An effective equity model allowing long term investments within the framework of Solvency II

Mohamed Majri, François-Xavier de Lauzon

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Mohamed Majri *François-Xavier de Lauzon †

Abstract

In this paper we propose an effective equity model (called the alternative model in this paper) adapted for medium term and long term risk assessment. It is a simplified discrete version of a model built and implemented by the SMABTP company . One of its specific aspects is to allow an asymetrical dampening of the equity risk (called the dampener effect) conditional to the cyclical level of equity prices and to enable accurate Value At Risk assessements for medium and long term horizons (1 year and beyond).

For a set of selected equity indexes we compare its relevancy for the 1-year 99.5% Value At Risk (VaR) assessment with the different releases of the Solvency II dampener equity models. In a second step we test its relevancy for VaR assessments beyond a 1 year investment horizon.

We show in our analysis that the alternative model we propose gives quite good results and outperforms widely the others tested. It appears particularly suitable for insurance companies and pension funds given their medium or long term asset management process. This model is by the way very simple to implement and to calibrate. The authors thinks that this alternative model could also be used by the supervisors to enhance the Solvency 2 equity standard formula.

Keywords

Solvency II, Equity risk, QIS, Dampener, symmetrical adjustment, standard formula, Value-At-Risk

Mathematics Subject Classification: ***
Journal of Economic Literature Classification: ***

*M Majri is Certificated Actuary fellow of the French Institute of Actuaries and head of Financial Modeling at SMABTP. mohamed_majri@smabtp.fr. Address: 114 Avenue Émile Zola, 75015, Paris, France. Phone: 33-(0)1-40-59-71-69

†FX de Lauzon is deputy head of Financial Modeling at SMABTP. francois-xavier_de_lauzon@smabtp.fr. Address: 114 Avenue Émile Zola, 75015, Paris, France. Phone: 33-(0)1-40-59-73-24

1 Introduction

With building the Solvency II new prudential framework directive the authorities have been confronted to the discontent of some insurance stakeholders regarding the high level of own funds requirement induced by the initial standard formula and also by its pernicious pro cyclical effects.

To deal with theses issues authorities have proposed several versions of the standard formula.

In this paper we focus on one specific component of the standard formula (the Equity SCR formula) which deals with the equity risk and the assessment of its marginal own funds requirement.

To respond to criticism of stakeholders and obtain a compromise between own funds cost and pro cyclical effects reduction the authorities have introduced the dampener mecanism in the Equity SCR formula.

The dampener mecanism aims at mitigating the equity cost of capital after a market shock so as to avoid forced equity sells in periods of distressed markets. The introduction of such a mecanism is empirally justified: usually market risk rises in the boom cycle and then falls after a market shock.

Nevertheless the implementation of the Equity dampener cyclical effect in the standard formula is a challenging work given the lake of literature about this subject.

In a first time we present the differents versions of the Equity SCR formulas named also Equity dampener formulas which have been proposed by the authorities.

We analyze them by extracting from historical datas (beginning from 1929 for some indexes) the following indicators for the different investment horizons tested: the Back-Testing performance indicators (back testing success rate and back testing overflow rate) and the DIFA indicator (the average relative difference between VaR estimated with and VaR estimated without the dampener mechanism).

We assume that a good equity risk model should be sufficiently prudent by maximizing the first indicator and minimizing the second one. And in the same time it should be quite incentive for long term equity investments with maximizing the last indicator.

In a second time we introduce a new simple non linear equity model (called the alternative model in this paper) adapted for medium term and long term risk assessment. It is a simplified discrete version of a model built and implemented by the SMABTP company . One of its specific aspects is to allow an asymetrical dampening of the equity risk (called the dampener effect) conditional to the cyclical level of equity prices.

Then for a set of selected equity indexes we test the relevancy of the 1-year 99.5% Value At Risk (VaR) obtained with this alternative model and also for the different releases of the Solvency II Dampener Equity Models.

We also test the relevancy of this non linear equity model for VaR assessments among a 1 year investment horizon.

We show in this analysis that the alternative model we propose gives really good results given these criterions and outperforms widely the others tested.

2 Solvency II Dampener Equity Models

The Equity Dampener adjustment introduced within the Solvency II directive aims to reduce the pro-cyclical effects of the standard formula. The Dampener adjustment finds its justification in historical market prices observation.

2.1 Definitions

In the solvency II prudential framework Equity SCR is defined as the marginal solvency capital requirement for Equity risk. It is defined as the 1 year 99.5% Value At Risk (VaR) estimated for the holding of Equities in the insurance company assets.

We define Own funds surplus as the difference between the Equity index value considered and its SCR value.

In every EIOPA works, the value at risk and symmetrical adjustment are calculated with daily data. However, as we will see in this work, we have similar results with monthly data. Using monthly data will ease the comparison with our model which needs a monte carlo simulation.

2.2 Methodology

The Dampener adjustment, as presented by the EIOPA, seems coming from empirical study and to still under construction. Today, we have three formula between its first appearance in QIS 4 (CEIOPS 2008) and its last definition in the draft of 2011 (EIOPA 2011).

To structure our analysis, we will confront each release of the EIOPA Equity formula with the following tests :

Performance of the Back Testing: For each past date to test (the test-date) we simulate the Equity SCR (with or without Dampener) that we should have obtained with using only the datas available before that test-date. Then we compare the Equity SCR assessed at the test-date with the effective one year lost. Two measures are then exhibited for the assessment of the Back Testing Performance :

- **Back Testing Success Rate (BTR):** The number of VaR exceeding the corresponding one year loses observed divided by the number of the VaR back tested.
- **Back Testing OverFlow (BTOF):** The average percentage of losses exceeding assessed VaR.

Dampener Impact on own funds surplus (DIFA): It is the average relative difference between SCR estimated with and without the Dampener mechanism.

All the figures will present results for a period from 01/01/2000 to 31/12/2012.

2.3 QIS 4 Dampener

Before the Solvency II directive publication, the Dampener adjustment appears in official document for the first time in (CEIOPS 2008). It is also in this publication where it has its most complex form by introducing the liabilities duration.

$$Mkt_{eqt,1} = MVEP * (\alpha(F(k) + G(k) * c_t) + (1 - \alpha) * 32\%)$$

$$c_t = \bar{Y_t^{10}} - \bar{Y_t^{261}}$$

$$\bar{Y_t^N} = \frac{\sum_{i=0}^{N-1} Ln(Y_{t-1})}{N}$$

MVEP : Market value of the equity portfolio

k : Duration of the liabilities

α **:** Share of the technical provisions accounting for more than 3 year commitments

F(k), G(k) : Coefficients depending on the duration k

c_t **:** Cyclical component

$\bar{Y_t^N}$ **:** Mean of the last N trading days of the equity index (MSCI Developed Markets index)

The justification of the coefficients F and G is unfortunately not given. So it is not possible to analyze this model.

2.4 First symmetric adjustment

2.4.1 Formula

In 2009, the CEIOPS promulgates the Solvency directives (European Commission 2009) which set up the basis for the Solvency Capital Requirement standard formula. In this text, the regulator takes into account cycle effect at the article 106:

- the standard formula will have a symmetric adjustment
- this adjustment cannot exceed ±10% of the equity capital charge
- this adjustment shall be based on a function of the current level of an appropriate equity index and a weighted average level of that index

The directive remains general and a clear formula will be given only in level 2 measure documents. However, the notion of symmetric adjustment appears and indicates that the previous formula won't be used anymore. One year later, the calibration paper (CEIOPS 2010c) (April 2010), based on consultation paper (CEIOPS 2010a) (January 2010), gives a more explicit formula (points 3.69 to 3.90) that was used after in (CEIOPS 2010b) (july 2010):

$$VaR_i^{afterDampener} = VaR_i^{beforeDampener} + A_i * \beta_i$$

$$A_i = \frac{I_i - \frac{1}{n}\sum_{s=t-1}^{i-n} I_s}{\frac{1}{n}\sum_{s=t-1}^{i-n} I_s}$$

I_i **:** Value of the MSCI Developed index at time i

β_i **:** Linear regression coefficient resulting from a fit of the equity index level on the weighted average equity index level

2.4.2 Baseline change

We can notice that the stress without considering the Dampener effect is equal to 44% in (CEIOPS 2010c) and equal to 39% in (CEIOPS 2010b). The 44% coefficient tie to the 99.5% percentile of the empirical profitability distribution. The 39% correspond to the 99.5% quantile of a Gaussian distribution calibrated on the corresponding sample.

2.4.3 Adjustment

CEIOPS advises a β equal to 1 and a period of one year for the assessment of the weighted average equity index level. The choice of MSCI Index as the equity index is understandable but is not adapted for companies holding mainly european equity assets. For this motive, we analyse both the model with considering the MSCI index and the DJ Eurostoxx 50 as the benchmark equity index. In order to simplify the annotation, we consider:

WDE : Equity SCR formula based on empirical distribution without the dampener adjustment

WDG : Equity SCR formula based on the gaussian distribution without the dampener adjustement

CP2010 : Equity SCR formula based on April calibration paper (empirical distribution plus symmetric adjustment)

QIS5 : Equity SCR formula based on july QIS5 technical specification (Gaussian distribution plus symmetric adjustment)

2.4.4 Results

Figure 1 and 2 exhibit the SCR evolution with or without the dampener mecanism in comparison with the one year lost for the MSCI and the DJ Eurostoxx 50 index. The noticeable points are the followings :

- The MSCI SCR is lower than the DJ Eurostoxx 50 SCR.
- The Equity SCR formula based on the gaussian distribution improves the back testing when considering the DJ Eurostoxx 50 but not if we consider the MSCI as the benchmark index.
- The Equity SCR obtained with the dampener adjustment covers less effectively the one year lost than the Equity SCR assessed without the dampener adjustment for the period 2007-2009.
- The Equity SCR can be higher with the dampener than without. Such a case means either a non reliable 99.5% VaR calibration or an unnecessary higher risk covering.
- The Dampener adjustment shows a strong volatility.

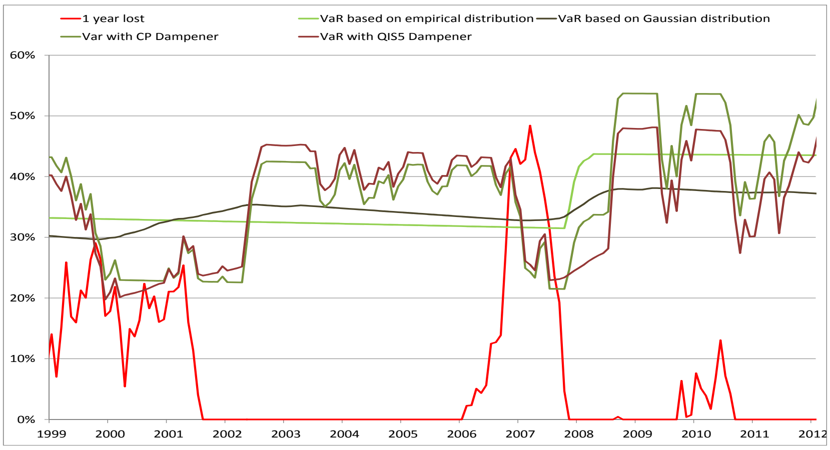

Figure 1: MSCI World Index SCR

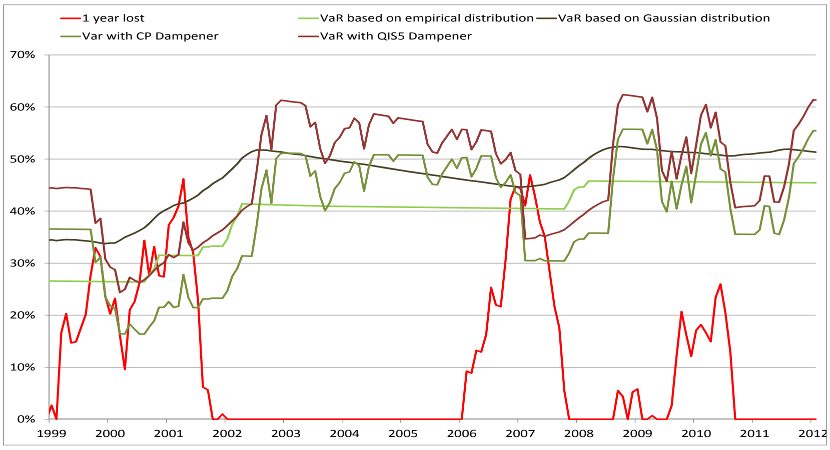

Figure 2: DJ Eurostoxx 50 Index SCR

In order to analyse the impact of the model over the market, figure 3 and 4 show the own funds surplus variation for MSCI and DJ Eurostoxx 50 indexes before and after introducing the dampener effects. The noticeable points are the followings:

– The Dampener has improved the situation for the 2001-2003.
– The Dampener has a cost for the 2003-2007 period.
– The adjustment has a counter cyclical impact for two periods: March 2010-November 2010 and August 2011-February 2012.

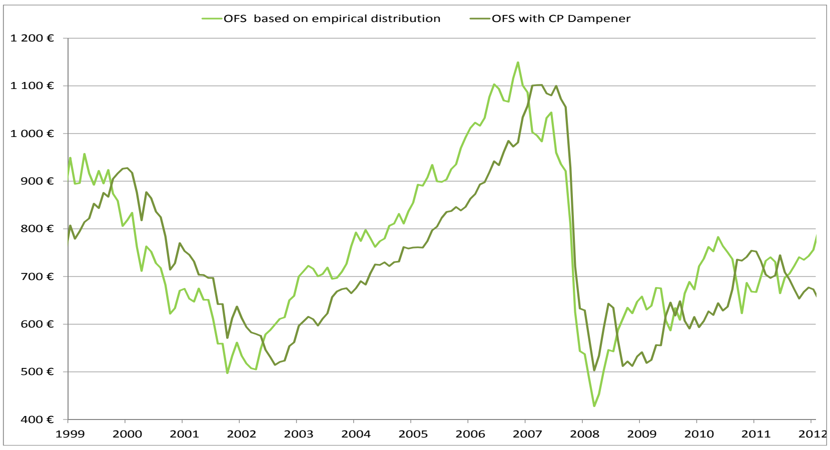

Figure 3: MSCI World Index impact on own funds surplus

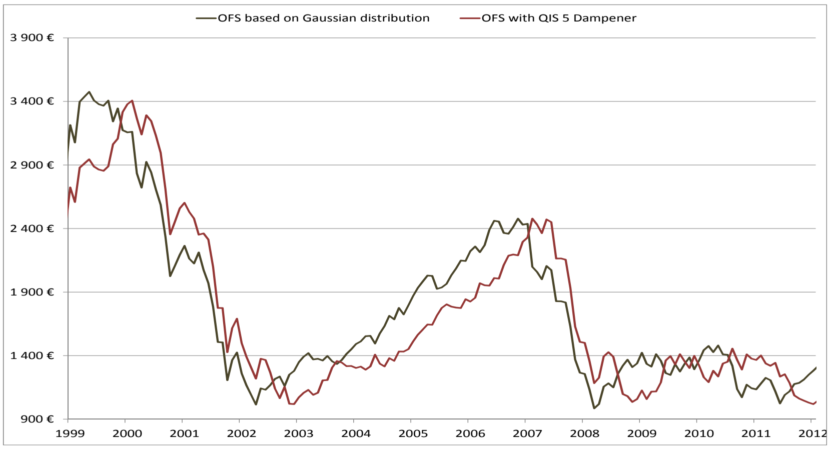

Figure 4: DJ Eurostoxx 50 Index impact on own funds surplus

Index	Period	Formula	BTR	BTOF	DIFA
DJ Index	01/01/2000	WDE	88%	13%	14%
		WDG	98%	5%	
		CP2010	82%	24%	13%
	31/12/2011	QIS 5	92%	15%	−1%
MSCI Index	01/01/2000	WDE	94%	26%	−1%
		WDG	94%	23%	
		CP2010	93%	27%	−4%
	31/12/2011	QIS5	91%	20%	−2%

Table 1: First adjustment formula impact

2.5 Second symmetric adjustment

2.5.1 Formula

In 2011, the regulator suggests a different formula in (EIOPA 2011).The 39% value in the model suggests that the gaussian distribution is used :

$$VaR_i^{afterDampener} = VaR_i^{beforeDampener} + SA$$

$$SA = \frac{1}{2}\left(\frac{CI - AI}{AI} - 8\%\right)$$

CI : Actual value of the considered index

AI : Moving average over 36 months of the index

2.5.2 Link with the first formula

FSB offers an explanation for the formula change from the first one to second one (FSB 2012). The two formulas can be written with the following closed form:

$$SA = a * \left(\frac{CI - AI}{AI} - b\right)$$

In QIS 5, we have $a = 1$ with $b = 0$ and in the 2011 draft we have $a = 0.5$ with $b = 8\%$.

2.5.3 Results

As we can see in figures 5, 6, 7 and 8 this new formula does cover the 1 year loses for the 2007-2008 period but shows improvements:

– the volatility of this new model is quite lower

– the 1 year risk overestimation is reduced for the quiet market periods

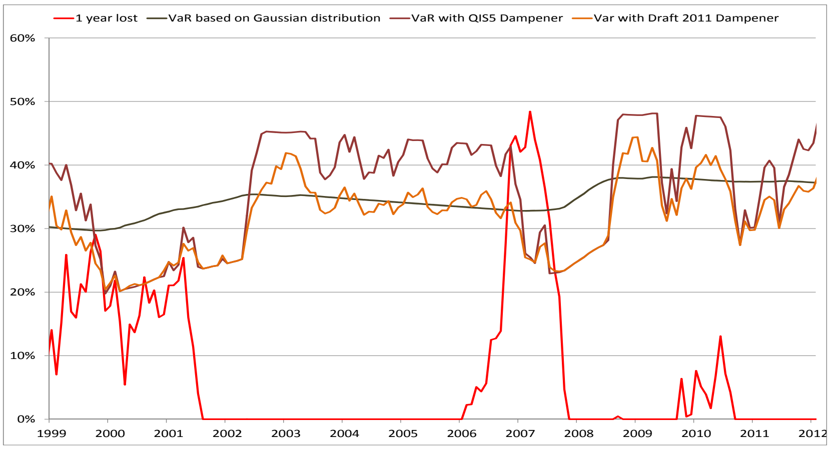

Figure 5: MSCI World Index SCR

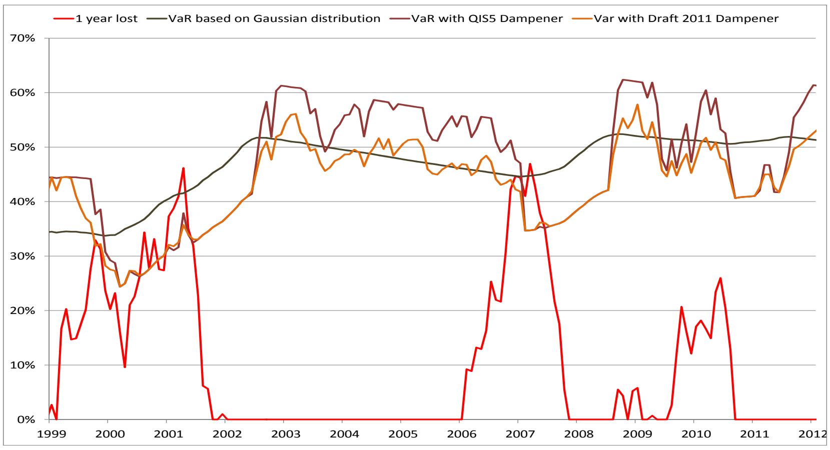

Figure 6: DJ Eurostoxx 50 SCR

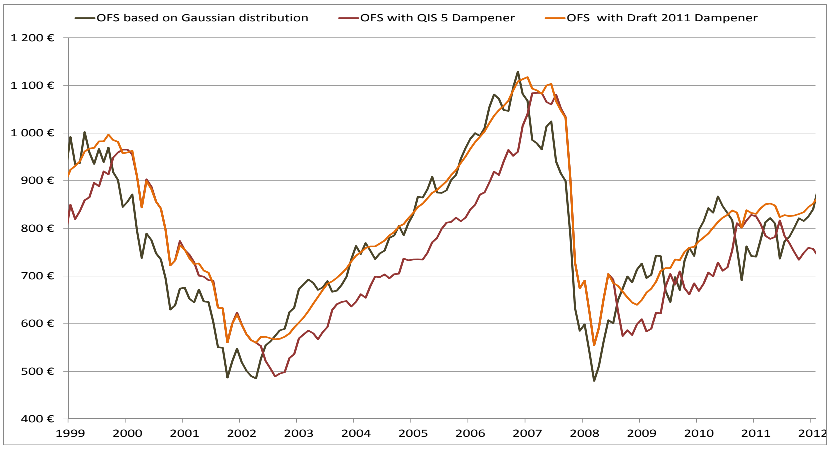

Figure 7: MSCI World Index impact on own funds surplus

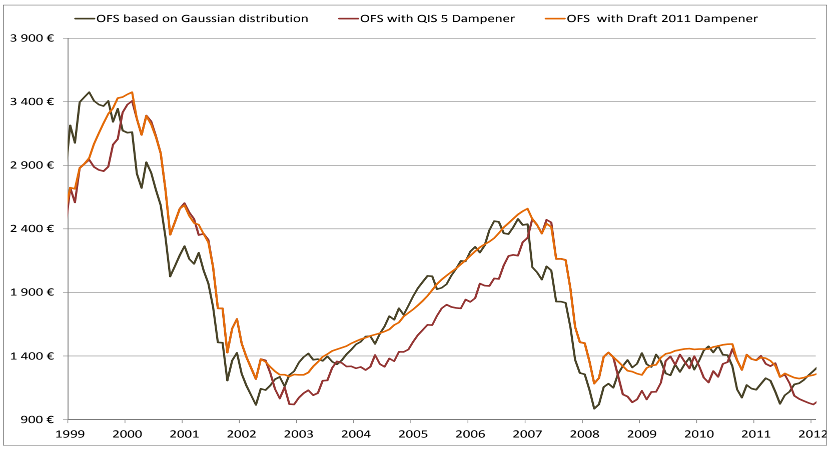

Figure 8: DJ Eurostoxx 50 Index impact on own funds surplus

Index	Period	Formula	BTR	BTOF	DIFA
DJ	01/01/2000	WDG	98%	5%	
		QIS 5	92%	15%	−1%
Index	31/12/2011	2011	90%	13%	7%
MSCI	01/01/2000	WDG	94%	23%	
		QIS5	91%	20%	−2%
Index	31/12/2011	2011	91%	25%	5%

Table 2: Second adjustment formula impact

The EIOPA Dampener adjustment is efficient for a short term period and for reduced falls : figures 7 and 8 demonstrate a good performance for the 2010-2013 period. However,

- the $\pm 10\%$ boundary appears as a limit for stressed periods.
- the adjustment is helpful for reduced market fall but not reliable for strong market crisis (like 2001 and 2007-2008)

3 Presentation of the alternative Equity model

For the concision of this paper the presentation of the continuous version of this model and its properties is not exposed here. The reader may refer to (Majri and Vehel 2013).

3.1 Presentation of the Alternative Model

For a given Equity or Equity index we introduce the largest sample of observed prices assumed to be relevant for the risk assessment. The observed prices are assumed to be extracted with a constant basic time step noted Δ (for instance Δ=1/12 of a year for a monthly price extraction).

This sample is noted $\{C_0, C_\Delta, C_{2\Delta} ..., C_{N\Delta}\}$ where N+1 is the number of the total observations C_i available.

We note $R_{i+\Delta}$ the arithmetic Equity yield for the basic Δ-period $[i, i+\Delta]$ where i is a basic time step ($i \in \{0, \Delta, 2\Delta, ...\}$). So we have:

$$R_{i+\Delta} = \frac{C_{i+\Delta} - C_i}{C_i}$$

The discret formula of the considered non linear model is:

$$ln(1 + R_{i+\Delta}) = F_a(\tilde{C}_i, \Delta) + r(\sigma_i(\Delta)) - r(\sigma_i(\Delta)).F_a(\tilde{C}_i, \Delta) \tag{1}$$

Where:

$r(\sigma_i(\Delta))$ is a gaussian variable with a nil average and a standard deviation $\sigma_i(\Delta)$,

$F_a(\tilde{C}_i, \Delta)$ is the rising component of the relative variation $R_{i+\Delta}$ defined by the following formula:

$$F_a(\tilde{C}_i, \Delta) = \Delta \frac{(S_i(\Delta) - C_i)^+}{S_i(\Delta)} \tag{2}$$

With:

$$S_i(\Delta) = 2.MM_i(l, \Delta) - MM_i(m, \Delta) \tag{3}$$

And:

$$MM_i(a) = \frac{1}{1+a} \sum_{k=0}^{a} C_{i-k\Delta} \tag{4}$$

The $p\%$ Value At Risk (VaR) calculated at a given time i for an investment horizon of T basic time step Δ can be obtained with a simple Monte Carlo simulation technique. So we have:

$$VaR_i(p, T\Delta) = quantile_{p\%}\left(1 - \left(\prod_{k=1}^{T}(1 + R_{i+k\Delta}(w)\right)\right) \tag{5}$$

Where the $R_{i+k\Delta}(w)$ are simulated values of the random variable $R_{i+k\Delta}$ obtained with the formula (1) assuming that $\sigma_{i+k\Delta}(\Delta) = \sigma_i(\Delta)$ for $k \in [1, T]$.We can notice that the assessment of the VaR is an iterative process. For a simulation w, $R_{i+\Delta}(w)$ is firstly simulated given the observed $C_{i-j\Delta}$ price values for $j \in [0, l]$. Then we have to proceed to the simulation of $R_{i+2\Delta}(w))$ which depend on the value of $C_{i+\Delta}(w)$ extracted from the simulated value of $R_{i+\Delta}(w)$ and so on until the simulation of $R_{i+T\Delta}(w))$.

3.2 Calibration

Choice of the values l and m for the assessment of $S_i(\Delta)$ In this paper we assume that in the formula 3 the values l and m are constant variables with $l = 7/\Delta$ and $m = 3/\Delta$ in any case. The justification of this choice is described in (Majri and Vehel 2013).

Calibration of σ_i The alternative model we present is based on a simple gaussian law for the innovation modelization while mostly observed equity yields exhibit a larger dispersion than the gaussian law is supposed to simulate (see (Courtois and Walter 2010)).

The aim of this simplification is to offer an alternative model for the assessment of the equity risk which is at the same time accurate, incentive for long term investments and simple to implement.

To address with a simple manner the viewed mismatch the calibration of $\sigma_i(\Delta)$ is done for a given date i in order to match up the Gaussian VaR with the historical VaR:

$$\sigma_i(\Delta) = \frac{quantile_{p\%}(R_k - mean(R_k))_{k=0}^{i}}{q_{99.5\%}}$$

Where:

$q_{99.5\%}$ is the normal standard quantile for a 99.5% confident level. So we have: $q_{99.5\%} = -2.5758$.

It is important to notice that for a given date i the only equity price observations past to the date i are used for the calibration. Indeed we consider for the estimation of $\sigma_i(\Delta)$ the values R_k, where k$\in \{0, \Delta, 2\Delta, ..., i\}$ and as already seen we have:

$$R_k = \frac{C_k - C_{k-\Delta}}{C_{k-\Delta}}$$

So the last price observation used for the calibration of the model at the date i is C_i.

In Particular the back testing tests for this alternative model (see below) are done with a great respect to this rule. The assessment of the VaR for any time horizon at a given date i do never use observed prices after this considered date i.

4 Comparaison between the alternative Equity model and the Solvency II Dampener Equity Models

4.1 Results

We show a comparison between the alternative model (obtained with monthly data prices) and the Solvency II dampener models (obtained with daily data prices as specified by the EIOPA).Figures 9, 10, 11 and 12 exhibit in particular VaR and Own funds surplus comparaison between the alternative model and the most recent Solvency II model (named the draft 2011 model) while considering MSCI World Index and also DJ Eurostoxx 50 index. For the two indexes tested the alternative model appears more prudent than the Solvency II equity models in the way that historical one year losses (in red) are better captured. In the same time the alternative model appears more sensitive for long term equity investments in the way that the VaR obtained decrease more strongly after a market shock to lead to more important own funds surplus than the Solvency II dampener models should allow.

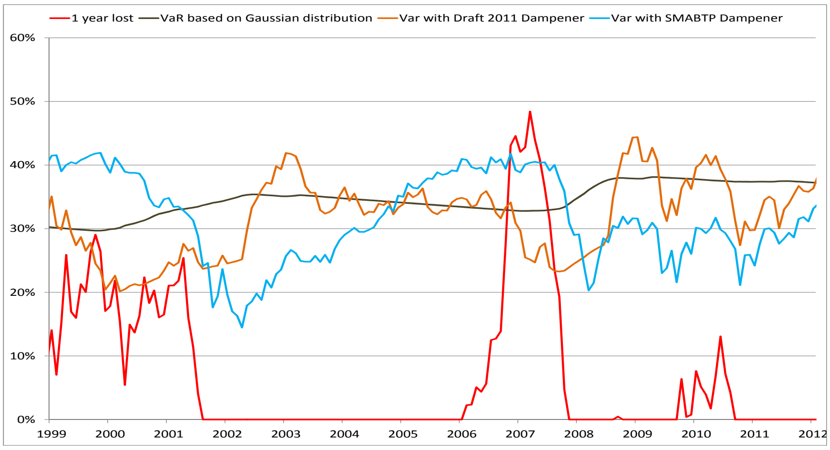

Figure 9: MSCI World Index SCR

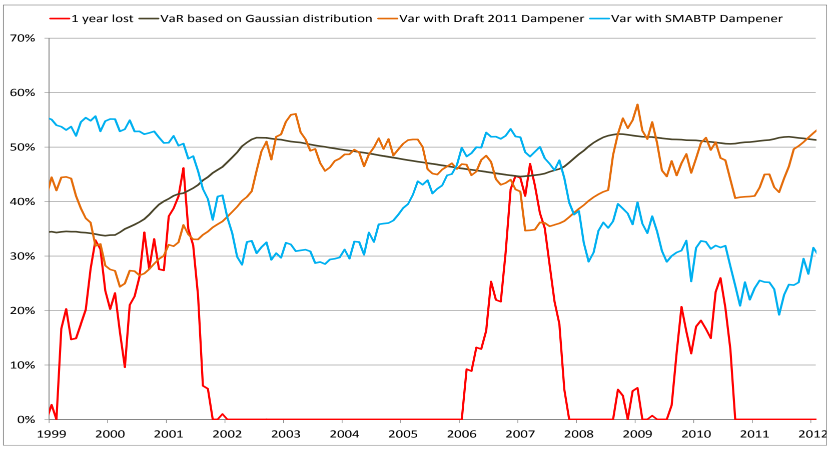

Figure 10: DJ Eurostoxx 50 SCR

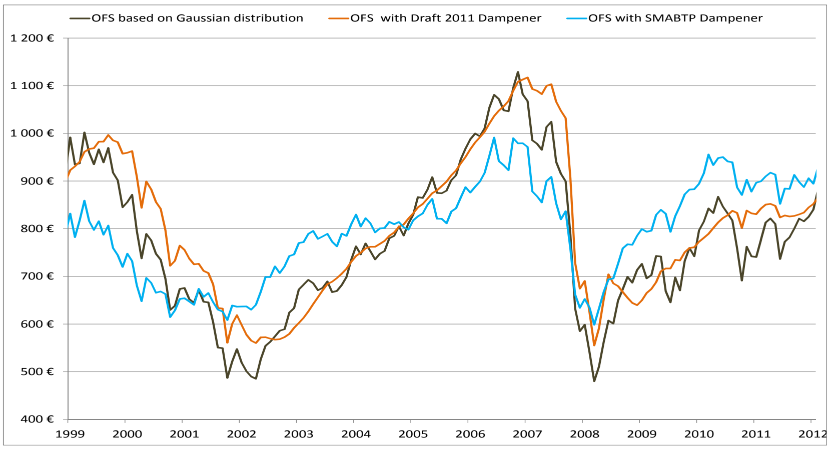

Figure 11: MSCI World Index impact on Own funds surplus

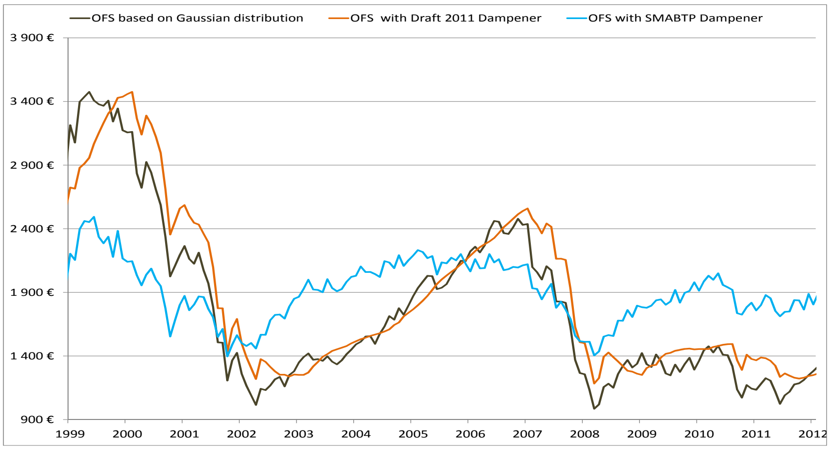

Figure 12: DJ Eurostoxx 50 impact on Own funds surplus

Index	Period	Formula	BTR	BTOF	DIFA
DJ	01/01/2000	QIS 5	92%	15%	−1%
		2011	90%	13%	7%
Index	31/12/2011	SMABTP	$99.7\% \pm 0.3\%$	$0.1\% \pm 0.1\%$	$0.6\% \pm 0.7\%$
MSCI	01/01/2000	QIS 5	91%	20%	−2%
		2011	91%	25%	5%
Index	31/12/2011	SMABTP	$95.5\% \pm 0.3\%$	$9.3\% \pm 1.9\%$	$4.7\% \pm 0.3\%$

Table 3: SMABTP Formula impact

4.2 Long term analysis

The previous analysis was focused only on the 2000-2012 period which is significant for the risk sensitivity test but not for long term resilience. Figure

13 offers a comparison between Solvency II models and the alternative model for the S&P500 Index. The Solvency II formulas are strongly impacted by the 1929 crisis. The empirical Var (without Dampener) is almost 70% in 1935 and goes under 55% only in 2001. Moreover, the S&P500 Index shows a limit for the Solvency II VaR that can go above 100% which does not make any sense. The alternative model is based on a conditional log normal price modelization that prevents that kind of effect. For the Solvency II formulas tested when the gaussian law gives values above 100%, the own funds surplus is therefore negative and the DIFA indicator has no sense anymore. For that reason, the measures showed in the following table concern a period starting in 1945 when all the Var are clearly under 100%. As already seen with the MSCI World index and the DJ Euro Stoxx 50 index the alternative model gives the better results in terms of BTR, BTOF and DIFA indicators.

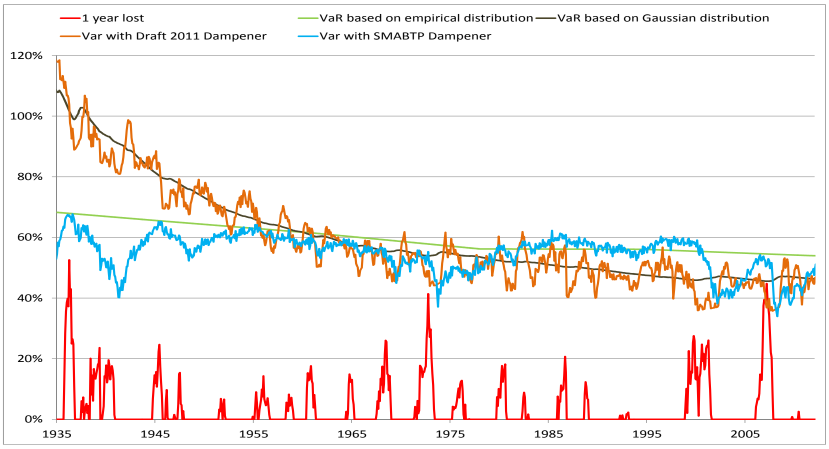

Figure 13: S&P 500 SCR

Period	Formula	BTR	BTOF	DIFA
04/01/1945	WDE	100%	0%	−1%
	WDG	100%	0%	
	CP2010	100%	0%	−9%
31/12/2011	QIS 5	99.6%	10%	−8%
	2011	99.6%	10%	4%
	SMABTP	100%	0%	$0.6\% \pm 0.6\%$

Table 4: Results for S&P 500

5 VaR estimation beyond a 1 year time horizon

The non linear model is presented as an alternative to the standard formula for the equity sub-module. With this prospect, the model has to cover the one year Equitiy risk. However, beyond the legal constraint, Insurers should have sufficient own funds to honor in the time the outflow of their liabilities. To do so, the model should also be tested for other time horizons.

Time step changes The model shows a interesting multifractal property for a large panel of Equities : The choice of the basic time step (using of biannual, quarterly, monthly or even weeekly equity price data) does not impact significantly the VaR obtained for all historical periods we have already tested. This point is developped in (Majri and Vehel 2013). For the concision of this paper we always use a monthly basic time scale to establish the alternative model results.

Back Testing For the three previous indexes the following table gives the back testing average rate of losses recovered by the VaR assessed with the alternative model. The Investment time horizon tested are included between 1 year and 7 years. The historical periods considered for the back testing are the longest available (see the figure belows).

Measure	Time frame	S&P 500	MSCI	Eurostoxx
BTR	1 year	100%	$98.6\% \pm 0.2\%$	$99.8\% \pm 0.2\%$
	2 years	100%	$99.9\% \pm 0.1\%$	100%
	3 years	100%	100%	100%
	4 years	100%	100%	100%
	5 years	100%	100%	$99.7\% \pm 0.3\%$
	6 years	100%	100%	100%
	7 years	100%	100%	$99.3\% \pm 0.7\%$
BTOF	1 year		$9.5\% \pm 2\%$	$0.5\% \pm 0.5\%$
	2 years		$3.2\% \pm 3.2\%$	
	3 years			
	4 years			
	5 years			$0.6\% \pm 0.6\%$
	6 years			
	7 years			$4\% \pm 4\%$

Table 5: Results for longer time frame VaR

As we can see in the figures below the Dampener appears to be the most efficient (in the way of following closely the highest losses while remaining above it) for an investment term included between 3 and 5 years.

S&P 500 results

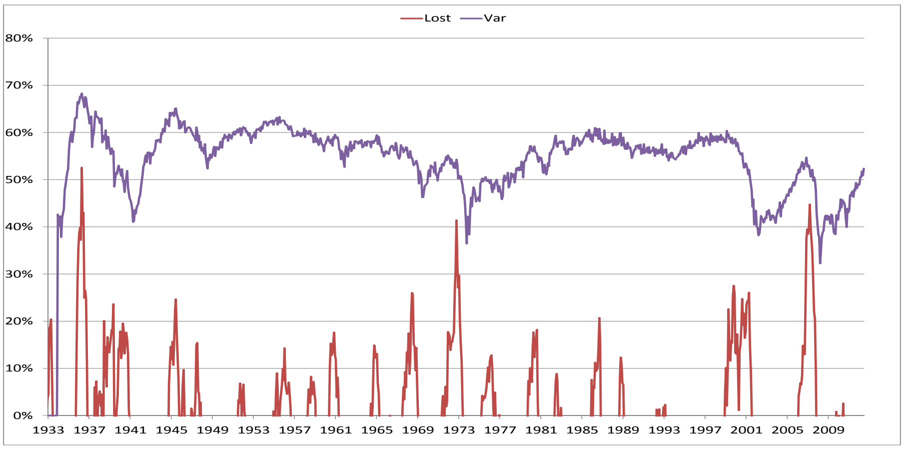

Figure 14: S&P 500 1 year VaR

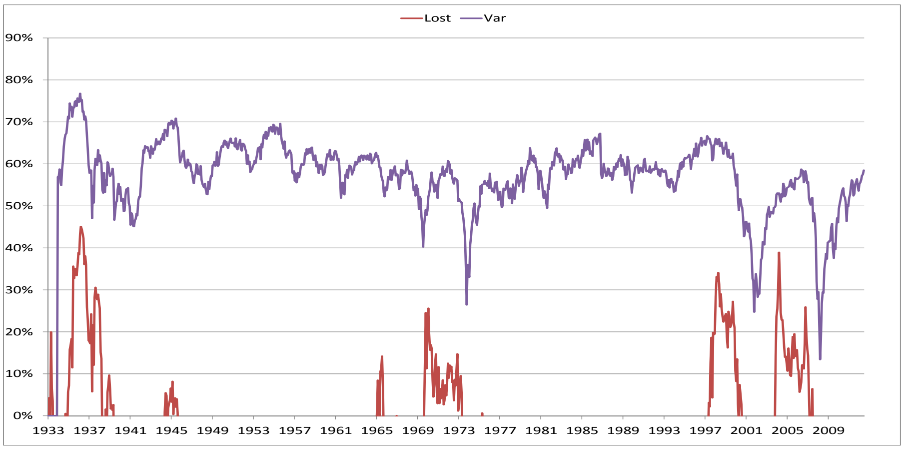

Figure 15: S&P 500 4 years VaR

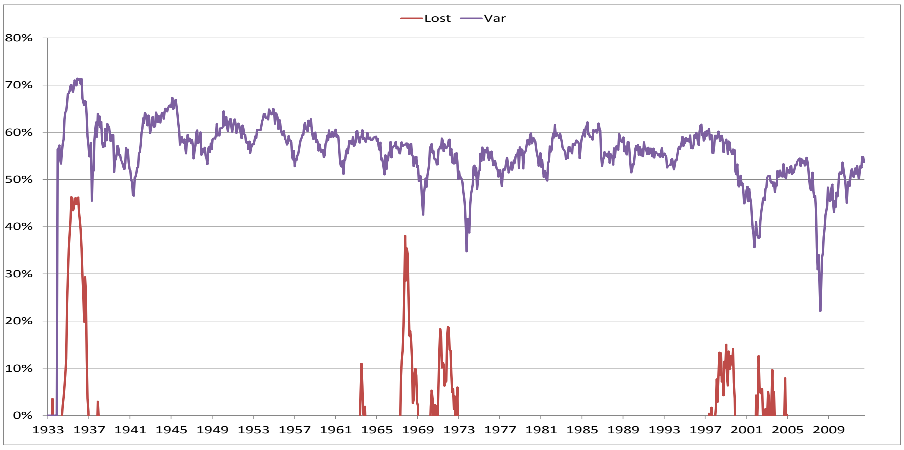

Figure 16: S&P 500 6 years VaR

MSCI results

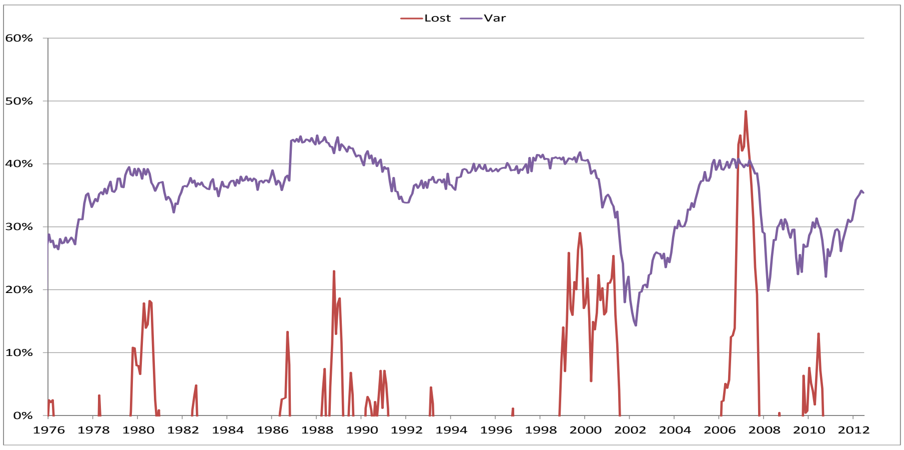

Figure 17: MSCI 1 year VaR

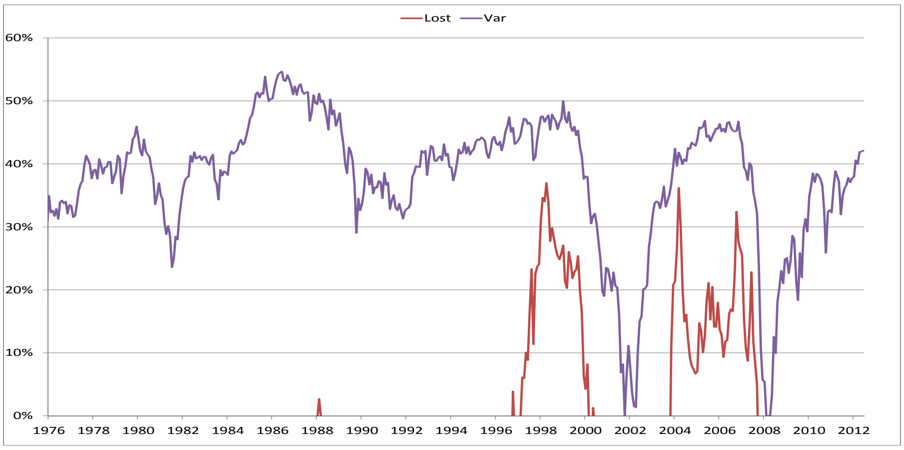

Figure 18: MSCI 4 years VaR

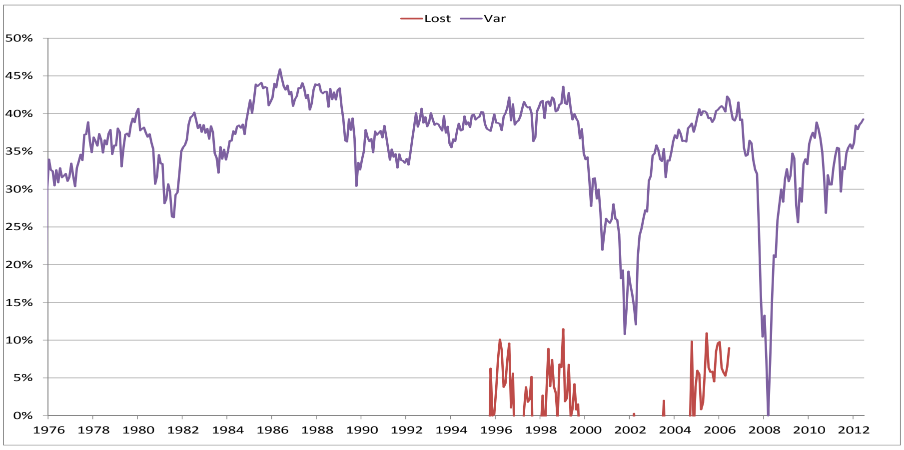

Figure 19: MSCI 6 years VaR

DJ Eurostoxx 50 results

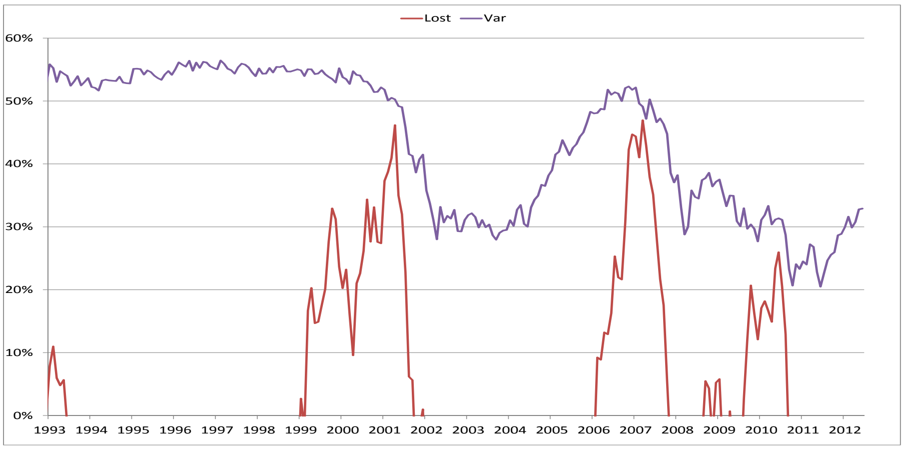

Figure 20: DJ Eurostoxx 50 1 year VaR

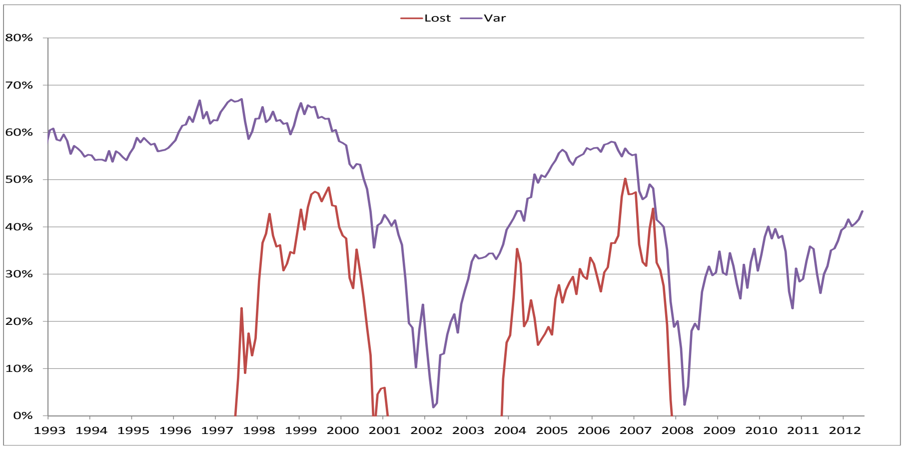

Figure 21: DJ Eurostoxx 50 4 years VaR

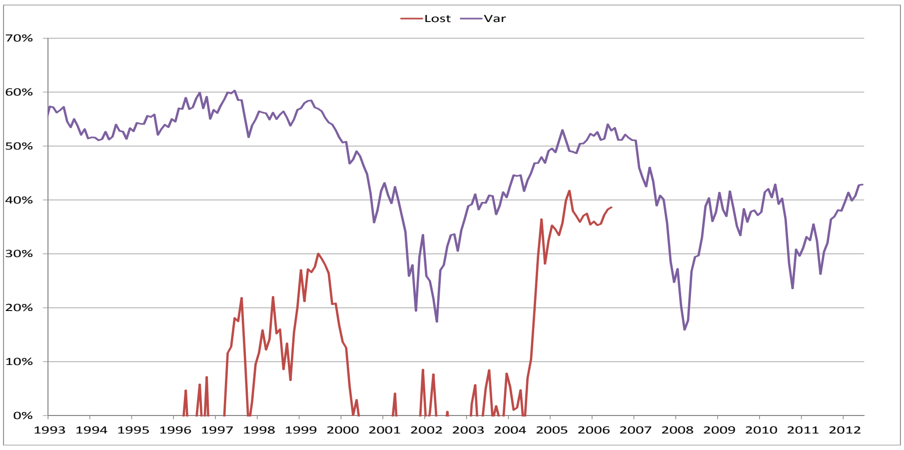

Figure 22: DJ Eurostoxx 50 6 years VaR

6 Others indexes results

For the Equity indexes tested the alternative model gives quite good results. The back testing average rate of losses recovered by VaR is above 95% for horizon time between 1 and 7 years.The model failure concerns Japaneses indexes (NIKKEI, MSCI Japan) for long time horizons.

Mesure	Index	1	2	3	5	7
BTR	MSCI World	98.6 ± 0.2	99.9 ± 0.1	100	100	100
	MSCI Europe	95.2 ± 0.8	97.8 ± 0.4	99.0 ± 1	98.7 ± 1.3	94.4 ± 1.9
	MSCI France	97.6 ± 0.8	99.6 ± 0.4	100	99.4 ± 0.6	95.4 ± 0.9
	MSCI Germany	97.6 ± 0.8	100	100	100	100
	MSCI Spain	98.8 ± 0.4	99.6 ± 0.4	99.5 ± 0.5	99.4 ± 0.6	100
	MSCI UK	100	100	100	100	100
	MSCI Japan	96.5 ± 0.5	93.7 ± 0.5	91.8 ± 0.7	90.1 ± 0.7	92.7 ± 0.9
	Eurostoxx	99.8 ± 0.2	100	100	99.7 ± 0.3	99.3 ± 0.7
	CAC	100	100	100	100	100
	DAX	99.2 ± 0.3	99 ± 0.3	99.5 ± 0.1	99.4 ± 0.2	100
	SP500	100	100	100	100	100
	NASDAQ	99.2 ± 0.1	99.4 ± 0.1	99.9 ± 0.1	100	100
	Dow Jones	98.6 ± 0.1	97.8 ± 0.2	96.9 ± 0.1	96.1 ± 0.1	96.8 ± 0.1
	IBEX	99.8 ± 0.2	100	100	100	100
	FTSE 100	100	100	100	100	100
	NIKKEI	96.5 ± 0.5	93.6 ± 0.4	92.8 ± 0.5	87.6 ± 0.8	84 ± 1
BTOF	MSCI World	9.5 ± 0.2	3.2 ± 3.2			
	MSCI Europe	7.3 ± 1.6	6 ± 2.4	2.2 ± 2.2	7 ± 7	12.9 ± 4.5
	MSCI France	8.1 ± 3.9	0.7 ± 0.7		1.6 ± 1.6	12.6 ± 4.2
	MSCI Germany	9.9 ± 3.2				
	MSCI Spain	11.7 ± 6.3	4.5 ± 4.5	3.8 ± 3.8	2.6 ± 2.6	
	MSCI UK					
	MSCI Japan	12.1 ± 2.3	30.2 ± 3.1	27.7 ± 3.7	15.1 ± 1	9 ± 1.2
	Eurostoxx	0.5 ± 0.5			0.6 ± 0.6	4 ± 4
	CAC					
	DAX	7.2 ± 2.2	4.6 ± 1.6	4.7 ± 2.3	4.9 ± 1.9	
	SP500					
	NASDAQ	8.1 ± 2.3	3.8 ± 2.2	0.6 ± 0.6		
	Dow Jones	24.5 ± 2.5	20.3 ± 1.4	27.3 ± 1.6	23.6 ± 0.8	13.4 ± 0.8
	IBEX	4.7 ± 4.7				
	FTSE 100					
	NIKKEI	18.6 ± 3	23.2 ± 2	22.9 ± 2.1	17.1 ± 1.1	19.4 ± 1.2

Table 6: Back Testing Rate and overflow for divers indexes (results are in percent)

7 Conclusion

The alternative model described in this paper is a simplified discrete version of a model implemented by the SMABTP company. We have compared its results with those obtained when using Solvency II equity formulas and showed that this alternative model is at the same time the most prudent and the most incentive for equity long term risk investments. This model is by the way very simple to implement and to calibrate. The authors thinks that it could also be used by the supervisors to enhance the Solvency 2 equity standard formula.

Beyond the 1 year equity risk forecast insurance companies and pension funds are also interested by the assessment of their market risks for a medium and long term horizon corresponding to their liabilities duration. The alternative model gives really good back testing results and can thus be considered as a good tool to achieve this aim.

8 Acknowledgment

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