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QUALITATIVE PROPERTIES OF A 3-STEPS MODEL OF ANAEROBIC DIGESTION INCLUDING HYDROLYSIS OF PARTICULATE MATTER

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Introduction. Anaerobic digestion is a biological process in which organic matter is transformed into methane and carbon dioxide (biogas) by microorganisms in the absence of oxygen. The search for models simple enough to be used for control design is of prior importance today to optimize fermentation processes and solve important problems such as the development of renewable energy from waste. Within the studies of microbiology, biochemistry and technology, the anaerobic digestion is generally considered as a three step process: hydrolysis and liquefaction of the large, insoluble organic molecules by extracellular enzymes, acid production by an acidogenic microbial consortium and a methane production stage realized by a methanogenic ecosystem. Several mathematical models describing these phenomena have been proposed in the literature. However, they are usually too complex to be used for control synthesis [1, 2, 6]. For control purposes, the most appropriate way to model the hydrolysis is still an open problem. Many chemists claim that this is a pure enzymatic phenomena while biologists emphasize the role of hydrolytic bacteria. So, there exist several choices :
- One may consider that the microbial enzymatic activity is constant without involving explicitly the biomass;
- One may divide the substrate compartment into two parts : slowly degradable substrates X_0 and readily biodegradable substrate S_1 which may be, for keeping the model simple enough considered to be degraded by the same biomass, *e.g.* a unique compartment grouping "hydrolitic and acidogenesis" bacteria.

The reactional scheme of the anaerobic digestion is given by :

Hydrolysis	Macromolecules X_0	$\xrightarrow{r_0}$	$k_0 S_1$
Acidogenesis	Monomers $k_1 S_1$	Acidogenic B.: X_1 $\xrightarrow{r_1=\mu_1(S_1)X_1}$	$X_1 + k_2 S_2 + CO_2$
Methanogenesis	Organic acids $k_3 S_2$	Methanogenic B.: X_2 $\xrightarrow{r_2=\mu_2(S_2)X_2}$	$X_2 + CO_2 + CH_4$

where $r_0 = k_{hyd} X_0$ or $r_0 = \mu_0(X_0)X_1$ and $r_i = \mu_i(S_i)X_i, i = 1,2$, denote the reaction rates, μ_0 is consumption rate of X_0 by X_1 and $\mu_i, i = 1,2$, are the specific growth rates of X_i on S_i. Finally, $k_i, i = 0 \ldots 3$, denote the pseudo-stoechiometric coefficients associated to the biological reactions.

Model. We consider a continuous culture, i.e the input flow rate is equal to the output flow rate [3]. The modified three step model is :

$$\left\{\begin{array}{rcl} \dot{X}_0 &=& DX_{0in} - \alpha D X_0 - r_0, \\ \dot{S}_1 &=& D(S_{1in} - S_1) + k_0 r_0 - k_1 \mu_1(S_1) X_1, \\ \dot{X}_1 &=& \Big[\mu_1(S_1) - \alpha D\Big] X_1, \\ \dot{S}_2 &=& D(S_{2in} - S_2) + k_2 \mu_1(S_1) X_1 - k_3 \mu_2(S_2) X_2, \\ \dot{X}_2 &=& \Big[\mu_2(S_2) - \alpha D\Big] X_2, \end{array}\right. \tag{1}$$

where $X_0(t)$ denotes the concentration of the slowly biodegradable substrate (typically the solid Chemical Oxygen Demand) at time t, with X_{0in} the concentration in the input, $S_j(t)$ denote the concentrations of the substrates in the effluent, $j = 1,2$ (the easily biodegradable COD and the Volatile Fatty Acids, respectively), at time t; with S_{jin} the input substrate concentrations j, $X_i(t)$ denote the concentrations of the ith population of microorganisms, $i = 1,2$, at time t (the hydrolitic and acidogens on the first side, and the methanogens on the other side). D denotes the dilution rate of the chemostat, $\alpha \in [0,1]$ represents the fraction of the biomass leaving the reactor and V denotes the volume of the bioreactor. According to the principle of conservation of matter within the reaction scheme we have

$$\int_{t_1}^{t_2} r_0 V d\tau \geqslant \int_{t_1}^{t_2} k_0 r_0 V d\tau \quad \text{i.e} \quad 1 \geqslant k_0,$$

which means that, the quantity of X_0 degraded is greater than or equal to the quantity of S_1 produced. Similarly, we have $k_1 \geqslant 1 + k_2$ and $k_3 \geqslant 1$, which means that, the quantity of S_1 degraded is greater than or equal to the quantity of X_1 and S_2 produced. The quantity of S_2 degraded is greater than or equal to the quantity of X_2 produced.

Results. In this work, we focus on the mathematical analysis of the model of chemostat with enzymatic degradation of a substrate (organic matter) that can partly be under a solid form [5]. We study the model (1) with a constant hydrolytic kinetics, $r_0 = k_{hyd} X_0$, in which the growth rate μ_1 is monotonic and the growth rate μ_2 is non monotonic. The first equation of system (1) depends only on the variable X_0, and X_0 globally converges towards its equilibrium

$$X_0^* = \frac{D}{k_{hyd} + \alpha D} X_{0in}.$$

If X_0 is put equal to its equilibrium, then the last four equations in system (1) reduce to the following AM2 model

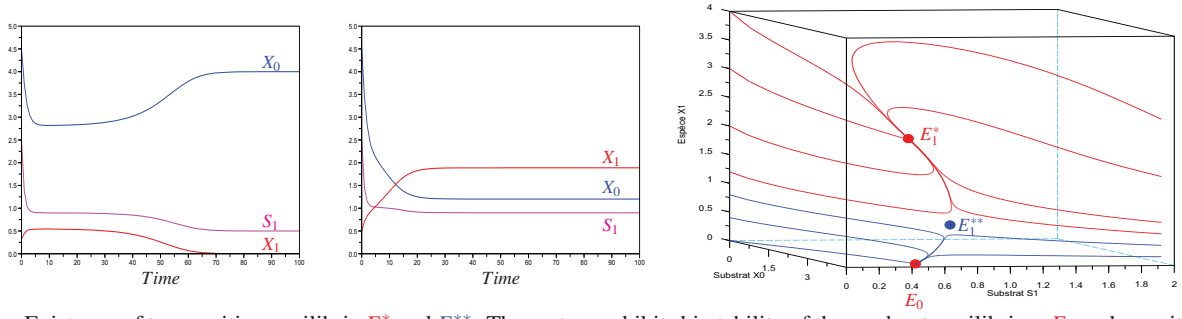
$$\begin{cases} \dot{S}_1 &= D(S_{1in}^* - S_1) - k_1 \mu_1(S_1) X_1, \\ \dot{X}_1 &= \Big[\mu_1(S_1) - \alpha D\Big] X_1, \\ \dot{S}_2 &= D(S_{2in} - S_2) + k_2 \mu_1(S_1) X_1 - k_3 \mu_2(S_2) X_2, \\ \dot{X}_2 &= \Big[\mu_2(S_2) - \alpha D\Big] X_2, \end{cases} \tag{2}$$

where

$$S_{1in}^* = S_{1in} + \frac{k_0 k_{hyd}}{k_{hyd} + \alpha D} X_{0in}.$$

The analysis of (2) was made in [2] and the condition of persistence of the species X_1 in the AM2 model is $\lambda_1 < S_{1in}$. Since S_{1in} was increased (by the effect of hydrolysis) to S_{1in}^*, the condition of persistence $\lambda_1 < S_{1in}^*$ of the species X_1 in the model with hydrolysis, shows that the species are favored by the addition of the hydrolysis term.

The study of model (1) with the hydrolytic activity related to biomass, $r_0 = \mu_0(X_0) X_1$, was given in [4]. It is derived from a smaller order sub-model since some variables can be decoupled from the others. We study the existence and the stability of equilibrium points of the sub-model considering Monod growth rates and distinct dilution rates. In the classical chemostat model with monotonic kinetics, it is well known that only one equilibrium point attracts all solutions and that bistability never occurs [7]. In the present study, although (i) only monotonic growth rates are considered and (ii) the concentrations of input substrate concentration is less than the break-even concentration, it is shown that the considered sub-model may exhibit bistability. Hence, the importance of hydrolysis in the appearance of positive equilibrium points and the bistability is pointed out.



Existence of two positive equilibria E_1^* and E_1^{**}. The system exhibits bi-stability of the washout equilibrium E_0 and a positive equilibrium E_1^*.

Acknowledgements. Part of This work has been realized within the framework of the 3+3 Euromed TREASURE project (cf. http://project.inria.fr/treasure/).

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