Stabilization of nonlinear systems using event-triggered output feedback laws

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***Abstract*— We design output-based event-triggered controllers to stabilize a class of nonlinear systems. We start from an output feedback law which stabilizes the plant in the absence of sampling and we then synthesize the event-triggering condition. The proposed event-triggering condition combines event-triggered and time-triggered techniques. The idea is to turn on the event-triggering mechanism only after a fixed amount of time has elapsed since the last transmission. This time is computed based on results on the stabilization of time-driven sampled-data systems. The overall strategy ensures an asymptotic stability property for the closed-loop system. Moreover, it has the advantage to enforce a (uniform) minimum amount of time between two transmissions which can be directly tuned. We show that the results are applicable to linear time-invariant systems as a particular case and we illustrate the approach for the stabilization of a nonlinear single-link robot arm model.**

I. Introduction

Networked control systems (NCSs) are systems in which the plant and the controller communicate with each other over a digital channel. NCSs are of great interest for a broad range of applications due to their advantages in terms of flexibility, cost and ease of maintenance. A major challenge in such systems is to achieve the control objectives despite the communication constraints induced by the network (like time-varying sampling, delay, packet drop-out, etc.). In conventional setups, data transmissions are time-driven and two successive transmission instants are constrained to be less than a fixed constant, called the *maximum allowable transmission interval* (MATI) (see *e.g.* [1], [2]). However, it is not clear that this paradigm is always suitable. Indeed, the same amount of transmissions per unit of time is generated in this case, even when transmissions are not necessary in view of the control objectives. To overcome this shortcoming, event-triggered control has been proposed as an alternative where the transmission instants are determined by the occurrence of an event, based on the measured output, and not a time-driven clock, see *e.g.* [3]–[8]. The main idea is to adapt transmissions to the state of the plant such that the loop is closed only when it is needed according to the stability or/and the performance requirements.

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Most existing results on event-triggered control assume that the full state measurement is available. However, this is not realistic for many applications where only a part of the plant state can be measured. Hence, the design of stabilizing event-triggered controllers based on output measurements has a significant practical interest. This problem has been first addressed in [9] to the best of our knowledge, and then in [10]–[16] for instance.

It is important to emphasize that the design of output feedback event-triggered controllers is particularly challenging because it is usually more difficult to ensure the existence of a minimum amount of time between two transmissions than with state feedbacks (see [10]); in particular when we aim to guarantee asymptotic stability properties. The existence of such a time is not only useful to prove stability but also because two triggering instants cannot occur arbitrarily close in time in practice due to hardware limitations. In [10], event-triggered controllers are proposed for linear time-invariant (LTI) systems which guarantee a uniform ultimate boundedness property. These controllers are such that the smaller the size of the ultimate bound, the shorter the guaranteed minimum inter-transmission time. In [17], event-triggered observer-based controllers have been designed to ensure an asymptotic stability property for three different architectures of LTI systems. More recently, a discrete-time event-triggering mechanism for LTI systems has been presented in [15] to asymptotically stabilize the system. To the best of our knowledge, output feedback event-triggered controllers for nonlinear systems have only been studied in [12] where passivity tools were used to derive triggering conditions which ensure an $\mathcal{L}_2$ stability property.

In this paper, we are interested in designing output feedback event-triggered controllers for nonlinear systems which ensure a (global) asymptotic stability property and which guarantee the existence of a uniform strictly positive lower bound on the inter-transmission times. We design the controller using the emulation approach (see *e.g.* [5], [7], [10]) as we assume that we know an output feedback law which stabilizes the system in the absence of network and we then take into account the communication constraints and construct the triggering condition. The overall problem is modeled as a hybrid system using the formalism of [18] like in [7], [10], [16]. The proposed strategy combines the event-triggering condition of [5] adapted to output measurements and the results on time-driven sampled-data systems in [19]. Indeed, the event-triggering condition is only (continuously) evaluated after T units of times have elapsed since the last transmission, where T corresponds to the MATI given by

[19]. This two-step procedure is justified by the fact that the adaption of the event-triggering condition of [3] to output feedback on its own may lead to Zeno phenomenon (see [10]). Although the rationale is intuitive, the analysis is not trivial as we show in the paper. This triggering mechanism has been used in [20] to stabilize nonlinear singularly perturbed systems under a different set of assumptions. Similar approaches have been followed in [16], [21], [22] to enforce a lower bound on the inter-transmission times in different contexts, mainly for linear systems. Note that the idea of enforcing a given time between two transmissions is linked to time regularization techniques, see [23].

Our results rely on similar assumptions as in [19] which allow us to derive local and global results. These conditions are shown to be always verified by LTI systems that are stabilizable and detectable, in which case these are reformulated as linear matrix inequalities (LMI). Contrary to [16], the approach is applicable to nonlinear systems and the output feedback law is not necessarily based on an observer. Compared to [12], we rely on a different set of assumptions and we conclude a different stability property. In addition, we show that our results are applicable to any LTI systems that are stabilizable and detectable and to a nonlinear robotic example for which the results in [12] are not applicable since the required conditions in Proposition 1 in [12] are not satisfied. Unlike [11], where LTI systems have been studied, we do not necessarily consider observer-based output feedbacks and the triggering condition does not necessarily rely on estimates of the unmeasured states. The latter has the advantage to lighten the implementation since the triggering mechanism only needs to have access to the output of the plant, and not the controller variable. Finally, in the particular case of LTI systems, we conclude asymptotic stability properties as opposed to ultimate boundedness in [10]. It has to be noted that the event-triggering mechanism that we propose is different from the periodic event-triggered control (PETC) paradigm, see *e.g.* [24], [25], where the triggering condition is verified only at some periodic sampling instants. In our case, the triggering mechanism is *continuously* evaluated, once T units of time have elapsed since the last transmission.

The remainder of the paper is organised as follows. Preliminaries are given in Section II. The system model is provided in Section III. In Section IV, we present the main results. In Section V, we show that the required conditions are always satisfied by stabilizable and detectable LTI systems. An illustrative example is given in Section VI and conclusions are proposed in Section VII. The proofs are given in the Appendix.

II. PRELIMINARIES

Notation. We denote $\mathbb{R} = (-\infty, \infty)$, $\mathbb{R}_{\geq 0} = [0, \infty)$ and $\mathbb{Z}_{\geq 0} = \{0, 1, 2, ..\}$. The Euclidean norm is denoted as $|.|$. We use the notation (x, y) to represent the vector $[x^T, y^T]^T$ for $x \in \mathbb{R}^n$ and $y \in \mathbb{R}^m$. A continuous function $\gamma : \mathbb{R}_{\geq 0} \to \mathbb{R}_{\geq 0}$ is of class $\mathcal{K}$ if it is zero at zero, strictly increasing, and it is of class $\mathcal{K}_\infty$ if in addition $\gamma(s) \to \infty$ as $s \to \infty$. A continuous function $\gamma : \mathbb{R}_{\geq 0} \times \mathbb{R}_{\geq 0} \to \mathbb{R}_{\geq 0}$ is of class $\mathcal{KL}$ if for each $t \in \mathbb{R}_{\geq 0}$, $\gamma(., t)$ is of class $\mathcal{K}$, and, for each $s \in \mathbb{R}_{\geq 0}$, $\gamma(s, .)$ is decreasing to zero. We denote the minimum and maximum eigenvalues of the symmetric positive definite matrix A as $\lambda_{\min}(A)$ and $\lambda_{\max}(A)$, respectively. We use $\mathbb{I}_n$ to denote the identity matrix of dimension n. We will consider locally Lipschitz Lyapunov functions (that are not necessarily differentiable everywhere), therefore we will use the generalized directional derivative of Clarke which is defined as follows. For a locally Lipschitz function $V : \mathbb{R}^n \to \mathbb{R}_{\geq 0}$ and a vector $\upsilon \in \mathbb{R}^n$, $V^\circ(x; \upsilon) := \limsup_{h \to 0^+,\, y \to x} (V(y + h\upsilon) - V(y))/h$. For a continuously differentiable function V, $V^\circ(x; \upsilon)$ reduces to the standard directional derivative $\langle \nabla V(x), \upsilon \rangle$, where $\nabla V(x)$ is the (classical) gradient. We will invoke the following result which corresponds to Proposition 1.1 in [26].

Lemma 1 (Lemma II.1 [26]). *Consider two functions $U_1 : \mathbb{R}^n \to \mathbb{R}$ and $U_2 : \mathbb{R}^n \to \mathbb{R}$ that have well-defined Clarke derivatives for all $x \in \mathbb{R}^n$ and $\upsilon \in \mathbb{R}^n$. Introduce three sets $A := \{x : U_1(x) > U_2(x)\}$, $B := \{x : U_1(x) < U_2(x)\}$, $\Gamma := \{x : U_1(x) = U_2(x)\}$. Then, for any $\upsilon \in \mathbb{R}^n$, the function $U(x) := \max\{U_1(x), U_2(x)\}$ satisfies $U^\circ(x; \upsilon) = U_1^\circ(x; \upsilon)$ for all $x \in A$, $U^\circ(x; \upsilon) = U_2^\circ(x; \upsilon)$ for all $x \in B$ and $U^\circ(x; \upsilon) \leq \max\{U_1^\circ(x; \upsilon), U_2^\circ(x; \upsilon)\}$ for all $x \in \Gamma$.* □

Basic definitions on hybrid systems. In this paper, we consider hybrid systems of the following form using the formalism of [18]

$$\dot{x} = F(x) \quad x \in C, \qquad x^+ = G(x) \quad x \in D, \tag{1}$$

where $x \in \mathbb{R}^n$ is the state, F is the flow map, C is the flow set, G is the jump map and D is the jump set. The vector fields F and G are assumed to be continuous and the sets C and D are closed. The solutions to system (1) are defined on so-called hybrid time domains. A set $E \subset \mathbb{R}_{\geq 0} \times \mathbb{Z}_{\geq 0}$ is called a *compact hybrid time domain* if $E = \underset{j \in \{0,\ldots,J\}}{\cup} ([t_j, t_{j+1}], j)$ for some finite sequence of times $0 = t_0 \leq t_1 \leq ... \leq t_J$ and it is a *hybrid time domain* if for all $(T, J) \in E, E \cap ([0, T] \times \{0, 1, ..., J\})$ is a compact hybrid time domain. A function $\phi : E \to \mathbb{R}^n$ is a hybrid arc if E is a hybrid time domain and if for each $j \in \mathbb{Z}_{\geq 0}, t \mapsto \phi(t, j)$ is locally absolutely continuous on $I^j := \{t : (t, j) \in E\}$. A hybrid arc ϕ is a solution to system (1) if: (i) $\phi(0, 0) \in C \cup D$; (ii) for any $j \in \mathbb{Z}_{\geq 0}$, $\phi(t, j) \in C$ and $\dot{\phi}(t, j) = F(\phi(t, j))$ for almost all $t \in I^j$; (iii) for every $(t, j) \in \operatorname{dom} \phi$ such that $(t, j + 1) \in \operatorname{dom} \phi$, $\phi(t, j) \in D$ and $\phi(t, j + 1) = G(\phi(t, j))$. A solution ϕ to system (1) is *maximal* if it cannot be extended, and it is *complete* if its domain, $\operatorname{dom} \phi$, is unbounded.

III. PROBLEM STATEMENT

Consider the nonlinear plant model

$$\begin{aligned} \dot{x}_p &= f_p(x_p, u) \\ y &= g_p(x_p), \end{aligned} \tag{2}$$

where $x_p \in \mathbb{R}^{n_p}$ is the plant state, $u \in \mathbb{R}^{n_u}$ is the control input and $y \in \mathbb{R}^{n_y}$ is the measured output of the plant. We assume that the dynamic controller below globally asymptotically stabilizes system (2)

$$\begin{aligned} \dot{x}_c &= f_c(x_c, y) \\ u &= g_c(x_c, y), \end{aligned} \tag{3}$$

where $x_c \in \mathbb{R}^{n_c}$ is the controller state. We emphasize that the x_c-system is not necessarily an observer. Moreover, (3) captures static output feedbacks as a particular case by setting $u = g_c(y)$.

We consider the scenario where controller (3) communicates with the plant via a digital channel. Hence, the plant output and the control input are sent only at transmission instants $t_i, i \in \mathbb{Z}_{\geq 0}$. We are interested in an event-triggered implementation (see Figure 1) in the sense that the sequence of transmission instants is determined by a criterion based on the output measurements. At each transmission instant, the plant output is sent to the controller which computes a new control input that is instantaneously transmitted to the plant. We assume that this process is performed in a synchronous manner and we ignore the computation times and the possible transmission delays[1]. In that way, we obtain

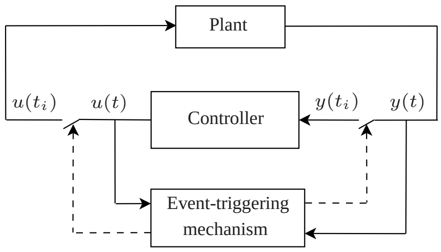

Fig. 1. Event-triggered control schematic [10]

$$\begin{array}{lll} \dot{x}_p = f_p(x_p, \hat{u}), & \dot{x}_c = f_c(x_c, \hat{y}) & t \in [t_i, t_{i+1}] \\ \dot{\hat{y}} = 0, & \dot{\hat{u}} = 0 & t \in [t_i, t_{i+1}] \\ \hat{y}(t_i^+) = y(t_i), & \hat{u}(t_i^+) = u(t_i), & u = g_c(x_c, \hat{y}), \end{array}$$

where $\hat{y}$ and $\hat{u}$ respectively denote the last transmitted values of the plant output and of the control input. We assume that zero-order-hold devices are used to generate the sampled values $\hat{y}$ and $\hat{u}$, which leads to $\dot{\hat{y}} = 0$ and $\dot{\hat{u}} = 0$. We introduce the network-induced error $e := (e_y, e_u) \in \mathbb{R}^{n_e}$, where $e_y := \hat{y} - y$ and $e_u := \hat{u} - u$ which are reset to 0 at each transmission instant.

We model the event-triggered control system using the hybrid formalism of [18] as in [10], [16], [7], for which a jump corresponds to a transmission. In that way, the system can be modeled as

$$\begin{aligned} \begin{pmatrix} \dot{x} \\ \dot{e} \\ \dot{\tau} \end{pmatrix} &= \begin{pmatrix} f(x,e) \\ g(x,e) \\ 1 \end{pmatrix} & (x, e, \tau) \in C \\ \begin{pmatrix} x^+ \\ e^+ \\ \tau^+ \end{pmatrix} &= \begin{pmatrix} x \\ 0 \\ 0 \end{pmatrix} & (x, e, \tau) \in D, \end{aligned} \tag{4}$$

[1] We think that the effect of computation and transmission delays can be studied by using similar arguments as in [5].

where $x := (x_p, x_c) \in \mathbb{R}^{n_x}$ and $\tau \in \mathbb{R}_{\geq 0}$ is a clock variable which describes the time elapsed since the last jump and

$$\begin{aligned} f(x,e) &= \begin{pmatrix} f_p\Big(x_p, g_c(x_c, y + e_y) + e_u\Big) \\ \\ f_c(x_c, y + e_y) \end{pmatrix} \\ g(x,e) &= \begin{pmatrix} -\frac{\partial}{\partial x_p} g_p(x_p) f_p\Big(x_p, g_c(x_c, y + e_y) + e_u\Big) \\ \\ -\frac{\partial}{\partial x_c} g_c(x_c, y + e_y) f_c(x_c, y + e_y) \end{pmatrix}. \end{aligned} \tag{5}$$

The flow and jump sets, respectively denoted C and D, are defined according to the triggering condition we will define. As long as the triggering condition is not violated, the system flows on C where no transmission occurs. Jumps occur only if the triggering condition is verified, *i.e.* $(x, e, \tau) \in D$. When $(x, e, \tau) \in C \cap D$, the system flows only if flowing keeps (x, e, τ) in C, otherwise the system experiences a jump. The functions f and g, defined in (5), are assumed to be continuous and the sets C and D will be closed (which ensure that system (4) is well-posed, see Chapter 6 in [18]).

The main objective of this paper is to design the flow and the jump sets of system (4), *i.e.* the triggering condition, to ensure a (global) asymptotic stability property for system (4).

IV. Main results

In this section, we first present the conditions that we impose for system (4), then we present the triggering technique and finally we state the main result. We make the following assumption on system (4), which is inspired by [19].

Assumption 1. *There exist $\Delta_x, \Delta_e > 0$, locally Lipschitz positive definite functions $V : \mathbb{R}^{n_x} \to \mathbb{R}_{\geq 0}$ and $W : \mathbb{R}^{n_e} \to \mathbb{R}_{\geq 0}$, continuous function $H : \mathbb{R}^{n_x} \to \mathbb{R}_{\geq 0}$, real numbers $\gamma, L \geq 0$, $\underline{\alpha}, \overline{\alpha} \in \mathcal{K}_\infty$ and continuous, positive definite functions $\delta : \mathbb{R}^{n_y} \to \mathbb{R}_{\geq 0}$ and $\alpha : \mathbb{R}_{\geq 0} \to \mathbb{R}_{\geq 0}$ such that, for all $x \in \mathbb{R}^{n_x}$*

$$\underline{\alpha}(|x|) \leq V(x) \leq \overline{\alpha}(|x|), \tag{6}$$

for all $|e| \leq \Delta_e$ and almost all $|x| \leq \Delta_x$

$$\langle \nabla V(x), f(x,e) \rangle \leq -\alpha(|x|) - H^2(x) - \delta(y) + \gamma^2 W^2(e) \tag{7}$$

and for all $|x| \leq \Delta_x$ and almost all $|e| \leq \Delta_e$

$$\langle \nabla W(e), g(x,e) \rangle \leq L W(e) + H(x). \tag{8}$$

We say that Assumption 1 holds globally if (7) and (8) hold for almost all $x \in \mathbb{R}^{n_x}$ and $e \in \mathbb{R}^{n_e}$. □

Conditions (6)-(7) imply that the system $\dot{x} = f(x,e)$ is $\mathcal{L}_2$-gain stable from W to $(H, \sqrt{\delta})$. This property can be analysed by investigating the robustness property of the closed-loop system (2)-(3) with respect to input and/or output measurement errors in the absence of sampling. Note that, since W is positive definite and continuous (since it is locally

Lipschitz), there exists $\chi \in \mathcal{K}_\infty$ such that $W(e) \leq \chi(|e|)$ (according to Lemma 4.3 in [27]) and hence (6), (7) imply that the system $\dot{x} = f(x, e)$ is input-to-state stable (ISS), see [28]. We also assume an exponential growth condition of the e-system on flows in (8) which is similarly used in [2], [19]. As we will show in Sections V-VI, Assumption 1 can always be satisfied by LTI systems that are stabilizable and detectable and is shown to hold for a nonlinear robotic system for example.

In view of Assumption 1, the adaptation of the idea of [5] leads to a triggering condition of the form

$$\gamma^2 W^2(e) \leq \delta(y). \tag{9}$$

Note that the terms $\alpha(|x|)$ and $H^2(x)$ cannot be used to define the triggering condition as these depend on the state x which is not known a priori. The problem is that the Zeno phenomenon may occur with (9). Indeed, when $y = 0$, an infinite number of jumps occurs for any value of x such that $g_p(x_p) = 0$. In [10], this issue was overcome by adding a constant to (9), which would lead to $\gamma^2 W^2(e) \leq \delta(y) + \varepsilon$ here for $\varepsilon > 0$, from which we can derive a practical stability property. The event-triggered mechanism that we propose allows us to guarantee an asymptotic stability property for the closed-loop while ensuring that the inter-transmission times are lower bounded by a strictly positive constant. The idea is to evaluate (9) only after T units have elapsed since the last transmission, where T corresponds to the MATI given by [19]. In that way, the triggering mechanism benefits from the advantages of the event-triggered [5] and time-triggered [19] strategies and allows the user to directly tune the minimum inter-jump interval, up to a specified bound given in the following. We thus redesign (9) as follows

$$\gamma^2 W^2(e) \leq \delta(y) \text{ or } \tau \in [0, T], \tag{10}$$

where we recall that $\tau \in \mathbb{R}_{\geq 0}$ is the clock variable introduced in (4). Consequently, the flow and jump sets of system (4) are

$$\begin{aligned} C &= \Big\{(x, e, \tau) : \gamma^2 W^2(e) \leq \delta(y) \text{ or } \tau \in [0, T]\Big\} \\ D &= \Big\{(x, e, \tau) : \Big(\gamma^2 W^2(e) = \delta(y) \text{ and } \tau \geq T\Big) \text{ or} \\ &\qquad \Big(\gamma^2 W^2(e) \geq \delta(y) \text{ and } \tau = T\Big)\Big\}. \end{aligned} \tag{11}$$

Hence, the inter-jump times are uniformly lower bounded by T. This constant is selected such that $T < \mathcal{T}(\gamma, L)$, where

$$\mathcal{T}(\gamma, L) := \begin{cases} \frac{1}{Lr} \arctan(r) & \gamma > L \\ \frac{1}{L} & \gamma = L \\ \frac{1}{Lr} \operatorname{arctanh}(r) & \gamma < L \end{cases} \tag{12}$$

with $r := \sqrt{\left|(\frac{\gamma}{L})^2 - 1\right|}$ and L, γ come from Assumption 1 as in [19]. We are ready to state the main result.

Theorem 1. *Consider system (4) with the flow and jump sets defined in (11) and suppose the following hold.*

- *Assumption 1 is satisfied.*
- *The constant T in (11) is such that $T \in (0, \mathcal{T}(\gamma, L))$.*

Then, there exist $\Delta > 0$ and $\beta \in \mathcal{KL}$ such that any solution $\phi = (\phi_x, \phi_e, \phi_\tau)$ with $|(\phi_x(0,0), \phi_e(0,0))| \leq \Delta$ satisfies

$$|\phi_x(t,j)| \leq \beta(|(\phi_x(0,0), \phi_e(0,0))|, t+j) \quad \forall (t,j) \in \text{dom}\, \phi, \tag{13}$$

furthermore, if ϕ is maximal, then it is complete. If Assumption 1 holds globally, then (13) holds globally. □

V. Linear systems

Consider a linear time-invariant plant of the form

$$\dot{x}_p = A_p x_p + B_p u, \qquad y = C_p x_p, \tag{14}$$

where $x_p \in \mathbb{R}^{n_p}$, $u \in \mathbb{R}^{n_u}$, $y \in \mathbb{R}^{n_y}$ and A_p, B_p, C_p are matrices of appropriate dimensions. We design the following dynamic controller

$$\dot{x}_c = A_c x_c + B_c y, \qquad u = C_c x_c + D_c y, \tag{15}$$

where $x_c \in \mathbb{R}^{n_c}$ and A_c, B_c, C_c, D_c are matrices of appropriate dimensions. We take into account the network-induced constraints. Then, the hybrid model (4) is

$$\begin{aligned} \begin{pmatrix} \dot{x} \\ \dot{e} \\ \dot{\tau} \end{pmatrix} &= \begin{pmatrix} \mathcal{A}_1 x + \mathcal{B}_1 e \\ \mathcal{A}_2 x + \mathcal{B}_2 e \\ 1 \end{pmatrix} \quad (x, e, \tau) \in C \\ \begin{pmatrix} x^+ \\ e^+ \\ \tau^+ \end{pmatrix} &= \begin{pmatrix} x \\ 0 \\ 0 \end{pmatrix} \qquad\qquad\quad (x, e, \tau) \in D, \end{aligned} \tag{16}$$

where $\mathcal{A}_1 := \begin{pmatrix} A_p + B_p D_c C_p & B_p C_c \\ B_c C_p & A_c \end{pmatrix}$, $\mathcal{B}_1 := \begin{pmatrix} B_p D_c & B_p \\ B_c & 0 \end{pmatrix}$, $\mathcal{A}_2 := \begin{pmatrix} -C_p(A_p + B_p D_c C_p) & -C_p B_p C_c \\ -C_c B_c C_p & -C_c A_c \end{pmatrix}$ and $\mathcal{B}_2 := \begin{pmatrix} -C_p B_p D_c & -C_p B_p \\ -C_c B_c & 0 \end{pmatrix}$. We obtain the following result.

Proposition 1. *Consider system (16). Suppose that there exist $\varepsilon_1, \varepsilon_2, \mu > 0$ and a positive definite symmetric real matrix P such that*

$$\begin{pmatrix} \mathcal{A}_1^T P + P\mathcal{A}_1 + \mathcal{A}_2^T \mathcal{A}_2 + \varepsilon_1 \overline{C}_p^T \overline{C}_p + \varepsilon_2 \mathbb{I}_{n_x} & P\mathcal{B}_1 \\ \mathcal{B}_1^T P & -\mu \mathbb{I}_{n_e} \end{pmatrix} \leq 0, \tag{17}$$

where $\overline{C}_p = [C_p \quad 0]$. Then Assumption 1 holds with $V(x) = x^T P x$, $\underline{\alpha}(|x|) = \lambda_{\min}(P)|x|^2$, $\overline{\alpha}(|x|) = \lambda_{\max}(P)|x|^2$, $W(e) = |e|$, $H(x) = |\mathcal{A}_2 x|$, $L = |\mathcal{B}_2| := \sqrt{\lambda_{\max}(\mathcal{B}_2^T \mathcal{B}_2)}$, $\gamma = \sqrt{\mu}$, $\alpha(|x|) = \varepsilon_2 |x|^2$ and $\delta(y) = \varepsilon_1 |y|^2$. □

The proof of Proposition 1 has been omitted due to space constraints. Proposition 1 provides a sufficient condition, namely (17), for the verification of Assumption 1, which thus allows us to use the results of Section IV for LTI systems. It has to be noted that the LMI (17) can always be satisfied when system (14) is stabilizable and detectable. Indeed, in this case, we can select the controller (15) such that $\mathcal{A}_1$ is Hurwitz. Noting that (17) is equivalent to the following inequalities, by using the Schur complement of (17) (see Section A.5.5 in [29]),

$$-\mu\mathbb{I}_{n_e} \leq 0$$
$$\mathcal{A}_1^T P + P\mathcal{A}_1 + \mathcal{A}_2^T\mathcal{A}_2 + \varepsilon_1\overline{C}_p^T\overline{C}_p + \varepsilon_2\mathbb{I}_{n_x} + \tfrac{1}{\mu}P\mathcal{B}_1\mathcal{B}_1^T P \leq 0. \tag{18}$$

We see that we can select the matrix P such that $\mathcal{A}_1^T P + P\mathcal{A}_1 + \mathcal{A}_2^T\mathcal{A}_2 + \varepsilon_1\overline{C}_p^T\overline{C}_p + \varepsilon_2\mathbb{I}_{n_x}$ is negative definite. It then suffices to select μ sufficiently large to ensure (18).

In view of Proposition 1, Assumption 1 holds with $\gamma^2 = \mu$. On the other hand, the smaller γ, the larger the upper-bound on T in (12). Hence, we can minimize μ under the linear constraint (17) to enlarge the constant T. Note that $L = |\mathcal{B}_2|$ is fixed, since $\mathcal{B}_2$ depends on the plant and the controller matrices and the controller is assumed to be known a priori, and hence, we can only play with γ to enlarge T.

VI. Illustrative example

A. Model

Consider the dynamics of a single-link robot arm

$$\begin{array}{rcl} \dot{x}_{p1} &=& x_{p2} \\ \dot{x}_{p2} &=& -\sin(x_{p1}) + u \\ y &=& x_{p1}, \end{array} \tag{19}$$

where x_{p1} denotes the angle, x_{p2} the rotational velocity and u the input torque. The system can be written as

$$\begin{array}{rcl} \dot{x}_p &=& Ax_p + Bu - \phi(y) \\ y &=& Cx_p, \end{array} \tag{20}$$

where $x_p := (x_{p1}, x_{p2})$ and

$$A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix},\; B = \begin{bmatrix} 0 \\ 1 \end{bmatrix},\; C = \begin{bmatrix} 1 \\ 0 \end{bmatrix}^T,\; \phi(y) = \begin{bmatrix} 0 \\ \sin(y) \end{bmatrix}. \tag{21}$$

In order to stabilize system (22), we first construct a state feedback controller of the form $u = Kx_p + B^T\phi(y)$. Hence, system (19) reduces to

$$\begin{array}{rcl} \dot{x}_p &=& (A + BK)x_p \\ y &=& Cx_p. \end{array} \tag{22}$$

We design the gain K such that the eigenvalues of the closed loop system (22) are $(-1, -2)$ (which is possible since the pair (A, B) is controllable). Hence, the gain K is selected to be $K = [-2 \quad -3]$. Next, since only the measurement of y is available, we construct a state-observer of the following form

$$\dot{x}_c = Ax_c + Bu - \phi(y) + M(y - Cx_c), \tag{23}$$

where $x_c \in \mathbb{R}^2$ is the estimated state and M is the observer gain matrix. We rewrite (23) as

$$\dot{x}_c = (A - MC)x_c + Bu - \phi(y) + My. \tag{24}$$

We design the gain matrix M such that the eigenvalues of $(A - MC)$ are $(-5, -6)$ (which is possible since the pair (A, C) is observable). Thus, the observer gain is selected to be $M = [11 \quad 30]^T$. As a result, the closed-loop system in the absence of sampling is given by

$$\begin{array}{rcl} \dot{x}_p &=& Ax_p + Bu - \phi(y) \\ y &=& Cx_p \\ \dot{x}_c &=& (A - MC)x_c + Bu - \phi(y) + My \\ u &=& Kx_c + B^T\phi(y). \end{array} \tag{25}$$

We now take into account the effect of the network. We consider the scenario where the controller receives the output measurements only at transmission instants $t_i, i \in \mathbb{Z}_{\geq 0}$ while the controller is directly connected to the plant actuators. We design a triggering condition of the form (10). As a consequence, the network-induced error is $e = e_y = \hat{y} - y$ and we obtain, for $t \in [t_i, t_{i+1})$

$$\begin{aligned} \dot{x}_p &= Ax_p + B\Big(Kx_c + B^T\phi(\hat{y})\Big) - \phi(y) \\ &= Ax_p + BKx_c + \phi(y + e) - \phi(y) \end{aligned} \tag{26}$$

and

$$\begin{aligned} \dot{x}_c &= (A - MC)x_c + B\Big(Kx_c + B^T\phi(\hat{y})\Big) - \phi(\hat{y}) + M\hat{y} \\ &= (A - MC + BK)x_c + MCx_p + Me_y. \end{aligned} \tag{27}$$

Let $x := (x_p, x_c)$. Then, system (26)-(27) has the following dynamics on flows

$$\begin{aligned} \dot{x} \;&=\; \begin{bmatrix} A & BK \\ MC & A - MC + BK \end{bmatrix}\begin{bmatrix} x_p \\ x_c \end{bmatrix} + \begin{bmatrix} 0 \\ M \end{bmatrix} e \\ &\quad + \begin{bmatrix} \phi(y + e) - \phi(y) \\ 0 \end{bmatrix} \\ &:= \; \mathcal{A}x + \mathcal{B}e + \psi(y, e). \end{aligned} \tag{28}$$

Since $e = \hat{y} - y$ and in view of (19), we have

$$\dot{e} = -\dot{y} = -x_{p2}. \tag{29}$$

Hence, in view of (28), (29), the functions f, g in (4) are $f(x, e) = \mathcal{A}x + \mathcal{B}e + \psi(y, e)$ and $g(x, e) = -x_{p2}$.

B. Verification of Assumption 1

We now verify Assumption 1. Let $W(e) := |e|$ for all $e \in \mathbb{R}$. Consequently, for almost all e and all x

$$\langle \nabla W(e), g(x, e)\rangle \leq |x_{p2}|. \tag{30}$$

Hence, condition (8) holds with $H(x) = |x_{p2}|$ and $L = 0$. Let $V(x) = x^T P x$, where P is a real positive definite symmetric matrix such that $\mathcal{A}^T P + P\mathcal{A} = -Q$ (such a matrix P always exist since $\mathcal{A}$ is Hurwitz) and Q is real positive definite and symmetric such that $\lambda_{\min}(Q) > 4$. We select Q as a block diagonal matrix with the diagonal elements equal to 4.2, thus $\lambda_{\min}(Q) = 4.2$. Then, we have, for all $e \in \mathbb{R}$ and almost all $x \in \mathbb{R}^4$

$$\begin{aligned} \langle \nabla V(x), f(x, e)\rangle &= x^T(\mathcal{A}^T P + P\mathcal{A})x + 2x^T P(\mathcal{B}e + \psi(y, e)) \\ &\leq -\lambda_{\min}(Q)|x|^2 + 2|P\mathcal{B}||x||e| \\ &\quad + 2|P||x||\psi(y, e)|. \end{aligned} \tag{31}$$

In view of (28),
$|\psi(y,e)| = |\phi(y+e) - \phi(y)| = |\sin(y+e) - \sin(y)| \leq |e|$.
As a consequence,

$$\langle \nabla V(x), f(x,e) \rangle \leq -\lambda_{\min}(Q)|x|^2 + 2(|P\mathcal{B}| + |P|)|x||e|. \tag{32}$$

Using the fact that $2(|P\mathcal{B}| + |P|)|x||e| \leq \frac{\lambda_{\min}(Q)}{2}|x|^2 + \frac{2(|P\mathcal{B}|+|P|)^2}{\lambda_{\min}(Q)}|e|^2$ and recalling that $\frac{\lambda_{\min}(Q)}{4} > 1$, we obtain

$$\begin{aligned}\langle \nabla V(x), f(x,e) \rangle &\leq -\frac{\lambda_{\min}(Q)}{2}|x|^2 + \frac{2(|P\mathcal{B}| + |P|)^2}{\lambda_{\min}(Q)}|e|^2 \\ &\leq -\frac{\lambda_{\min}(Q)}{4}|x|^2 - |x_{p2}|^2 - \frac{\lambda_{\min}(Q)}{4}y^2 + \frac{2(|P\mathcal{B}|+|P|)^2}{\lambda_{\min}(Q)}|e|^2.\end{aligned} \tag{33}$$

Thus, condition (7) is verified with $\alpha(|x|) = \frac{\lambda_{\min}(Q)}{4}|x|^2$, $\delta(y) = \frac{\lambda_{\min}(Q)}{4}y^2$ and $\gamma^2 = \frac{2(|P\mathcal{B}|+|P|)^2}{\lambda_{\min}(Q)}$.

C. Simulation results

We obtain the numerical value $\gamma = 26.5333$, which gives, in view of (12), $\mathcal{T} = 0.0592$. We take $T = 0.059$. Figure 2 shows that the plant and the estimated state asymptotically converge to the origin as expected. The generated inter-transmission times by the proposed mechanism (10) are shown in Figure 3 where we can observe the interaction between the time-triggered [19] and the event-triggered [5] techniques. Table I gives the minimum and the average inter-sampling times for the proposed triggering mechanism (11) for 200 randomly distributed initial conditions such that $|(x(0,0), e(0,0))| \leq 100$ and $\tau(0,0) = 0$ and for different values of T. We can see that the average and the minimum inter-transmission times, respectively denoted as τ_{avg} and $\tau_{\min}$, increase when T increases as shown in Table I. To justify the proposed triggering mechanism, Figure 4 presents the inter-transmission times with the triggering condition $\gamma^2 W^2(e) \leq \delta(y)$ without enforcing a constant time T between transmissions (*i.e.* $T = 0$ in (10), (11)). We note that Zeno phenomenon occurs in this case, as discussed in Section IV.

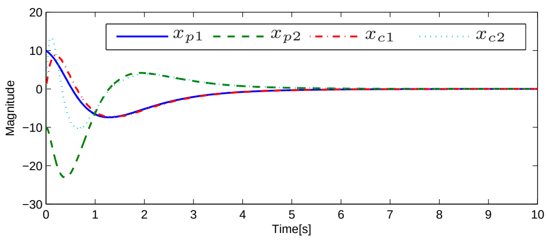

Fig. 2. Actual and estimated states of the plant

VII. Conclusion

An emulation-based approach has been presented for the design of output-based event-triggered controllers for nonlinear systems. The results apply to a class of nonlinear systems, which includes stabilizable and detectable LTI systems as a particular case. The proposed technique ensures an asymptotic stability property and enforces the existence of a (uniform) strictly positive lower bound on the inter-transmission times. The structure of the proposed mechanism provides insight into the interaction between event-triggered and time-triggered approaches which opens the door for future developments.

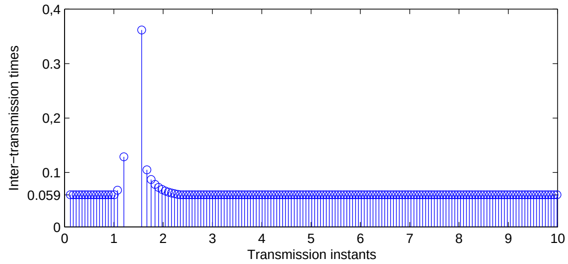

Fig. 3. Inter-transmission times.

	$T = 0.01$	$T = 0.04$	$T = 0.059$
$\tau_{\min}$	0.01	0.04	0.059
τ_{avg}	0.0489	0.0567	0.0625

TABLE I

Minimum and average inter-transmission times for 200 randomly distributed initial conditions such that $|(x(0,0), e(0,0))| \leq 100$ and $\tau(0,0) = 0$ and for a simulation time of 10s.

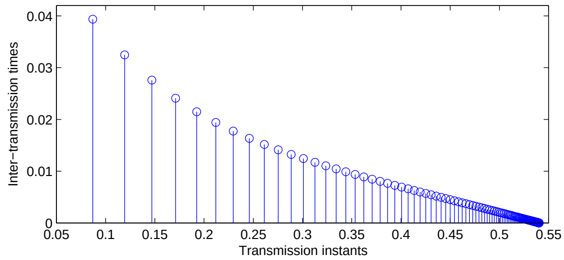

Fig. 4. Inter-transmission times generated by a triggering-condition of the form $\gamma^2 W^2(e) \leq \delta(y)$.

Appendix

Proof of Theorem 1. First, we prove the result when Assumption 1 holds globally. Let $\zeta : \mathbb{R}_{\geq 0} \to \mathbb{R}$ be the solution to

$$\dot{\zeta} = -2L\zeta - \lambda(\zeta^2 + 1) \qquad \zeta(0) = \theta^{-1}, \tag{34}$$

where $\theta \in (0,1)$, $\lambda := \sqrt{\gamma^2 + \eta}$ for some $\eta > 0$ and γ comes from Assumption 1. We denote $\widetilde{\mathcal{T}}(\theta,\eta,\gamma,L)$ the time it takes for ζ to decrease from θ^{-1} to θ. This time $\widetilde{\mathcal{T}}(\theta,\eta,\gamma,L)$ is a continuous function of (θ,η) which is decreasing in θ and η (by invoking the comparison principle). On the other hand, we note that $\widetilde{\mathcal{T}}(\theta,\eta,\gamma,L) \to \mathcal{T}(\gamma,L)$ as (θ,η) tends to $(0,0)$ (where $\mathcal{T}(\gamma,L)$ is defined in (12)). As a consequence, since $T < \mathcal{T}$, there exists (θ,η) such that $T < \widetilde{\mathcal{T}}(\theta,\eta,\gamma,L)$. We fix the couple (θ,η). Let $q := (x,e,\tau)$.

We define for all $q \in C \cup D$

$$R(q) := V(x) + \max\{0, \lambda\zeta(\tau)W^2(e)\}. \qquad (35)$$

Let $q \in D$, we obtain, in view of (4) and the fact that W is positive definite,

$$\begin{aligned} R(G(q)) &= V(x) + \max\{0, \lambda\zeta(0)W^2(0)\} \\ &= V(x) \leq R(q), \end{aligned} \qquad (36)$$

where $G(q) := (x, 0, 0)$. Let $q \in C$ and suppose that $\zeta(\tau) < 0$. As a consequence it holds that $\tau > T$. Indeed, $\zeta(\tau)$ is strictly decreasing in τ, in view of (34), and $\zeta(T) > \zeta(\widetilde{\mathcal{T}}) = \theta > 0$ as $T < \widetilde{\mathcal{T}}$. As a consequence $\zeta(\tau) < 0$ implies that $\tau > T$. Hence, $\gamma^2 W^2(e) \leq \delta(y)$ in view of (11) since $q \in C$. Consequently, in view of page 100 in [30], Lemma 1, Assumption 1 and (35)

$$R^\circ(q; F(q)) = V^\circ(x, f(x, e)) \leq -\alpha(|x|), \qquad (37)$$

where $F(q) := (f(x, e), g(x, e), 1)$. Hence, by following similar arguments as in the proof of Theorem 1 in [19] since α is continuous and positive definite and V is positive definite and radially unbounded, there exists a continuous positive definite function ρ_1 such that

$$R^\circ(q; F(q)) \leq -\rho_1(V(x)) = -\rho_1(R(q)). \qquad (38)$$

When $q \in C$ and $\zeta(\tau) > 0$, we have

$$R(q) = V(x) + \lambda\zeta(\tau)W^2(e). \qquad (39)$$

As above, in view of Lemma 1, Assumption 1 and (34) and by following the same lines as in the proof of Theorem 1 in [19], we obtain

$$\begin{aligned} R^\circ(q;F(q)) \leq& -\alpha(|x|) - H^2(x) - \delta(y) + \gamma^2 W^2(e) \\ &+ 2\lambda\zeta(\tau)W(e)H(x) - \lambda^2\zeta^2(\tau)W^2(e) - \lambda^2 W^2(e). \end{aligned} \qquad (40)$$

Using the fact that $2\lambda\zeta(\tau)W(e)H(x) \leq \lambda^2\zeta^2(\tau)W^2(e) + H^2(x)$,

$$R^\circ(q; F(q)) \leq -\alpha(|x|) + \gamma^2 W^2(e) - \lambda^2 W^2(e). \qquad (41)$$

Recall that $\lambda^2 = \gamma^2 + \eta$, it holds that

$$R^\circ(q; F(q)) \leq -\alpha(|x|) - \eta W^2(e). \qquad (42)$$

By using the same argument as in (38), we derive that

$$\begin{aligned} R^\circ(q; F(q)) &\leq -\rho_1(V(x)) - \eta W^2(e) \\ &= -\rho_1(V(x)) - \frac{\eta\theta}{\lambda}\lambda\theta^{-1}W^2(e) \\ &= -\rho_1(V(x)) - \rho_2(\lambda\theta^{-1}W^2(e)), \end{aligned} \qquad (43)$$

where $\rho_2 : s \mapsto \frac{\eta\theta}{\lambda}s \in \mathcal{K}_\infty$. Since $\zeta(\tau) \leq \theta^{-1}$ for all $\tau \geq 0$ in view of (34), it holds that

$$R^\circ(q; F(q)) \leq -\rho_1(V(x)) - \rho_2(\lambda\zeta(\tau)W^2(e)). \qquad (44)$$

We deduce that there exists a continuous positive definite function ρ_3 such that

$$R^\circ(q; F(q)) \leq -\rho_3(V(x) + \lambda\zeta(\tau)W^2(e)) = -\rho_3(R(q)), \qquad (45)$$

In view of (38), (45) and Lemma 1, when $\zeta(\tau) = 0, R^\circ(q; F(q)) \leq \max\{-\rho_1(R(q)), -\rho_3(R(q))\}$. Consequently, it holds that, for all $q \in C$

$$R^\circ(q; F(q)) \leq -\rho(R(q)) \qquad (46)$$

where $\rho := \min\{\rho_1, \rho_3\}$ is continuous and positive definite. Let ϕ be a solution to (4), (11). In view of (46) and by definition of the Clarke's derivative (see for instance page 99 in [30]), it holds that, for all j and for almost all $t \in I^j$ (where $I^j = \{t : (t, j) \in \operatorname{dom}\phi\}$)

$$\dot{R}(\phi(t, j)) \leq R^\circ(\phi(t, j); F(\phi(t, j))) \leq -\rho(R(\phi(t, j))). \qquad (47)$$

Thus, in view of (36), (47) and since inter-jump times are lower bounded by T in view of (11), we conclude that, by following the same lines as in the end of the proof of Theorem 1 in [19], there exists $\tilde{\beta} \in \mathcal{KL}$ such that for any solution ϕ to (4), (11) and any $(t, j) \in \operatorname{dom}\phi$,

$$R(\phi(t, j)) \leq \tilde{\beta}(R(\phi(0, 0)), 0.5t + 0.5Tj). \qquad (48)$$

In view of Assumption 1 and since W is continuous (since it is locally Lipschitz) and positive definite, there exists $\overline{\alpha}_W \in \mathcal{K}_\infty$ such that $W(e) \leq \overline{\alpha}_W(|e|)$ for all $e \in \mathbb{R}^{n_e}$ according to Lemma 4.3 in [27]. As a result, in view of Assumption 1, (34) and (35), it holds that, for all $q \in C \cup D$,

$$\begin{aligned} V(x) &\leq R(q) \leq V(x) + \frac{\lambda}{\theta}W^2(e) \\ \underline{\alpha}(|x|) &\leq R(q) \leq \overline{\alpha}(|x|) + \frac{\lambda}{\theta}\overline{\alpha}_W(|e|) \\ \underline{\alpha}(|x|) &\leq R(q) \leq \overline{\alpha}_R(|(x, e)|), \end{aligned} \qquad (49)$$

where $\overline{\alpha}_R : s \mapsto \overline{\alpha}(s) + \frac{\lambda}{\theta}\overline{\alpha}_W(s) \in \mathcal{K}_\infty$. Hence, in view of (48) and (49), we deduce that for any solution ϕ to (4), (11) and for all $(t, j) \in \operatorname{dom}\phi$

$$\begin{aligned} \underline{\alpha}(|\phi_x(t, j)|) &\leq R(\phi(t, j)) \\ &\leq \tilde{\beta}\Big(\overline{\alpha}_R(|(\phi_x(0, 0), \phi_e(0, 0))|), 0.5t + 0.5Tj\Big). \end{aligned} \qquad (50)$$

Consequently,

$$|\phi_x(t, j)| \leq \beta(|(\phi_x(0, 0), \phi_e(0, 0))|), t + j), \qquad (51)$$

where $\beta : (s_1, s_2) \mapsto \underline{\alpha}^{-1}(\tilde{\beta}(\overline{\alpha}_R(s_1), s_2)) \in \mathcal{KL}$. Thus, (13) holds.

We now investigate the completeness of the maximal solutions to system (4), (11). Let ϕ be a maximal solution to (4), (11). We first show that ϕ is nontrivial, *i.e.* its domain contains at least two points (see Definition 2.5 in [18]). According to Proposition 6.10 in [18], it suffices for that purpose to prove that $F(q) \in T_C(q)$ for any $q = (x, e, \tau) \in C \backslash D$, where $T_C(q)$ is the tangent cone[2] to C at q. Let $q \in C \backslash D$. If q is in the interior of C, $T_C(q) = \mathbb{R}^{n_x + n_e + 1}$ and the required condition holds. If q is not in the interior

[2]The tangent cone to a set $S \subset \mathbb{R}^n$ at a point $x \in \mathbb{R}^n$, denoted $T_S(x)$, is the set of all vectors $\omega \in \mathbb{R}^n$ for which there exist $x_i \in S, \tau_i > 0$ with $x_i \to x, \tau \to 0$ as $i \to \infty$ such that $\omega = \lim_{i\to\infty}(x_i - x)/\tau_i$ (see Definition 5.12 in [18]).

of C, necessarily $\tau = 0$ as $q \in C \backslash D$, in this case $T_C(q) = \mathbb{R}^{n_x+n_e} \times \mathbb{R}_{\geq 0}$ and we see that $F(q) \in T_C(q)$, in view of (4). Hence, ϕ is nontrivial according to Proposition 6.10 in [18]. In view of (4), (11) and (51), ϕ_x and ϕ_τ cannot explode in finite time. Recall that the network-induced error is $\phi_e = (\phi_{e_y}, \phi_{e_u})$ with $\phi_{e_y} = \phi_y(t_j, j) - \phi_y(t, j)$, $\phi_{e_u} = \phi_u(t_j, j) - \phi_u(t, j)$ for $j > 0$ and $(t, j) \in \operatorname{dom} \phi$ where we write $\operatorname{dom} \phi = \cup_{j \in \{0,\ldots,J\}}([t_j, t_{j+1}], j)$ with some abuse of notation. Hence, in view of (2), (3), (51) and since g_p, g_c are continuous, it holds that, for all $j > 0$ and $(t, j) \in \operatorname{dom} \phi$

$$\begin{aligned} |\phi_{e_y}(t,j)| &= |g_p(\phi_{x_p}(t_j,j)) - g_p(\phi_{x_p}(t,j))| \\ &\leq |g_p(\phi_{x_p}(t_j,j))| + |g_p(\phi_{x_p}(t,j))| \\ &\leq 2 \max_{|z| \leq \beta(|(\phi_x(0,0),\phi_e(0,0))|,0)} |g_p(z)|. \end{aligned} \tag{52}$$

Similarly, we obtain, for all $j > 0$ and $(t, j) \in \operatorname{dom} \phi$

$$\begin{aligned} |\phi_{e_u}(t,j)| &\leq |g_c(\phi_{x_c}(t_j,j), \phi_y(t_j,j))| \\ &\quad + |g_c(\phi_{x_c}(t,j), \phi_y(t_j,j))| \\ &= |g_c(\phi_{x_c}(t_j,j), g_p(\phi_{x_p}(t_j,j))| \\ &\quad + |g_c(\phi_{x_c}(t,j), g_p(\phi_{x_p}(t_j,j))| \\ &\leq 2 \max_{\substack{|z_1| \leq \beta(|(\phi_x(0,0),\phi_e(0,0))|,0) \\ |z_2| \leq \max |g_p(z_1)|}} |g_c(z_1,z_2)|. \end{aligned} \tag{53}$$

When $j = 0$, we have that $|\phi_{e_y}(t,0)| \leq |\phi_{e_y}(0,0)| + |g_p(\phi_{x_p}(0,0)) - g_p(\phi_{x_p}(t,0))|$ and $|\phi_{e_u}(t,0)| \leq |\phi_{e_u}(0,0)| + |g_c(\phi_{x_c}(0,0), \phi_y(0,0)) - g_c(\phi_{x_c}(t,0), \phi_y(0,0))|$ and we can derive similar bounds on the interval $[0, t_1]$. Thus, in view of (52) and (53) and since ϕ_e is reset to 0 at each jump, ϕ_e cannot blow up in finite time. As a consequence, ϕ cannot explode in finite time. Let $G(x, e, \tau) := (x, 0, 0)$ denotes the jump map in (11). The solutions to (4), (11) cannot leave the set $C \cup D$ after a jump since $G(D) \subset C$ in view of (4), (11). Thus, we conclude that maximal solutions to (4), (11) are complete according to Proposition 6.10 in [18]. Finally, we note that if Assumption 1 holds locally, then there exists $\Delta > 0$ such that (36) and (47) hold on the invariant set $|(x, e)| \leq \Delta$ and consequently (13) holds locally. □

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