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Abstract

A large body of theoretical and empirical research focusses on two rationales for government subsidies to college students: positive fiscal externalities when subsidies lead to greater human capital accumulation and a larger income tax base, and liquidity constraints among students. In this paper, I calculate the optimal subsidy in a simple model that incorporates both fiscal externalities and liquidity constraints. I use two approaches in which outcomes of the model are matched to US data: calibration of a simple structural model, and a "sufficient statistics" approach in which I derive an equation for the welfare impact of tuition subsidies as a function of a few empirical statistics. Both approaches lead to the striking result that optimal subsidies should be increased to the point of completely offsetting average tuition at public universities. This finding is driven by fiscal externalities, and is not sensitive to the extent of liquidity constraints, indicating that we do not need to know their magnitude to make welfare statements about tuition subsidy policy.

Keywords: college tuition subsidies, fiscal externality, liquidity constraints, sufficient statistics, free tuition

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1 Introduction

The affordability of a college education and appropriate government education policy is an important and widely-studied subject.[1] A large body of theoretical and empirical research focusses on two rationales for government subsidies to students: positive fiscal externalities from the higher income tax base that results when subsidies lead to greater human capital accumulation, and liquidity constraints in the market for student borrowing. In this paper, I contribute to a small numerical literature by calculating the optimal tuition subsidy in a simple model that incorporates both fiscal externalities and liquidity constraints, using two approaches in which outcomes of the model are matched to US data.

The economic intuition behind a fiscal externality is simple: greater educational attainment leads to higher wages and higher tax revenues, but individuals do not internalize the benefits of higher tax revenues when making decisions about investments in education; thus, a subsidy to education offsets the pre-existing tax distortion and can increase efficiency. In other words, in the presence of income taxes, the social return to education can be significantly higher than the private return. Simulations in Trostel (1993) show that proportional income taxation could have a significant negative effect on investment in human capital, and Trostel (2010) estimates that net government spending per degree is negative in the United States.[2] The literature that considers optimal tuition subsidies, however, is mostly theoretical: Bovenberg and Jacobs (2005) present a model in which it is optimal to use a subsidy to education to perfectly offset income taxes, returning human capital investment to the first-best amount, and numerous recent papers (including Richter (2009), Richter and Braun (2010), and Braun (2010)) qualify this finding by seeking conditions under which the quantity of education should be induced to move above or below the first-best amount. These papers focus on providing general insights rather than numerical evidence on optimal policy.

Liquidity constraints also provide an obvious reason for government intervention, as discussed in Kane (1999), and they have therefore been the subject of a large empirical litera-

[1]See, for example, the surveys in Kane (2006) and McPherson and Schapiro (2006).

[2]Trostel finds that direct expenditures of about \$71000 (in present-value 2005 dollars) per degree are more than offset by savings of \$56000 from reduced expenditures on programs such as social assistance and corrections, and increased tax revenues amounting to \$197000. Similar findings for high school graduation are discussed in a New York Times editorial, Levin and Rouse (January 25, 2012), which points out that reducing undereducation in that context would pay for itself.

ture, often aiming to estimate causal effects of family income on enrollment. However, the existence of liquidity constraints among students remains the subject of a persistent empirical controversy. Several papers find little evidence of constraints, including Cameron and Taber (2004) and a series of papers by James Heckman and various co-authors summarized in Cunha, Heckman, Lochner, and Masterov (2006), while others find evidence of positive effects of income on enrollment, most notably Belley and Lochner (2007) who indicate that income has become a much more important determinant of enrollment in recent decades, perhaps due to rising tuition and reductions in spending on Pell Grants.

Despite these large theoretical and empirical literatures, very few papers have calculated numerical estimates of optimal tuition subsidies. Trostel (1996), Akyol and Athreya (2005), and Bohacek and Kapicka (2008) evaluate optimal subsidies to education in a context without liquidity constraints, while Caucutt and Kumar (2005) focusses on liquidity constraints and ignores income taxes and therefore fiscal externalities. However, not only do these papers focus on only one of the two motivations for tuition subsidies discussed above, they each produce results from simulation of a structural model for which the parameter values are selected in accordance with values used in related literatures rather than through a calibration to important moments of data from the education sector.[3] It is thus unclear how sensitive existing results are to the parameters used and the way those parameters are chosen.

The current paper contributes to this literature by calculating optimal subsidies in a simple model which incorporates both fiscal externalities and liquidity constraints. To do so, I make use of two approaches in which the relevant parameters are chosen to ensure a good fit with evidence from the education sector: a "sufficient statistics" approach, and the calibration of a simple structural model, approaches which have both been widely used in the extensive literature on optimal unemployment insurance. In the sufficient statistics method, I derive an equation for the derivative of social welfare with respect to student grants as a function of a few empirical statistics, which are therefore the sufficient statistics for welfare analysis (see Chetty (2009) for a detailed discussion of the method). Specifically, the effect of income on enrollment, which depends on the magnitude of liquidity constraints, is weighed against the fiscal benefits of a larger tax base.[4]

[3]For example, Akyol and Athreya (2005) choose parameters of a production function and individual income persistence process from the macro labour literature.

[4]This discussion indicates why it is important that I model both the fiscal externality and liquidity constraint motives for subsidies: my numerical conclusions depend directly on the empirical values of both, and it is contrary to the idea of sufficient statistic analysis to arbitrarily abstract from one or the other.

The sufficient statistics approach does not require me to specify the underlying structural parameters and functional forms; empirical measurement of the sufficient statistics is all that is needed to make welfare predictions. However, the welfare derivative is only valid locally, and in order to make out-of-sample predictions and solve for the optimal policy, a statistical extrapolation of the sufficient statistics is required. Such an extrapolation can be viewed as ad-hoc, and so I complement this analysis by using the sufficient statistics to calibrate and simulate a structural model, to demonstrate the robustness of my results to alternative assumptions.

Both approaches indicate that tuition subsidies should be significantly increased. In fact, the optimal subsidy in the baseline case is sufficient to completely offset average tuition at public universities.[5] This finding is driven by fiscal externalities: higher tuition subsidies lead to increased enrollment and a larger tax base, so that a relatively small tax increase is needed to pay for the increased subsidies.

This result is not sensitive to the extent of liquidity constraints: fiscal externalities on their own are sufficient to justify zero tuition, and enacting such a generous policy would render any liquidity constraints largely irrelevant. This insensitivity of my general results to liquidity constraints is an important discovery, given that the empirical literature studying liquidity constraints has failed to reach a consensus about their magnitude: my results indicate that we do not need to know the answer in order to make welfare statements about tuition subsidy policy.[6] Additionally, the results are not sensitive to the degree of risk-aversion in the population.

The key elements to which my results appear to be sensitive are general equilibrium effects on relative and absolute wages: if increased enrollment significantly reduces the college wage premium, then tuition subsidies will ultimately be unsuccessful at increasing enroll-

Furthermore, the effect of income on enrollment is not zero in the absence of liquidity constraints, and thus there is no obvious value to choose if I make that abstraction.

[5]This result is similar to the finding in Saez (2002) that an Earned Income Tax Credit is optimal when low-income behavioural responses to taxation are concentrated on the extensive margin: the education decision is an important extensive margin, and thus a large transfer to individuals on that margin may be efficient.

[6]Discussions of optimal policy have been limited in papers that discuss liquidity constraints, and often it is simply argued that guaranteed loans can solve whatever liquidity constraints may exist. However, Keane and Wolpin (2001) and Johnson (2012) argue that raising borrowing limits in guaranteed loan programs will have very little effect on enrollment or graduation, because of a precautionary savings motive. Therefore, I abstract from changes in loan policy, assuming a fixed borrowing constraint, and focus my analysis on grants to students, as Johnson (2012) specifically argues that tuition subsidies will be much more effective than loans in raising college completion.

ment, whereas positive wage spillovers from the educated to other individuals could justify stipends to students of around $10000 per year above and beyond free tuition. This analysis indicates that future study of the general equilibrium effects of education subsidies would be of particular policy relevance.

The result of zero tuition, or a 100% subsidy to investments in college education, contrasts with previous finding on optimal tuition subsidies: the optimal subsidy rate has varied from around 40% in Trostel (1996) to 25-80% in Akyol and Athreya (2005) to around 70% in Caucutt and Kumar (2005). In the first two papers, the parameters chosen imply that enrollment is relatively unresponsive to subsidy rates, to an extent that appears to be at odds with empirical findings on the subject. Meanwhile, the abstraction from fiscal externalities in Caucutt and Kumar (2005) is responsible for the difference between their results and mine. Bohacek and Kapicka (2008), meanwhile, characterize their result as an optimal subsidy rate of between 0 and 20%, but they use a model in which there are no direct costs of education and specify the "subsidy rate" as a reduction in taxes for a one-unit increase in education, so it is difficult to translate their results into the usual framework.

I use a simple and intuitive model in order to highlight the essential tradeoffs of financial aid policy. However, my results hold in a far more general analysis, as demonstrated in Lawson (2014), where I produce an expression that simplifies to exactly the same form. Additionally, to be conservative, I abstract away from non-monetary motivations for government support of students, such as social benefits from better-educated citizens, as well as other potential positive externalities from education such as effects on growth.

The rest of the paper is organized as follows. Section 2 lays out my simple model, and solves it for a sufficient statistics expression for the derivative of social welfare with respect to student grants. Section 3 provides the main numerical results. Section 4 then performs the experiment of shutting down the liquidity constraint and fiscal externality motivations for financial aid one by one, to examine the robustness of the results. Section 5 extends the model to include general equilibrium effects, and section 6 provides a conclusion.

2 A Simple Model of College Education

In this section, I present my model of college education, followed by the calculations leading to an expression for the derivative of social welfare with respect to student grants.

2.1 Model Setup

Time is finite, and is divided into two parts: in the first, each individual has a choice of attending college or working, while in the second, individuals work at wages which depend on their education level. Since the second part will be far longer than the first, the model will consist of 12 periods, 1 in the first part and 11 in the second, where each period corresponds to 4 years, thus representing a normal working life of 48 years (say, from age 18 to 65 inclusive). This is equivalent in practical terms to a two-period model, but since discounting and comparison of quantities across periods is of great importance in my analysis, the notation and intuition are both simplified when periods of equal length are used.

In the first period, the individual chooses between attending college and working at wage Y_{01}, and this choice is represented by $s = \{0, 1\}$, where 1 indicates college attendance. In periods $t = 2, ..., 12$, the individual works at a wage Y_{st} that depends on the education choice in the first period, where $Y_{1t} > Y_{0t}$.[7] I assume that the real interest and discount rates are both equal to 3% per year, and since a period is equal to 4 years, I will use $r = 0.12$ for the interest rate and $\beta = \frac{1}{1.12} \simeq 0.893$ for the discount rate. I also allow for real wage growth, calculated from the average net compensation series used by the Social Security Administration for the computation of the national average wage index, deflated using the CPI; the average real growth rate over 1991-2008 is almost exactly 1%, so I allow wages to grow at $g = 0.04$ per period.

The individual's utility from consumption c while in college is $u(c)$, whereas it is $v(c)$ while employed, allowing for direct utility or disutility from college attendance as well as different utility from consumption in the two states. Both utility functions obey the usual properties of $u', v' > 0$ and $u'', v'' < 0$, and I denote individuals by i. If an individual chooses not to attend college, then since the interest and discount rates are equal, they will simply set consumption to a constant value c^0_{vi} in each period, and receive lifetime utility of $U_{0i} = \sum_{t=1}^{12} \beta^{t-1} v(c^0_{vi})$. If they do attend college, they will set per-period post-schooling consumption c^1_{vi} to some constant value, and choose some value c_{ui} of consumption while in school, receiving lifetime utility of $U_{1i} + \eta_i$, where $U_{1i} = u(c_{ui}) + \sum_{t=2}^{12} \beta^{t-1} v(c^1_{vi})$ and where η_i captures any idiosyncratic portion of the utility or disutility from schooling.[8]

[7] These wages are assumed to be exogenous, and there is no uncertainty. I allow for uncertainty in future incomes in an extension in appendix C.3.

[8] I therefore consider a "representative-agent" setting in which there is heterogeneity in taste for schooling but not income or returns to education; heterogeneity in the latter dimension will be considered in appendix

The individual's budget constraints, for $s = 0$ and $s = 1$ respectively, can be written in the following way:

$$\sum_{t=1}^{12} \left(\frac{1}{1+r}\right)^{t-1} c_{vi}^0 = (1-\tau) \sum_{t=1}^{12} \left(\frac{1}{1+r}\right)^{t-1} Y_{0t}$$

$$c_{ui} + \sum_{t=2}^{12} \left(\frac{1}{1+r}\right)^{t-1} c_{vi}^1 = (b-e) + (1-\tau) \sum_{t=2}^{12} \left(\frac{1}{1+r}\right)^{t-1} Y_{1t}$$

where e is the direct cost of college to the individual,[9] τ is the marginal tax rate, and b is the government grant given to students. For simplicity, I restrict attention to a lump-sum grant for now, though I will consider a 2-tier grant scheme in appendix C.4.[10] To simplify the notation, let me define $R_x = \sum_{t=x}^{12} \left(\frac{1}{1+r}\right)^{t-1}$ and $\gamma_x = \sum_{t=x}^{12} \left(\frac{1+g}{1+r}\right)^{t-1}$; then the budget constraints can be written as:

$$R_1 c_{vi}^0 = (1-\tau)\gamma_1 Y_{01}$$

$$c_{ui} + R_2 c_{vi}^1 = (b-e) + (1-\tau)\gamma_2 Y_{11}.$$

I also allow students to face a liquidity constraint, which will take the form of a limit A_i to the debt that a student may accumulate:

$$c_{ui} - (b-e) \leq A_i.$$

Given the lack of uncertainty in the labour market, this constraint will never bind on individuals who have completed their education.

Therefore, the individual's maximization problem is to choose $\{s_i, c_{vi}^0, c_{vi}^1, c_{ui}\}$ to maximize $V_i = s_i (U_{1i} + \eta_i) + (1-s_i)U_{0i}$:

$$V_i = s_i[u(c_{ui}) + R_2 v(c_{vi}^1) + \eta_i - \lambda_{1i}(c_{ui} + R_2 c_{vi}^1 - (b-e) - (1-\tau)\gamma_2 Y_{11}) - \mu_i(c_{ui} - (b-e) - A_i)]$$

$$+(1-s_i)[R_1 v(c_{vi}^0) - \lambda_{0i}(R_1 c_{vi}^0 - (1-\tau)\gamma_1 Y_{01})].$$

The government chooses b and τ subject to a budget constraint:

$$Sb + G = \tau[S\gamma_2 Y_{11} + (1-S)\gamma_1 Y_{01}] = \tau\bar{Y}$$

C.5.

[9]This cost is assumed to be exogenous and constant.

[10]Partly due to the rising importance of merit aid and tax credits, government financial aid is not universally directed at low-income families; McPherson and Schapiro (2006) state that governments provide "rather little" in the form of grants to low-income students.

where $S = E(s_i)$ is the mean of s_i across the population, or the fraction of the population attending college, G is the discounted total of other (exogenous) government spending over the 12 periods,[11] and $\bar{Y}$ is mean total discounted lifetime income. If V_i is total lifetime utility of individual i, and social welfare V is utilitarian with equal weights, then $V = E(V_i)$ and the social welfare gain from increasing the student grant b is:

$$\begin{aligned}\frac{dV}{db} &= \frac{\partial V}{\partial b} + \frac{\partial V}{\partial \tau}\frac{d\tau}{db} \\ &= E\left(\frac{\partial V_i}{\partial b}\right) + E\left(\frac{\partial V_i}{\partial \tau}\right)\frac{d\tau}{db}.\end{aligned} \tag{1}$$

2.2 Welfare Calculations

I will now solve the model for an empirically-implementable version of (1). First, I evaluate the terms in (1), making use of the (unwritten) first-order conditions of the individual's maximization problem:

$$\frac{\partial V_i}{\partial b} = s_i(\lambda_{1i} + \mu_i) = s_i u'(c_{ui})$$

$$\frac{\partial V_i}{\partial \tau} = -s_i\lambda_{1i}\gamma_2 Y_{11} - (1-s_i)\lambda_{0i}\gamma_1 Y_{01} = -s_i\gamma_2 Y_{11} v'(c_{vi}^1) - (1-s_i)\gamma_1 Y_{01} v'(c_{vi}^0)$$

$$\frac{d\tau}{db} = \frac{S}{\bar{Y}}\left[1 + \varepsilon_{Sb} - \left(1 + \frac{G}{Sb}\right)\varepsilon_{\bar{Y}b}\right] \tag{2}$$

where ε_{ab} represents the (total derivative) elasticity of a with respect to b, and thus ε_{Sb} measures the effect of student grants on college enrollment, while $\varepsilon_{\bar{Y}b}$ measures the total effects of grants on average income, and thus on the tax base. Defining $E_0[\cdot]$ and $E_1[\cdot]$ as expectations over individuals for whom $s_i = 0$ and $s_i = 1$ respectively, the welfare derivative is:

$$\frac{dV}{db} = SE_1[u'(c_{ui})] - [S\gamma_2 Y_{11} E_1[v'(c_{vi}^1)] + (1-S)\gamma_1 Y_{01} E_0[v'(c_{vi}^0)]]\frac{S}{\bar{Y}}\left[1 + \varepsilon_{Sb} - \left(1 + \frac{G}{Sb}\right)\varepsilon_{\bar{Y}b}\right].$$

Next, I use the fact that η_i, the idiosyncratic taste for schooling, only affects the choice of s_i, and that the debt limit has no effect on the consumption of those who do not attend college; therefore, $c_{vi}^0 = c_v^0$ is constant across individuals and $E_0[v'(c_{vi}^0)] = v'(c_v^0)$. Next, for some intermediate value of consumption c^*, I can write $S\gamma_2 Y_{11} E_1[v'(c_{vi}^1)] + (1-S)\gamma_1 Y_{01} v'(c_v^0) = \bar{Y}v'(c^*)$; presumably $c_{vi}^1 > c_v^0$, and therefore by the intermediate value theorem $c_{vi}^1 > c^* > c_v^0$

[11] Because G is exogenously fixed, I do not need to account for it in individual utility.

and $v'(c^*) < v'(c_v^0)$. The expression for $\frac{dV}{db}$ thus becomes:

$$\frac{dV}{db} = SE_1[u'(c_{ui})] - Sv'(c^*)\left[1 + \varepsilon_{Sb} - \left(1 + \frac{G}{Sb}\right)\varepsilon_{\bar{Y}b}\right].$$

Finally, I normalize the welfare gain into a dollar amount, to facilitate a comparison to the size of tuition subsidies and a simple expression of the economy-wide gain. Define $\frac{dW}{db} \equiv \frac{\frac{dV}{db}}{v'(c_v^0)}$; this expresses the welfare gain in terms of an equivalent amount of additional consumption among non-graduates.[12] Therefore:

$$\begin{aligned}\frac{dW}{db} &= S\frac{E_1[u'(c_{ui})]}{v'(c_v^0)} - S\frac{v'(c^*)}{v'(c_v^0)}\left[1 + \varepsilon_{Sb} - \left(1 + \frac{G}{Sb}\right)\varepsilon_{\bar{Y}b}\right] \\ &\simeq S\left[\frac{E_1[u'(c_{ui})] - v'(c_v^0)}{v'(c_v^0)} - \varepsilon_{Sb} + \left(1 + \frac{G}{Sb}\right)\varepsilon_{\bar{Y}b}\right]. \end{aligned} \tag{3}$$

By making the assumption that $v'(c^*) \simeq v'(c_v^0)$ above, I am overstating the relative importance of the derivative of the government budget constraint $\frac{d\tau}{db}$; given that the optimum will occur where $\frac{d\tau}{db}$ is positive, this will lead to an underestimate of the optimal b.[13]

Equation (3) provides a simple and intuitive illustration of the welfare consequences of tuition subsidies in terms of the magnitudes of liquidity constraints and fiscal externalities. The first term in the square brackets, the ratio of marginal utilities, measures the welfare effect of taking a dollar from workers and giving it to students, which depends on the magnitude of liquidity constraints: if students are highly constrained, their marginal utility of income will be large and redistribution will generate welfare gains. Meanwhile, the remaining terms measure the fiscal impact of tuition subsidies: if higher subsidies raise average incomes, the resulting increase in tax revenues will also raise welfare. In fact, equation (3) is more general than the current setting, and I demonstrate in Lawson (2014) that a formula of this sort could be applied to any government transfer program, as the welfare gain or loss from taking a dollar from one group and giving it to another is weighed against the revenue cost or saving to the government from behavioural responses.[14]

Although this result is very general, this model has been conspicuous in its simplicity, for two reasons: ease of interpretation, and a starting point for the next step in my analysis, which is the replacement of the ratio of marginal utilities with some empirically observable

[12]Since presumably $v'(c_v^0) > E[v'(c_{vi}^1)]$, this is less than the dollar amount I would get if I normalized by the mean marginal utility while employed.

[13]When I produce results using a structural approach, I will no longer need to make this assumption.

[14]In the terminology of Okun (1975), this revenue effect measures the "leakiness of the bucket" used to transfer resources.

quantity. Similar to the analysis of unemployment insurance in Chetty (2008), I will decompose the marginal utility term into two effects, which I will call the liquidity and substitution effects.[15]

For simplicity, let me first assume that debt limits are the same for all individuals; the result is robust to a distribution of debt limits under certain assumptions, as I show in appendix A, but the intuition is clearer in the simplest case. Thus, since the only heterogeneity enters in the form of the personal taste for schooling η_i, consumption choices if schooling is undertaken are identical for all individuals, i.e. $c_{ui} = c_u, c_{vi}^1 = c_v^1$ for all i. An individual chooses to attend college if $U_1 + \eta_i \geq U_0$, or equivalently if the taste for schooling exceeds a critical value:

$$\eta_i \geq R_1 v(c_v^0) - u(c_u) - R_2 v(c_v^1).$$

I assume that the taste for schooling η_i follows some continuously differentiable distribution $F(\eta)$, with a density given by $f(\eta)$. It follows that $S = 1 - F[R_1 v(c_v^0) - u(c_u) - R_2 v(c_v^1)]$, and therefore:

$$\frac{\partial S}{\partial b} = f(\eta^*) u'(c_u)$$

$$\frac{\partial S}{\partial a_1} = f(\eta^*)[u'(c_u) - v'(c_v^0)]$$

where η^* is the critical value, and where a_1 is an additional lump-sum of cash in the first period, representing changes in initial assets. It follows that I can rewrite (3) in the following way:

$$\frac{dW}{db} \simeq S\left[L - \varepsilon_{Sb} + \left(1 + \frac{G}{Sb}\right)\varepsilon_{\bar{Y}b}\right] \tag{4}$$

where $L = \frac{\frac{\partial S}{\partial a_1}}{\frac{\partial S}{\partial b} - \frac{\partial S}{\partial a_1}}$.

The $\frac{\partial S}{\partial a_1}$ in the numerator of L is the liquidity effect, as it is the effect on enrollment of changing initial assets, whereas I call the $\frac{\partial S}{\partial b} - \frac{\partial S}{\partial a_1}$ in the denominator the substitution effect, as it represents the effect on enrollment of changing relative prices without providing immediate income to students. L is therefore the liquidity-substitution ratio, and a higher value of L indicates more severe liquidity constraints and a greater welfare gain from redistributing to students.[16] (4) is the equation I will use in my sufficient statistics analysis in

[15] Chetty (2008) refers to the latter as a moral hazard effect in the context of unemployment insurance.

[16] Notice that if there is no causal effect of income on enrollment, i.e. $\frac{\partial S}{\partial a_1} = 0$, it must be that $u'(c_u) = v'(c_v^0)$, and I expect that $v'(c_v^0) > v'(c_v^1)$, so therefore $u'(c_u) > v'(c_v^1)$. However, the absence of liquidity constraints requires $u'(c_u) = v'(c_v^1)$; therefore, a precisely-estimated zero effect of income on enrollment is

the next section, as it allows me to estimate the welfare gain from a marginal change in b, given values of the quantities which appear in the equation.

3 Numerical Results

In this section, I will focus on providing numerical results, starting with equation (4). First of all, using estimates of the current values of each of the sufficient statistics in (4), I calculate an estimated value of $\frac{dW}{db}$ and thereby determine if financial aid ought to be increased or decreased. To go beyond this local derivative, I must make additional assumptions, and I begin by performing statistical extrapolations of the quantities in (4), predicting their values out of sample. As an alternative, I then use the sufficient statistics to calibrate my model and simulate to find the optimum, to demonstrate that similar results can be obtained from both methods.

3.1 Sufficient Statistics Method

To evaluate (4), I must specify values for a number of quantities; these values are summarized in Table 1. Throughout the numerical analysis, I use estimates derived from American data. To begin with, I use $S = 0.388$, which is the estimate of the enrollment rate of 18-24-year-olds in 2007 from NCES (2011).[17] I also specify a baseline grant of $b = 2$ (defining monetary amounts as thousands of dollars per year), using data on federal aid and state grants in 2007-08 from NCES (2011) and applying the formula of Epple, Romano, and Sieg (2006) for turning loans and work-study into grant equivalents.[18]

in fact evidence in favour of liquidity constraints. If individuals were unconstrained, income should have a negative causal effect on enrollment, because a dollar of income would be more valuable to those who do not attend college, as argued by Belley and Lochner (2007). To illustrate this point, when I study a case without liquidity constraints in section 4.1, I find that the model implies that each additional \$1000 of initial assets should reduce enrollment by between 0.12 and 0.25 percentage points. Therefore, my results support the argument of Belley and Lochner (2007) that a zero causal effect of income on enrollment should not necessarily be taken as evidence against liquidity constraints.

[17] I intend this value as a compromise. Given that the population of 18-24-year-olds includes individuals who have already completed or dropped out of college, this is an underestimate of the proportion of individuals ever enrolled; Lovenheim (2011) finds that 52% of his sample has completed more than 12 years of schooling. However, it is also an overestimate of the proportion actually completing a degree, which was 28.7% in 2007 according to NCES (2011).

[18] In 2007-08, 27.6% of undergraduates received federal grants averaging \$2800, 34.7% received federal loans averaging \$5100, 5.6% received federal work-study averaging \$2300, and 16.4% received state grants averaging \$2500; Epple, Romano, and Sieg (2006) suggest using a formula of $aid = grants + 0.25 \times loans + 0.5 \times workstudy$, which gives a per-person average of \$1690. Lacking data on other government aid and tax credits, I round this total up to \$2000.

In choosing $b = 2$, I am assuming that the out-of-pocket tuition cost e prior to government aid covers the marginal cost of college education. At public institutions, this may not be accurate if colleges' need for public funding increases with enrollment, but increased education should also lead to reductions in government spending on programs such as corrections, social insurance and social assistance. In fact, Trostel (2010) finds that additional expenditures on appropriations per degree are roughly offset by reductions in other government spending,[19] supporting my assumption that the marginal cost of education is well captured by e; in that case, increased education appropriations and reduced social spending cancel out of the government budget constraint and I can ignore both in my analysis. My conclusions, however, are not sensitive to this assumption, as the effects of education on income are considerably larger than the effects on other program expenditures; in appendix C.2, I redo all the calculations using Trostel's most pessimistic estimates, and the results are only slightly changed.

Deming and Dynarski (2009) summarize the literature on the price response of college attendance, and conclude that the general consensus is that a \$1000 increase in price leads to a 4 percentage point decline in attendance, with a similar proportional impact on completion; this implies an elasticity of $\varepsilon_{Sb} \simeq 0.2$, which will be my baseline case. However, Dynarski (2008) estimates that \$2500 of financial aid leads to a 4 percentage point increase in degree completion from a base of 27%, which suggests a value closer to 0.1, so I will present results for this case as well.

As discussed in the introduction, numerous papers argue that income has no causal effect on enrollment, so $L = 0$ will be my preferred estimate. However, several papers do find a positive income effect, the largest of which is Coelli (2011), whose results imply an effect on enrollment 25% as large as my preferred estimate of $\frac{\partial S}{\partial b}$, suggesting $L = \frac{1}{3}$; I will therefore present results for both values.

For $\frac{G}{Sb}$, the ratio of the exogenous government spending to spending on grants, I begin with my estimates of $b = 2$ and $S = 0.388$; I then need to estimate $\tau\bar{Y}$ in order to be able to compute G. I use a value of $\tau = 0.23$, which incorporates a 15% federal tax rate, a 5%

[19] Trostel (2010) liberally estimates the cost to government per degree at \$71400 in present value, or about \$16000 per year of education beyond direct subsidies; meanwhile, he conservatively estimates the reduction in expenditures on such things as Medicaid, UI, and corrections per degree as \$55800 in present value, or about \$14000 per year. On its own, however, this does not imply that increasing tuition subsidies is nearly costless, as increased payments must also be made to inframarginal students.

state tax, and 3% for the Medicare tax.[20] For $\bar{Y}$, I turn to the CPS 2008 Annual Social and Economic Supplement, which estimates the mean earnings of a high school graduate in 2007 to be \$33609, which I round up to $Y_{01} = 34$, meaning that $Y_{11} = 34(1.08)^4 = 46.26$ and $\bar{Y} = 301.661$. Therefore, $G = 68.606$, and the ratio of G to grant spending is $\frac{G}{Sb} = 88.410$.

Finally, to calculate a value for the elasticity of income with respect to grants, I assume that each year of schooling increases earnings by a constant 8%,[21] and that the elasticity of taxable income to the net-of-tax rate is 0.4, as found by Gruber and Saez (2002).[22] Utilizing these estimates, appendix B demonstrates that the elasticity is:

$$\varepsilon_{\bar{Y}b} = \left[\frac{[\gamma_2(1.08)^4 - \gamma_1]\, S}{\gamma_2(1.08)^4 S + \gamma_1(1-S)} - \frac{0.4\tau}{1-\tau}\left(1 + \frac{G}{Sb}\right)^{-1} \right] \frac{1-\tau}{1-1.4\tau}\varepsilon_{Sb} - \frac{0.4\tau}{1-1.4\tau}\left(1 + \frac{G}{Sb}\right)^{-1}. \tag{5}$$

Therefore, at baseline, the elasticity takes a value of 0.0142.

Table 1: Baseline Values of Sufficient Statistics

Statistic	Definition	Value
S	enrollment rate	0.388
b	per-year student grant	2
ε_{Sb}	elasticity of enrollment w.r.t. b	$\{0.1, 0.2\}$
L	liquidity ratio	$\{0, \frac{1}{3}\}$
$\frac{G}{Sb}$	ratio of exogenous spending to grant spending	88.410
$\varepsilon_{\bar{Y}b}$	elasticity of mean income w.r.t. b	0.0142

Plugging in each of these values, I find values for $\frac{dW}{db}$ as displayed in Panel A of Table 2. They are positive and substantial, suggesting that a one dollar per year increase in student grants from the baseline of \$2000 would provide a welfare increase equivalent to between 18 and 54 cents per year for 4 years. Spreading these gains out over an entire lifetimes, this means that a 1% increase in b to \$2020 would provide an average individual with an annual welfare gain of between \$0.52 and \$1.57 over their working lifetime. Aggregated up to an

[20] The federal and state rates are chosen to be appropriate for the typical high school graduate; they will be conservative for many college graduates. In my analysis of heterogeneous returns to education in appendix C.5, I model the tax system in more detail.

[21] The bulk of the estimates summarized in Card (1999) are between 6% and 11%, but more recent estimates are higher: Dynarski (2008) summarizes several higher estimates of returns to college education, and Heckman, Lochner, and Todd (2006) and Carneiro, Heckman, and Vytlacil (2010) estimate "policy relevant treatment effects" of tuition subsidies that range from 9% to 25%, making 8% a conservative estimate.

[22] I adjust equilibrium earnings for changes in taxes according to this elasticity, but I do not directly model the labour supply decision; in this way, I will tend to produce an underestimate of the optimal b, since I overstate the cost of tax increases by ignoring increases in leisure.

economy-wide level, this corresponds to a net welfare gain of \$101 to \$305 million per year for an increase in yearly grant spending of about \$126 million,[23] indicating a very large return to public investments in education, as the fiscal benefits of subsidizing college education are substantial even if gains from redistribution to constrained individuals are zero.

Table 2: Results from Sufficient Statistics and Extrapolation using (4)

	$\hat{L}$	
Value of ε_{Sb}	0	$\frac{1}{3}$
	A. Estimate of $\frac{dW}{db}$ at $b=2$	
0.1	0.1811	0.3104
0.2	0.4148	0.5442
	B. Optimal Student Grants	
0.1	\$5843	\$13718
0.2	\$8093	\$9075
	C. Welfare Gains from Moving to Optimum	
0.1	\$947 (30.5%)	\$3836 (123.6%)
0.2	\$3138 (101.1%)	\$4935 (159.0%)

Panel A presents the one-period per-year increase in welfare, expressed in dollars of consumption from a per-year one dollar increase in b. Panel C expresses welfare gains as a one-time lump-sum increase in consumption, and as a percentage of baseline spending on student grants. This format is used throughout all subsequent tables of this form.

3.2 Statistical Extrapolation

To make predictions out of sample, and thereby to produce an estimate of the optimal level of student grants, I need to make some functional form assumptions. In the current subsection, I make functional form assumptions about the sufficient statistics themselves; this approach is proposed in Chetty (2009), and has previously been used in sufficient statistic studies of unemployment insurance, including Baily (1978) and Lawson (2013).

First, let me now denote the baseline values of quantities using hats, i.e. $\hat{b} = 2$ and $\hat{S} = 0.388$. I assume a constant elasticity of enrollment with respect to grants, $\varepsilon_{Sb} = \{0.1, 0.2\}$, implying that $S = \phi b^{\varepsilon_{Sb}}$, where $\phi = \frac{\hat{S}}{\hat{b}^{\varepsilon_{Sb}}}$. I can then write the government spending ratio as $\frac{G}{Sb} = 88.41\frac{\hat{S}\hat{b}}{Sb}$, and $\varepsilon_{\bar{Y}b}$ is given by (5) as before but with τ held fixed at 0.23 for simplicity.

This leaves only the liquidity-substitution ratio L to be extrapolated. Belley, Frenette, and Lochner (2011) find a 16 percentage point gap in attendance at 4-year colleges between

[23] I use the Census Bureau's April 2010 estimate of the 18-64 population as 194,296,087.

the highest and lowest family income quartiles, so I follow their simple approach and assume that a 16 percentage point increase in enrollment is needed to eliminate the liquidity effect. I assume that the effect of income on enrollment (as a fraction of the effect of b on enrollment) declines linearly in enrollment until it reaches zero; therefore, if the initial L is $\frac{1}{3}$, I assume $\frac{\partial S}{\partial a_1} = \frac{1}{4}\frac{0.16-(S-\hat{S})}{0.16}\frac{\partial S}{\partial b}$. Then, plugging this equation into $L = \frac{\frac{\partial S}{\partial a_1}}{\frac{\partial S}{\partial b}-\frac{\partial S}{\partial a_1}}$, I find that $L = \max\{\frac{\frac{25}{16}(0.16-(S-\hat{S}))}{1-\frac{25}{16}(0.16-(S-\hat{S}))}, 0\}$.

Putting all of this together, I obtain the results displayed in Panels B and C of Table 2; to estimate the net welfare gain from moving to the optimum, I numerically integrate $\frac{dW}{db}$ from $b = 2$ to the optimum. I then express the welfare gain in two ways: I multiply by 4 to calculate the dollar amount of an equivalent one-year per-person consumption increase, and I also divide by $\hat{S}\hat{b}$ to express the gain as a percentage of the initial size of the student grant program; the latter values are shown in brackets in Panel C.

The results indicate that student grants should be increased substantially, by at least \$3800 per year. NCES (2011) estimates that median tuition at public 4-year universities was \$5689 in 2007-08, so my results could be interpreted to mean that net tuition should be eliminated, and government appropriations increased accordingly.[24] With a larger responsiveness of enrollment to tuition or more serious borrowing constraints, optimal grants are even larger, corresponding to not only free tuition but also a yearly stipend which reaches as high as \$8000; with the preferred estimates of $\varepsilon_{Sb} = 0.2$ and $\hat{L} = 0$, the optimal stipend is about \$2400 per year above and beyond free tuition.

Meanwhile, the estimated welfare gains are substantial, particularly in comparison to the size of the policy change; aggregated to an economy-wide level, they indicate annual net welfare improvements of between \$6.6 billion and \$34.6 billion, or as much as 0.24% of GDP.[25]

[24] Although this is outside the scope of the current analysis, a policy of abolishing tuition is likely to be more effective than offsetting tuition with financial aid, for reasons of salience and reduced administration. Courant, McPherson, and Resch (2006) argue that the "old tradition of making public higher education 'free' has much to recommend it," and claim that this policy might be efficient if enrollment is sufficiently sensitive to tuition, but they do not evaluate the welfare implications of the policy themselves.

[25] As a comparison, I can approximate the welfare gains from solving the educational mismatch found by Robst (2008), who evaluates wage penalties from mismatch between education and occupation on dimensions of both quantity and type of education. Robst finds little evidence of inefficient private decision-making, as the majority of mismatched individuals state supply-side reasons for their situation; in that case, the only welfare losses are from the fiscal externality, and an upper bound on the welfare gain can be computed under the assumption that mismatched individuals are indifferent between their current situation and an "appropriate" education-occupation match (so that there are no personal utility losses from being forced to take a different degree or job). In this case, the upper bound of the welfare gain is 0.36% of GDP,

3.3 Simulation of Structural Model

As discussed in the introduction, previous welfare analyses of college tuition subsidies have relied on simulations of parameterized structural models; the previous subsection therefore represents the first application of the method of extrapolation of sufficient statistics to the search for optimal subsidies. In this subsection, I will demonstrate that the results above are not an artifact of the method, by calculating optimal subsidies from a calibrated version of my simple structural model. However, unlike previous analyses, I will calibrate the model parameters with the use of the sufficient statistics, to ensure that the moments of the model match the most important empirical quantities in the education sector.

I begin by assuming CRRA utility, so $u(c) = \frac{c^{1-\theta}}{1-\theta}$ and $v(c) = \frac{c^{1-\rho}}{1-\rho}$. I specify starting incomes as $Y_{01} = 34y$ and $Y_{11} = 34(1.08)^4 y$, where $y = \alpha(1-\tau)^{ETI}$ and $\alpha = \frac{1}{(1-\hat{\tau})^{ETI}}$, so $y = 1$ at baseline and shifts with τ to capture the distortionary effects of taxation. I also assume that η follows a logistic distribution with mean μ and scale parameter σ.

I use $e = 5.7$ to represent public tuition, and exogenous spending $G = 68.606$ as described earlier. I assume that all individuals face the same debt limit A, so I have to solve for 5 parameters: $\{A, \theta, \rho, \mu, \sigma\}$.[26] However, I only have three sufficient statistic conditions: $\hat{S} = 0.388$, $\varepsilon_{Sb} = \{0.1, 0.2\}$ and $\hat{L} = \{0, \frac{1}{3}\}$, so I need to incorporate additional data.

One piece of data I can use is some comparison of the values of consumption: $\hat{c_u}$, $\hat{c_v^0}$ and $\hat{c_v^1}$. Any ratio of two of these, along with the equation for the debt limit and the first-order conditions, will define all three. One possibility is to use consumption values from the Consumer Expenditure Survey, where I find that, on average, college graduates consumed 73.9% of their pre-tax income and high school graduates consumed 83.4% in 2007. The NBER's TAXSIM calculator for 2007 allows me to transform these into percentages of after-tax income (ignoring state taxes and assuming a single-earner married couple), and if I then apply those values to my estimates of Y_{01} and Y_{11}, I find that consumption of college graduates is 27.58% higher than that of high school graduates: $\hat{c_v^1} = 1.2758\hat{c_v^0}$. An alternative is to use results in Keane and Wolpin (2001) implying student consumption (not including room and board) of $8077 in 1987, plus the estimate from NCES (2011) of average room and board expenses in 1987-88 of $3037, compared to average per-equivalent-person

demonstrating that welfare gains from the source I am considering are of at least the same order of magnitude. All calculations are available upon request.

[26] μ is not normalized to zero because $u(c)$ and $v(c)$ both have zero intercepts, so μ represents the difference in intercepts, or the mean direct utility or disutility from schooling.

consumption of \$15816 in 1988 as estimated by Cutler and Katz (1991). I then find that student consumption is 73.18% of average consumption across a steady-state of individuals, which implies $\hat{c_v^1} = 1.2437\hat{c_v^0}$. These estimates are very similar, and so I will take a value halfway in between, specifically $\hat{c_v^1} = 1.26\hat{c_v^0}$.

Finally, I can use an external estimate of relative risk-aversion to pin down one of θ and ρ. A CRRA parameter of 1 is typical, so I assume $v(c) = \ln(c)$, but I also try $\rho = 2$ in appendix C.1.

My calibration method begins by using $\hat{c_v^1} = 1.26\hat{c_v^0}$ to solve for the debt limit A; the condition $u'(c_u) = (\hat{L}+1)v'(c_v^0)$ then allows me to solve for θ, and I can use these results and the conditions that $\hat{S} = 0.388$ and $\varepsilon_{Sb} = \{0.1, 0.2\}$ to find the parameters of the preference distribution, μ and σ. I then simulate the model for various values of b to find the optimum, and the results are displayed in Table 3.

Table 3: Results from Calibration and Simulation

	$\hat{L}$	
Value of ε_{Sb}	0	$\frac{1}{3}$
	A. Numerical Estimate of $\frac{dW}{db}$ at $b = 2$	
0.1	0.1246	0.2537
0.2	0.4210	0.5505
	B. Optimal Student Grants	
0.1	\$4016	\$6408
0.2	\$6151	\$7945
	C. Welfare Gains from Moving to Optimum	
0.1	\$500 (16.1%)	\$2214 (71.3%)
0.2	\$3538 (114.0%)	\$6667 (214.8%)

Panel A presents the one-period per-year increase in welfare, expressed in dollars of consumption from a per-year one dollar increase in b. Panel C expresses welfare gains as a one-time lump-sum increase in consumption, and as a percentage of baseline spending on student grants. This format is used throughout all subsequent tables of this form.

The results for the optimal level of b are somewhat smaller than those in Table 2, by less than \$2000 except for a much larger drop in the case where $\varepsilon_{Sb} = 0.1$ and $\hat{L} = \frac{1}{3}$, in which my statistical extrapolations implied that the liquidity effect goes away very slowly. This is primarily the result of the statistical extrapolation of L, where I assumed that L does not drop below zero; in simulations of the model, L can drop below -0.2 at high values of b. However, the qualitative conclusions are similar, in that eliminating tuition remains the optimal policy in all but one case. The estimated welfare gains, meanwhile, are actually larger than before when $\varepsilon_{Sb} = 0.2$, reaching \$24.8 billion in the baseline case;[27] overall, they vary from a low of \$3.5 billion to a high of \$46.7 billion or 0.32% of GDP.

Figure 1 displays the values of the enrollment rate S over the relevant range for $\hat{L} = 0$ (the results are almost identical for positive L). In both cases, but especially with $\varepsilon_{Sb} = 0.2$, the optimal policy (indicated by the squares) involves inducing significant increases in the fraction of the population that attends college. Figure 2 displays the budget-balancing tax rates, and it is remarkable (though hard to see) that, when $\varepsilon_{Sb} = 0.2$, a small increase in b from the current level leads to a lower tax rate, because average income increases enough that the increased grants more than pay for themselves. This quickly ceases to be true as grants increase further, but if this standard estimate of the responsiveness of enrollment to tuition is correct, then at present we are slightly on the wrong side of a "financial aid Laffer curve," and thus there are Pareto improvements available from a small increase in tuition subsidies: taxes do not have to rise until the grant level reaches about \$2570. Beyond that, the tax rate does rise, which means that there is redistribution away from high school graduates, which is socially costly, and yet the losses of high school graduates are more than offset by the considerable gains of college graduates until b is over \$6000.

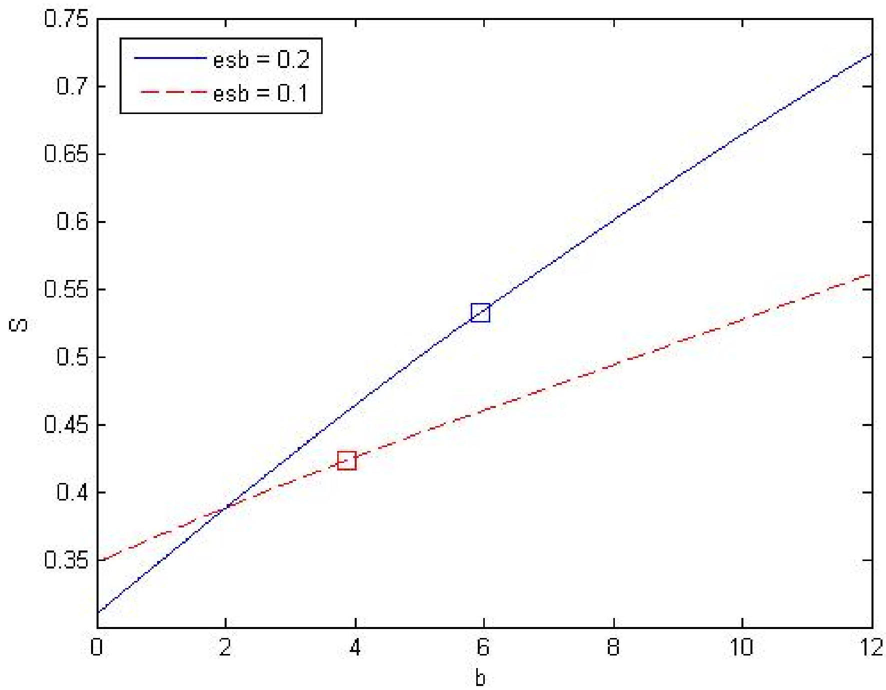
To further test the robustness of my results, I perform several extensions and alterations to the model in appendix C. A higher degree of risk-aversion slightly lowers optimal grants, while uncertainty about future income raises them; an alternative specification of government spending has varying effects depending on whether the statistical extrapolation or calibration method is used. I also find that heterogeneity in liquidity constraints has small effects on the results, while heterogeneous returns to education tend to lead to somewhat lower optimal subsidies but considerably larger welfare gains. The qualitative conclusions, however, are very similar across all extensions, and correspond closely to those found in this section.

4 Comparing the Effects of Liquidity Constraints and Fiscal Externalities

In this section, as a sensitivity analysis, I perform the experiment of "switching off" the liquidity constraints and the fiscal externalities one at a time, to determine which contributes

[27]This occurs both because S rises faster with b in the calibration case and because I no longer make the assumption implicit in equation (3) which overstates the importance of $\frac{d\tau}{db}$.

Figure 1: Enrollment Rates as Function of b for $\hat{L} = 0$

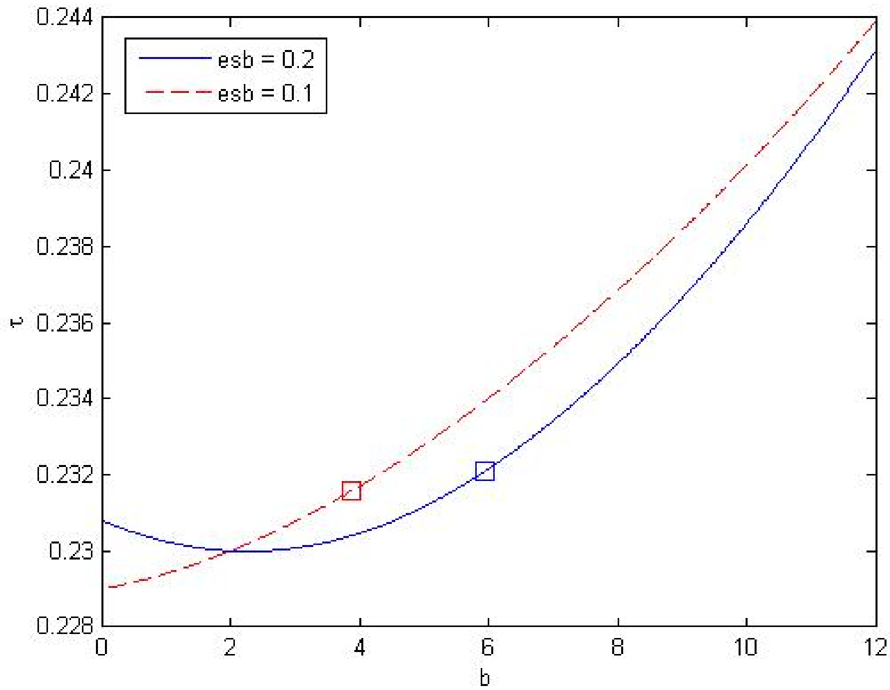


more to the argument for more generous financial aid.

4.1 No Liquidity Constraints

I begin by assuming away liquidity constraints. This is difficult to accomplish with the sufficient statistics appraoch, as there is no simple way to impose a zero-liquidity-constraint condition; as noted in footnote 16, $L = 0$ does not correspond to no liquidity constraints, since an absence of liquidity constraints actually requires $v'(c_v^1) = u'(c_u)$, which implies a negative effect of income on enrollment. In order to impose the theoretical condition of no liquidity constraints in the sufficient statistics approach, I need to know what empirical value of L this would correspond to. One way to calculate the implied L is by simulating a structural model, and the results later on in this section suggest a value of $L = -0.2063$ in the absence of liquidity constraints; therefore I will proceed to estimate the optimal subsidies assuming a fixed value of $L = -0.2063$. The results are found in Table 4, and although the optimal subsidies are noticeably smaller in this case, significant increases in generosity are still called for, and in the baseline case the optimal policy nearly corresponds to completely offsetting tuition.

Figure 2: Budget-Balancing Tax Rate as Function of b for $\hat{L} = 0$



For the structural analysis, I begin by using $\hat{c_v^1} = 1.26\hat{c_v^0}$ to solve for values of consumption, and then I use $v'(c_v^1) = u'(c_u)$, the no-liquidity-constraint condition, to solve for θ. The rest of the calibration procedure continues as before, and the results are displayed in Table 5. The results are similar to before; the values of $\frac{dW}{db}$ are somewhat smaller than in Table 3, but in the baseline case the optimal benefit level is actually higher than in the $\hat{L} = 0$ case, and the welfare gain is nearly identical. It therefore appears that my general conclusion is not sensitive to the existence of liquidity constraints.

4.2 No Fiscal Externalities

Next, I instead shut off the fiscal externality, in the sense that I ignore G and assume that τ_t is a lump-sum tax imposed on employed workers to pay for tuition subsidies, growing at rate g per period, so $\tau_t = (1 + g)^{t-1}\tau$. Re-doing my initial analysis in this context is straightforward, and the resulting equation for the welfare gain from increasing b is:

$$\frac{dW}{db} \simeq S\left(L - \frac{\gamma_1}{\gamma_1 - S}\varepsilon_{Sb}\right).$$

Table 4: Results from Sufficient Statistics and Extrapolations with No Liquidity Constraints

Value of ε_{Sb}	
	A. Numerical Estimate of $\frac{dW}{db}$ at $b = 2$
0.1	0.1010
0.2	0.3348
	B. Optimal Student Grants
0.1	\$3156
0.2	\$5075
	C. Welfare Gains from Moving to Optimum
0.1	\$202 (6.5%)
0.2	\$1533 (49.4%)

Table 5: Results from Calibration and Simulation with No Liquidity Constraints

Value of ε_{Sb}	
	A. Numerical Estimate of $\frac{dW}{db}$ at $b = 2$
0.1	0.0451
0.2	0.3412
	B. Optimal Student Grants
0.1	\$3314
0.2	\$6635
	C. Welfare Gains from Moving to Optimum
0.1	\$120 (3.9%)
0.2	\$3352 (108.0%)

This result also follows directly from (4) if I assume $G = 0$ and take $\varepsilon_{\bar{Y}b}$ to be the elasticity of the tax base $\gamma_1 - S$. Implementing this formula, I get the results in Table 6. The values of $\frac{dW}{db}$ are much smaller, as are most of the optimal values of b; if $L = 0$ then there is no reason whatsoever to subsidize education (I set $b = 0$ as a lower bound). In the case of $\varepsilon_{Sb} = 0.1$ and $\hat{L} = \frac{1}{3}$, the estimated optimal b is very large, but this is an anomaly resulting from the assumption that the fiscal externality is negative but small whereas the liquidity effect is significant and takes a long time to completely dissipate.

The structural approach also follows in the usual way. The results can be found in Table 7, and present conclusions that are similar to those in Table 6, with slightly larger values for $\frac{dW}{db}$ and optimal grants, with the exception of a much smaller optimal b in the $\varepsilon_{Sb} = 0.1$, $\hat{L} = \frac{1}{3}$ case. Therefore, using both approaches, I find that fiscal externalities are important to establishing beneficial effects of significantly increased grants to students; severe liquidity constraints would otherwise be required in order to support significant grant increases, and

Table 6: Results from Sufficient Statistics and Extrapolation with no Fiscal Externalities

	$\hat{L}$	
Value of ε_{Sb}	0	$\frac{1}{3}$
	A. Estimate of $\frac{dW}{db}$ at $b = 2$	
0.1	-0.0407	0.0886
0.2	-0.0814	0.0479
	B. Optimal Student Grants	
0.1	$0	$19218
0.2	$0	$3600
	C. Welfare Gains from Moving to Optimum	
0.1	$295 (9.5%)	$1859 (59.9%)
0.2	$539 (17.4%)	$143 (4.6%)

in the baseline case, the optimal policy would involve reducing or even abolishing tuition subsidies in the absence of fiscal externalities.[28]

Table 7: Results from Calibration and Simulation with No Fiscal Externalities

Value of ε_{Sb}	$\hat{L} = 0$	$\hat{L} = \frac{1}{3}$
	A. Numerical Estimate of $\frac{dW}{db}$ at $b = 2$	
0.1	-0.0094	0.1200
0.2	-0.0473	0.0821
	B. Optimal Student Grants	
0.1	$1706	$6304
0.2	$1014	$4029
	C. Welfare Gains from Moving to Optimum	
0.1	$5 (0.2%)	$1040 (33.5%)
0.2	$91 (2.9%)	$338 (10.9%)

4.3 Relative Importance of Fiscal Externalities and Liquidity Constraints

To summarize my results in this section, it appears that the liquidity term makes relatively little difference to the optimal level of b; eliminating all liquidity constraints may moderately reduce the optimal b but does not change its qualitative implications. Fiscal externalities, on the other hand, seem to be far more important. The logic is that, while liquidity constraints may well be a motivation for financial aid, fiscal externalities on their own can justify eliminating tuition and possibly providing stipends in most cases, by which point any liquidity

[28]An indirect subsidy from state appropriations may still exist; thus, it is perhaps more appropriate to speak here of reducing the total effective subsidy rather than of abolishing subsidies.

constraints will have ceased to be a major concern. Thus, liquidity constraints appear to be of second-order importance when designing optimal financial aid policy for college students.

5 General Equilibrium Effects

In this section, I model general equilibrium effects and demonstrate how sensitive my results are to their existence and magnitude. I begin by looking at how the wage premium may shift with the supply of college graduates, using two different estimates of the elasticity of substitution. Then I consider the possibility of spillovers, or positive externalities of education onto the wages of other workers.

5.1 GE Effects on College Wage Premium

Analysis in papers such as Katz and Murphy (1992) suggests that changes in the supply of college graduates may have significant effects on relative wages. In Heckman, Lochner, and Taber (1998a) it is shown that this has consequences for the effectiveness of tuition subsidies: if increased attendance lowers the college wage premium, then grants to students can only induce a small increase in attendance before declines in the wage premium completely offset the increased incentives to attend. Heckman, Lochner, and Taber (1998a) estimate an elasticity of substitution between high school and college graduates of 1.441, and show that this means that the effect of a tuition subsidy on enrollment in general equilibrium is about one-tenth the size of the partial equilibrium effect.

However, this conclusion is sensitive to assumptions about the usage of skill in the economy, as Lee (2005) finds general equilibrium effects of tuition subsidies that are more than 90% as large as the partial equilibrium values. Also, there is reason to believe that the short-run effects on relative wages of an increase in supply of college graduates may overstate the long-run effect if increased supply of skills leads to technological change to take advantage of those skills. Acemoglu (1998), Kiley (1999) and Acemoglu (2002) present models in which an increased supply of skilled workers leads to technological adjustment that creates more jobs designed for skilled workers, with the skill premium then increasing over time, possibly above the original level. The magnitude of long-run general equilibrium effects therefore remains an unanswered question, and one that is deserving of further study; in the present analysis I will simply present results corresponding to both the Heckman, Lochner, and Taber (1998a) and Lee (2005) cases.

I begin by assuming a CES production function over high school and college graduates, specifically:

$$Y_t = \zeta_t \left(aS_{1t}^{\kappa} + (1-a)S_{0t}^{\kappa}\right)^{\frac{1}{\kappa}}$$

where $S_{1t} = S$ and $S_{0t} = 1 - S$. I assume that wages and the production function are specific to the generation in question, i.e. that vintage effects make the human capital of different cohorts perfectly non-substitutable, thereby producing an upper bound on general equilibrium effects.[29] Therefore the wage of a college graduate is $Y_{1t} = \frac{\partial Y_t}{S_{1t}}$ and the wage of a high school graduate is $Y_{0t} = \frac{\partial Y_t}{S_{0t}}$, and a is chosen to make $\frac{Y_{1t}}{Y_{0t}} = 1.08^4$ at baseline. Calibration proceeds in the same way as before, since the only derivative used there is $\frac{dS}{db}$, which I assume is evaluated at constant wages.

I produce results for two different values of the elasticity of substitution, which can be written as $\frac{1}{1-\kappa}$, namely 1.441 as in Heckman, Lochner, and Taber (1998a), which is a typical value in the literature, and a much higher value of 375, which generates a ratio of general equilibrium to partial equilibrium effects that is comparable to Lee (2005).[30] These results are displayed in Tables 8 and 9.

Table 8: Results from Calibration and Simulation with Elasticity of Substitution = 1.441

	$\hat{L}$	
Value of ε_{Sb}	0	$\frac{1}{3}$
	A. Numerical Estimate of $\frac{dW}{db}$ at $b = 2$	
0.1	-0.0567	0.1078
0.2	-0.0532	0.1137
	B. Optimal Student Grants	
0.1	\$315	\$5277
0.2	\$426	\$5431
	C. Welfare Gains from Moving to Optimum	
0.1	\$195 (6.3%)	\$681 (21.9%)
0.2	\$171 (5.5%)	\$751 (24.2%)

If high- and low-education workers are not good substitutes for each other, as argued by Heckman, Lochner, and Taber (1998a), then my findings confirm those of the latter paper in

[29] That is, I assume that the population share of college graduates adjusts immediately to that of the current generation, rather than allowing for gradual adjustment to a new long-run equilibrium. In this I follow the approach of Heckman, Lochner, and Taber (1998b), who state that short-run general equilibrium effects on enrollment with rational expectations are also very small.

[30] The average of the ratio for men $\left(\frac{1.05}{1.12}\right)$ and for women $\left(\frac{1.52}{1.66}\right)$ in Lee (2005) is 0.9266; the average ratio across the four cases displayed in Table 9 is 0.9248.

Table 9: Results from Calibration and Simulation with Elasticity of Substitution = 375

	$\hat{L}$	
Value of ε_{Sb}	0	$\frac{1}{3}$
	A. Numerical Estimate of $\frac{dW}{db}$ at $b = 2$	
0.1	0.1133	0.2466
0.2	0.3668	0.5114
	B. Optimal Student Grants	
0.1	\$3879	\$6349
0.2	\$5902	\$7802
	C. Welfare Gains from Moving to Optimum	
0.1	\$424 (13.7%)	\$2120 (68.3%)
0.2	\$2879 (92.8%)	\$6011 (193.6%)

that the role of tuition subsidies in increasing enrollment is minimal in the absence of liquidity constraints; in order to justify substantial increases in grants, significant liquidity constraints are required. However, with a much higher elasticity of substitution as in Lee (2005), the results are nearly identical to those from the baseline analysis. The magnitude of these general equilibrium effects, therefore, is of considerable importance, clearly demonstrating the importance of future work that can shed more light onto this phenomenon.

5.2 Wage Spillovers

A number of papers have sought evidence of positive wage spillovers from college education, i.e. a positive externality of education manifesting itself in higher wages for other workers, resulting from off-the-job interactions or some form of social capital. Moretti (2004a) and Moretti (2004b) represent two prominent examples that do find significant effects, whereas Ciccone and Peri (2006) do not. Damon and Glewwe (2011) evaluate the literature and conclude that a "very conservative" estimate of this effect is that a one percentage point increase in the population with a bachelor's degree increases average wages by 0.2% within education groups, with other estimates often in the range of 1%. I will therefore proceed by using this estimate that each percentage point increase in enrollment raises average wages within education group by 0.2%.

This effect can easily be incorporated into the simulation to find a numerical estimate of $\frac{dW}{db}$, but moving away from $\hat{b} = 2$, it does not seem plausible that this spillover would remain at the same level as S increases. Therefore, in Table 10 below I present results where the effect declines with S, so that the wage increase per percentage point of attendance is $\frac{\delta}{S^2}$, where $\delta = 0.002(0.388^2)$.[31]

Table 10: Results from Calibration and Simulation with Spillovers

	$\hat{L}$	
Value of ε_{Sb}	0	$\frac{1}{3}$
	A. Numerical Estimate of $\frac{dW}{db}$ at $b = 2$	
0.1	1.6665	1.8309
0.2	3.2238	3.4802
	B. Optimal Student Grants	
0.1	\$18187	\$16869
0.2	\$15609	\$15401
	C. Welfare Gains from Moving to Optimum	
0.1	\$44377 (1429.7%)	\$49706 (1601.3%)
0.2	\$74695 (2406.4%)	\$85018 (2739.0%)

Even with this "very conservative" assumption about wage spillovers, the welfare gain from increasing student grants is now enormous; a 1% increase in b to \$2020 generates an annual economy-wide gain of \$1.81 billion in the baseline case of $\varepsilon_{Sb} = 0.2$ and $\hat{L} = 0$. Furthermore, even with spillovers that diminish at the rate of S^2, the optimal grants are very large, higher than the median value of tuition, room and board at public universities of \$13035 in 2007-08, and the welfare gains are also very large, with a value of \$523.0 billion in the baseline case, or 3.6% of GDP. Given the increased spending of \$196.8 billion per year that such a policy change would imply, the return to investment is a very large 266%. Also, because of the spillovers to uneducated individuals, there is considerable scope for Pareto improvements; in all cases, Pareto gains can be obtained from marginal increases in b up to at least \$10000, and at the optimum, both high school and college graduates are better off than when $b = 2$.

The results for the optimum can only be a rough approximation, given the lack of evidence on how spillovers would change with S, but the magnitude of the welfare derivative alone indicates that wage spillovers that might have been considered small in previous work are actually extremely important, indicating a need for further work in this area.

[31] This implies a spillover that declines from 0.2% at $S = 0.388$ to 0.12% at $S = 0.5$ and to 0.06% at $S = 0.7$.

6 Conclusion

In this paper, I have presented a simple model of college education, and performed an analysis of optimal tuition subsidies in the presence of fiscal externalities and liquidity constraints using both a sufficient statistics method and a simple calibrated model. My results indicate that fiscal externalities on their own provide justification for increased government support for students. The preferred estimates indicate as a first-order policy recommendation the elimination of tuition (at least at public schools, with perhaps a stipend of equal value for private institutions); some cases also recommend a partial stipend for living expenses. These results are robust to a number of sensitivity analyses and extensions of the basic model. In particular, although the previous research on liquidity constraints has been enveloped in controversy about their magnitude and even their existence, my conclusions are driven by a strong fiscal externality from an increased tax base resulting from increased college enrollment, and are largely robust to an elimination of liquidity constraints, suggesting that the latter are of second-order importance to policy.

The one factor which can alter these conclusions is the existence of significant general equilibrium effects of tuition subsidies on wages. If effects on relative wages are as severe as those estimated by Heckman, Lochner, and Taber (1998a), then the case for abolishing tuition rests entirely on the existence of significant liquidity constraints. On the other hand, even modest wage spillovers could make a case for large stipends on top of free tuition. Thus, further work that models and estimates wage formation in general equilibrium is called for.

When interpreting the results, we must acknowledge that there is more that we need to know. However, my analysis does suggest a baseline conclusion of eliminating public tuition, and provides a clear guide to future research by highlighting the areas where we need to know more. This paper also provides a methodological advance through a novel application of the sufficient statistics method to the area of college education, demonstrating how this approach can be used outside of the usual contexts of social insurance and optimal income taxation.

A Liquidity Term with Heterogeneous Constraints

To be as general as possible, let me allow for the possibility that η_i and A_i are jointly distributed according to some bivariate distribution function $F(\eta, A)$. Let me define $S_A(A)$ to be the probability of college attendance for an individual with debt limit A; this can be written as:

$$S_A(A) = 1 - F_{\eta|A}[R_1 v(c_v^0) - u(c_u(A)) - R_2 v(c_v^1(A))|A]$$

where $F_{\eta|A}$ represents the conditional cdf. Then the overall probability of college attendance is simply $S = \int_A S_A(A) f_A(A) dA$, where f_A is the marginal density of A.

Next, observe that:

$$\frac{\partial S}{\partial b} = \int_A \frac{\partial S_A(A)}{\partial b} f_A(A) dA = \int_A f_{\eta|A}(\eta_A^*|A) f_A(A) u'(c_u(A)) dA$$

$$\frac{\partial S}{\partial a_1} = \int_A \frac{\partial S_A(A)}{\partial a_1} f_A(A) dA = \int_A f_{\eta|A}(\eta_A^*|A) f_A(A) [u'(c_u(A)) - v'(c_v^0)] dA$$

where η_A^* is the critical value for $A_i = A$. Therefore, using the definition of L from the text:

$$L = \frac{\int_A f_{\eta|A}(\eta_A^*|A) f_A(A) [u'(c_u(A)) - v'(c_v^0)] dA}{\int_A f_{\eta|A}(\eta_A^*|A) f_A(A) v'(c_v^0) dA}.$$

Meanwhile, the term I wish to replace is $\frac{E_1[u'(c_{ui})] - v'(c_v^0)}{v'(c_v^0)}$; this is greater or less than L as:

$$E_1[u'(c_{ui})] \gtreqless \frac{\int_A f_{\eta|A}(\eta_A^*|A) f_A(A) u'(c_u(A)) dA}{\int_A f_{\eta|A}(\eta_A^*|A) f_A(A) dA}$$

$$\frac{\int_A [1 - F_{\eta|A}(\eta_A^*|A)] f_A(A) u'(c_u(A)) dA}{\int_A [1 - F_{\eta|A}(\eta_A^*|A)] f_A(A) dA} \gtreqless \frac{\int_A f_{\eta|A}(\eta_A^*|A) f_A(A) u'(c_u(A)) dA}{\int_A f_{\eta|A}(\eta_A^*|A) f_A(A) dA}.$$

If the conditional hazard rate $\frac{f_{\eta|A}(\eta_A^*|A)}{1 - F_{\eta|A}(\eta_A^*|A)}$ is constant, these two terms will be equal, and I can safely replace $\frac{E_1[u'(c_{ui})] - v'(c_v^0)}{v'(c_v^0)}$ with L in (3). More generally, let me continue by substituting $h(A)$ for the conditional hazard rate, and let me also write $k(A) = [1 - F_{\eta|A}(\eta_A^*|A)] f_A(A)$ to represent the measure of enrollees at a particular value of A_i; then the comparison becomes:

$$\frac{\int_A k(A) u'(c_u(A)) dA}{\int_A k(A) dA} \gtreqless \frac{\int_A k(A) h(A) u'(c_u(A)) dA}{\int_A k(A) h(A) dA}$$

$$\frac{\int_A k(A) h(A) dA}{\int_A k(A) dA} \frac{\int_A k(A) u'(c_u(A)) dA}{\int_A k(A) dA} \gtreqless \frac{\int_A k(A) h(A) u'(c_u(A)) dA}{\int_A k(A) dA}$$

$$E_1[h(A)] E_1[u'(c_u(A))] \gtreqless E_1[h(A) u'(c_u(A))]$$

$$0 \gtreqless Cov_1[h(A), u'(c_u(A))].$$

Therefore, if the covariance of the hazard and the marginal utility among students is close to zero, it will be a reasonable approximation to insert L into (3). Meanwhile, I will tend to underestimate the liquidity effect if the covariance is negative, which would follow, for instance, if $h(A)$ is increasing in A (given that $u'(c_u(A))$ should be non-increasing in A).

Given that η_A^* is decreasing in A, I would want the hazard to be decreasing in η, which would be the case for distributions such as the Pareto and the χ^2 for degrees of freedom less than 2 (with 2 degrees of freedom, the hazard is constant). However, many other distributions, including the logistic that I use in my calibration, feature an increasing hazard, in which case my overestimate of L would tend to offset the conservative assumptions elsewhere in the model.

B Calculation of $\varepsilon_{\bar{Y}b}$

First, assuming that the only effects of b on $\bar{Y}$ are from b's effect on schooling and from the effect of the tax change on earnings Y_{01} and Y_{11}, I can write:

$$\varepsilon_{\bar{Y}b} = \frac{b}{\bar{Y}}\frac{d\bar{Y}}{db} = \frac{b}{\bar{Y}}\left[\frac{\partial \bar{Y}}{\partial S}\frac{dS}{db} + \frac{\partial \bar{Y}}{\partial \tau}\frac{d\tau}{db}\right].$$

It is clear that $\frac{\partial \bar{Y}}{\partial S} = \gamma_2 Y_{11} - \gamma_1 Y_{01} = \left[\gamma_2(1.08)^4 - \gamma_1\right] Y_{01}$, and given that I assume that the elasticity of taxable income is 0.4, I have $\frac{\partial \bar{Y}}{\partial \tau} = -0.4\frac{\bar{Y}}{1-\tau}$. Using (2) for $\frac{d\tau}{db}$, the equation for $\varepsilon_{\bar{Y}b}$ becomes:

$$\varepsilon_{\bar{Y}b} = \left[\gamma_2(1.08)^4 - \gamma_1\right] Y_{01}\frac{S}{\bar{Y}}\varepsilon_{Sb} - 0.4\frac{Sb}{(1-\tau)\bar{Y}}\left[1 + \varepsilon_{Sb} - \left(1 + \frac{G}{Sb}\right)\varepsilon_{\bar{Y}b}\right]$$

and rearranging, I arrive at:

$$\varepsilon_{\bar{Y}b} = \left[\frac{\left[\gamma_2(1.08)^4 - \gamma_1\right] S}{\gamma_2(1.08)^4 S + \gamma_1(1-S)} - \frac{0.4\tau}{1-\tau}\left(1 + \frac{G}{Sb}\right)^{-1}\right]\frac{1-\tau}{1-1.4\tau}\varepsilon_{Sb} - \frac{0.4\tau}{1-1.4\tau}\left(1 + \frac{G}{Sb}\right)^{-1}.$$

C Sensitivity Analyses

This section will be devoted to an examination of the robustness of my results. I begin with an analysis of the sensitivity of my results to the coefficient of relative risk-aversion, and then I use the estimates of fiscal costs and benefits from Trostel (2010) to assess the impact on my conclusions of how these fiscal effects are modelled. I also extend the model to consider uncertainty about future incomes, as well as heterogeneity in liquidity constraints and returns to education. The quantitative results are only slightly altered in each case, and the qualitative conclusions remain very similar.

C.1 Sensitivity of Results to Risk-Aversion

My first sensitivity analysis considers how the results change when I specify a coefficient of relative risk-aversion of $\rho = 2$ for the employed state. Since I only need to specify this parameter when using the structural method, it will only affect my simulation results. Calibration proceeds as before, and simulation yields the results displayed in Table 11. The optimal values of b and welfare effects are a bit smaller in most cases, but the conclusion of abolishing tuition continues to hold in the baseline case.

C.2 Evidence from Trostel (2010) on Fiscal Effects of Education

In this subsection, I will test the robustness of my results to a different choice of $\hat{b}$; specifically, I perform my analysis again using the most pessimistic estimates from Trostel (2010), in which he concludes that each year of college costs the government \$17850 and saves expenditures amounting to \$13950 in present value. I therefore select $\hat{b} = 18$, increasing e to 21.7 to correspond, and I assume that each year of schooling also saves expenditures amounting to $p = 14$.[32] This changes

[32] If instead I set $p = 16$ to correspond to the baseline case in which I assume that government appropriations for education are exactly offset by reductions in other expenditures, all results are identical to those in section 3 except that the optimal grants and welfare gains are almost all larger using statistical extrapolations, due to functional form assumptions.

Table 11: Results from Calibration and Simulation for $\rho = 2$

Value of ε_{Sb}	$\hat{L}$: 0	$\hat{L}$: $\frac{1}{3}$
	A. Numerical Estimate of $\frac{dW}{db}$ at $b = 2$	
0.1	0.1485	0.2766
0.2	0.4188	0.5476
	B. Optimal Student Grants	
0.1	\$3835	\$5415
0.2	\$5703	\$6934
	C. Welfare Gains from Moving to Optimum	
0.1	\$536 (17.3%)	\$1837 (59.2%)
0.2	\$3079 (99.2%)	\$5396 (173.8%)

the government budget constraint: I now divide G into two components, one exogenous component denoted by G_1, and one component $G_2 = (1 - S)p$ representing the expenditures which can be eliminated with increased schooling. Therefore, the derivative of the budget constraint is:

$$\frac{d\tau}{db} = \frac{S}{\bar{Y}} \left[1 + \left(\frac{b-p}{b} \right) \varepsilon_{Sb} - \left(1 + \frac{G}{Sb} \right) \varepsilon_{\bar{Y}b} \right]$$

and inserting this into $\frac{dV}{db}$, I derive the following variant of (4):

$$\frac{dW}{db} \simeq S \left[L - \left(\frac{b-p}{b} \right) \varepsilon_{Sb} + \left(1 + \frac{G}{Sb} \right) \varepsilon_{\bar{Y}b} \right]. \tag{6}$$

Because the baseline value of b is 9 times larger, the earlier values of $\varepsilon_{Sb} = \{0.1, 0.2\}$ are now replaced by $\varepsilon_{Sb} = \{0.9, 1.8\}$. For the optimal grants, let me write them as $\tilde{b} = b - 16$ to make them comparable to earlier results; evaluating (6) and using the same statistical extrapolations as before leads to the results displayed in Table 12. The values of $\frac{dW}{db}$ are smaller now, but the optimal grants are generally larger, as are the welfare gains at the optimum, due to the assumptions involved, particularly that of a constant value of ε_{Sb}. The baseline result involves an optimal stipend of over \$3600 and a welfare gain amounting to \$36.6 billion.

Calibration and simulation follows the same procedure as before, and the results are found in Table 13. In every case, the welfare derivative at baseline is smaller, as are the optimal grants and the welfare gains from moving to the optimum; the optimal grants drop about \$1000, and the baseline result no longer involves the complete abolition of tuition, but still calls for significantly reduced out-of-pocket costs.

C.3 Income Uncertainty

Next, I consider a case with uncertainty about future incomes. To keep the problem simple, I assume that all uncertainty is resolved after the first period. Thereafter, educated individuals receive either $Y_{1tH} = (1+g)^{t-1} Y_{11H}$ in each period or $Y_{1tL} = (1+g)^{t-1} Y_{11L}$, each with probability 0.5, where $Y_{11H} > Y_{11L}$ and $\frac{Y_{11H}+Y_{11L}}{2} = Y_{11}$. Meanwhile, uneducated workers begin with Y_{01} in the first period, and thereafter receive $Y_{0tH} = (1+g)^{t-1} Y_{01H}$ or $Y_{0tL} = (1+g)^{t-1} Y_{01L}$, each with probability 0.5, where $\frac{Y_{01H}+Y_{01L}}{2} = Y_{01}$. The corresponding consumption values will be denoted as

Table 12: Results from Sufficient Statistics and Extrapolation using (6)

Value of ε_{Sb}	$\hat{L}$ = 0	$\hat{L}$ = $\frac{1}{3}$
	A. Estimate of $\frac{dW}{db}$ at $b = 18$	
0.9	0.1370	0.2664
1.8	0.3267	0.4560
	B. Optimal Student Grants $\tilde{b} = b - 16$	
0.9	\$7795	\$8883
1.8	\$9343	\$9343
	C. Welfare Gains from Moving to Optimum	
0.9	\$1563 (5.6%)	\$3648 (13.1%)
1.8	\$5220 (18.7%)	\$6252 (22.4%)

Table 13: Results from Calibration and Simulation for $\hat{b} = 18$

Value of ε_{Sb}	$\hat{L}$ = 0	$\hat{L}$ = $\frac{1}{3}$
	A. Numerical Estimate of $\frac{dW}{db}$ at $b = 18$	
0.9	0.0684	0.1975
1.8	0.3094	0.4387
	B. Optimal Student Grants $\tilde{b} = b - 16$	
0.9	\$3127	\$5483
1.8	\$5149	\$6895
	C. Welfare Gains from Moving to Optimum	
0.9	\$154 (0.6%)	\$1365 (4.9%)
1.8	\$1968 (7.0%)	\$4358 (15.6%)

c_{vH}^1 and c_{vL}^1 for educated workers and c_{vH}^0 and c_{vL}^0 for uneducated workers, with c_{v1}^0 representing the consumption of first-period workers.

In deriving $\frac{dW}{db}$, the only meaningful change will come from the fact that $\frac{\partial V}{\partial \tau}$ takes a different form, specifically:

$$\frac{\partial V}{\partial \tau} = -\frac{\gamma_2}{2} S \left(v'(c_{vL}^1) Y_{11L} + v'(c_{vH}^1) Y_{11H} \right) - \frac{1-S}{2} \left(v'(c_{vL}^0)(Y_{01} + \gamma_2 Y_{01L}) + v'(c_{vH}^0)(Y_{01} + \gamma_2 Y_{01H}) \right).$$

However, this equation cannot be used in its current form, and the most reasonable simplification is still $\bar{Y} v'(c^*)$, where $\bar{Y}$ remains equal to $S\gamma_2 Y_{11} + (1-S)\gamma_1 Y_{01}$, so that (4) holds in this case as well, and the results are unchanged.

I will therefore focus on the structural analysis. The calibration proceeds largely as before, except that A and θ must be chosen simultaneously to generate consumption choices which match $E(c_v^1) = 1.26 E(c_v^0)$ and $u'(c_u) = (\hat{L} + 1) v'(c_{v1}^0)$. For the variability of of income, I collect data on the median and interquartile range of income for high school and college graduates from the CPS in the 4th quarter of 2012. Then I consider three cases: one case in which I choose the values of $\{Y_{0L}, Y_{0H}, Y_{1H}, Y_{1L}\}$ that produce the same interquartile range, specifically 74.3% for high school graduates and 81.5% for college graduates, one case in which I cut the high school IQR in half, and one in which I cut the college IQR in half. The results are displayed in Table 14, and the optimal grants and welfare gains are larger in every case, though there does not appear to be one unambiguous pattern of results across the three cases. The baseline results feature the abolition of tuition accompanied by a stipend of about \$1000 to \$2200 per year.

Table 14: Results from Calibration and Simulation with Uncertain Income

	CPS Variance		Low HS Variance		Low College Variance	
	$\hat{L}$					
Value of ε_{Sb}	0	$\frac{1}{3}$	0	$\frac{1}{3}$	0	$\frac{1}{3}$
	A. Numerical Estimate of $\frac{dW}{db}$ at $b=2$					
0.1	0.1552	0.2844	0.1302	0.2594	0.1570	0.2862
0.2	0.4172	0.5467	0.4204	0.5498	0.4169	0.5464
	B. Optimal Student Grants					
0.1	\$4815	\$7422	\$5394	\$6545	\$4800	\$7526
0.2	\$7674	\$8676	\$7920	\$8007	\$6660	\$8787
	C. Welfare Gains from Moving to Optimum					
0.1	\$839	\$3034	\$777	\$2331	\$869	\$3111
0.2	\$4235	\$7398	\$4899	\$6717	\$3914	\$7520

C.4 Heterogeneity in Liquidity Constraints and Two-Tier Grants

In appendix A, I examined how robust the sufficient statistics condition in (4) is to a distribution of debt limits; an alternative examination of the robustness of the results to heterogeneous liquidity constraints can be performed using a structural approach. I allow for two groups, each representing half of the population,[33] one of which is unconstrained while the other faces a debt limit A. I calibrate the model for $\{A, \theta, \mu, \sigma\}$ using the sufficient statistics as averages, and then solve for the optimal lump-sum student grant, with the results displayed in Table 15. The values of $\frac{dW}{db}$ are slightly smaller than in Table 3, which is to be expected because the logistic distribution for η has an increasing hazard (see appendix A), but the rest of the results are generally close to those from the baseline calculations; in most cases, including the baseline case, the optimal level of b is actually higher.

With this calibrated model in hand, I can go one step further and consider what policy the government would want to set if they could observe individuals' debt limits; with two types of individuals, the government could introduce a two-tier grant system, with one grant amount b_1 for the constrained group and another amount b_2 for unconstrained students. It is straightforward to numerically maximize welfare (still measured as equally-weighted utilitarian social welfare) over the pair (b_1, b_2), and the results for this exercise can be found in Table 16. Not surprisingly, it is always optimal to provide more generous aid to the constrained group, but substantial grants to the unconstrained group are still optimal with the standard estimate of $\varepsilon_{Sb} = 0.2$, as the fiscal externality motive remains strong; in the baseline case with $L = 0$, it remains optimal to abolish tuition, plus a stipend of about \$1400 for the constrained group. The welfare gains over and above those from the lump-sum policy in Table 15 are relatively small in most cases.

[33] Brown, Scholz, and Seshadri (2012) find that approximately half of the children in their sample did not receive post-schooling cash transfers from their parents, which they claim as an indicator for student liquidity constraints.

Table 15: Results from Calibration and Simulation with Heterogeneous Liquidity Constraints

	$\hat{L}$	
Value of ε_{Sb}	0	$\frac{1}{3}$
	A. Numerical Estimate of $\frac{dW}{db}$ at $b = 2$	
0.1	0.1221	0.2292
0.2	0.4161	0.5005
	B. Optimal Student Grants	
0.1	\$4358	\$6279
0.2	\$6674	\$8024
	C. Welfare Gains from Moving to Optimum	
0.1	\$575 (18.5%)	\$1938 (62.4%)
0.2	\$3992 (128.6%)	\$6368 (205.2%)

Table 16: Results from Calibration and Simulation with Two-Tier Grants

	$\hat{L}$	
Value of ε_{Sb}	0	$\frac{1}{3}$
	A. Optimal Two-Tier Student Grants (b_1/b_2)	
0.1	\$5475/\$1763	\$8678/\$0
0.2	\$7174/\$5830	\$9716/\$4446
	B. Welfare Gains from Moving to Optimum	
0.1	\$844 (27.2%)	\$3622 (116.7%)
0.2	\$4072 (131.2%)	\$7475 (240.8%)

C.5 Heterogeneous Returns to Education

I now investigate how sensitive the results are to allowing for heterogeneous returns to education. I assume that the college wage premium P (where $Y_{11} = PY_{01}$) follows some distribution $G(P)$, and to be precise I use a quadratic approximation to the marginal treatment effect distribution presented in Figure 4 of Carneiro, Heckman, and Vytlacil (2011). I divide the population into 100 equal masses denoted by $j = \{1, 2, ..., 100\}$, with wage premia equal to $\{G^{-1}(0.005), G^{-1}(0.015), ..., G^{-1}(0.995)\}$, and then I allow for a distribution of η for each mini-population, where η is allowed to be correlated with P. In particular, I let $\eta_{ij} = \bar{\eta_j} + \eta_i$, where $\bar{\eta_j}$ is deterministic for each j and η_i comes from a logistic distribution with mean 0 and scale parameter σ. I specify $\bar{\eta_j} = U_0 - U_{1j} + z - \mu_s \left(\frac{j-1}{j}\right)^{1.2}$, where $U_{1j} = u(c_{uj}) + R_2 v(c^1_{vj})$, as this generates a pattern of responsiveness to b which is consistent with that found in Carneiro, Heckman, and Vytlacil (2011).

Allowing for a distribution of wage premia makes it important to model the tax system more realistically: I assume that the state and Medicare tax rates do not vary with income, but I use an approximation to the US federal system in 2008, with a 15% marginal rate up to \$41500 and a 25% rate beyond. To account for the personal exemption of \$3500 and the standard deduction of \$5450, as well as the fact that the first \$8025 of taxable income is only taxed at a 10% rate, I provide a universal tax refund of \$1743.75. To avoid discontinuities in the marginal tax schedule, I use a smoothed approximation to the tax rate between \$39000 and \$44000, specifically a sine connecting $\tau = 0.23$ at \$39000 to $\tau = 0.33$ at \$44000. I assume that the tax rate threshold moves up with wage growth, and that when taxes need to adjust to balance the budget, the base (state and Medicare) tax rate is the one that moves.

When calibrating, I select values for $\{A, \theta, \mu_s, \sigma, z\}$ in order to match five quantities, three of which are familiar: $E_1[u'(c_{ui})] = (\hat{L}+1)v'(c_v^0)$, $\hat{S} = 0.388$, and $\varepsilon_{Sb} = \{0.1, 0.2\}$, although in this case ε_{Sb} is interpreted as a partial derivative. I also choose z to generate a probability of attendance of 95% for the highest-return group, and I use the fact that college graduates consume 73.9% of their pre-tax income and high school graduates consume 83.4% to motivate setting $\frac{\frac{E_1(c_v^1)}{E_1(Y_1)}}{\frac{c_v^0}{Y_0}} = \frac{0.739}{0.834}$.

This leads to the results presented in Table 17. The striking finding is that the welfare derivative at baseline is significantly larger, because the average return to education among those induced to go to school is higher using the estimates from Carneiro, Heckman, and Vytlacil (2011). However, there are diminishing returns to inducing college attendance, because increasingly generous grants induce students with lower monetary returns to go to school; therefore, optimal grants are lower when $\varepsilon_{Sb} = 0.2$, though they are larger when $\varepsilon_{Sb} = 0.1$ and $\hat{L} = \frac{1}{3}$ because the returns to inducing college attendance do not decline as quickly in that case. In the baseline case, if heterogeneous returns of this magnitude do exist, it may no longer be optimal to completely eliminate tuition, but a significant increase in the generosity of grants is still indicated, and the welfare gains are significantly larger than before, amounting to \$103.4 billion per year.

Table 17: Results from Calibration and Simulation with Heterogeneous Returns to Education

	$\hat{L}$	
Value of ε_{Sb}	0	$\frac{1}{3}$
	A. Numerical Estimate of $\frac{dW}{db}$ at $b = 2$	
0.1	1.1593	1.4041
0.2	2.9816	3.3447
	B. Optimal Student Grants	
0.1	\$5248	\$6376
0.2	\$4574	\$3345
	C. Welfare Gains from Moving to Optimum	
0.1	\$7036 (226.7%)	\$11043 (355.8%)
0.2	\$14767 (475.7%)	\$19805 (638.1%)

This analysis provides us with a sense of how heterogeneity in returns can affect the results, but naturally has more of a "black box" character than the baseline analysis. The current method is not as well suited to answering questions about how financial aid could be better targetted at students on the margin of attending college or from groups with high returns, and therefore future work using structural models with observed and unobserved heterogeneity could be useful in providing answers to such questions.

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