



**HAL**  
open science

# Adaptive observer based fault diagnosis applied to differential-algebraic systems

Abdouramane Moussa Ali, Qinghua Zhang

► **To cite this version:**

Abdouramane Moussa Ali, Qinghua Zhang. Adaptive observer based fault diagnosis applied to differential-algebraic systems. 5th Symposium on System Structure and Control, IFAC Joint Conference 2013 SSSC, TDS, FDA, Feb 2013, Grenoble, France. pp.1. hal-00988291

**HAL Id: hal-00988291**

**<https://hal.science/hal-00988291>**

Submitted on 12 May 2014

**HAL** is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

# Adaptive observer based fault diagnosis applied to differential-algebraic systems<sup>★</sup>

Abdouramane Moussa Ali\* Qinghua Zhang\*

\* Project-team SISYPHE of INRIA, Campus de Beaulieu,  
35042 Rennes, France  
(e-mail: {amoussaa, zhang}@irisa.fr)

---

## Abstract

Some engineering systems are naturally described by differential-algebraic equations (DAE), whereas it may be difficult or impossible to model them with ordinary differential equations (ODE). This paper proposes an approach to fault diagnosis for systems described by DAEs. Through a particular discretization method and under realistic assumptions, the considered continuous time DAE model is transformed to an explicit state space model in discrete time. An adaptive observer is then applied to the discretized system for monitoring faults possibly affecting the system and represented by changes in model parameters. As an illustrative example, the diagnosis of faults in a heat exchanger modeled by nonlinear DAEs is studied by numerical simulations.

*Keywords:* fault diagnosis, nonlinear systems, differential-algebraic equations, adaptive observer, extended Kalman filter.

---

## 1. INTRODUCTION

Many modern engineering systems can be modeled by an explicit Ordinary Differential Equation (ODE) of the form

$$\dot{x} = f(x, u, t) \quad (1)$$

where  $x$  and  $u$  represent respectively the (vectorial) state and input of the system, and the dot over  $x$  denotes the derivative in the time  $t$ . This equation has a long-term mathematical history, and a large number of analytical and numerical tools have been developed for its study.

However, in some cases such an explicit state space model for the dynamics of a given system is not available. The system may instead be described by an implicit differential equation of the form

$$F(\dot{x}, x, u, t) = 0 \quad (2)$$

This class of systems includes and is broader than state space systems in the ODE form (1). Some of the relations within this implicit model may not involve at all the time derivative  $\dot{x}$ , hence being purely algebraic equations. This motivates calling (2) a Differential-Algebraic Equation (DAE), a descriptor or singular system.

The theory on DAEs goes back a long time ago. The origins of this theory can be traced back to the work of K. Weierstrass (Weierstrass (1868)) and L. Kronecker (Kronecker (1891)) on parameterized families of bilinear forms. The term *algebraic-differential system* was used in the circuit context by Brown (Brown (1963)).

Some DAEs can be simply regarded as implicitly written ODEs, this is the case of DAEs in which the matrix of partial derivatives  $\partial F/\partial \dot{x}$  has full column rank. In principle, the theory developed in the framework of ODEs

can be applied in this case. However, after repeated failure of numerical integration methods on certain DAEs, researchers initiated discussions on treatment of DAEs (Petzold (1982), Lewis (1986)).

In contrast to systems modeled by ODEs, where the theory for fault diagnosis is well-established (Ding (2008), Isermann (2006) and references therein), for DAE systems such study is not as well developed. The few existing studies on fault diagnosis of DAE systems deal with linear case (Benveniste et al. (1993), Duan et al. (2002)), a certain class of nonlinearities (Shields (1997), Zhang et al. (1998)) or systems with all states measurable (Polycarpou et al. (1997), Vemuri et al. (2001)).

This paper considers the problem of nonlinear fault diagnosis for a large class of systems modeled by nonlinear DAEs. The main idea used in this paper for dealing with DAEs is based on a particular method for the discretization of DAEs (Milne (1949), Petzold (1982)). This discretization method results in explicit discrete time state space equations. The proposed fault diagnosis methodology is then based on an adaptive observer applied to the discrete time state space system. This adaptive observer based approach is an integration and extension of the observer based and parameter estimation approaches. The core of such fault diagnosis methods consists of an adaptive observer which is used both to estimate the monitored faults and to improve the robustness against model uncertainty due to parameter changes.

The paper is organized as follows. Different assumptions on the system and the fault are needed to solve the fault diagnosis problem. Section 2 is devoted to the outline of the approach discussed in this paper. We detail the assumptions made and the motivations of this contribution

---

<sup>★</sup> This work was supported by the ASTech MODIPRO project.

and explain how adaptive observer based approach is used for the generation of features, and applied to the detection and diagnosis of faults. Section 3 presents a simulation model of a heat exchanger by discarding flow approximations in favor of one-dimensional solutions of the Navier-Stokes in a device with flow ports in the ends such as a pipe. Then, in section 4 the proposed model is used to illustrate the proposed approach.

## 2. FAULT DIAGNOSIS OF NONLINEAR DAE SYSTEMS

Consider the DAE system

$$0 = F_c(\dot{x}, x, u, \theta) \quad (3a)$$

$$y(t) = h(x, \theta) \quad (3b)$$

where  $x \in \mathbb{R}^n$  is the state vector,  $u \in \mathbb{R}^m$  is the known variables vector (typically the control input vector),  $y \in \mathbb{R}^p$  is the output vector,  $F_c$  and  $h$  are known smooth vector-valued functions (linear or nonlinear) respectively in  $\mathbb{R}^n$  and  $\mathbb{R}^p$ . The parameter vector  $\theta$  represents some characteristics of the system subject to changes caused by faults possibly affecting the system. When the system is operating normally,  $\theta$  is at its nominal value  $\theta_0$ .

With the above formulation, the diagnosis of fault can be achieved by estimating the parameter vector  $\theta$ . For this purpose, the input  $u$  and the output  $y$  are the only available signals. The hidden variable vector  $x$  has to be dealt with in some manner during the estimation of the parameter vector  $\theta$ .

The first difficulty of the problem formulated in this section is the implicit form of the state equation, or rather the DAE, in (3). This difficulty is dealt with by adopting a particular discretization of the DAE leading to a discrete time explicit state equation.

After discretizing the continuous time  $t$  to the discrete instants  $(t_k = k\Delta)_{k=0,1,2,\dots}$ , the continuous time state vector  $x(t)$  is discretized accordingly as  $x_k = x(t_k)$ , and the derivative  $\dot{x}(t_k)$  at time  $t_k$  is approximated by a backward difference of  $x(t)$ . The resulting discretized system equations write

$$F_d(x_k, x_{k-1}, u_k, \theta) \triangleq F_c\left(\frac{x_k - x_{k-1}}{\Delta}, x_k, u_k, \theta\right) = 0 \quad (4)$$

$$y_k = h(x_k, \theta) \quad (5)$$

Assume that the matrix of partial derivatives  $\frac{\partial F_d}{\partial x_k}$  is non singular, equation (4) implicitly defines a function (Krantz and Parks (2002))

$$x_k = f_d(x_{k-1}, u_k, \theta) \quad (6)$$

This non singular  $\frac{\partial F_d}{\partial x_k}$  assumption is equivalent to assume that

$$\frac{1}{\Delta} \frac{\partial F_c}{\partial \dot{x}} + \frac{\partial F_c}{\partial x} \quad (7)$$

is non singular, with  $\frac{\partial F_c}{\partial \dot{x}}$  denoting the matrix of partial derivatives of  $F_c$  with respect to its first argument. Alternatively, to directly transform the DAE (3) to an explicit continuous time state equation, a non singular matrix of partial derivatives  $\partial F_c / \partial \dot{x}$  would be required. This condition is different from the one on (7), which

concerns two matrices of partial derivatives of  $F_c$  and is more likely to be satisfied in practice. For example, if some of the equations in (3a) are purely algebraic, or in other words, some components of  $F_c$ , say grouped in  $\check{F}_c$ , do not include  $\dot{x}$ , then matrix  $\partial \check{F}_c / \partial \dot{x}$  is clearly zero. As  $\check{F}_c$  must contain  $x$  (otherwise the corresponding equations would not contain any information about the state vector  $x$ ), the matrix of partial derivatives  $\partial \check{F}_c / \partial x$  is not generally zero.

The implicitly defined function  $f_d(x, u, \theta)$  can rarely be found analytically and one usually implements an quasi-Newton iterative method (Dennis and Morée (1977)) to compute  $x_k$  from  $x_{k-1}$ ,  $u_k$  and  $\theta$ , by solving equation (4) for  $x_k$ . If the implicit function value defined by (4) is not unique, the value the closest to the value of  $x_{k-1}$  is chosen. This choice should correspond to the true system trajectory if the discretization step size  $\Delta$  is small enough and if the system trajectory is continuous in time.

The partial derivatives of  $x_k$  with respect to  $x_{k-1}$  and  $\theta$  can be computed by the implicit function theorem

$$\frac{\partial x_k}{\partial x_{k-1}} = \frac{\partial f_d}{\partial x} = \left[ \frac{\partial F_c}{\partial \dot{x}} + \Delta \frac{\partial F_c}{\partial x} \right]^{-1} \frac{\partial F_c}{\partial \dot{x}} \quad (8)$$

$$\frac{\partial x_k}{\partial \theta} = \frac{\partial f_d}{\partial \theta} = - \left[ \frac{1}{\Delta} \frac{\partial F_c}{\partial \dot{x}} + \frac{\partial F_c}{\partial x} \right]^{-1} \frac{\partial F_c}{\partial \theta} \quad (9)$$

where the matrices of partial derivatives are evaluated at the point  $\left( \frac{x_k - x_{k-1}}{\Delta}, x_k, u_k, \theta \right)$ .

From the relation (6), one commonly used method to solve the problem of diagnosis is to augment the state  $x_k$  with the parameter vector  $\theta$  and to implement an extended Kalman filter (EKF) (Cox (1964)). While this approach has proved effective in some applications, at least in the case of state space systems, it has also some well known drawbacks. In particular, the fact of treating equally the state vector  $x_k$  and the parameter vector  $\theta$  as if they had similar dynamics may make the tuning of the EKF delicate.

The approach proposed in this section relies on an adaptive observer. Conceptually similar to the EKF applied to the augmented system, the adaptive observer has the advantage of being able to be designed in two steps:

- first a state estimator by assuming that the parameter vector  $\theta$  is known,
- then the parameter estimator, which is coupled with the state estimator.

In the case of linear time varying systems, a comparison in some other aspects between the EKF applied to the augmented system and the adaptive observer can be found in the Appendix A of (Li et al. (2011)).

### 2.1 Adaptive observer

After the previously presented discretization, the continuous time DAE system (3) becomes the discrete time state space system

$$x_k = f_d(x_{k-1}, u_k, \theta) + w_k \quad (10a)$$

$$y_k = h(x_k, \theta) + v_k \quad (10b)$$

where  $w_k$  and  $v_k$  represent errors caused by discretization and modeling/measurement errors ignored in the continuous time model. The purpose of this subsection is to

propose an adaptive observer for joint estimation of the state  $x$  and the parameter  $\theta$  in this discrete time state space system. Though adaptive observers with formally proved convergence have been developed through a constructive approach for some particular class of nonlinear systems (Xu and Zhang (2004), Zhang and Besançon (2008), Farza et al. (2009)), the design of such algorithms for wider classes of nonlinear systems remains a difficult task. Here we propose to follow an approach in the spirit of the EKF, in the sense that we apply an adaptive observer originally designed for linear systems to the nonlinear system (10) by linearizing it around the last values of the state and parameter estimates. While the convergence of this adaptive observer applied to linear systems has been proved formally (Guyader and Zhang (2003), Li et al. (2011)), the study on its convergence for general nonlinear systems, like the convergence of the EKF in general, remains an open problem, except in the particular cases studied in (Xu and Zhang (2004), Zhang and Besançon (2008), Farza et al. (2009)).

The main idea for designing this adaptive observer, as originally proposed in Zhang (2002) and for its nonlinear generalizations, consists of a two-step procedure: first design a state estimator by assuming that the parameter vector  $\theta$  is known, then modify and associate the state estimator with an parameter estimation algorithm. In this design procedure the difference between the dynamics of the state variables (varying and governed by state equations) and of the parameters (typically constant) is explicitly taken into account.

Following this procedure, it is first assumed that the parameter vector  $\theta$  is known in (10). A classical state estimator for general nonlinear systems of this form is the EKF, that can be formulated as follows:

$$\begin{cases} \hat{x}_k = f_d(\hat{x}_{k-1} + K_{k-1}(y_{k-1} - \hat{y}_{k-1}), u_k, \theta) \\ \hat{y}_k = h(\hat{x}_k, \theta) \end{cases} \quad (11)$$

where  $\hat{x}_k$  is the state estimate,  $\hat{y}_k$  is the output estimate, the computation of the Kalman gain  $K_k$  will be detailed later.

Remark that here the term EKF is in the sense that the Kalman filter is applied to the nonlinear system (10) by linearizing the system, at each time instant  $t_k$ , around the last state estimate  $\hat{x}_{k-1}$ . This is a widely used practice, though in general no convergence proof is formally established. The EKF in this sense is to be distinguished from the application of the (extended) Kalman filter to the augmented system by viewing the parameter vector  $\theta$  as extra states. For the moment  $\theta$  is simply assumed known.

Now let us consider the estimation of the parameter vector  $\theta$ . Again inspired by the adaptive observer for linear systems (Guyader and Zhang (2003), Li et al. (2011)), the parameter estimate  $\hat{\theta}_k$  is recursively updated from the output error of the current model:

$$\hat{\theta}_k = \hat{\theta}_{k-1} + \Theta_k (y_k - \hat{y}_k)$$

where  $\Theta_k$  is a gain matrix to be specified. At the same time, the parameter vector  $\theta$  in the state estimation equation (11) should be replaced by  $\hat{\theta}_{k-1}$ . After this substitution, the state estimate becomes somehow biased because

of the difference between  $\hat{\theta}_{k-1}$  and the true  $\theta$ . It is shown in (Guyader and Zhang (2003), Li et al. (2011)) that, in the case of linear systems, this bias should be compensated by an additional term in the state estimation equation in order to establish the convergence proof of the adaptive observer. Similarly, an additional term will be added to the modified state estimation equation (11). The resulting state and parameter estimation equations constitute the adaptive observer for the nonlinear system (10):

$$\hat{\theta}_k = \hat{\theta}_{k-1} + \Theta_k (y_k - \hat{y}_k) \quad (12a)$$

$$\begin{aligned} \hat{x}_k = f_d(\hat{x}_{k-1} + K_{k-1}(y_{k-1} - h(\hat{x}_{k-1}, \hat{\theta}_{k-1})), u_k, \hat{\theta}_{k-1}) \\ + \Upsilon_k (\hat{\theta}_k - \hat{\theta}_{k-1}) \end{aligned} \quad (12b)$$

$$\hat{y}_k = h(\hat{x}_k, \hat{\theta}_{k-1}) \quad (12c)$$

with the gain matrices  $\Theta_k$ ,  $K_k$  and  $\Upsilon_k$  computed from the following equations:

$$P_k = F_k[(I - K_{k-1}H_{k-1})P_{k-1}]F_k^T + Q_{k-1} \quad (13)$$

$$K_k = P_k H_k^T (H_k P_k H_k^T + R_k)^{-1} \quad (14)$$

$$\Upsilon_k = F_k(I - K_{k-1}H_{k-1})\Upsilon_{k-1} + N_{k-1} \quad (15)$$

$$\Omega_k = H_k \Upsilon_k + L_k \quad (16)$$

$$S_k = \frac{1}{\lambda} S_{k-1} - \frac{1}{\lambda} S_{k-1} \Omega_{k-1}^T \Gamma_{k-1} \Omega_{k-1} S_{k-1} \quad (17)$$

$$\Gamma_k = (\lambda R_k + \Omega_k S_k \Omega_k^T)^{-1} \quad (18)$$

$$\Theta_k = S_k \Omega_k^T \Gamma_k \quad (19)$$

$$F_k \triangleq \frac{\partial f_d}{\partial x}(\hat{\mathbf{x}}_{k-1}) \quad (20)$$

$$G_k \triangleq \frac{\partial f_d}{\partial \theta}(\hat{\mathbf{x}}_{k-1}) \quad (21)$$

$$H_k \triangleq \frac{\partial h}{\partial x}(\hat{x}_k, \hat{\theta}_k) \quad (22)$$

$$L_k \triangleq \frac{\partial h}{\partial \theta}(\hat{x}_k, \hat{\theta}_k) \quad (23)$$

where

$\lambda > 0$  is a forgetting factor,  $Q_k \in \mathbb{R}^{n \times n}$  and  $R_k \in \mathbb{R}^{p \times p}$  are symmetric positive definite matrices corresponding to the covariance matrices of  $w_k, v_k$  when they are modeled as random noises and in (20) and (21),

$$\hat{\mathbf{x}}_{k-1} \triangleq (\hat{x}_{k-1} + K_{k-1}(y_{k-1} - h(\hat{x}_{k-1}, \hat{\theta}_{k-1})), u_k, \hat{\theta}_{k-1}).$$

Notice that, compared to (11), the term  $\Upsilon_k(\hat{\theta}_k - \hat{\theta}_{k-1})$  has been added to the state estimation equation (12b) to compensate the difference between  $\theta$  and  $\hat{\theta}_{k-1}$ .

Let us remark again that the formulation of this nonlinear adaptive observer has been in the same spirit as the EKF, by applying the adaptive observer initially designed for linear systems (Guyader and Zhang (2003), Li et al. (2011)) to the nonlinear system(10) linearized around the last state and parameter estimates at each time instant  $t_k$ .

For the convenience of the reader, we recall the outline of the convergence proof in the linear case. Necessary conditions for the existence of the observer are the asymptotical convergence to zero of the state and the parameter

estimation errors, respectively defined by  $\tilde{x}_k = x_k - \hat{x}_k$  and  $\tilde{\theta}_k = \theta - \hat{\theta}_k$  in the noise-free case ( $w_k = 0$  and  $v_k = 0$ ).

The dynamics of the error  $\tilde{x}_k$  in the linear case is as follows

$$\tilde{x}_k = F_k (I - K_{k-1}H_{k-1}) \tilde{x}_{k-1} + (G_k - F_k K_{k-1}L_k) \tilde{\theta}_{k-1} - \Upsilon_k (\hat{\theta}_k - \hat{\theta}_{k-1}) \quad (24)$$

Define the variable  $z_k = \tilde{x}_k - \Upsilon_k \tilde{\theta}_k$ . From the above relations, the dynamics of  $z_k$  is simplified to an exponentially stable system

$$z_k = F_k (I - K_{k-1}H_{k-1}) z_{k-1} \quad (25)$$

and from (17)-(19), the homogeneous part of the parameter estimation error governed by

$$\tilde{\theta}_k = (I - \Theta_k \Omega_k) \tilde{\theta}_{k-1} - \Theta_k H_k z_k \quad (26)$$

tends exponentially to zero under some persistent excitation condition (Guyader and Zhang (2003), Narendra and Annaswamy (2005)). Thus, by construction, we have

$$\begin{cases} \lim_{k \rightarrow +\infty} z_k = 0 \\ \lim_{k \rightarrow +\infty} \tilde{\theta}_k = 0 \end{cases} \Rightarrow \lim_{k \rightarrow +\infty} \tilde{x}_k = 0$$

It is also possible to apply the EKF to the augmented system (treating  $\theta$  as extra states) for joint estimation of  $x$  and  $\theta$ . The proposed adaptive observer has two practical advantages. The gains  $K_k$  for state estimation and  $\Theta_k$  for parameter estimation can be tuned in two steps in simulation studies. In the first step the parameter  $\theta$  is assumed known, hence the tuning of  $K_k$  is like in the case of the classical EKF for state estimation and afterward, in the second step,  $\Theta_k$  is tuned while the tuning of  $K_k$  is fixed. Another advantage of the adaptive observer is that the recursive computations of  $K_k$  and  $\Theta_k$  are separated, implying a lower numerical cost, compared to the fully coupled gain matrix computation in the EKF applied to the augmented system.

### 3. HEAT EXCHANGER MODEL EQUATIONS

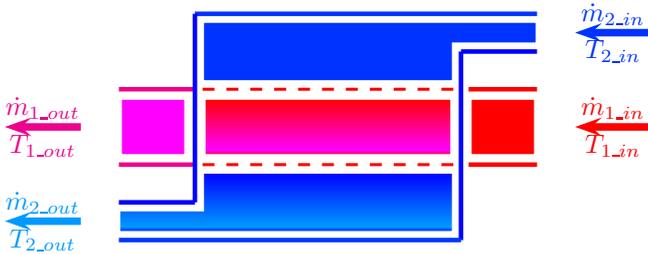


Figure 1. A heat exchanger

To illustrate the fault diagnosis method presented in this paper, the monitoring of a heat exchanger is considered in this section. Heat exchangers are typical devices for heating, cooling, refrigeration and air conditioning. Their task is to transfer energy in the form of heat from one medium (e.g. a gas or liquid) to another. The heat transfer is from a hot stream with entering and exiting temperatures  $T_{1,in}$  and  $T_{1,out}$  and incoming and outgoing mass flow rates  $\dot{m}_{1,in}$  and  $\dot{m}_{1,out}$ , to a cold stream with entering and exiting temperatures  $T_{2,in}$  and  $T_{2,out}$  and incoming and outgoing mass flow rates  $\dot{m}_{2,in}$  and  $\dot{m}_{2,out}$ . A schematic of

a heat exchanger with these temperatures and mass flow rates is shown on figure 1. The mathematical model of heat exchanger used in this study has been developed in the CSDL (Complex Systems Design Lab) project<sup>1</sup>

The actual heat transfer rate is given by the equalities

$$\dot{Q} = C_h (T_{1,in} - T_{1,out}) \quad (27)$$

$$= -C_c (T_{2,in} - T_{2,out}) \quad (28)$$

where  $C_h$  and  $C_c$  denote respectively the heat capacity rates (product of specific heat capacity and incoming mass flow rate) of the hot and cold stream.

If the heat exchanger effectiveness, specific heat capacities and incoming temperatures are known, the exit temperatures can be obtained as following :

$$T_{1,out} = T_{1,in} - \epsilon \frac{\min\{C_h, C_c\}}{C_h} (T_{1,in} - T_{2,in}) \quad (29)$$

$$T_{2,out} = T_{2,in} + \epsilon \frac{\min\{C_h, C_c\}}{C_c} (T_{1,in} - T_{2,in}) \quad (30)$$

where  $\epsilon$  is the heat exchanger effectiveness coefficient.

A heat exchanger can be described mathematically in terms of Navier-Stokes and heat transfer equations that are partial differential equations. We consider the hot fluid as working fluid with a quasi-one dimensional flow along a cylindrical tube of length  $L$  and lateral area  $A$ .

Following the finite-volume method, the computational domain is approximated by two cells of volumes  $v_1$  and  $v_2$ , each one characterized at fixed  $t$  by a temperature  $T_i$  and pressure  $p_i$  ( $i = 1, 2$ ). Using the well-known ideal gas law  $\rho = \frac{p}{RT}$ , we obtain the thermodynamic heat exchanger model given by the following DAEs :

$$\begin{cases} 0 = \frac{v_1}{R} \frac{\dot{p}_1 T_1 - p_1 \dot{T}_1}{T_1^2} + \frac{v_2}{R} \frac{\dot{p}_2 T_2 - p_2 \dot{T}_2}{T_2^2} - \dot{m}_1 + \dot{m}_2 \\ 0 = v_1 \left[ \dot{p}_1 \left( \frac{h(T_1)}{RT_1} - 1 \right) + \frac{p_1 \dot{T}_1}{R} \left( \frac{\partial h(T_1)}{\partial T} T_1 - h(T_1) \right) \right] \\ + v_2 \left[ \dot{p}_2 \left( \frac{h(T_2)}{RT_2} - 1 \right) + \frac{p_2 \dot{T}_2}{R} \left( \frac{\partial h(T_2)}{\partial T} T_2 - h(T_2) \right) \right] \\ - \dot{m}_1 h(T_{1,in}) + \dot{m}_2 h(T_2) + \dot{Q} \\ 0 = \dot{m}_1^2 \frac{T_1}{p_1} - \dot{m}_2^2 \frac{T_2}{p_2} - \frac{AL}{2R} (\dot{m}_1 + \dot{m}_2) \end{cases} \quad (31)$$

where  $h(T)$  is the specific enthalpy of a fluid with temperature  $T$ ,  $R$  is the specific gas constant,  $\dot{m}_1$  and  $\dot{m}_2$  are respectively incoming and outgoing mass flow rate of working fluid.

By assuming known the following variable

- the mass flow rates  $\dot{m}_1, \dot{m}_2$
- incoming temperatures  $T_{1,in}, T_{2,in}$
- outgoing temperature  $T_{1,out} = T_2$

<sup>1</sup> The CSDL (Complex Systems Design Lab) project funded by FUI (2009-2012) is for the purpose of developing a collaborative platform for the design of complex systems.

<http://www.systematic-paris-region.org/fr/projets/csdl>

and supposing the pressure  $p_2$  (in the cell 2) measured, the model used for the diagnosis is given by system (32) below

$$\begin{cases} 0 = F_c(\dot{x}, x, u, \phi) \\ y = [0 \ 1 \ 0] x + \psi \end{cases} \quad (32)$$

where the state  $x \in \mathbb{R}^3$  and the output  $y \in \mathbb{R}$  are

$$x = \begin{bmatrix} p_1 \\ p_2 \\ T_1 \end{bmatrix}, \quad y = p_2,$$

the input vector  $u$  contains all known variables i.e.  $\dot{m}_{1,2}$ ,  $\ddot{m}_{1,2}$ ,  $T_{1,2,in}$ ,  $T_2$ ,  $\dot{Q}$ , the scalar values  $\phi$  and  $\psi$  represent respectively the eventual efficiency loss and the pressor sensor bias. The parameters  $\phi$  and  $\psi$  are written in the vectorial form as

$$\theta = \begin{bmatrix} \phi \\ \psi \end{bmatrix}.$$

The function  $F_c$  is defined by

$$F_c(\dot{x}, x, u, \phi) = \begin{bmatrix} f_1(\dot{x}, x, u, \phi) \\ f_2(\dot{x}, x, u, \phi) \\ f_3(\dot{x}, x, u, \phi) \end{bmatrix}$$

with

$$\begin{aligned} f_1(z, x, u, \phi) &= \frac{v_1}{R} \frac{z_1 x_3 - x_1 z_3}{x_3^2} + \frac{v_2}{R} \frac{z_2 T_2 - x_2 \dot{T}_2}{T_2} - \dot{m}_1 + \dot{m}_2 \\ f_2(z, x, u, \phi) &= v_1 \left[ z_1 \left( \frac{h(x_3)}{R x_3} - 1 \right) + \frac{x_1 z_3}{R} \left( \frac{\partial h(x_3)}{\partial T} x_3 - h(x_3) \right) \right] \\ &\quad + v_2 \left[ z_2 \left( \frac{h(T_2)}{R T_2} - 1 \right) + \frac{x_2 \dot{T}_2}{R} \left( \frac{\partial h(T_2)}{\partial T} T_2 - h(T_2) \right) \right] \\ &\quad - \dot{m}_1 h(T_{1,in}) + \dot{m}_2 h(T_2) + \phi \dot{Q} \\ f_3(z, x, u, \phi) &= \dot{m}_1^2 \frac{x_3}{x_1} - \dot{m}_2^2 \frac{T_2}{x_2} - \frac{AL}{2R} (\dot{m}_1 + \dot{m}_2) \end{aligned}$$

When the process operates under normal operating conditions, the nominal parameter value

$$\theta = \theta_0 \triangleq \begin{bmatrix} 1 \\ 0 \end{bmatrix}.$$

In the presence of a a loss of efficiency (i.e  $0 < \phi < 1$ ) and/or a sensor bias (i.e  $\psi \neq 0$ ), the parameters vector  $\theta$  will deviate from the nominal value  $\theta_0$ .

#### 4. SIMULATION EXAMPLE

In this section, we use the adaptive observer to monitor faults in a heat exchanger modeled by (32). The simulated faults are

- a degradation of the efficiency coefficient,
- a sensor bias.

The sampling period for the discretize time model is 1s and the simulation is performed during 1000s. Centered gaussian white noise is added to the input and output to satisfy the condition of persistent excitation required by the adaptive observer.

The recursively estimated values of  $\phi$  and  $\psi$  are respectively illustrated in figures 2 and 3, where the dotted lines represent the true simulated parameter values, and the solid lines represent the estimated values.

The estimated states are compared with the simulated states in figures 4, 5 and 6, where dotted lines represent the true simulated state variables, and the solid lines represent the estimated values.

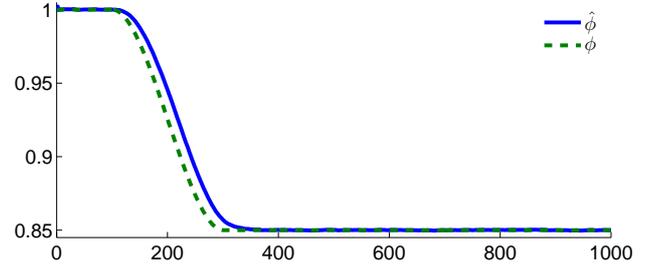


Figure 2. Graphical representations of the simulated parameter  $\phi$  in dotted line and its estimate  $\hat{\phi}$  in solid line, over time (unit : second).

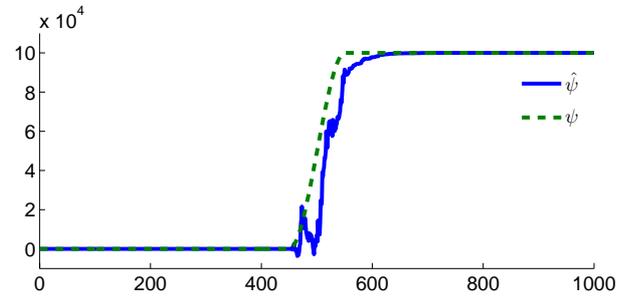


Figure 3. Graphical representations of the simulated parameter  $\psi$  (unit : pascal) in dotted line and its estimate  $\hat{\psi}$  in solid line, over time (unit : second).

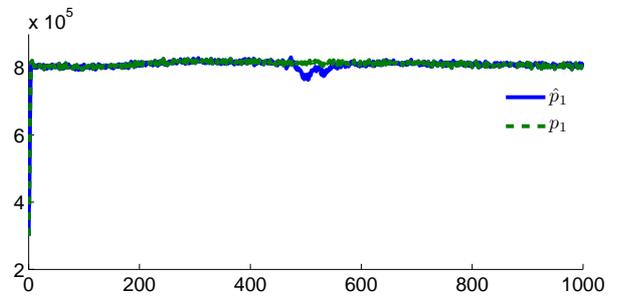


Figure 4. Graphical representations of the simulated state  $p_1$  (unit : pascal) in dotted line and its estimate in solid line, over time (unit : second).

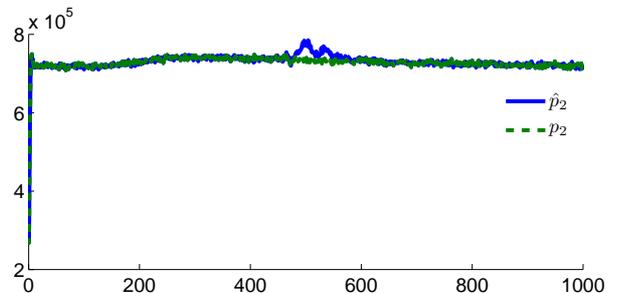


Figure 5. Graphical representations of the simulated state  $p_2$  (unit : pascal) in dotted line and its estimate in solid line, over time (unit : second).

These results show that, after the transient time of about 100s corresponding to the transient time (unmodelled part) of the bias  $\psi$ , the parameter estimates follow closely the evolution of the simulated parameters. It is then possible to detect the simulated faults and estimate their severity.

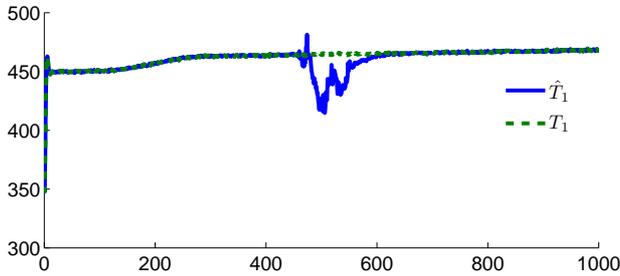


Figure 6. Graphical representations of the simulated state  $T_1$  (unit : kelvin) in dotted line and its estimate in solid line, over time (unit : second).

## 5. CONCLUSION

This paper has dealt with fault diagnosis in DAE systems. We have focused our study on the diagnosis of faults modeled as parameter changes in a class of dynamic systems modeled by implicit nonlinear DAEs. The adaptive observer technique is used to accomplish the fault diagnosis task. We have shown that the method initially developed for linear systems based on adaptive observers can be extended to general nonlinear systems. The decision for fault diagnosis is based on the time evolution of parameter estimates. Simulation results are produced to illustrate the ability of the proposed approach to detect faults in a heat exchanger between two streams of dry air treated as an ideal gas. Like EKF-based methods, further studies should be made on the robustness of the proposed method to the severity of nonlinearities.

## ACKNOWLEDGEMENTS

The authors would like to thank Dassault Aviation and Dassault Systèmes for having provided the CSDL model in Dymola and for their assistance.

## REFERENCES

Benveniste, A., Basseville, M., El Ghaoui, L., Nikoukhah, R., and Willsky, Alan, S. (1993). On the use of descriptor systems for failure detection and isolation. In *Proceedings of the 12th IFAC World Congress*, 499–502. Sydney, AU.

Brown, D.P. (1963). Derivative-explicit differential equations for rlc graphs. *Journal of the Franklin Institute*, 275, 503–514.

Cox, H. (1964). On the estimation of state variables and parameters for noisy dynamic systems. *Automatic Control, IEEE Transactions on*, 9(1), 5 – 12.

Dennis, J. E., J. and Morée, J.J. (1977). Quasi-newton methods, motivation and theory. *SIAM Review*, 19(1), pp. 46–89.

Ding, S.X. (2008). *Model-Based Fault Diagnosis Techniques - Design Schemes Algorithms and tools*. Springer-Verlag.

Duan, G.R., Howe, D., and Patton, R.J. (2002). Robust fault detection in descriptor linear systems via generalized unknown input observers. *International Journal of Systems Science*, 33(5), 369–377.

Farza, M., M'Saada, M., Maatouga, T., and Kamounb, M. (2009). Adaptive observers for nonlinearly parameterized class of nonlinear systems. *Automatica*, 45(10), 2292–2299.

Guyader, A. and Zhang, Q. (2003). Adaptive observer for discrete time linear time varying systems. In *13th IFAC/IFORS Symposium on System Identification (SYSID)*. Rotterdam.

Isermann, R. (2006). *Fault-Diagnosis System*. Springer, Berlin.

Krantz, S. and Parks, H. (2002). *The Implicit Function Theorem: History, Theory, and Applications*. Birkhäuser.

Kronecker, L. (1891). *Algebraische Reduction der Schaaren bilinearer Formen: Algebraische Reduction der Schaaren quadratischer Formen*.

Lewis, F.L. (1986). A survey of linear singular systems. *Circuits, Systems, and Signal Processing*, 5, 3–36.

Li, X., Zhang, Q., and Su, H. (2011). An adaptive observer for joint estimation of states and parameters in both state and output equations. *International Journal of Adaptive Control and Signal Processing*, 25(9), 831–842.

Milne, W. (1949). *Numerical Calculus*. Princeton University Press.

Narendra, K. and Annaswamy, A. (2005). *Stable Adaptive Systems*. Dover Publications.

Petzold, L. (1982). Differential/algebraic equations are not ode's. *SIAM Journal on Scientific and Statistical Computing*, 3(3), 367–384.

Polycarpou, M.M., Vemuri, A.T., and Ciric, A.R. (1997). Nonlinear fault diagnosis in differential algebraic systems. In *In Proceedings of IFAC Symposium on Fault Detection, Supervision and Safety for Technical Processes*, 510–515.

Shields, D.N. (1997). Observer design and detection for nonlinear descriptor systems. *International Journal of Control*, 67(2), 153–168.

Vemuri, A., Polycarpou, M., and Ciric, A. (2001). Fault diagnosis of differential-algebraic systems. *Systems, Man and Cybernetics, Part A: Systems and Humans, IEEE Transactions on*, 31(2), 143 – 152.

Weierstrass, K. (1868). *Zur Theorie der bilinearen und quadratischen Formen*.

Xu, A. and Zhang, Q. (2004). Nonlinear system fault diagnosis based on adaptive estimation. *Automatica*, 40(7), 1181–1193.

Zhang, Q. (2002). Adaptive observer for multiple-input-multiple-output (mimo) linear time-varying systems. *Automatic Control, IEEE Transactions on*, 47(3), 525 – 529.

Zhang, Q., Basseville, M., and Benveniste, A. (1998). Fault detection and isolation in nonlinear dynamic systems: A combined input-output and local approach. *Automatica*, 34(11), 1359–1373.

Zhang, Q. and Besançon, G. (2008). An adaptive observer for sensor fault estimation in a class of uniformly observable nonlinear systems. *International Journal of Modelling, Identification and Control*, 4(1), 37 – 43. Special Issue on Observers for Nonlinear Systems.