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# On equilibrium payoffs in wage bargaining with discount rates varying in time

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**Abstract.** We provide an equilibrium analysis of a wage bargaining model between a union and a firm in which the union must choose between strike and holdout in case of a disagreement. While in the literature it is assumed that the parties of wage bargaining have constant discount factors, in our model preferences of the union and the firm are expressed by sequences of discount rates varying in time. First, we describe necessary conditions under arbitrary sequences of discount rates for the supremum of the union's payoffs and the infimum of the firm's payoffs under subgame perfect equilibrium in all periods when the given party makes an offer. Then, we determine the equilibrium payoffs for particular cases of sequences of discount rates varying in time. Besides deriving the exact bounds of equilibrium payoffs, we also characterize the equilibrium strategy profiles that support these extreme payoffs.

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**Keywords**: union - firm bargaining, varying discount rates, subgame perfect equilibrium, equilibrium payoffs

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## 1 Introduction

There are numerous works in the literature that generalize Rubinstein's bargaining model of alternative offers (Rubinstein (1982)); for surveys, see e.g. Osborne and Rubinstein (1990), Muthoo (1999). One extension of that model concerns incorporating the choice of going on strike in union-firm negotiations. Such a generalized model with the same discount rate $\delta$ for both parties is presented e.g. in Haller and Holden (1990) and Holden (1994). It is assumed that in each period until an agreement is reached, the union must decide whether it will strike or hold out in that period. The same wage bargaining but with the union and the firm having different discount rates $\delta_u$ and $\delta_f$ is studied in Fernandez and Glazer (1991). In this model, referred here as the *F-G model*, the union achieves the maximum-wage contract by threatening an alternating strike strategy (to go on strike when the firm rejects an offer but to continue working at the old contract wage when the firm makes an unacceptable offer). As shown by Bolt (1995), this subgame perfect equilibrium (SPE) only holds if $\delta_u \leq \delta_f$. If, however, $\delta_u > \delta_f$, then the firm can increase its payoff by playing the so called no-concession strategy (reject all offers of the union and make always unacceptable offers). In this case, the SPE is restored by modifying the alternating strike strategies. Houba and Wen (2008) apply the method of Shaked and Sutton (1984) to derive the exact bounds of equilibrium payoffs in the original F-G model and characterize the equilibrium strategy profiles that support these extreme equilibrium payoffs for all discount factors.

While several works in the literature concern the F-G model, it is usually assumed that the parties have constant discount rates. However, usually in real life discount rates of the parties are not really constant. Patience of bargainers may be changing over time due to many circumstances (e.g., political, economic, financial, social, environmental). To the best of our knowledge, our previous work on wage bargaining (Ozkardas and Rusinowska (2014)) is the first one in which the issue of changing patience of the union and the firm in the F-G model is addressed. More precisely, in Ozkardas and Rusinowska (2014) we generalize the F-G model to the union-firm wage bargaining in which both parties have preferences expressed by *sequences* $(\delta_{u,t})_{t\in\mathbb{N}}$ and $(\delta_{f,t})_{t\in\mathbb{N}}$ of discount factors *varying in time*. We determine SPE for three cases when the strike decision of the union is *exogenous*: the case when the union is committed to go on strike in each period in which there is a disagreement, the case when the union is committed to go on strike only when its own offer is rejected, and the case when the union is supposed to go never on strike. We show that while in the original F-G model the exogenous 'always-strike' case coincides with Rubinstein's model, it is not the case in the generalized framework. We present the unique SPE for this case and also for each of the remaining two cases. Furthermore, we consider the general model with no assumption on the commitment to strike and find subgame perfect equilibria for some particular cases.

The *aim of the present paper* is to continue our research on the wage bargaining initiated in Ozkardas and Rusinowska (2014) and to find the extreme payoffs under SPE in the wage bargaining with discount rates varying in time. In order to achieve that, we apply to our generalized model the method used in Houba and Wen (2008). First, we describe necessary conditions under arbitrary sequences of discount rates for the supremum of the union's SPE payoffs and the infimum of the firm's SPE payoffs in all periods when the given party makes an offer. Then, we determine the extreme payoffs under SPE for particular cases of sequences of discount rates varying in time. Apart from deriving the exact bounds of the equilibrium payoffs, we also characterize the equilibrium strategy profiles that support these extreme payoffs. Our findings for the model with varying discount rates generalize the results of Houba and Wen (2008) obtained for the model with constant discount rates.

The remainder of the paper is structured as follows. In Section 2 we briefly present the generalized wage bargaining model with discount rates varying in time. In Section 3 necessary conditions for the supremum of the union's SPE payoffs and the infimum of the firm's SPE payoffs are determined. In Section 4 we calculate the extreme payoffs for particular cases of the sequences of discount rates varying in time. We also present equilibrium strategy profiles that support these payoffs. Section 5 contains some concluding remarks. Proofs of all results are presented in the Appendix.

## 2 Wage bargaining with discount rates varying in time

The point of departure of our study is the following bargaining procedure between the union and the firm, presented in Fernandez and Glazer (1991), and Haller and Holden (1990). There is an existing wage contract, that specifies the wage that a worker is entitled to per day of work, which has come up for renegotiation. Two parties (union and firm) bargain sequentially over discrete time and a potentially infinite horizon. They alternate in making offers of wage contracts that the other party can either accept or reject. When a party rejects a proposed wage contract, the union must decide whether or not to strike in that period. Under the previous contract $w_0$, where $w_0 \in [0, 1]$, the union and the firm gets $w_0$ and $1 - w_0$, respectively. By a new contract $W \in [0, 1]$, the union will receive $W$ and the firm $1 - W$. Figure 1 presents the first three periods of this wage bargaining.

ABOUT HERE FIGURE 1

More precisely, the bargaining procedure is the following. The union proposes $x_0$. If the firm accepts the new wage contract, the agreement is reached and the payoffs are $(x_0, 1 - x_0)$. If the firm rejects it, then the union can either strike, and then both parties obtain $(0, 0)$ in the current period, or continue with the previous contract with payoffs $(w_0, 1-w_0)$. If the union goes on strike, it is the firm's turn to make a new offer $y_1$, where $y_1$ is assigned to the union and $(1 - y_1)$ to the firm. This procedure continues until an agreement is reached, where $x_{2t}$ denotes the offer of the union made in an even-numbered period $2t$, and $y_{2t+1}$ denotes the offer of the firm made in an odd-numbered period $(2t+1)$.

Our generalization of the original F-G model concerns preferences of the union and the firm and, as a consequence, the payoff functions of both parties. While Fernandez and Glazer (1991) assume stationary preferences described by constant discount rates $\delta_u$ and $\delta_f$, we analyze a wage bargaining in which preferences of the union and the firm are described by *sequences of discount factors (rates) varying in time*, $(\delta_{u,t})_{t\in\mathbb{N}}$ and $(\delta_{f,t})_{t\in\mathbb{N}}$, respectively, where

$$\delta_{u,t} = \text{ discount factor of the union in period } t \in \mathbb{N}, \quad \delta_{u,0} = 1,\ 0 < \delta_{u,0} < 1 \text{ for } t \geq 1$$

$$\delta_{f,t} = \text{ discount factor of the firm in period } t \in \mathbb{N}, \quad \delta_{f,0} = 1,\ 0 < \delta_{f,0} < 1 \text{ for } t \geq 1$$

The *result* of the wage bargaining is either a pair $(W, T)$, where $W$ is the wage contract agreed upon and $T \in \mathbb{N}$ is the number of proposals rejected in the bargaining, or a *disagreement* denoted by $(0, \infty)$ and meaning the situation in which the parties never reach an agreement. We introduce the following notation. Let for each $t \in \mathbb{N}$

$$\delta_u(t) := \prod_{k=0}^{t} \delta_{u,k}, \quad \delta_f(t) := \prod_{k=0}^{t} \delta_{f,k} \tag{1}$$

and for $0 < t' \leq t$

$$\delta_u(t', t) := \frac{\delta_u(t)}{\delta_u(t'-1)} = \prod_{k=t'}^{t} \delta_{u,k}, \quad \delta_f(t', t) := \frac{\delta_f(t)}{\delta_f(t'-1)} = \prod_{k=t'}^{t} \delta_{f,k} \tag{2}$$

The utility of the result $(W, T)$ for the union is equal to

$$U(W,T) = \sum_{t=0}^{\infty} \delta_u(t) u_t \tag{3}$$

where $u_t = W$ for each $t \geq T$, and if $T > 0$ then for each $0 \leq t < T$

$u_t = 0$ if there is a strike in period $t \in \mathbb{N}$
$u_t = w_0$ if there is no strike in period $t$.

The utility of the result $(W, T)$ for the firm is equal to

$$V(W,T) = \sum_{t=0}^{\infty} \delta_f(t) v_t \tag{4}$$

where $v_t = 1 - W$ for each $t \geq T$, and if $T > 0$ then for each $0 \leq t < T$

$v_t = 0$ if there is a strike in period $t$
$v_t = 1 - w_0$ if there is no strike in period $t$.

The utility of the disagreement is equal to

$$U(0, \infty) = V(0, \infty) = 0 \tag{5}$$

We assume that the infinite series in (3) and (4) are convergent. A sufficient condition for the convergence of (3) and (4) is to assume that the sequences of discount rates $(\delta_{u,t})_{t \in \mathbb{N}}$ and $(\delta_{f,t})_{t \in \mathbb{N}}$ are bounded by a certain number smaller than 1, i.e.,

$$\text{there exist } a < 1 \text{ and } b < 1 \text{ such that } \delta_{u,t} \leq a \text{ and } \delta_{f,t} \leq b \text{ for each } t \in \mathbb{N} \tag{6}$$

Let $\Delta_u(t)$ and $\Delta_f(t)$ denote the *generalized discount factors* of the union and the firm in period $t$, respectively. They take into account the sequences of discount rates varying in time and the fact that the utilities are defined by the discounted streams of payoffs. They are defined as follows, for every $t \in \mathbb{N}_+$:

$$\Delta_u(t) := \frac{\sum_{k=t}^{\infty} \delta_u(t,k)}{1 + \sum_{k=t}^{\infty} \delta_u(t,k)}, \quad \Delta_f(t) := \frac{\sum_{k=t}^{\infty} \delta_f(t,k)}{1 + \sum_{k=t}^{\infty} \delta_f(t,k)} \tag{7}$$

and consequently, for every $t \in \mathbb{N}_+$

$$1 - \Delta_u(t) = \frac{1}{1 + \sum_{k=t}^{\infty} \delta_u(t,k)}, \quad 1 - \Delta_f(t) = \frac{1}{1 + \sum_{k=t}^{\infty} \delta_f(t,k)} \tag{8}$$

Note that for every $t \in \mathbb{N}_+$

$$\sum_{k=t}^{\infty} \delta_f(t,k) \geq \sum_{k=t}^{\infty} \delta_u(t,k) \quad \text{if and only if} \quad \Delta_f(t) \geq \Delta_u(t)$$

For the special case of constant discount rates, i.e., if $\delta_{u,t} = \delta_u$ and $\delta_{f,t} = \delta_f$ for every $t \in \mathbb{N}_+$, $\Delta_u(t) = \delta_u$ and $\Delta_f(t) = \delta_f$.

## 3 Necessary conditions in the generalized wage bargaining

Houba and Wen (2008) apply the method of Shaked and Sutton (1984) to the F-G model to derive the supremum of the union's SPE payoffs in any even period and the infimum of the firm's SPE payoffs in any odd period. We generalize their method to the wage bargaining with sequences of discount rates varying in time. Let for $t \in \mathbb{N}$

$M_u^{2t}$ = supremum of the union's SPE payoffs in any even period $2t$ where the union makes an offer

$m_f^{2t+1}$ = infimum of the firm's SPE payoffs in any odd $2t+1$ period where the firm makes an offer

$M_u^{2t}$ and $m_f^{2t+1}$ depend on the sequences $(\delta_{u,t})_{t\in\mathbb{N}}$, $(\delta_{f,t})_{t\in\mathbb{N}}$, and $0 \leq w_0 \leq 1$. Since in this model $w_0$ is the union's worst SPE payoff (see Ozkardas and Rusinowska (2014)), we have for each $t \in \mathbb{N}$

$$w_0 \leq M_u^{2t} \leq 1 \quad \text{and} \quad w_0 \leq 1 - m_f^{2t+1} \leq 1$$

In this section, we determine necessary conditions for $M_u^{2t}$ and $m_f^{2t+1}$, where $t \in \mathbb{N}$. The analysis is practically the same as the one given in Houba and Wen (2008) for constant discount rates, except that we consider periods $2t$ and $2t+1$, and the generalized discount factors $\varDelta_i(t)$ in a given period $t \in \mathbb{N}$ instead of constant discount rates $\delta_i$ for $i = u, f$. For coherence of the exposition, we present this analysis for the generalized case. We have for all $(\delta_{u,t})_{t\in\mathbb{N}}$, $(\delta_{f,t})_{t\in\mathbb{N}}$, $0 \leq w_0 \leq 1$ and $t \in \mathbb{N}$

$$M_u^{2t} \leq \max \begin{cases} w_0(1-\varDelta_f(2t+1)) + (1-m_f^{2t+1})\varDelta_f(2t+1) & (9a) \\ w_0(1-\varDelta_u(2t+1)) + (1-m_f^{2t+1})\varDelta_u(2t+1) & (9b) \\ 1 - m_f^{2t+1}\varDelta_f(2t+1) \ \text{ subject to } \ (1-m_f^{2t+1})\varDelta_u(2t+1) \geq w_0 & (9c) \end{cases} \qquad (9)$$

To see that, consider an arbitrary even period $2t$, $t \in \mathbb{N}$. First of all, note that for $i = u, f$

$$1-(1-w_0)(1-\varDelta_i(2t+1)) - m_f^{2t+1}\varDelta_i(2t+1) = w_0(1-\varDelta_i(2t+1)) + (1-m_f^{2t+1})\varDelta_i(2t+1)$$

(1) If the union holds out after its offer is rejected, the firm will get at least

$$(1-w_0)(1-\varDelta_f(2t+1)) + m_f^{2t+1}\varDelta_f(2t+1)$$

by rejecting the union's offer. Hence, the union's SPE payoffs must be at most

$$1-(1-w_0)(1-\varDelta_f(2t+1)) - m_f^{2t+1}\varDelta_f(2t+1) \qquad (10)$$

from making the least acceptable offer, or

$$w_0(1-\varDelta_u(2t+1)) + (1-m_f^{2t+1})\varDelta_u(2t+1) \qquad (11)$$

from making an unacceptable offer.

(2) The union may threaten to strike in period $2t$ if the firm rejects its offer, which is credible if and only if $(1-m_f^{2t+1})\sum_{k=2t+1}^{\infty}\delta_u(2t+1,k) \geq w_0 + w_0\sum_{k=2t+1}^{\infty}\delta_u(2t+1,k)$, i.e., if and only if

$$(1-m_f^{2t+1})\varDelta_u(2t+1) \geq w_0$$

In this case, the union's SPE payoffs must be smaller than or equal to

$$1-m_f^{2t+1}\varDelta_f(2t+1) \tag{12}$$

from making the least acceptable offer, or

$$(1-m_f^{2t+1})\varDelta_u(2t+1) \tag{13}$$

from making an unacceptable offer. Note that we have always

$$1-\varDelta_u(2t+1) \geq m_f^{2t+1}\left(\varDelta_f(2t+1)-\varDelta_u(2t+1)\right)$$

which is equivalent to

$$1-m_f^{2t+1}\varDelta_f(2t+1) \geq (1-m_f^{2t+1})\varDelta_u(2t+1)$$

This means that if the union threatens to strike, it will not make an unacceptable offer in period $2t$. Hence, the union's SPE payoffs cannot be greater than the maximum of the three cases (10), (11) and (12), and therefore we obtain (9).

From (9) we get the necessary conditions for the supremum of the union's SPE payoffs in an even period:

**Proposition 1** *We have for all* $(\delta_{u,t})_{t\in\mathbb{N}}$, $(\delta_{f,t})_{t\in\mathbb{N}}$, $0 \leq w_0 \leq 1$ *and* $t \in \mathbb{N}$

$$M_u^{2t} \leq \begin{cases} 1-m_f^{2t+1}\varDelta_f(2t+1) & \textit{if } (15) \\ w_0(1-\varDelta_f(2t+1)) + (1-m_f^{2t+1})\varDelta_f(2t+1) & \textit{if } (16) \\ w_0(1-\varDelta_u(2t+1)) + (1-m_f^{2t+1})\varDelta_u(2t+1) & \textit{if } (17) \end{cases} \tag{14}$$

*where*

$$(1-m_f^{2t+1})\varDelta_u(2t+1) \geq w_0 \tag{15}$$

$$(1-m_f^{2t+1})\varDelta_u(2t+1) < w_0 \quad \textit{and} \quad \varDelta_f(2t+1) \geq \varDelta_u(2t+1) \tag{16}$$

$$(1-m_f^{2t+1})\varDelta_u(2t+1) < w_0 \quad \textit{and} \quad \varDelta_f(2t+1) < \varDelta_u(2t+1) \tag{17}$$

For the proof of Proposition 1, see the Appendix.

Similarly, we have for all $(\delta_{u,t})_{t\in\mathbb{N}}$, $(\delta_{f,t})_{t\in\mathbb{N}}$, $0 \leq w_0 \leq 1$ and $t \in \mathbb{N}$

$$m_f^{2t+1} \geq \min \begin{cases} \max \begin{cases} 1-w_0(1-\varDelta_u(2t+2)) - M_u^{2t+2}\varDelta_u(2t+2) & (18a) \\ 1-w_0(1-\varDelta_f(2t+2)) - M_u^{2t+2}\varDelta_f(2t+2) & (18b) \end{cases} \\ 1-M_u^{2t+2}\varDelta_u(2t+2) \;\text{ subject to }\; M_u^{2t+2}\varDelta_u(2t+2) \geq w_0 \quad (18c) \end{cases} \tag{18}$$

Consider an arbitrary odd period $2t+1$, $t \in \mathbb{N}$.

(1) If the union holds out after rejecting the firm's offer, the union will get at most

$$w_0(1-\varDelta_u(2t+2))+M_u^{2t+2}\varDelta_u(2t+2)$$

Hence, the firm could get at least

$$1-w_0(1-\varDelta_u(2t+2))-M_u^{2t+2}\varDelta_u(2t+2) \tag{19}$$

from making the least irresistible offer. The firm could receive at least

$$(1-w_0)(1-\varDelta_f(2t+2))+(1-M_u^{2t+2})\varDelta_f(2t+2)=1-w_0(1-\varDelta_f(2t+2))-M_u^{2t+2}\varDelta_f(2t+2) \tag{20}$$

from making any unacceptable offer. The firm will make either the least irresistible offer or an unacceptable offer, depending on whether (19) or (20) is greater. For the union, it is always credible to holdout after rejecting a firm's offer, as the union gets at most $M_u^{2t+2}\varDelta_u(2t+2)$ when it strikes after the firm's offer is rejected.

(2) If the union strikes after rejecting the firm's offer, then the firm will get at least

$$1-M_u^{2t+2}\varDelta_u(2t+2)$$

from making the least irresistible offer, or

$$(1-M_u^{2t+2})\varDelta_f(2t+2)$$

from making an unacceptable offer. Since $M_u^{2t+2}\leq 1$, note that

$$M_u^{2t+2}\left(\varDelta_u(2t+2)-\varDelta_f(2t+2)\right)\leq 1-\varDelta_f(2t+2)$$

which is equivalent to

$$1-M_u^{2t+2}\varDelta_u(2t+2)\geq(1-M_u^{2t+2})\varDelta_f(2t+2)$$

This implies that the firm will never make an unacceptable offer if the union threatens to strike after rejecting the firm's offer. Strike in period $2t+1$ is credible if and only if $M_u^{2t+2}\sum_{k=2t+2}^{\infty}\delta_u(2t+2,k)\geq w_0+w_0\sum_{k=2t+2}^{\infty}\delta_u(2t+2,k)$, i.e., if and only if

$$M_u^{2t+2}\varDelta_u(2t+2)\geq w_0$$

Hence, we obtain (18).

From (18) we get the necessary conditions for the infimum of the firm's SPE payoffs in an odd period:

**Proposition 2** *We have for all $(\delta_{u,t})_{t\in\mathbb{N}}$, $(\delta_{f,t})_{t\in\mathbb{N}}$, $0\leq w_0\leq 1$ and $t\in\mathbb{N}$*

$$m_f^{2t+1}\geq\begin{cases}1-M_u^{2t+2}\varDelta_u(2t+2) & \text{if } (22)\\ 1-w_0(1-\varDelta_f(2t+2))-M_u^{2t+2}\varDelta_f(2t+2) & \text{if } (23)\\ 1-w_0(1-\varDelta_u(2t+2))-M_u^{2t+2}\varDelta_u(2t+2) & \text{if } (24)\end{cases} \tag{21}$$

*where*

$$M_u^{2t+2}\left(\varDelta_u(2t+2)-\varDelta_f(2t+2)\right)\geq w_0(1-\varDelta_f(2t+2)) \tag{22}$$

$$M_u^{2t+2}\left(\varDelta_u(2t+2)-\varDelta_f(2t+2)\right)<w_0(1-\varDelta_f(2t+2)) \quad \text{and} \quad \varDelta_u(2t+2)>\varDelta_f(2t+2) \tag{23}$$

$$\varDelta_u(2t+2)\leq\varDelta_f(2t+2) \tag{24}$$

For the proof of Proposition 2, see the Appendix.

Note that our Propositions 1 and 2 generalize the corresponding results on necessary conditions for $M_u$ and $m_f$ for the model with constant discount rates presented in Houba and Wen (2008) (Propositions 2 and 1).

From Propositions 1 and 2 we can write the following fact which will be useful for determining $M_u^{2t}$ and $m_f^{2t+1}$ for particular cases of the generalized discount factors.

**Fact 1** *Let* $t \in \mathbb{N}$.

*(i) If* $\Delta_u(2t+1) \leq \Delta_f(2t+1)$, *then*

$$M_u^{2t} \leq \begin{cases} 1 - m_f^{2t+1}\Delta_f(2t+1) & \text{if } (1-m_f^{2t+1})\Delta_u(2t+1) \geq w_0 \\ w_0(1-\Delta_f(2t+1)) + (1-m_f^{2t+1})\Delta_f(2t+1) & \text{if } (1-m_f^{2t+1})\Delta_u(2t+1) < w_0 \end{cases}$$

*(ii) If* $\Delta_u(2t+1) > \Delta_f(2t+1)$, *then*

$$M_u^{2t} \leq \begin{cases} 1 - m_f^{2t+1}\Delta_f(2t+1) & \text{if } (1-m_f^{2t+1})\Delta_u(2t+1) \geq w_0 \\ w_0(1-\Delta_u(2t+1)) + (1-m_f^{2t+1})\Delta_u(2t+1) & \text{if } (1-m_f^{2t+1})\Delta_u(2t+1) < w_0 \end{cases}$$

*(iii) If* $\Delta_u(2t+2) \leq \Delta_f(2t+2)$, *then*

$$m_f^{2t+1} \geq 1 - w_0(1-\Delta_u(2t+2)) - M_u^{2t+2}\Delta_u(2t+2)$$

*(iv) If* $\Delta_u(2t+2) > \Delta_f(2t+2)$, *then*

$$m_f^{2t+1} \geq \begin{cases} 1 - M_u^{2t+2}\Delta_u(2t+2) & \text{if } (25) \\ 1 - M_u^{2t+2}\Delta_f(2t+2) - w_0(1-\Delta_f(2t+2)) & \text{if } (26) \end{cases}$$

*where*

$$M_u^{2t+2}\left(\Delta_u(2t+2) - \Delta_f(2t+2)\right) \geq w_0(1-\Delta_f(2t+2)) \quad (25)$$

$$M_u^{2t+2}\left(\Delta_u(2t+2) - \Delta_f(2t+2)\right) < w_0(1-\Delta_f(2t+2)) \quad (26)$$

Since there exist infinitely many cases of the relation between the generalized discount factors of the union and the firm in even and odd periods, and consequently, infinitely many combinations of the necessary conditions, we cannot fully determine $M_u^{2t}$ and $m_f^{2t+1}$ for all possibilities. However, given the sequences of discount rates, the corresponding necessary conditions can be used to find $M_u^{2t}$ and $m_f^{2t+1}$, if they exist.

# 4 Maximum wage contract in the generalized model

From the necessary conditions presented in the previous section, we now determine $M_u^{2t}$ and $m_f^{2t+1}$ for $t \in \mathbb{N}$ for some particular cases of the discount rates varying in time. Let $\Delta_u(t)$ and $\Delta_f(t)$ for $t \in \mathbb{N}$ be the generalized discount rates of the union and the firm, respectively, as defined in (7).

In order to simplify the presentation of the results, first we introduce the notation for different sums of the generalized discount rates. We have for each $t \in \mathbb{N}$:

$$\widetilde{\Delta}(t) := 1 - \Delta_f(2t+1) + \sum_{m=t}^{\infty}(1 - \Delta_f(2m+3)) \prod_{j=t}^{m} \Delta_u(2j+2)\Delta_f(2j+1) \tag{27}$$

$$\overline{\Delta}(t) := 1 - \Delta_u(2t+2) + \sum_{m=t}^{\infty}(1 - \Delta_u(2m+4)) \prod_{j=t}^{m} \Delta_u(2j+2)\Delta_f(2j+3) \tag{28}$$

$$\widehat{\Delta}(t) := w_0+(1-w_0) \left(1 - \Delta_f(2t+1) + \sum_{m=t}^{\infty}(1 - \Delta_f(2m+3)) \prod_{j=t}^{m} \Delta_f(2j+1)\Delta_f(2j+2)\right) \tag{29}$$

$$\breve{\Delta}(t) := (1 - w_0) \left(1 - \Delta_f(2t+2) + \sum_{m=t}^{\infty}(1 - \Delta_f(2m+4)) \prod_{j=t}^{m} \Delta_f(2j+2)\Delta_f(2j+3)\right) \tag{30}$$

When we consider the model with constant discount rates, i.e., $\delta_{u,t} = \delta_u$ and $\delta_{f,t} = \delta_f$ for each $t \in \mathbb{N}$, we get for every $t \in \mathbb{N}$

$$\widetilde{\Delta}(t) = \frac{1-\delta_f}{1-\delta_u\delta_f}, \quad \overline{\Delta}(t) = \frac{1-\delta_u}{1-\delta_u\delta_f}, \quad \widehat{\Delta}(t) = \frac{1+w_0\delta_f}{1+\delta_f}, \quad \breve{\Delta}(t) = \frac{1-w_0}{1+\delta_f}$$

Our first results present the supremum of the union's SPE payoffs in any even period and the infimum of the firm's SPE payoffs in any odd period for the particular cases with $\Delta_u(2t+2) \leq \Delta_f(2t+2)$ for every $t \in \mathbb{N}$: when either the strike is always credible (Proposition 3(i)) or the strike is never credible (Proposition 3(ii)).

**Proposition 3** *Let $\Delta_u(2t+2) \leq \Delta_f(2t+2)$ for every $t \in \mathbb{N}$.*

*(i) If for every $t \in \mathbb{N}$*

$$\left[w_0 + (1-w_0)\Delta_u(2t+2)\widetilde{\Delta}(t+1)\right] \Delta_u(2t+1) \geq w_0 \tag{31}$$

*then*

$$M_u^{2t} = w_0 + (1-w_0)\widetilde{\Delta}(t) \tag{32}$$

$$m_f^{2t+1} = (1-w_0)\left[1 - \Delta_u(2t+2)\widetilde{\Delta}(t+1)\right] \tag{33}$$

*The SPE strategy profile that supports these $M_u^{2t}$ and $m_f^{2t+1}$ defined in (32) and (33) is given by the following 'generalized alternating strike strategies':*

- *In period $2t$ the union proposes $w_0 + (1-w_0)\widetilde{\Delta}(t)$, in period $2t+1$ it accepts an offer $y$ if and only if $y \geq w_0 + (1-w_0)\Delta_u(2t+2)\widetilde{\Delta}(t+1)$, it goes on strike after rejection of its own proposals and holds out after rejecting firm's offers.*
- *In period $2t+1$ the firm proposes $w_0 + (1-w_0)\Delta_u(2t+2)\widetilde{\Delta}(t+1)$, in period $2t$ it accepts $x$ if and only if $x \leq w_0 + (1-w_0)\widetilde{\Delta}(t)$.*

– *If, however, at some point, the union deviates from the above rule, then both parties play thereafter according to the following 'minimum-wage strategies':*
  - *The union always proposes $w_0$, accepts $y$ if and only if $y \geq w_0$, and never goes on strike.*
  - *The firm always proposes $w_0$ and accepts $x$ if and only if $x \leq w_0$.*

*(ii) If for every $t \in \mathbb{N}$*

$$\left[w_0 + (1 - w_0)\varDelta_u(2t+2)\widetilde{\varDelta}(t+1)\right] \varDelta_u(2t+1) < w_0 \tag{34}$$

*then*

$$M_u^{2t} = w_0 \quad \text{and} \quad m_f^{2t+1} = 1 - w_0 \tag{35}$$

*The SPE strategy profile that supports these $M_u^{2t}$ and $m_f^{2t+1}$ defined in (35) is given by the minimum-wage strategies.*

For the proof of Proposition 3, see the Appendix.

Note that our Proposition 3 generalizes the corresponding results on $M_u$ and $m_f$ for the model with constant discount rates presented in Houba and Wen (2008) (Proposition 3). When we consider the model with constant discount rates, i.e., we put $\delta_{u,t} = \delta_u$ and $\delta_{f,t} = \delta_f$ for each $t \in \mathbb{N}$, and we assume that $\delta_u \leq \delta_f$, we get for every $t \in \mathbb{N}$

$$M_u^{2t} = w_0 + \frac{(1 - w_0)(1 - \delta_f)}{1 - \delta_u \delta_f}, \quad m_f^{2t+1} = \frac{(1 - w_0)(1 - \delta_u)}{1 - \delta_u \delta_f}$$

and the strike credibility condition (31) is equivalent to

$$(1 - w_0)\delta_u^2 + w_0\delta_u - w_0 \geq \delta_u\delta_f(\delta_u - w_0)$$

Our next results concern some particular cases when the the generalized discount rate of the union is always greater than the generalized discount rate of the firm in the same even period. Three particular cases are considered.

**Proposition 4** *Let $\varDelta_u(2t+2) > \varDelta_f(2t+2)$ for every $t \in \mathbb{N}$.*

*(i) If for every $t \in \mathbb{N}$*

$$\left(1 - \overline{\varDelta}(t)\right) \varDelta_u(2t+1) \geq w_0 \tag{36}$$

*and*

$$\widetilde{\varDelta}(t+1)\left(\varDelta_u(2t+2) - \varDelta_f(2t+2)\right) \geq w_0(1 - \varDelta_f(2t+2)) \tag{37}$$

*then*

$$M_u^{2t} = \widetilde{\varDelta}(t) \quad \text{and} \quad m_f^{2t+1} = \overline{\varDelta}(t) \tag{38}$$

*The SPE strategy profile that supports these $M_u^{2t}$ and $m_f^{2t+1}$ defined in (38) is given by the following 'always strike strategies':*

– *In period $2t$ the union proposes $\widetilde{\varDelta}(t)$, in period $2t+1$ it accepts an offer $y$ if and only if $y \geq 1 - \overline{\varDelta}(t)$, it always goes on strike if there is a disagreement.*
– *In period $2t+1$ the firm proposes $1 - \overline{\varDelta}(t)$, in period $2t$ it accepts $x$ if and only if $x \leq \widetilde{\varDelta}(t)$.*
– *If, however, at some point, the union deviates from the above rule, then both parties play thereafter according to the 'minimum-wage strategies'.*

*(ii) If for every $t \in \mathbb{N}$*

$$\left(1 - \check{\Delta}(t)\right) \Delta_u(2t+1) \geq w_0 \tag{39}$$

*and*

$$\widehat{\Delta}(t+1)\left(\Delta_u(2t+2) - \Delta_f(2t+2)\right) < w_0(1 - \Delta_f(2t+2)) \tag{40}$$

*then*

$$M_u^{2t} = \widehat{\Delta}(t) \quad \textit{and} \quad m_f^{2t+1} = \check{\Delta}(t) \tag{41}$$

*The SPE strategy profile that supports these $M_u^{2t}$ and $m_f^{2t+1}$ defined in (41) is given by the following 'modified generalized alternating strike strategies':*

- *In period $2t$ the union proposes $\widehat{\Delta}(t)$, in period $2t+1$ it accepts an offer $y$ if and only if $y \geq (1 - \Delta_u(2t+2))w_0 + \Delta_u(2t+2)\widehat{\Delta}(t+1)$, it strikes in even periods and holds out in odd periods if no agreement is reached.*
- *In period $2t+1$ the firm proposes $0$, in period $2t$ it accepts $x$ if and only if $x \leq \widehat{\Delta}(t)$.*
- *If, however, at some point, the union deviates from the above rule, then both parties play thereafter according to the 'minimum-wage strategies'.*

*(iii) If for every $t \in \mathbb{N}$*

$$M_u^{2t+2}(\Delta_u(2t+2) - \Delta_f(2t+2)) < w_0(1 - \Delta_f(2t+2))$$

*and*

$$(1 - m_f^{2t+1})\Delta_u(2t+1) < w_0$$

*then for each $t \in \mathbb{N}$*

$$M_u^{2t} = w_0 \quad \textit{and} \quad m_f^{2t+1} = 1 - w_0 \tag{42}$$

*The SPE strategy profile that supports these $M_u^{2t}$ and $m_f^{2t+1}$ defined in (42) is given by the minimum-wage strategies.*

For the proof of Proposition 4, see the Appendix.

Note that our Proposition 4 generalizes the corresponding results on $M_u$ and $m_f$ for the model with constant discount rates presented in Houba and Wen (2008) (Proposition 4). Consider the model with constant discount rates, i.e., let $\delta_{u,t} = \delta_u$ and $\delta_{f,t} = \delta_f$ for each $t \in \mathbb{N}$, and assume that $\delta_u > \delta_f$. Then for Proposition 4(i), we get for every $t \in \mathbb{N}$

$$M_u^{2t} = \frac{1 - \delta_f}{1 - \delta_u \delta_f}, \quad m_f^{2t+1} = \frac{1 - \delta_u}{1 - \delta_u \delta_f}$$

and the strike credibility conditions (36) and (37) are equivalent to the set $C$ in Houba and Wen (2008):

$$(\delta_u - w_0)\delta_f \leq \frac{\delta_u^2 - w_0}{\delta_u} \quad \text{and} \quad \delta_f \leq \frac{\delta_u - w_0}{1 - w_0 \delta_u}$$

respectively. For Proposition 4(ii), we get for every $t \in \mathbb{N}$

$$M_u^{2t} = \frac{1 + w_0 \delta_f}{1 + \delta_f}, \quad m_f^{2t+1} = \frac{1 - w_0}{1 + \delta_f}$$

and the conditions (39) and (40) are equivalent to the set $B$ in Houba and Wen (2008):

$$\delta_f(\delta_u - w_0) \geq w_0(1 - \delta_u) \quad \text{and} \quad \delta_f > \frac{\delta_u - w_0}{1 - \delta_u w_0}$$

In Propositions 3 and 4, $M_u^{2t}$ and $m_f^{2t+1}$ for every $t \in \mathbb{N}$ are determined for several cases where particular conditions on the discount rates of both parties are satisfied. In order to calculate $M_u^{2t}$ and $m_f^{2t+1}$ for an arbitrary case, we can proceed as follows. Given the sequences of discount rates $(\delta_{u,t})_{t\in\mathbb{N}}$ and $(\delta_{f,t})_{t\in\mathbb{N}}$, we also obtain the sequences of the generalized discount rates $(\Delta_u(t))_{t\in\mathbb{N}}$ and $(\Delta_f(t))_{t\in\mathbb{N}}$. Depending on which conditions hold, we apply Fact 1 to determine the infinite sequence of necessary conditions for $M_u^{2t}$ and $m_f^{2t+1}$ for every $t \in \mathbb{N}$. Note that we get always an infinite regular triangular system of equations which has a unique solution, being the sequence $(M_u^{2t}, m_f^{2t+1})_{t\in\mathbb{N}}$. However, the solution does not always satisfy the required conditions. To see that consider the case where for every $t \in \mathbb{N}$,

$$\Delta_u(t) > \Delta_f(t), \quad (1 - m_f^{2t+1})\Delta_u(2t+1) < w_0 \quad \text{and}$$

$$M_u^{2t+2}\left(\Delta_u(2t+2) - \Delta_f(2t+2)\right) \geq w_0(1 - \Delta_f(2t+2))$$

Then, solving for every $t \in \mathbb{N}$

$$M_u^{2t} = w_0(1 - \Delta_u(2t+1)) + (1 - m_f^{2t+1})\Delta_u(2t+1) \quad \text{and} \quad m_f^{2t+1} = 1 - M_u^{2t+2}\Delta_u(2t+2)$$

leads to

$$M_u^{2t} = w_0\left(1 - \Delta_u(2t+1) + \sum_{m=t}^{\infty}(1 - \Delta_u(2m+3))\prod_{j=t}^{m}\Delta_u(2j+1)\Delta_u(2j+2)\right)$$

but this means that $M_u^{2t} < w_0$, and therefore we get a contradiction.

Similarly, consider the case where for every $t \in \mathbb{N}$,

$$\Delta_u(2t+1) \leq \Delta_f(2t+1), \quad \Delta_u(2t+2) > \Delta_f(2t+2), \quad (1 - m_f^{2t+1})\Delta_u(2t+1) < w_0 \quad \text{and}$$

$$M_u^{2t+2}\left(\Delta_u(2t+2) - \Delta_f(2t+2)\right) \geq w_0(1 - \Delta_f(2t+2))$$

Then, solving for every $t \in \mathbb{N}$

$$M_u^{2t} = w_0(1 - \Delta_f(2t+1)) + (1 - m_f^{2t+1})\Delta_f(2t+1) \quad \text{and} \quad m_f^{2t+1} = 1 - M_u^{2t+2}\Delta_u(2t+2)$$

leads to $M_u^{2t} = w_0\widetilde{\Delta}(t) < w_0$, and therefore we get again a contradiction.

## 5 Conclusion

We calculated the equilibrium payoffs for the wage bargaining model between the union and the firm with preferences of the parties expressed by discount rates varying in time. We extended the analysis presented in Houba and Wen (2008) for the wage bargaining with constant discount rates. We focused on determining the supremum of the union's payoff and the infimum of the firm's payoff under SPE in all periods when the given party makes its offer. While we described the necessary conditions for these payoffs for arbitrary sequences of discount rates, we determined them and the supporting equilibria only for some particular (representative) cases of sequences of discount rates varying in time. In all these equilibria, the agreement is reached immediately in period 0. In a follow-up research on the wage bargaining with sequences of discount rates varying in time it would be interesting to analyze the existence of inefficient SPE with a strike for some periods followed by an agreement. Another issue in our future research agenda is to see how our results would be affected if not only the union could go on strike or holdout, but also the firm could lock out. We would like also to apply the model to one of the important economic issues – pharmaceutical product price determination; see e.g. Jelovac (2005); Garcia-Marinoso et al. (2011).

## Appendix - Proofs

### Proof of Proposition 1

Consider an arbitrary $t \in \mathbb{N}$.
(1) Suppose that strike is not credible, i.e., $(1 - m_f^{2t+1})\varDelta_u(2t+1) < w_0$.
We have (9a) $\geq$ (9b) if and only if

$$w_0(1-\varDelta_f(2t+1))+(1-m_f^{2t+1})\varDelta_f(2t+1) \geq w_0(1-\varDelta_u(2t+1))+(1-m_f^{2t+1})\varDelta_u(2t+1) \quad \Leftrightarrow$$

$$(1 - m_f^{2t+1} - w_0)\varDelta_f(2t+1) \geq (1 - m_f^{2t+1} - w_0)\varDelta_u(2t+1)$$

As $1 - m_f^{2t+1} - w_0 \geq 0$, this establishes the second and the third cases of (14).

(2) Suppose that strike is credible, i.e., $(1 - m_f^{2t+1})\varDelta_u(2t+1) \geq w_0$. Then, (9c) $\geq$ (9a). Moreover, (9c) $\geq$ (9b), because (9c) $\geq$ (9b) if and only if

$$1 - m_f^{2t+1}\varDelta_f(2t+1) \geq w_0(1 - \varDelta_u(2t+1)) + (1 - m_f^{2t+1})\varDelta_u(2t+1) \quad \Leftrightarrow$$

$$(1 - w_0)(1 - \varDelta_u(2t+1)) \geq m_f^{2t+1}\left(\varDelta_f(2t+1) - \varDelta_u(2t+1)\right)$$

which is always true, since $m_f^{2t+1} \leq 1 - w_0$. Then, we obtain the first case of (14). ∎

### Proof of Proposition 2

Consider an arbitrary $t \in \mathbb{N}$.
(1) Assume that $\varDelta_u(2t+2) \leq \varDelta_f(2t+2)$. We have

$$1 - w_0(1 - \varDelta_f(2t+2)) - M_u^{2t+2}\varDelta_f(2t+2) = 1 - M_u^{2t+2} + (M_u^{2t+2} - w_0)(1 - \varDelta_f(2t+2)) \leq$$

$$1 - M_u^{2t+2} + (M_u^{2t+2} - w_0)(1 - \varDelta_u(2t+2)) = 1 - w_0(1 - \varDelta_u(2t+2)) - M_u^{2t+2}\varDelta_u(2t+2)$$

Hence, (18a) $\geq$ (18b). Moreover, (18c) $>$ (18a), and we get the third case of (21).

(2) Assume that $\varDelta_u(2t+2) > \varDelta_f(2t+2)$. Then (18b) $>$ (18a). Moreover, (18c) $>$ (18b) if and only if

$$w_0(1 - \varDelta_f(2t+2)) + M_u^{2t+2}\varDelta_f(2t+2) > M_u^{2t+2}\varDelta_u(2t+2) \quad \Leftrightarrow$$

$$M_u^{2t+2}\left(\varDelta_u(2t+2) - \varDelta_f(2t+2)\right) < w_0(1 - \varDelta_f(2t+2))$$

which gives the second case of (21). On the other hand, if

$$M_u^{2t+2}\left(\varDelta_u(2t+2) - \varDelta_f(2t+2)\right) \geq w_0(1 - \varDelta_f(2t+2))$$

then (18c) $\leq$ (18b) and the strike is credible, because

$$M_u^{2t+2}\Delta_u(2t+2) - w_0 \geq M_u^{2t+2}\Delta_f(2t+2) + w_0(1-\Delta_f(2t+2)) - w_0 =$$

$$= \Delta_f(2t+2)(M_u^{2t+2} - w_0) \geq 0$$

We get then the first case of (21). ■

## Proof of Proposition 3

Let $\Delta_u(2t+2) \leq \Delta_f(2t+2)$ for every $t \in \mathbb{N}$.
(i) Consider the case when the strike is always credible, i.e., $(1-m_f^{2t+1})\Delta_u(2t+1) \geq w_0$ for every $t \in \mathbb{N}$. From Fact 1 we have for every $t \in \mathbb{N}$:

$$M_u^{2t} + m_f^{2t+1}\Delta_f(2t+1) = 1 \quad \text{and} \quad m_f^{2t+1} + M_u^{2t+2}\Delta_u(2t+2) = 1 - w_0(1-\Delta_u(2t+2))$$

which is a regular triangular system $AX = Y$, with $A = [a_{ij}]_{i,j\in\mathbb{N}^+}$, $X = [(x_i)_{i\in\mathbb{N}^+}]^T$, $Y = [(y_i)_{i\in\mathbb{N}^+}]^T$, where for each $t, j \geq 1$

$$a_{t,t} = 1,\ a_{t,j} = 0 \text{ for } j < t \text{ or } j > t+1$$

and for each $t \in \mathbb{N}$

$$a_{2t+1,2t+2} = \Delta_f(2t+1), \quad a_{2t+2,2t+3} = \Delta_u(2t+2)$$

$$x_{2t+1} = M_u^{2t}, \quad x_{2t+2} = m_f^{2t+1}, \quad y_{2t+1} = 1, \quad y_{2t+2} = 1 - w_0(1-\Delta_u(2t+2))$$

Any regular triangular matrix $A$ possesses the (unique) inverse matrix $B$, i.e., there exists $B$ such that $BA = I$, where $I$ is the infinite identity matrix. The matrix $B = [b_{ij}]_{i,j\in\mathbb{N}^+}$ is also regular triangular, and its elements are the following:

$$b_{t,t} = 1,\ b_{t,j} = 0 \text{ for each } t, j \geq 1 \text{ such that } j < t$$

$$b_{2t+1,2t+2} = -\Delta_f(2t+1),\ b_{2t+2,2t+3} = -\Delta_u(2t+2) \text{ for each } t \in \mathbb{N}$$

and for each $t, m \in \mathbb{N}$ and $m > t$

$$b_{2t+1,2m+1} = \prod_{j=t}^{m-1} \Delta_f(2j+1)\Delta_u(2j+2), \quad b_{2t+1,2m+2} = -\prod_{j=t}^{m-1} \Delta_f(2j+1)\Delta_u(2j+2)\Delta_f(2m+1)$$

$$b_{2t+2,2m+2} = \prod_{j=t}^{m-1} \Delta_u(2j+2)\Delta_f(2j+3), \quad b_{2t+2,2m+3} = -\prod_{j=t}^{m-1} \Delta_u(2j+2)\Delta_f(2j+3)\Delta_u(2m+2)$$

Next, by applying $X = BY$ we get $M_u^{2t}$ as given in (32) and $m_f^{2t+1}$ as given in (33). The strike credibility condition $(1-m_f^{2t+1})\Delta_u(2t+1) \geq w_0$ for every $t \in \mathbb{N}$ is then written as in (31). In Ozkardas and Rusinowska (2014) (Proposition 4) we show that under an equivalently expressed condition (31) and $\Delta_u(2t+2) \leq \Delta_f(2t+2)$ for every $t \in \mathbb{N}$, the proposed strategy profile (formed by the generalized alternating strike strategies) is a SPE.

(ii) Consider the case when the strike is never credible, i.e., $(1 - m_f^{2t+1})\Delta_u(2t+1) < w_0$ for every $t \in \mathbb{N}$. Then we have the infinite system for $t \in \mathbb{N}$

$$M_u^{2t} + m_f^{2t+1}\Delta_f(2t+1) = w_0(1 - \Delta_f(2t+1)) + \Delta_f(2t+1)$$

or

$$M_u^{2t} + m_f^{2t+1}\Delta_u(2t+1) = w_0(1 - \Delta_u(2t+1)) + \Delta_u(2t+1)$$

and

$$m_f^{2t+1} + M_u^{2t+2}\Delta_u(2t+2) = 1 - w_0(1 - \Delta_u(2t+2))$$

which as a regular triangular system possesses a unique solution. This solution is given by (35). It is supported by the minimum-wage strategies profile which is a SPE as shown in Ozkardas and Rusinowska (2014) (Fact 3). ■

## Proof of Proposition 4

Let $\Delta_u(2t+2) > \Delta_f(2t+2)$ for every $t \in \mathbb{N}$.

(i) Consider the case when for every $t \in \mathbb{N}$, $(1 - m_f^{2t+1})\Delta_u(2t+1) \geq w_0$ (i.e., strike is credible in period $2t$) and condition (25) holds. If (25) is satisfied, then strike is credible in period $2t+1$. From Fact 1 we get the infinite system for every $t \in \mathbb{N}$

$$M_u^{2t} + m_f^{2t+1}\Delta_f(2t+1) = 1 \quad \text{and} \quad m_f^{2t+1} + M_u^{2t+2}\Delta_u(2t+2) = 1$$

which is is a regular triangular system $AX = Y$, with $A = [a_{ij}]_{i,j\in\mathbb{N}^+}$ and $X = [(x_i)_{i\in\mathbb{N}^+}]^T$ the same as in the proof of Proposition 3, and with $Y = [(y_i)_{i\in\mathbb{N}^+}]^T$ such that $y_{2t+1} = y_{2t+2} = 1$. The (unique) inverse matrix $B$ is the same as before, and by applying $X = BY$ we get $M_u^{2t}$ and $m_f^{2t+1}$ as given by (38). The conditions $(1 - m_f^{2t+1})\Delta_u(2t+1) \geq w_0$ and (25) are equivalent to (36) and (37). In Ozkardas and Rusinowska (2014) (Proposition 3) we show that the proposed strategy profile (formed by the 'always strike strategies') is a SPE under an equivalently expressed condition (36).

(ii) Consider the case when for every $t \in \mathbb{N}$, $(1 - m_f^{2t+1})\Delta_u(2t+1) \geq w_0$ (i.e., strike is credible in period $2t$) and condition (26) holds. Then, we solve the infinite system for every $t \in \mathbb{N}$

$$M_u^{2t} + m_f^{2t+1}\Delta_f(2t+1) = 1 \quad \text{and} \quad m_f^{2t+1} + M_u^{2t+2}\Delta_f(2t+2) = 1 - w_0(1 - \Delta_f(2t+2))$$

which is a regular triangular system $AX = Y$. By applying $X = BY$ we get $M_u^{2t}$ and $m_f^{2t+1}$ as given by (41). The conditions $(1-m_f^{2t+1})\Delta_u(2t+1) \geq w_0$ and (26) are equivalent to (39) and (40). In Ozkardas and Rusinowska (2014) (Theorem 3) we show that if $\Delta_u(2t+2) > \Delta_f(2t+2)$ for each $t \in \mathbb{N}$, then the proposed strategy profile (formed by the 'modified generalized alternating strike strategies') is a SPE under the following condition:

$$w_0 \leq \Delta_u(2t+1)\left((1 - \Delta_u(2t+2))w_0 + \Delta_u(2t+2)W^{2t+2}\right) \tag{43}$$

where

$$W^{2t} = \frac{1 + \sum_{m=t}^{\infty} \delta_f(2t+1, 2m+2) + w_0 \sum_{m=t}^{\infty} \delta_f(2t+1, 2m+1)}{1 + \sum_{m=2t+1}^{\infty} \delta_f(2t+1, m)}$$

is the SPE offer of the union proposed in period $2t$. One can show that $W^{2t} = M_u^{2t} = \widehat{\Delta}(t)$:

$$W^{2t} = w_0 + (1 - w_0)\left(\frac{1 + \sum_{m=t}^{\infty} \delta_f(2t+1, 2m+2)}{1 + \sum_{m=2t+1}^{\infty} \delta_f(2t+1, m)}\right) =$$

$$= w_0 + (1 - w_0)\left(1 - \Delta_f(2t+1) + \frac{\sum_{m=t}^{\infty} \delta_f(2t+1, 2m+2)}{1 + \sum_{m=2t+1}^{\infty} \delta_f(2t+1, m)}\right) =$$

$$= w_0 + (1 - w_0)\left(1 - \Delta_f(2t+1) + \sum_{m=t}^{\infty}(1 - \Delta_f(2m+3)) \prod_{j=t}^{m} \Delta_f(2j+1)\Delta_f(2j+2)\right) = \widehat{\Delta}(t)$$

Moreover, note that (39) implies condition (43):

$$w_0 \leq \Delta_u(2t+1)\left(1 - \breve{\Delta}(t)\right) =$$

$$= \Delta_u(2t+1)\left[w_0 + (1 - w_0)\left(\Delta_f(2t+2) - \sum_{m=t}^{\infty}(1 - \Delta_f(2m+4)) \prod_{j=t}^{m} \Delta_f(2j+2)\Delta_f(2j+3)\right)\right]$$

$$= \Delta_u(2t+1)\left[w_0 + (1 - w_0)\Delta_f(2t+2)\left(1 - \Delta_f(2t+3) + \sum_{m=t+1}^{\infty}(1 - \Delta_f(2m+3)) \prod_{j=t+1}^{m} \Delta_f(2j+1)\Delta_f(2j+2)\right)\right]$$

$$= \Delta_u(2t+1)\left(w_0 + \Delta_f(2t+2)(\widehat{\Delta}(t+1) - w_0)\right) <$$

$$< \Delta_u(2t+1)\left(w_0 + \Delta_u(2t+2)(\widehat{\Delta}(t+1) - w_0)\right) =$$

$$= \Delta_u(2t+1)\left((1 - \Delta_u(2t+2))w_0 + \Delta_u(2t+2)\widehat{\Delta}(t+1)\right)$$

(iii) Consider the case when for every $t \in \mathbb{N}$,

$(1 - m_f^{2t+1})\Delta_u(2t+1) < w_0$ and $M_u^{2t+2}(\Delta_u(2t+2) - \Delta_f(2t+2)) < w_0(1 - \Delta_f(2t+2))$

Then, from Fact 1 we get the infinite system for every $t \in \mathbb{N}$

$$M_u^{2t} + m_f^{2t+1}\Delta_u(2t+1) = w_0 + \Delta_u(2t+1)(1 - w_0)$$

or

$$M_u^{2t} + m_f^{2t+1}\Delta_f(2t+1) = w_0 + \Delta_f(2t+1)(1 - w_0)$$

and

$$m_f^{2t+1} + M_u^{2t+2}\Delta_f(2t+2) = 1 - w_0(1 - \Delta_f(2t+2))$$

which is is a regular triangular system $AX = Y$ with the solution $M_u^{2t} = w_0$ and $m_f^{2t+1} = 1 - w_0$ for each $t \in \mathbb{N}$. The SPE supporting this solution is the minimum-wage strategies profile. ∎

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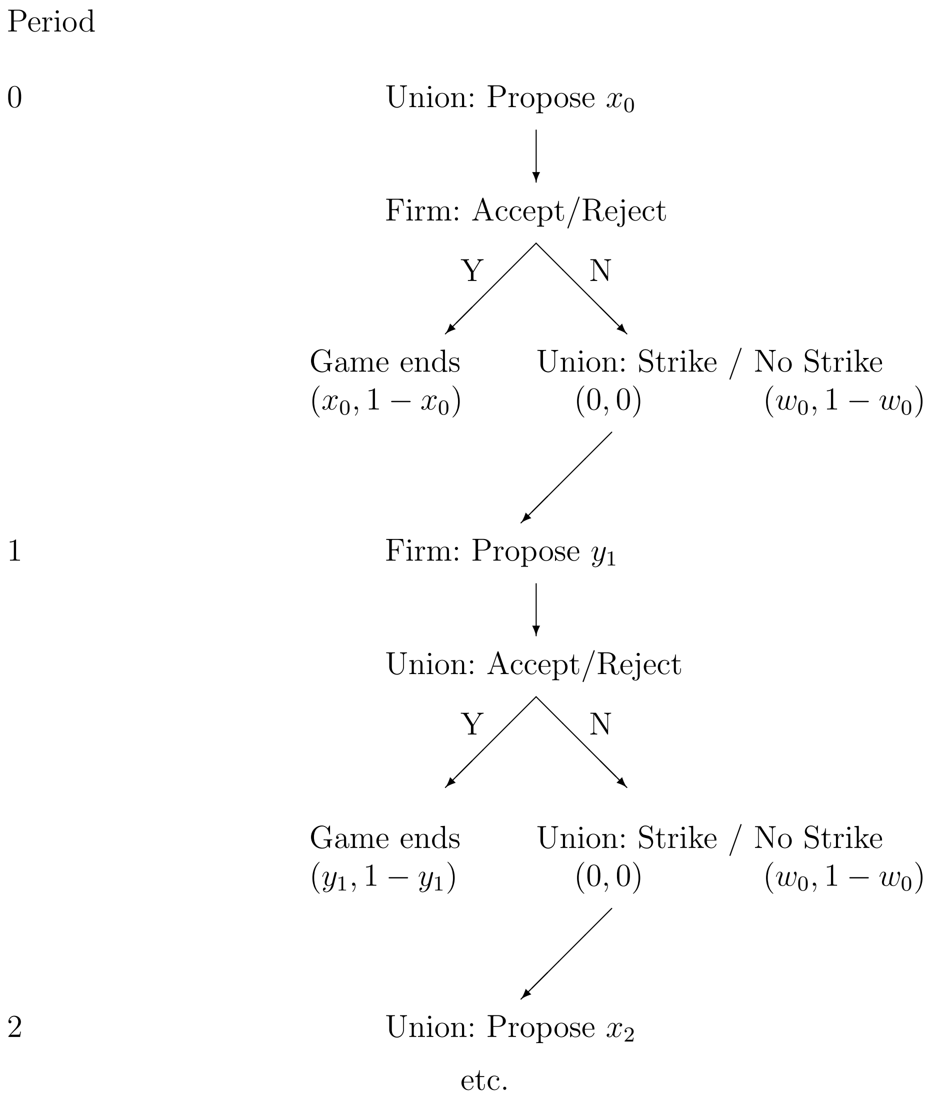


**Figure 1**: Non-cooperative bargaining game between the union and the firm