameter, the Young
 256 modulus and the density: 32.7 μm , 70.0 GPa and 2700 kg/m³. We found that the best fit is
 257 obtained with $\sigma_{res} = 0.47$ MHz when the Poisson ratio $\nu = 0.3496$. If the diameter and the
 258 density are fixed to the preceding values, the best fit is $E = 69.6$ GPa and $\nu = 0.350$, which
 259 is obtained with $\sigma_{res} = 0.17$ MHz. No significant improvement of the fit can be obtained by
 260 varying the density.

Table 1: Measured and calculated eigenfrequencies in MHz, corresponding to the mode shapes of Figure 3.
 The right column shows the differences: $\Delta f = f_{exp} - f_{calc}$.

	mode	f_{exp}	f_{calc}	Δf		mode	f_{exp}	f_{calc}	Δf
1	R2,0	70.54	70.67	-0.13	11	R6,0	204.71	204.44	0.27
2	WG1,1	86.15	86.54	-0.39	12	WG1,3	233.02	233.37	-0.35
3	R3,0	109.07	108.99	0.08	13	WG4,1	234.28	233.55	0.73
4	WG2,1	134.72	135.10	-0.38	14	R7,0	235.11	234.33	0.79
5	L0,1	136.97	136.92	0.06	15	WG2,2	241.98	240.70	1.28
6	R4,0	142.47	142.49	-0.01	16	T0,2	NA	253.17	NA
7	T0,1	NA	154.47	NA	17	R8,0	264.02	263.83	0.19
8	R5,0	173.87	173.97	-0.10	18	WG5,1	280.83	280.62	0.22
9	WG3,1	184.64	184.69	-0.05	19	WG3,2	281.62	282.05	-0.43
10	WG1,2	198.33	198.27	0.05	20	R9,0	292.70	293.09	-0.39

261 In order to evaluate the measurement uncertainty for the Young modulus and the Poisson
 262 ratio, due to the measurement uncertainty of the fiber diameter, we supposed that the fiber

263 diameter is now $32.8\ \mu\text{m}$. The best evaluation of the parameters is $E = 70.0\ \text{GPa}$ and
 264 $\nu = 0.350$ with $\sigma_{res} = 0.17\ \text{MHz}$. Therefore, an error in the evaluation of the fiber diameter
 265 of $0.1\ \mu\text{m}$ propagates an error of $0.4\ \text{GPa}$ in the Young modulus. But it is remarkable that
 266 the Poisson ratio is not sensitive to the diameter uncertainty. The evaluation of the Young
 267 modulus is also sensitive to the density, with a sensitivity $\partial E/\partial\rho = 0.026\ \text{GPa}/(\text{kg}/\text{m}^3)$.
 268 If the uncertainty of the fiber density is neglected, the result is $E = (69.6 \pm 0.4)\ \text{GPa}$ and
 269 $\nu = 0.350 \pm 0.001$.

270 The Young modulus and the Poisson ratio were determined by considering only 9 exper-
 271 imental frequencies below $210\ \text{MHz}$. We must verify now if determined parameters give the
 272 correct prediction of the other eigenfrequencies. Table 1 shows the comparison between the
 273 experimental and calculated frequencies up to $300\ \text{MHz}$ with the parameters $d = 32.7\ \mu\text{m}$,
 274 $E = 69.6\ \text{GPa}$, $\nu = 0.350$ and $\rho = 2700\ \text{kg}/\text{m}^3$. Each experimental frequency f_{exp} is an
 275 average of several measured eigenfrequencies. The relative errors $\Delta = (f_{exp} - f_{calc})/f_{calc}$
 276 remain under 0.5% for all the calculated frequencies below $300\ \text{MHz}$. The $WG(2, 1)$ mode
 277 is located before the "breathing" $L(0, 1)$ mode, in agreement with our initial guess.

278 Moreover, the correlation between the calculated and the experimental frequencies is
 279 quite good for 31 modes up to the frequency of the Rayleigh mode $R(15, 0)$, which is calcu-
 280 lated at $466\ \text{MHz}$ and measured at $464\ \text{MHz}$. The relative residuals $\Delta = (f_{exp} - f_{calc})/f_{calc}$
 281 are plotted in Fig. 4. For all the measured eigenfrequencies the relative error is below 0.5% .

282 3.5. The limits of the 2-dimensional approach

283 Though the 2D model is capable to fit rather well most of the measured eigenfrenquen-
 284 cies, it cannot explain some particular features observed in the experimental spectrum, in
 285 particular the unexpected weakness of the "breathing mode" line at $137\ \text{MHz}$.

286 In the 2D approach, the propagation of acoustic waves along the fiber axis is neglected.
 287 We implicitly assume that the group velocities of the excited guided modes are zero. In
 288 order to evaluate an upper limit for the group velocity, we must take into account of the
 289 finite length of the pump line, which determines a cutoff in the wave vector spectrum of
 290 the excited guided modes. The cutoff wave number is roughly $1/L_p = 1/0.1 = 10\ \text{mm}^{-1}$,

291 where L_p is the length of the pump spot. The propagation of guided modes can be ignored
292 during the acquisition time $T_a = 2 \mu\text{s}$ only if their group velocity is much lower than $v_g^{max} =$
293 $L_p/T_a = 50 \text{ m s}^{-1}$. In consequence, the 2D approach is valid only if the group velocity is
294 inferior to 50 m s^{-1} for all wave vectors $k \lesssim 2\pi \cdot 10 \text{ mm}^{-1}$.

295 The group velocity is zero for pure cross-section modes, *i.e.* with the $k = 0$ guided
296 modes. But for some modes, the variation of the group velocity with k may be too rapid
297 between the zero and the cutoff wave vector $k_{max} = 2\pi \cdot 10 \text{ mm}^{-1}$ and this would result in a
298 quick vanishing signal and thus a strong apparent damping of the measured vibrations. In
299 the spectral domain, this will result in weak and broad spectral line.

300 The fact that most of the spectral lines displayed in Fig. 2 have relatively high amplitudes
301 proves *a posteriori* that the group velocities of most guided modes are not much larger than
302 the above estimated value 50 m s^{-1} . However, the condition $v_g < 50 \text{ m s}^{-1}$ may not be
303 fulfilled for some modes, in particular for the breathing mode.

304 The 2D model does not take into account the possible Zero Group Velocity modes (ZGV)
305 that may be excited in a cylinder. Such modes are known in structures capable of guiding
306 acoustic waves, such as plates [29]. The existence of ZGV modes in a cylinder is predicted
307 theoretically [30]. The ZGV modes can exhibit very high quality factors in the range of
308 $Q = 1000 - 10000$. Moreover, it is known that ZGV eigenfrequencies are different from
309 the cross-section eigenfrequencies. The differences between cross-section and ZGV eigen-
310 frequencies could explain some discrepancies between the experimental and the calculated
311 eigenfrequencies in the 2D modeling.

312 4. Conclusion

313 This paper presents a method to evaluate the transverse Young modulus and Poisson
314 ratio of a micrometric metallic wire by using laser resonant ultrasound spectroscopy. A
315 precision of 0.7% for the Young modulus and 0.3% for the Poisson ratio of an aluminum
316 fiber was achieved.

317 The pump laser was shaped as a line spot parallel to the fiber axis in order to excite
318 cross section eigenmodes. Using a stabilized homodyne Michelson interferometer, the cross

319 section eigenfrequencies were measured up to 500 MHz. The 2D vibration model of the wire
320 proved to be in good agreement for about 30 measured eigenfrequencies within an accuracy
321 of 0.5% in the 50-500 MHz range.

322 Nevertheless, the spectral resolution can be improved by increasing the acquisition time
323 windows, as some modes are not completely damped 2 μ s after the excitation. The improved
324 resolution could be useful to detect minute geometrical defects of the cylinder through the
325 observation of the degeneracy lifting of some modes. Furthermore, the measurement of high
326 frequency Rayleigh modes could be useful to probe the mechanical properties of the material
327 in the vicinity of the surface, as the displacements of high frequency Rayleigh modes are
328 localized mainly near the surface.

329 In order to achieve ultimate precision for the evaluation of the elastic parameters of
330 fibers, it is essential to take into account the propagation of guided modes along the fiber
331 and therefore to carry out the 3-dimensional modeling of the mechanical system and then to
332 determine the eigenfrequencies of the Zero Group Velocity modes (ZGV). Such modes could
333 be useful to improve the precision in the determination of the elastic parameters and could
334 give access to the determination of the intrinsic damping of the material, *i.e.* the imaginary
335 part of the Young modulus.

336 The application of the LRUS method is not restricted to metallic fibers. We intend to
337 apply more extensively our LRUS method to characterize fibers used in reinforced composite
338 materials, such as carbon, glass, kevlar and vegetal fibers. The study of vegetal fibers, in
339 particular flax fiber, which is a hollow structure approximately cylindrical, requires the
340 association of LRUS with a method to determine the actual geometry of the fiber, such as
341 digital holographic tomography.

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Figure 3: Mode shapes of the first 20 calculated eigenmodes of an aluminum cylinder. Numbers correspond to the ordering numbers of Table 1. Category of each eigenmode: Rayleigh (R), Wave Gallery (WG), longitudinal (L) or torsional (T) is indicated. Rayleigh modes are characterized by predominant radial components. Colors represent the magnitude of the displacements: the deep blue color means a zero displacement, *i.e.* the nodal areas of the mode shape and the red color is for the maximum magnitude. The arrows indicate both the direction and the magnitude of displacements.

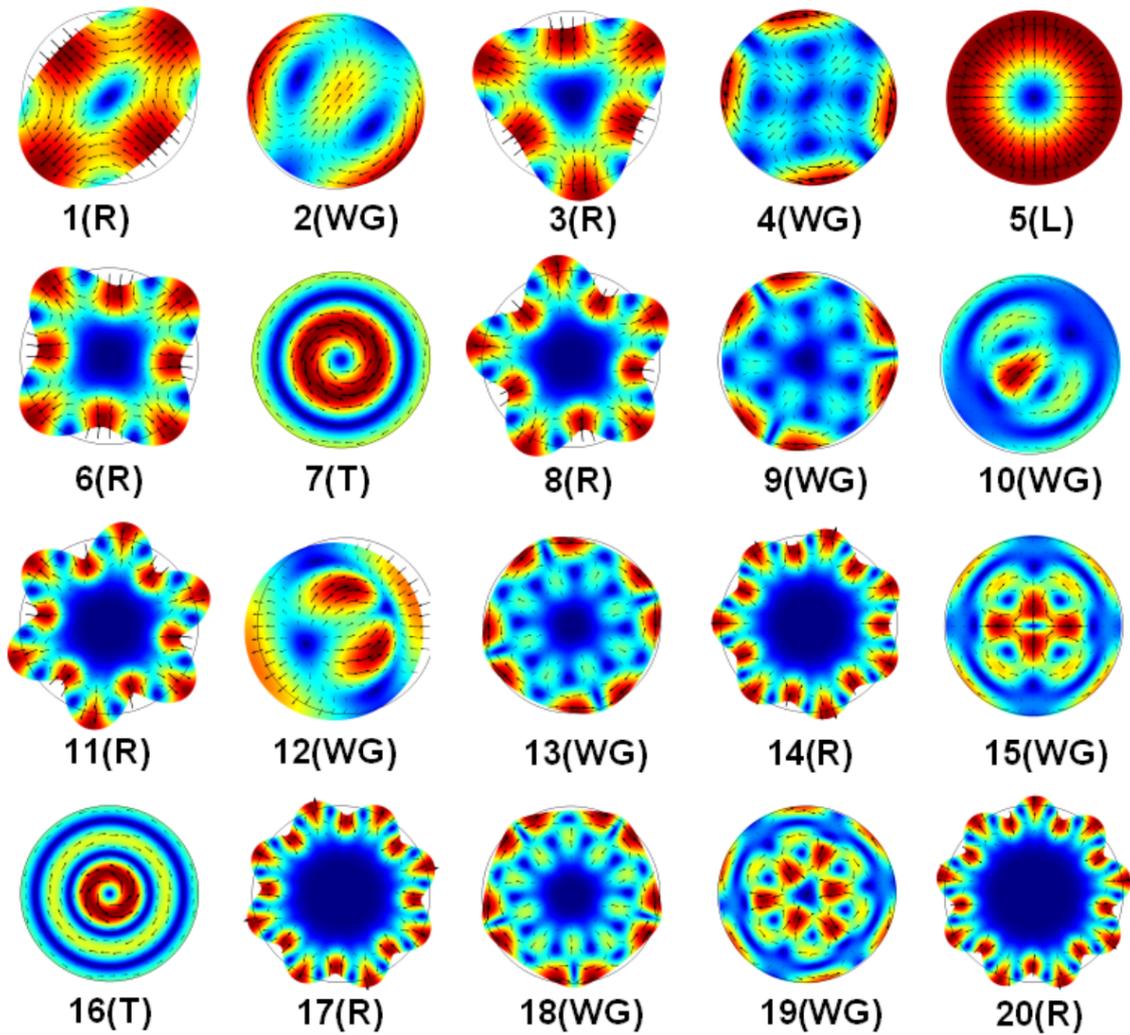


Figure 4: Percentage of relative error between experimental and calculated eigenfrequencies with the parameters: $d = 32.7 \mu\text{m}$, $E = 69.6 \text{ GPa}$, $\nu = 0.350$ and $\rho = 2700 \text{ kg/m}^3$.

