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### ► To cite this version:

| Lionel Zoghaib, Pierre-Olivier Mattei. Damping analysis of a free aluminum plate. Journal of Vibration and Control, 2013, pp.1077546313507098. 10.1177/1077546313507098 . hal-01024479

**HAL Id:** hal-01024479

<https://hal.science/hal-01024479>

Submitted on 17 Jul 2014

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<sup>1</sup> **Damping analysis of a free aluminum plate**

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<sup>7</sup> **Abstract**

<sup>9</sup> An analysis of the energy dissipation sources acting in a vibrating aluminum plate is pre-  
<sup>10</sup> sented in this paper. In a first step, the contact-free modal analysis of a suspended plate  
<sup>11</sup> is conducted using a laser vibrometer and an acoustic excitation to obtain reference data.  
<sup>12</sup> The thin nylon suspension set-up guarantees a low boundary damping, which is assumed  
<sup>13</sup> to be negligible. In a second step, a number of damping sources are modeled. Acoustic  
<sup>14</sup> damping due to the noise radiation of the non-baffled plate is computed using the boundary  
<sup>15</sup> integral method and a light fluid approximation to express the vibroacoustic coupling in  
<sup>16</sup> analytical terms. The damping due to the sheared air flow along the free plate borders is  
<sup>17</sup> determined on the basis of a simple two-dimensional boundary layer model. Thermoelastic  
<sup>18</sup> damping is assessed using a Fourier series expression for the temperature field along with a  
<sup>19</sup> perturbation technique to take thermoelastic coupling into account. Since no robust model  
<sup>20</sup> is available so far to quantify viscoelastic material damping in aluminum, it is determined  
<sup>21</sup> in a last step by subtracting measured values of damping to the one that have previously  
<sup>22</sup> been computed. Aluminum viscoelastic damping turns out to be very small and almost  
<sup>23</sup> independent of frequency.

<sup>24</sup> **Keyword**

<sup>25</sup> Damping, thermoelastic damping, viscoelastic damping, material damping, acoustic radia-  
<sup>26</sup> tion damping

<sup>28</sup> **1 Introduction**

<sup>29</sup> While a broad literature exists on the subject of damping, a few authors only have per-  
<sup>30</sup> formed a systematic analysis of all dissipation sources in structures. Among them, Cuesta  
<sup>31</sup> and Valette (1993) have made an interesting contribution on strings damping by quantify-  
<sup>32</sup> ing several dissipation sources such as thermoelastic damping, viscoelastic material damping  
<sup>33</sup> and air flow damping. Lambourg and Chaigne (2001) have also published a theoretical and  
<sup>34</sup> experimental study on the damping in wood and metal plates. The authors have modeled  
<sup>35</sup> and identified a number of dissipation sources for the application field of sound synthesis.  
<sup>36</sup> In the present paper, which focuses on the processes that most contribute to damping in  
<sup>37</sup> an aluminum plate, the strategy to model rather than to identify them has been chosen  
<sup>38</sup> whenever possible. The aim, indeed, is to give a better insight into the physics of dissipa-  
<sup>39</sup> tion. The drawback of this strategy, however, is that a complete quantification of the  
<sup>40</sup> main dissipative phenomena is required to be able to validate the model predictions, since  
<sup>41</sup> the only accessible quantity experimentally is the overall damping. A systematic evaluation  
<sup>42</sup> of all damping components involves a number of disciplines such as tribology, acoustics or  
<sup>43</sup> thermomechanics, which make a detailed analysis particularly challenging.

<sup>44</sup> A first step is to draw up an inventory of all dissipation sources. This has already been  
<sup>45</sup> carried out some time ago by Zener (1948) as far as aluminum plates concerns. According  
<sup>46</sup> to him, thermoelastic damping is the main damping component in aluminum due to the

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48 material high conductivity and compressibility. Cremer et al. (1988) have also pointed out  
49 the existence of a damping component related to microstructural viscoelasticity. Its value is  
50 generally assumed to be rather small and constant in most studies. From a structural per-  
51 spective, the boundary area where the plate is being fixed can heavily contribute to damping.  
52 Physical phenomena involved in this area such as friction, thermoelasticity or energy leakage  
53 are particularly complex and have a great influence on the overall structural behavior. The  
54 first two of them require a local description of the physics at the interface, whereas energy  
55 leakage can only be handled properly by modeling the neighborhood and trimming it where  
56 the energy transmission is found negligible, for instance where the vibration energy starts  
57 being very low. In the present paper, a drastic simplification of the boundary modeling has  
58 been aimed at by suspending the plate with two thin nylon wires. These wires are poor  
59 thermal conductors and are also assumed to be not very dissipative. They can be idealized  
60 by a string model, for which the transverse stiffness is determined by the weight of the  
61 plate, and consequently apply a very low restoring force to the plate boundary. Boundary  
62 dissipation is thus taken to be negligible and the structural configuration considered as free.  
63

64 Apart from material damping, two additional boundary damping components involving  
65 the air surrounding the structure have also been investigated here. The first one, called  
66 acoustic damping, accounts for the vibrational energy being lost due to the noise radiation  
67 in the air. It requires an extension of the domain to take the exterior fluid into account.  
68 Boundary conditions such as Sommerfeld far field condition are also needed to trim the fluid  
69 domain. The second damping is due to the air flow along the structure edges. Large bending  
70 displacements take place at the edges of the suspended plate, since it is almost free. The  
71 fluid is sheared upon by flowing tangentially to the plate thickness and thus causes energy  
72 dissipation to occur.

73 The paper is constructed as following. First, data from an experimental modal analysis  
74 of an aluminum plate is gathered and presented. The measured modal damping is consid-  
75 ered as a reference value of the total damping since it results from the contribution of a  
76 variety of dissipation sources. The rest of the paper is then aiming at understanding what  
77 this total damping is essentially made of. The fluid-structure interactions are first evaluated  
78 ; air flow damping as well as acoustic radiation damping, considered as the most impor-  
79 tant sources, are modeled and computed. Two main sources of material damping are then  
80 considered in a final part: thermoelastic and viscoelastic damping. While thermoelastic  
81 damping can still be modeled and assessed, no satisfactory model is currently available to  
82 estimate the viscoelastic damping precisely. This final dissipation source, due to friction  
83 micromechanisms, has thus to be identified. It is deduced in a last step from all modeled  
84 and measured values.

## 86 **2 Measurement of the overall damping of a suspended 87 plate**

88 The modal analysis of a 35 cm × 40 cm × 2 mm ( $\pm 0.1$  mm) aluminum plate, weighting  
89 190.5 g, has been carried out to obtain a reference frequency and an overall value of damping  
90 for each mode. To be able to suspend the plate, two tiny bores have been drilled out on one  
91 edge; a thin nylon wire has then been embedded and bonded at each bore. A special care  
92 has been taken to reduce as much as possible any source of joint damping at the attach-  
93 ment. Since rigid modes can easily be excited in configurations of this kind, a contact-free  
94 set-up has been implemented with a laser vibrometer (polytech OFV 303) to measure the  
95 plate velocity and a loudspeaker to excite it (figure 1). Signals have been generated and  
96 processed by a Hewlett Packard acquisition system (paragon HP356xa). A high-pass filter

97 and an amplifier have been set between the signal generator and the loudspeaker in order  
 98 to suitably tune the excitation signal and to remove damaging low frequency components.

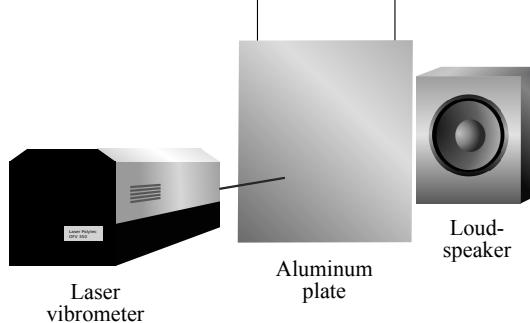


Figure 1: *Contact-free modal analysis set-up*

99 The classical decay method, particularly appropriate to make accurate measurements of  
 100 damping in frequency dependent systems, has been used (Rao, 2010; Nashif et al., 1985).  
 101 The method consists of two main steps. In a first step, a broadband excitation is generated  
 102 to detect the plate resonance frequencies, which are the free solutions of the vibroacoustic  
 103 system. A very good approximation of these frequencies can be estimated in the low fre-  
 104 quency range where peaks appear on the spectrum, at least for low damping cases. For  
 105 larger modal overlap configurations, the contribution of other modes can pollute the fre-  
 106 quency identification by slightly shifting the peak location. The effect is considered to be  
 107 negligible here due to the very low damping values and the wide gap between modal fre-  
 108 quencies. In a second step, the damping of each low frequency resonance mode is studied  
 109 one after the other by emitting a pure sine wave. The frequency of the sine is tuned to that  
 110 of the considered resonance mode. After a short pause, the excitation signal is switched off  
 111 and the velocity decay observed. Using a logarithmic scale, the signal envelope is found to  
 112 take the form of a straight line, the slope of which is a measure of damping. No pollution  
 113 from close modes has been observed and only clean unequivocal measurements have been  
 114 reported. The procedure has been repeated for several measurement points without any  
 115 noticeable change. The relation between the measured damping  $\alpha$  (the straight line slope)  
 116 and the complex resonance angular frequency  $\Omega$ , which can be computed numerically using  
 117 an eigenvalue solver, is simply given by  $\Omega = \omega + i\alpha$ , where  $\omega$  is the real angular frequency  
 118 in  $\text{rad.s}^{-1}$  and  $\alpha$  the imaginary part of the angular frequency expressed in Hz. This damping  
 119 definition, although quite unusual, will be used throughout the article since it has a strong  
 120 numerical and experimental meaning. It is also a direct measure of the amplitude weighting  
 121 applied by dissipation at the resonance. Classical measures of damping, such as the loss  
 122 factor  $\eta$  or the inverse of the quality factor  $Q^{-1}$  can easily be deduced using the formula  
 123  $\eta = Q^{-1} = \alpha / \pi f$ . The shape, frequency and damping of 26 out-of-plane modes ranging  
 124 from 43.4 Hz to 839 Hz have been identified. Their main characteristics are summarized in  
 125 table 1. This frequency domain will be considered throughout the article and sets validity  
 126 bounds for the analysis presented here. Frequencies have been measured with a 0.125 Hz  
 127 resolution, while shapes have been obtained by scanning the whole plate with two meshes  
 128 of either 16 x 16 or 32 x 32 points, depending on the shape pattern complexity. Damping  
 129 measurements are displayed in figure 2. It can be noticed that damping varies quite sig-  
 130 nificantly from a mode to another. This modal behavior, due to the fact that some areas  
 131 damp more than others within the structure, will be identified later as resulting from the  
 132 plate thermoelasticity. A consequence of damping non-proportionality is the existence of  
 133 complex modes. A complex eigensolver or any alternative technique (Adhikari, 2011; Cha,  
 134 2005; Cortés and Elejabarrieta, 2006) such as the perturbation method used in this article

Mode	(1,1)	(0,2)	(2,0)	(1,2)	(2,1)	(0,3)	(2,2)
Frequency [Hz]	43.40	62.25	93.75	111.75	124.38	191.13	218.38
Damping [Hz]	0.13	0.37	1.11	0.49	0.51	1.17	0.74
Mode	(1,3)	(3,0)	(3,1)	(2,3)	(0,4)	(3,2)	(1,4)
Frequency [Hz]	226.13	248.13	288.00	352.25	372.00	376.50	419.50
Damping [Hz]	0.73	1.28	1.29	0.93	1.32	1.01	1.33
Mode	(4,0)	(4,1)	(3,3)	(2,4)	(0,5)	(4,2)	(1,5)
Frequency [Hz]	487.25	518.88	530.75	538.75	619.13	624.25	654.38
Damping [Hz]	1.71	1.50	1.19	1.30	1.74	1.62	1.61
Mode	(3,4)	(4,3)	(2,5)	(5,0)	(5,1)		
Frequency [Hz]	727.00	765.13	795.63	809.50	839.00		
Damping [Hz]	1.41	1.43	1.67	2.11	1.99		

Table 1: Identified modes of the 35 cm × 40 cm × 2 mm free aluminum plate

needs thus to be implemented to properly account for the structure dynamic behavior.

### 3 Fluid-structure interaction damping

#### 3.1 Airflow damping

##### 3.1.1 Introduction

The viscosity of air causes energy to be dissipated via compression and shear. While compression dissipation is assumed to be negligible here, shear dissipation could possibly be important in the vicinity of the plate free edges. In this area, indeed, air flows tangentially to the border. Air viscosity can play a significant role in a number of situations, for instance in the case of wave propagation in ducts. It has also been identified as a major source of dissipation in strings in the low frequency range by Cuesta and Valette (1993). The authors have used Stokes formalism to model the laminar flow around a cylinder with a low Reynolds number hypothesis. Landau and Lifschitz' equations (Landau and Lifschitz, 1986) have been used here instead, which provide with a viscous friction caused by the motion of a flat infinite surface in its plane. The extension to the finite surface case is considered by including a correction term. As opposed to Blasius solution (Cousteix, 1988), which describes the boundary layer created by a two-dimensional incompressible flow on a half-infinite plane, non-linear terms are neglected but inertial terms kept. Blasius solution gives a variable boundary layer thickness, whereas the present model gives a boundary layer with a constant thickness. Based on the hypothesis of stationary motion, it results in a linear viscous damping mechanism.

##### 3.1.2 Two-dimensional analytical flow modeling

The method description is illustrated by the simplified geometry shown in figure 3. The plate edge is associated with an infinite line  $L$  that corresponds to an infinitely thick plate. The edge has an up-right stationary motion along  $L$  of amplitude  $u(s)$  in Laplace domain. Laplace parameter  $s$  is linked to the angular frequency  $\omega$  via  $s = i\omega$ . Pressure  $p(x, y, z, s)$  and velocity  $v_i(x, y, z, s)$  ( $i \in \{x, y, z\}$ ) of an air particle are the unknowns of the problem. By assuming that the system is invariable with respect to any translation along  $z$ , that the motion in the fluid is driven by the fluid-structure coupling only, and incompressibility, the components of Navier-Stokes equation become:

$$\begin{cases} p_{,x}(x, s) = 0 \\ s v_z(x, s) = \nu v_{z,xx}(x, s) \end{cases} \quad (1)$$

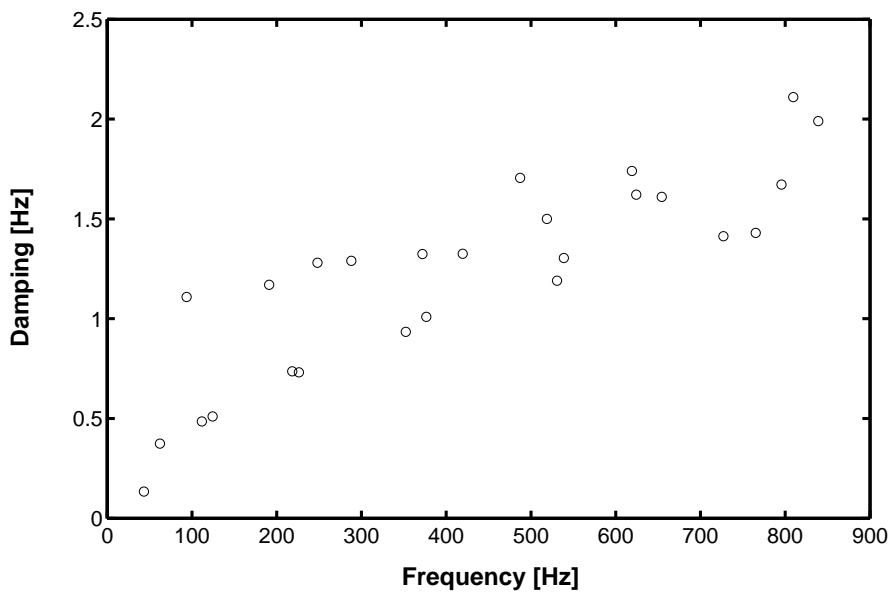


Figure 2: Aluminum plate measured damping

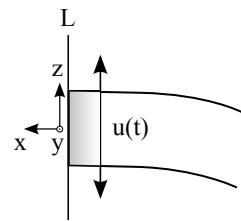


Figure 3: 2-D flow model along the edge of a free plate experiencing bending motion

165 where  $\nu$  is the kinematic viscosity. All details of the calculation can be found in Landau and  
 166 Lifschitz' textbook (Landau and Lifschitz, 1986). The first equation shows that pressure is  
 167 constant. The second one is a diffusion equation. If  $v_z(x, s)$  is given the form  $v_z(x, s) = e^{ikx} u(s)$  and inserted in equation (1), following result is obtained:

$$169 \quad \left\{ \begin{array}{l} k = \pm i \sqrt{\frac{s}{\nu}} \\ v_z(x, s) = e^{\sqrt{\frac{s}{\nu}} x} u(s) = e^{\sqrt{\frac{\omega}{2\nu}} (1+i) x} u(s) \end{array} \right. \quad (2)$$

170 The complex plane offers two solutions. The only physical one is associated to a highly  
 171 damped shear wave that propagates in the fluid. The wave penetration depth  $\delta = \sqrt{\frac{2\nu}{\omega}}$   
 172 can be compared to Cuesta and Valette's result  $\delta = \sqrt{\frac{\nu}{2\omega}}$  obtained in the case of a flow  
 173 along a cylinder. These expressions for  $\delta$  are consistent in terms of kinematic viscosity and  
 174 frequency dependence. Blasius solution, which assumes a incompressible non-linear flow  
 175 with no inertia, gives a boundary layer thickness  $\delta = 5\sqrt{\frac{\nu z}{u}}$  that depends also on  $z$  and  
 176 the flow constant velocity  $u$ . An illustration of the boundary layer thicknesses obtained  
 177 with Blasius solution and the present model, based on Landau and Lifschitz equations, is  
 displayed in figure 4. The following tangential shear stress results in the fluid of density  $\rho_f$ :

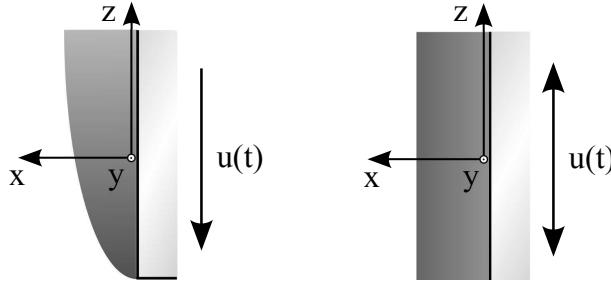


Figure 4: *Boundary layer obtained with Blasius solution (half-plane, non-stationary flow) and with Landau and Lifschitz's approach (infinite plane, stationary flow)*

178

$$179 \quad \sigma_{zx}^F(x, s) = \rho_f \nu v_{z,x}(x, s) = \rho_f \sqrt{\nu s} e^{\sqrt{\frac{s}{\nu}} x} u(s) \quad (3)$$

180 As a reaction, a transverse shear stress is generated within the solid along the interface  $L$ :

181

$$\sigma_{zx}^S(z, s) = \sigma_{zx}^S(s) = \rho_f \sqrt{\nu s} u(s) \quad (\text{over } L) \quad (4)$$

182 In the case of a half-plane oscillation, a border correction term equivalent to an increase  
 183 of  $\delta/2$  in the area swept by the half-plane is added by Landau and Lifschitz. Cuesta and  
 184 Valette (1993) have also made a correction of Stokes solution after noting that the boundary  
 185 layer thickness  $\delta$  is similar to the cylinder radius.

### 186 3.1.3 Numerical results

187 This analytical solution can easily be introduced into a modal analysis program based on the  
 188 finite element method and a standard eigenvalue solver, which computes the eigenpairs  $(\lambda, u)$   
 189 of the system formed by the stiffness matrix  $K$  and the mass matrix  $M$  of the structure. A  
 190 flow stress matrix  $K^{flow}$ , built up according to equation (4), can be introduced in a new  
 191 eigenvalue problem associated to new eigenpairs  $(\Lambda, U)$ :

192

$$193 \quad (K + K^{flow} - \Lambda M) U = 0 \quad (5)$$

Assuming that  $K^{flow}$  has only a slight influence on the overall system, it can be regarded as a perturbation matrix (Woodhouse, 1988). The eigenvectors of the system in vacuum are used as a projection basis without the need for a complex solver and a specific strategy to handle frequency dependence. The new resonance values are deduced from those of the unperturbed system by using following expression:

$$\Lambda \sim \frac{u^T(K + K^{flow})u}{u^T M u} = \lambda + u^T K^{flow} u \quad (6)$$

where vectors have first been mass-orthonormalized. The method's precision is not known *a priori* but it can be very high if the perturbation is small, as confirmed by results *a posteriori*. The computation has been carried out with a finite element program that implements a 27-nodes solid element and a Gauss full integration scheme. Convergence checks have been performed with various mesh densities and have shown that the selected element size is satisfactory for all considered modes. A summary of the model physical characteristics is displayed in table 2. Numerical air flow damping values expressed in Hz are given in

Dimensions	35 cm × 40 cm × 2 mm
Young's modulus	70 GPa
Poisson ratio	0.3
aluminum density	2700 kg.m <sup>-3</sup>
Air density	1.3 kg.m <sup>-3</sup>
Sound velocity	340 m.s <sup>-1</sup>
Kinematic viscosity	1.385 10 <sup>-5</sup> m <sup>2</sup> .s <sup>-1</sup>

Table 2: *Model characteristics*

figure 5. These values have been found to be several orders lower than other damping sources. Although air flow damping -when expressed as a loss factor- is stronger in the low frequency range, in line with Cuesta and Valette's findings for the case of strings, it can be said to be negligible in the current configuration.

## 3.2 Acoustic radiation damping

### 3.2.1 Introduction

Bending motion can induce dissipation by transmitting vibrational energy to the surrounding air via noise radiation. To model this phenomenon, the finite element program described previously has been used to model the dynamics of the plate in vacuum in combination with a boundary element program. Based on the indirect integral formulation and applied to a thin finite screen in an infinite fluid medium, it is able to compute the dipole part of the acoustic radiation. Both routines have been linked via a perturbation technique that accounts for the vibroacoustic coupling and that eventually determines an acoustic radiation damping.

### 3.2.2 Radiation of a non-baffled plate

Many authors, including Laulagnet (1998), Côté et al. (1998) and Atalla et al. (1996) have studied the radiation of non-baffled plates, using Helmholtz equation and the associated Green integral formulation of the acoustic pressure. A similar computation, adapted to the current situation that aims at assessing damping, is presented now. Assuming that the plate

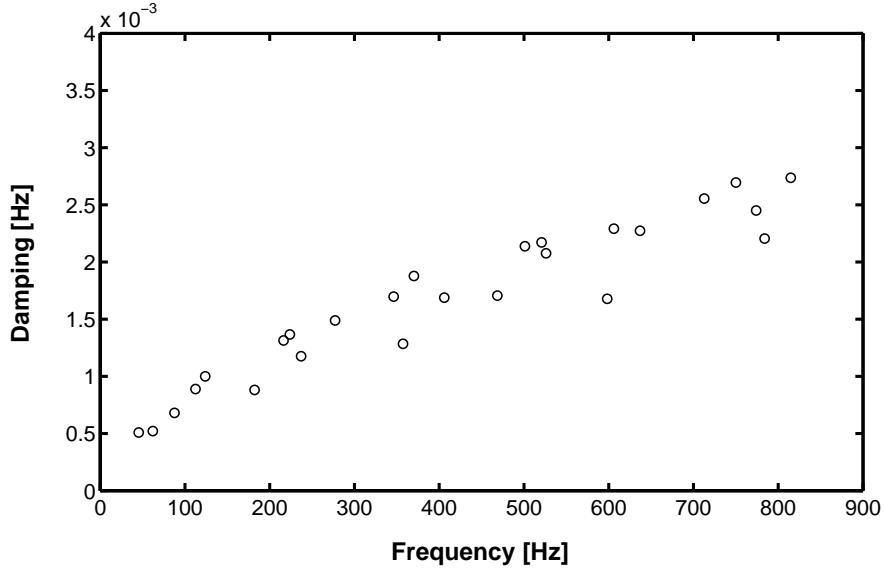


Figure 5: Airflow damping of a free 35 cm × 40 cm × 2 mm aluminum plate

is thin and oriented perpendicularly to the direction  $z$ , an indirect integral formulation of the problem can be written as following (Filippi et al., 1999):

$$p(M_0, s) = \rho_f s^2 \int_S (u_+(M, s) - u_-(M, s)) G(M_0, M, s) dS(M) - \int_S \mu(M, s) \partial_{z(M)} G(M_0, M, s) dS(M) \quad (7)$$

where  $p(M_0, s)$  is the pressure at point  $M_0$ ,  $\rho_f$  is the fluid density,  $s$  is Laplace parameter,  $G(M_0, M, s)$  Green's elementary solution in an infinite medium, and  $\mu(M, s) = p_+(M, s) - p_-(M, s)$  a double layer potential, which expresses the pressure difference between the bottom and the top face of the plate at point  $M$ . The monopole part of the radiation is often neglected, since the normal velocity difference  $u_+(M, s) - u_-(M, s)$  caused by the transverse strain along the thickness is found negligible in thin plates for bending modes. The dipole radiation term containing the diffraction effect is computed using the fluid-structure boundary condition  $\partial_z p_\pm(M, s) = -\rho_f s^2 u_\pm(M, s)$ . This requires to evaluate the normal derivative of equation (7) when  $M_0$  is located over the plate face:

$$\partial_{z(M_0)} p(M_0, s) = \rho_f s^2 \int_S (u_+(M, s) - u_-(M, s)) \partial_{z(M_0)} G(M_0, M, s) dS(M) - P.F. \int_S \mu(M, s) \partial_{z(M_0)} \partial_{z(M)} G(M_0, M, s) dS(M) \quad (8)$$

The second integral diverges and must be calculated as the finite part of Hadamard integral (P.F.). The first integral, which is weakly singular, is taken in the sense of Cauchy principal values (Filippi et al., 1999). It can be shown that

$$-\rho_f s^2 u_+(M_0, s) = \rho_f s^2 (u_-(M_0, s) - u_+(M_0, s)) / 2 - P.F. \int_S \mu(M, s) \partial_{z(M_0)} \partial_{z(M)} G(M_0, M, s) dS(M) \quad (9)$$

Finally,

$$\rho_f s^2 \frac{u_-(M_0, s) + u_+(M_0, s)}{2} = P.F. \int_S \mu(M, s) \partial_z^2 G(M_0, M, s) dS(M) \quad (10)$$

This equation is a Fredholm integral equation of the first kind similar to classical results (Laulagnet, 1998) when the bottom and top face displacement is equal. The equation has

been solved using a one-point collocation method. The boundary and finite element meshes coincide in the xy-plane so that a collocation point can be centered in each finite element. The displacement required by the boundary element program over each face is evaluated with the finite element solution and quadratic interpolation functions. In the idealized baffled configuration, based on the hypothesis of symmetry  $u_+ = -u_-$ , the left hand side of equation (10) is equal to zero. The double layer potential  $\mu(M, s)$  is therefore also equal to zero and no boundary element program is needed.

### 3.2.3 The vibroacoustic problem

Continuity of normal stresses reads

$$\begin{cases} \sigma_{33}(M, s) = p(M, s) & \text{on } S_+ \\ \sigma_{33}(M, s) = -p(M, s) & \text{on } S_- \end{cases} \quad (11)$$

A vibroacoustic operator  $\mathcal{P}(w, u, s)$  can be constructed by using the weak formulation of the classical finite element method. The product of the parietal pressure with a normal displacement  $w$  is integrated over the plate faces:

$$\mathcal{P}(w, u, s) = \int_{S_+} w(M_0) p(M_0, s) dS - \int_{S_-} w(M_0) p(M_0, s) dS \quad (12)$$

When  $M_0$  belongs to the top face, the indirect integral equation (7) is given by

$$\begin{aligned} p(M_0, s) &= -\mu(M_0, s) / 2 \\ &+ \rho_f s^2 \int_S (u_+(M, s) - u_-(M, s)) G(M_0, M, s) dS(M) \end{aligned} \quad (13)$$

$\mu(M_0, s)$  changes of sign if  $M_0$  belongs to the bottom face. Using (13), the vibroacoustic operator becomes

$$\begin{aligned} \mathcal{P}(w, u, s) &= \int_S -(w_+(M_0, s) + w_-(M_0, s)) \mu(M_0, s) / 2 \\ &+ \rho_f s^2 \int_S \int_S (w_+(M_0, s) - w_-(M_0, s)) \\ &\times G(M_0, M, s) (u_+(M, s) - u_-(M, s)) dS(M) dS(M_0) \end{aligned} \quad (14)$$

The vibroacoustic system can finally be expressed in the following matrix form

$$w^T [s^2 M + K] u(s) - P^h(w, u, s) = w^T (F(s) + sI_0 + V_0) \quad (15)$$

where  $P^h(w, u, s)$  is the discrete approximation of  $\mathcal{P}(w, u, s)$ .  $K$  and  $M$  are the structure stiffness and mass matrices,  $u$  is displacement,  $F(s)$  the frequency dependent excitation,  $sI_0$  and  $V_0$  the initial displacement and velocity, and  $w$  is an arbitrary displacement vector.

### 3.2.4 The perturbation method

A weighting coupling parameter  $\epsilon$  that varies from 0 in the uncoupled case to 1 in the fully coupled case is introduced. It is comparable, to some extent, with the small parameter introduced in the classical formalism of the light fluid approximation (Filippi et al., 2001), equal to the ratio between the fluid and solid density. The resonance value problem is written as

$$w^T [s_k(\epsilon)^2 M + K] u_k(\epsilon) = \epsilon P^h(w, u_k(\epsilon), s_k(\epsilon)), \quad (16)$$

279 It consists in searching the resonance modes  $u_k$  and their associated resonance values  $s_k$ .  
 280 Deriving by  $\epsilon$  yields

$$\begin{aligned} & w^T [s_k(\epsilon)^2 M + K] \frac{\partial}{\partial \epsilon} (u_k(\epsilon)) + w^T [\frac{\partial}{\partial \epsilon} (s_k(\epsilon)^2) M] u_k(\epsilon) \\ &= P^h(w, u_k(\epsilon), s_k(\epsilon)) + \epsilon \frac{\partial}{\partial \epsilon} (P^h(w, u_k(\epsilon), s_k(\epsilon))) \end{aligned} \quad (17)$$

282 By setting  $\epsilon = 0$ ,  $w = u_k(0)$  and reminding that  $(s_k(0), u_k(0))$  is the solution of the resonance  
 283 value problem in vacuum, equation (17) becomes

$$284 u_k^T(0) [\frac{\partial}{\partial \epsilon} (s_k(0)^2) M] u_k(0) = P^h(u_k(0), u_k(0), s_k(0)) \quad (18)$$

285 And thus

$$286 \frac{\partial s_k(0)}{\partial \epsilon} = P^h(u_k(0), u_k(0), s_k(0)) / [2 s_k(0) u_k^T(0) M u_k(0)] \quad (19)$$

287 The first order approximation of the vibroacoustic resonance values is given by

$$288 s_k(\epsilon) = s_k(0) + \epsilon \frac{\partial s_k(0)}{\partial \epsilon} \quad (20)$$

289 In the fully coupled case ( $\epsilon = 1$ ),

$$290 s_k(1) = s_k(0) + P^h(u_k(0), u_k(0), s_k(0)) / 2 s_k(0) \quad (21)$$

291 The acoustic radiation damping  $\alpha$ , expressed in Hz, is finally deduced by computing the real  
 292 part of  $-s_k(1)$ .

### 293 3.2.5 Numerical results

294 The acoustic radiation damping of the free aluminum plate studied in this paper has been  
 295 computed. The plate main characteristics are summarized in table 2. The numerical pro-  
 296 cedure has consisted of several steps. The finite element program introduced before and  
 297 based on quadratic solid elements is used in a first step with a classical solver to compute  
 298 the eigenpairs of the plate. In a second step, the complex scalar quantity  $P^h$  is calculated  
 299 from equation (14). This involves a boundary element program and integration routines  
 300 that handle Green kernel singularity. A regularization is carried out by using a local ele-  
 301 ment cylindrical coordinate system instead of the cartesian one. The integration is based on  
 302 a Gauss-Legendre scheme. In a last step, frequencies in the air and acoustic damping are  
 303 evaluated using equation (21). Numerical results of the 26 modes are displayed in figure 6.  
 304 It can be observed that acoustic damping is very low or even negligible for most of them.  
 305 This behavior is expected since the coincidence frequency of the plate is close to 5970 Hz for  
 306 the plate under consideration. However, radiation efficiency increases with frequency and  
 307 some higher frequency modes experience damping by radiating.

## 308 4 Material damping

309 As pointed out by Cremer et al. (1988), two sources of dissipation mainly contribute to  
 310 damping in aluminum. The first one, due to the thermoelastic behavior of the material,  
 311 is quantified here numerically. The second one, due to friction micromechanisms that con-  
 312 tribute to the viscoelastic behavior of the material, is then quantified by identification.

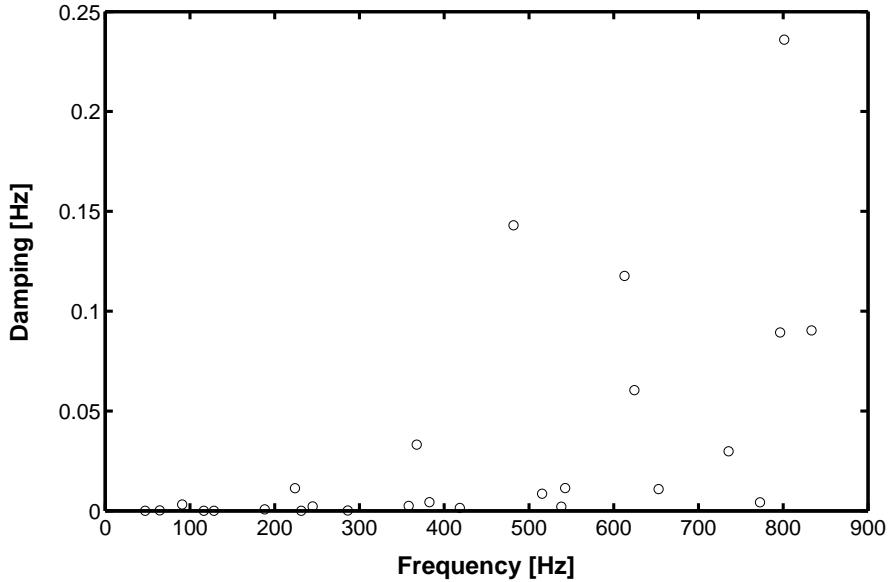


Figure 6: *Computed acoustic damping of the 35 cm × 40 cm × 2 mm free aluminum plate*

313    **4.1 Thermoelastic damping**

314    **4.1.1 Introduction**

315    Like acoustic radiation damping, thermoelastic damping is a coupling-related type of damping.  
 316    It requires the analysis of an additional physical phenomenon, acoustic or thermal for  
 317    instance, that alters the dynamic behavior of the structure. It also requires a new definition  
 318    or an extension of the domain just as in vibroacoustics when the fluid domain is added  
 319    to the solid domain. In the current thermoelastic case though, temperature and dynamic  
 320    domains are identical since thermal effects occur within the plate. Thermoelastic damping  
 321    is associated with an irreversible process during which heat flows by conduction from the  
 322    hottest areas to the coldest ones. In line with the second thermodynamic law, entropy is  
 323    created and vibratory energy is converted into heat. In the field of dynamics, isothermal  
 324    elastic constants are usually used and no local temperature variation is observed. In acous-  
 325    tics, on the contrary, processes are regarded as adiabatic and a local temperature variation  
 326    occurs but results in neither a heat transfer nor an irreversible process. Determining whether  
 327    thermoelastic mechanisms can be regarded as adiabatic or isothermal is a difficult task; it is  
 328    often somewhere in between, depending on the wave type and the geometry involved. The  
 329    process can be considered as isothermal or relaxed when thermal relaxation occurs during  
 330    a vibration period, while it is adiabatic if no equilibrium can be reached during this time.  
 331    A key parameter governing the phenomenon is the distance between hot and cold regions.  
 332    According to Lifshitz and Roukes (1999), this distance is related to the wavelength for longi-  
 333    tudinal mechanical waves and to the thickness of the structure for flexural mechanical waves.  
 334    Longitudinal waves are therefore associated with an adiabatic process in the lower frequency  
 335    range and with an isothermal one in the upper frequency range, whereas the opposite is true  
 336    for flexural waves. Shear waves are not coupled to the thermal field.

337 **4.1.2 Equations**

338 After linearization, the classical thermoelasticity equations (CCT) of an isotropic material  
 339 such as aluminum are given by the following expressions, in which Einstein summation  
 340 convention is used (Nowacki, 1975):

$$341 \quad \begin{cases} \sigma_{ij} = \delta_{ij} \lambda \epsilon_{kk} + 2\mu \epsilon_{ij} - \delta_{ij} (3\lambda + 2\mu) \alpha (T - T_A) & \text{(elasticity eq.)} \\ k T_{,ii} = \rho c_V \dot{T} + \alpha T_A (3\lambda + 2\mu) \epsilon_{kk} & \text{(thermal eq.)} \end{cases} \quad (22)$$

342 where  $\lambda, \mu$  are Lamé coefficients,  $T$  is the temperature,  $\sigma_{ij}$  the stress tensor,  $\epsilon_{ij}$  the strain  
 343 tensor,  $T_A$  the ambient temperature constant,  $\rho$  the material density,  $k$  the thermal con-  
 344 ductivity,  $c_V$  the specific heat per volume unit at constant strain,  $\alpha$  the thermal expansion  
 345 linear coefficient and  $\delta_{ij}$  Kronecker symbol.

346 **4.1.3 Zener's thermoelastic model**

347 Zener (1948) has investigated dissipation in metals in great detail in the thirties. He has  
 348 developed a model of thermoelastic damping for simply supported Euler-Bernoulli beams.  
 349 Many authors are still using this model that can be considered as a standard reference in  
 350 the field. It is based on the fundamental hypothesis that dissipation is mainly due to the  
 351 first transverse thermal mode, which accounts for heat transfers within the beam thickness  
 352  $h$ . The characteristic distance  $d$  between hot and cold parts is thus unique ( $d = h / \pi$ ) and  
 353 associated with a unique relaxation time constant  $\tau$  ( $\tau = d^2 c_V / k$ ). Zener's approximation  
 354 therefore transforms the coupled thermoelastic system of equations into a unique equation  
 355 of dynamics with a dissipation term. This one is characterized by Zener's rheological model,  
 356 written as following:

$$357 \quad \begin{cases} Q^{-1} = \Delta_E \frac{\omega \tau}{1 + (\omega \tau)^2} \\ \Delta_E = \frac{E_a - E_i}{E_a E_i} = \frac{E \alpha^2 T_A}{c_V} \end{cases} \quad (23)$$

359 where  $E_a$  is the adiabatic or unrelaxed modulus,  $E_i$  the isothermal or relaxed modulus, and  
 360  $c_V$  the heat capacity per unit volume. The value  $E$  is the default isothermal value ( $E = E_i$ ).  
 361 It is valid for flexural waves in the lower frequency range only. Zener's thermoelastic model  
 362 is proportional: the computed loss factor applies to the whole strain field without any  
 363 distinction, whereas the equations show that it should depend on the normal strain field.  
 364 No modal dependence can therefore be expected in the computed damping values, only  
 365 a smooth frequency one. More sophisticated analytical models have been developed since  
 366 then, like the one by Li et al. (2012). According to this author though, it is not able  
 367 to handle free boundary conditions properly as required here. An interesting discussion  
 368 about four thermoelastic models of a thin plate with various degrees of approximation of  
 369 the temperature field, in particular, can also be found in the paper by Norris (2006). The  
 370 free boundary conditions case is not investigated directly yet, although some of the presented  
 371 models may handle this specific boundary type.

372 **4.1.4 Numerical method**

373 As thermoelastic coupling is weak, it has been chosen to solve the thermomechanical problem  
 374 by dealing with the thermal and dynamic equations separately, one after another. In order  
 375 to gain some physical insights, the thermal problem has been investigated using a modal  
 376 approach similar to the one used by Zener to model the temperature field. As almost no  
 377 heat flows from the plate to the air due to its very low thermal conductivity, the temperature

378 field in the plate can be expressed easily with a Fourier series. Thermoelastic coupling is  
 379 handled using the following three steps procedure:

- 380     1     | Solve the uncoupled dynamic problem based on equation  
            $\sigma_{ij} = \delta_{ij} \lambda \epsilon_{kk} + 2\mu \epsilon_{ij}$   
    381     | Compute  $\epsilon_{kk}$  in the form of a Fourier series (projection)
- 383     2     | Solve the heat equation with a second member  
            $k T_{,ii} - \rho c_V \frac{\partial T}{\partial t} = \alpha T_A (3\lambda + 2\mu) \frac{\partial \epsilon_{kk}}{\partial t}$   
    384     | Deduce thermal stresses and build up a thermal stress matrix  
            $\sigma_{ij}^{th} = -(3\lambda + 2\mu) \alpha (T - T_A) \delta_{ij}$
- 386     3     | Compute the thermoelastic strain energy  $E_{th}$   
            $E_{th} = \int_V \epsilon_{ii} \sigma_{kk}^{th}$   
    387     | Deduce the thermomechanic system resonance values  $\lambda_{th}$   
            $\lambda_{th} = \lambda + E_{th}$     (after mass-orthonormalization, see eq. (6))

389 In step 1, the dynamic problem is solved using classical tools such as the finite element  
 390 program based on the solid 27-nodes element presented before and a real eigenvalue solver.  
 392 Fourier coefficients are evaluated by projecting the computed normal strain on the Fourier  
 393 basis. It is a time-consuming operation of numerical integration that requires a modal series  
 394 truncation. A sufficient number of modes has been selected to observe a good convergence of  
 395 the results. Once Fourier coefficients are known, the analytical solution of step 2 is straight-  
 396 forward to evaluate. The following non-homogeneous heat equation is solved in Laplace  
 397 domain with homogeneous initial conditions:

$$398 \quad k T_{,ii}(s) - \rho c s T(s) = \alpha T_A s (3\lambda + 2\mu) \epsilon_{kk} \quad (24)$$

399 By positioning the plate normally to the z-axis so that all its coordinates are positive and  
 400 a corner is located at point  $(x = 0, y = 0, z = 0)$ , the zero heat flow boundary conditions  
 401 result in a simple temperature field expression with cosines only:

$$402 \quad T(s) = \sum_{m,n,q}^{\infty} T_{mnq}(s) \cos(m\pi x/l) \cos(n\pi y/L) \cos(q\pi z/e) \quad (25)$$

403 Each mode has its own frequency dependence, as required to be able to model non-proportionality  
 404 correctly. A proportional damping such as the one described by Zener would have been  
 405 modeled using a single frequency function for all modes. By inserting the temperature  
 406 expression (25) into the homogeneous form of equation (24), thermal eigenvalues can be  
 407 obtained:

$$408 \quad s_{mnq} = -\frac{k\pi^2}{\rho c} \left( \left(\frac{m}{l}\right)^2 + \left(\frac{n}{L}\right)^2 + \left(\frac{q}{e}\right)^2 \right) \quad (26)$$

409 A Fourier series of the normal strain can be written as following:

$$410 \quad \epsilon_{kk} = \sum_{m,n,q} A_{mnq} \cos(m\pi x/l) \cos(n\pi y/L) \cos(q\pi z/e) \quad (27)$$

411 Numerical values of  $\epsilon_{kk}$ , obtained during step 1, are used to compute the coefficients  $A_{mnq}$   
 412 by projection. The coefficients of the temperature series can easily be deduced using

$$413 \quad T_{mnq}(s) = \frac{\alpha T_A s (3\lambda + 2\mu) A_{mnq}}{\rho c (s_{mnq} - s)} \quad (28)$$

414  $s_{mnq}$  has real values, while  $s$  is imaginary. This expression has thus no pole and no resonance behavior can be observed in the temperature field. A thermal stress as well as an  
415 associated finite element matrix are then computed. Each resonance value is finally updated  
416 by projecting this matrix onto the mode subspace of the uncoupled system following the  
417 perturbation approach described previously (equation (6)).

419 **4.1.5 Numerical results**

420 Numerical simulations of thermoelastic damping have been carried out for the considered  
421 plate as well as for a number of interesting configurations. The model main characteristics  
422 are summarized in table 3. Figure 7 displays computed thermoelastic damping values of

<b>Plate dimensions (x × y × z)</b>	35 cm × 40 cm × 2 mm
<b>Mesh (x × y × z)</b>	30 × 30 × 3
<b>Number of thermal modes (x × y × z)</b>	10 × 10 × 6
<b>Thermal expansion linear coefficient <math>\alpha</math></b>	$23.0 \cdot 10^{-6} K^{-1}$
<b>Ambient temperature <math>T_A</math></b>	295.15 K
<b>Heat capacity at constant pressure <math>C_p</math></b>	900.0 J/(K.kg)
<b>Thermal conductivity <math>k</math></b>	237.0 W/(m.K)

Table 3: *Thermomechanical properties of the aluminum plate model*

422 about thirty modes in the clamped, simply supported and free boundary conditions cases.  
423 Zener's model is also represented. It is worth noting that Zener's damping model is accu-  
424 rate for clamped or simply supported boundary conditions but gives poor approximations  
425 of damping in the lower frequency range for free boundary conditions. The absence of link

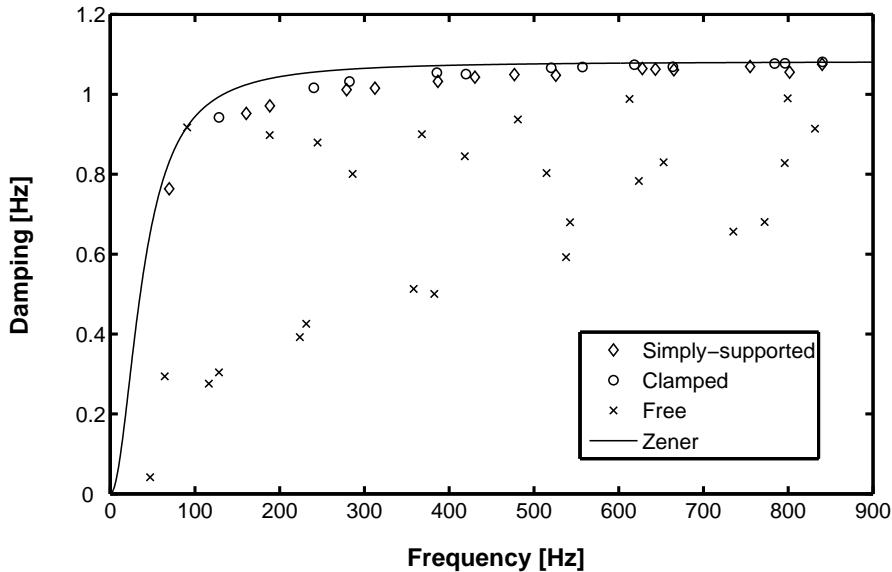


Figure 7: *Comparison of the thermoelastic damping computed with various boundary conditions. Simply supported, clamped and free 35 cm × 40 cm × 2 mm aluminum plate*

426 between Zener's model of damping and the normal strain is noteworthy, since this last one is  
427

in fact directly associated to the local temperature variation. A better thermoelastic damping model inspired from the modal strain energy analysis could probably be implemented from the normal strain energy knowledge. Figure 8 illustrates this possibility in the free boundary conditions case and displays the ratio of the normal strain to the total energy as well as computed thermoelastic damping values. Simply-supported and clamped boundary conditions give an almost constant ratio of about 17%. This is why a proportional damping model such as Zener's model can be applied in these cases.

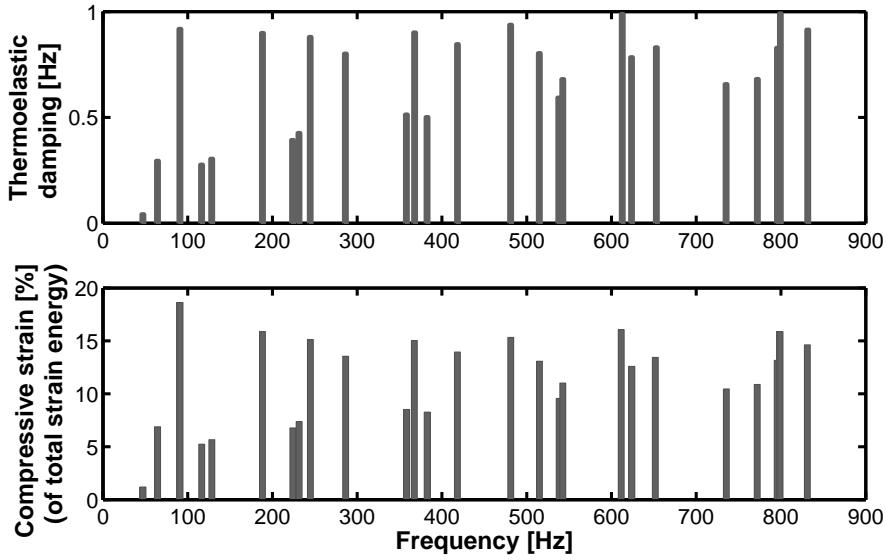


Figure 8: Comparison between the modeled thermoelastic damping and the ratio between the normal strain energy and the total strain energy. Free  $35\text{ cm} \times 40\text{ cm} \times 2\text{ mm}$  aluminum plate

Figure 9 shows numerical estimations of the frequency shift due to thermoelastic coupling. The shift increases roughly linearly with respect to the frequency for all boundary conditions types. Thermoelastic coupling has thus a stiffening effect on the structure as pointed out also by Prabhakar et al. (2009) for the case of cantilever and doubly-clamped thermoelastic beams. Figure 10 and 11 present the contribution in percent of the most important thermal modes to the damping of the first structural modes. While figure 10 focuses on the first four modes of the free boundary conditions case, figure 11 compares the thermal modes contribution for various boundary conditions in the first structural mode case. The great majority of thermal modes have a component equal to one in the z-direction associated to a wavelength equal to twice the thickness. This confirms Zener's prediction that the first thermal transverse mode is responsible for most of the thermoelastic dissipation. It is also worth noting that each structural mode is associated with a very specific combination of thermal modes that strongly varies depending on the boundary conditions. Figure 12 finally gives an illustration of the influence of the thickness  $h$ . Zener pointed out that thermoelastic damping has a  $1/h^2$  dependence by carrying out a limit calculation. The approximation is only valid in the upper frequency range above the characteristic frequency  $f_c$  ( $f_c = \pi^2 k / h^2 c_V$ ). The numerical results confirm the approximation quality, since a damping value of about 0.2, 0.8 and roughly 3.5 is obtained for 4-mm, 2-mm and 1-mm thick plates, respectively.

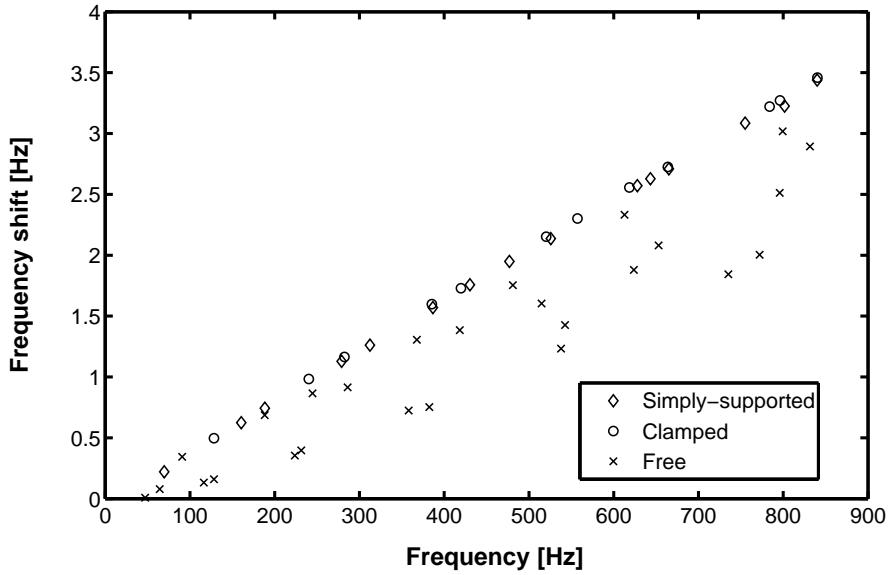


Figure 9: Comparison of the frequency shift due to the thermoelastic coupling for various boundary conditions. Simply supported, clamped and free  $35\text{ cm} \times 40\text{ cm} \times 2\text{ mm}$  aluminum plate

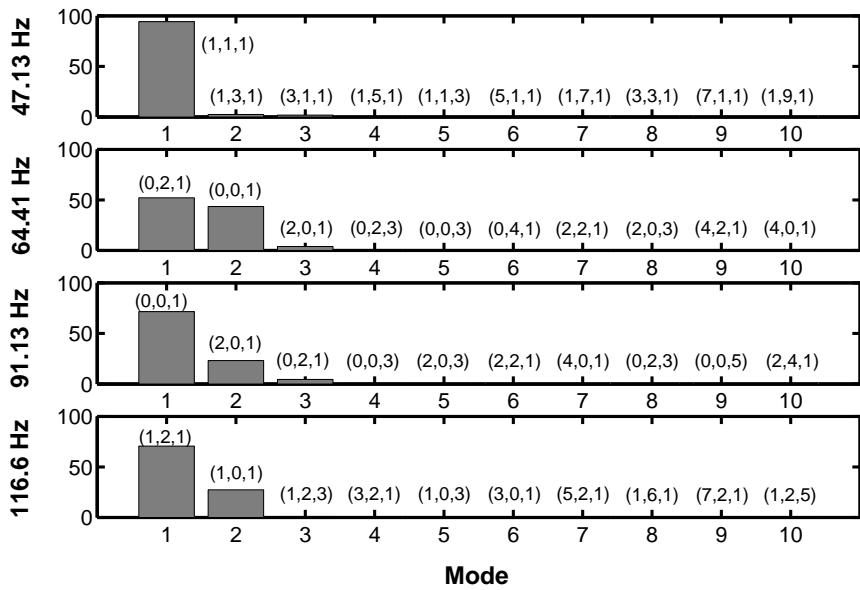


Figure 10: Thermal modes that most contribute most to the thermoelastic damping of a free  $35\text{ cm} \times 40\text{ cm} \times 2\text{ mm}$  aluminum plate. Contributions as a percent of total damping

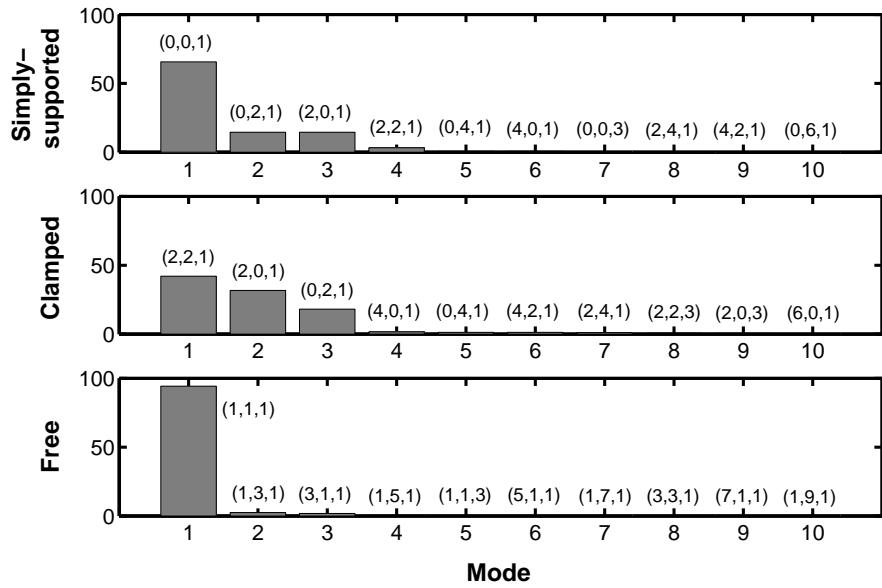


Figure 11: Thermal modes that most contribute to the first mode thermoelastic damping. 35 cm × 40 cm × 2 mm aluminum plate with free, simply-supported and clamped boundary conditions. Contributions expressed as a percent of total damping

454

## 455 4.2 Viscoelastic damping

456 The remaining main damping component of an aluminum plate is referred to as viscoelastic  
 457 damping. It is due to a local viscoelastic dissipation process caused by crystallographic  
 458 defects or irregularities such as dislocations, interstices or grain boundaries, that essentially  
 459 depend on the material and the manufacturing process. Despite the existence of fine mi-  
 460 cromechanical models (Granato-Lücke model (1956)), it is still impossible to quantify this  
 461 component correctly without making any measurement. Many references can provide with  
 462 damping values identified experimentally in the literature. However, most of them focus on  
 463 the effects of temperature on the aluminum in the very low frequency range rather than  
 464 on the frequency itself, probably because measurements in higher frequency ranges are par-  
 465 ticularly difficult to carry out. Rivière (2004) has obtained a damping of  $Q^{-1} \sim 0.003$  at  
 466 0.01 Hz in a polycrystalline aluminum at room temperature. Wei et al. (2002) have mea-  
 467 sured internal friction values of about  $Q^{-1} \sim 0.001$  in aluminum at 1 Hz, and Wang et al.  
 468 (2000) a value of  $Q^{-1} \sim 0.0036$  at room temperature. In a broader frequency range, Cremer  
 469 et al. (1988) have reported a constant frequency value of  $Q^{-1} \sim 0.0001$  at room temperature.

470

471 In order to analyze the remaining part of damping in aluminum, a direct comparison be-  
 472 tween computed damping (thermoelastic and acoustic damping) and the measured one is  
 473 displayed in figure 13. Since all other main dissipation sources (air flow and attachment  
 474 dissipation) have been found or are assumed negligible, the difference between these quan-  
 475 tities is supposed to be a good approximation of viscoelastic damping. It is confirmed, at  
 476 least partially, by observing that the difference matches a straight line. If another damping  
 477 unit such as the loss factor or the inverse quality factor is used, the damping is found to

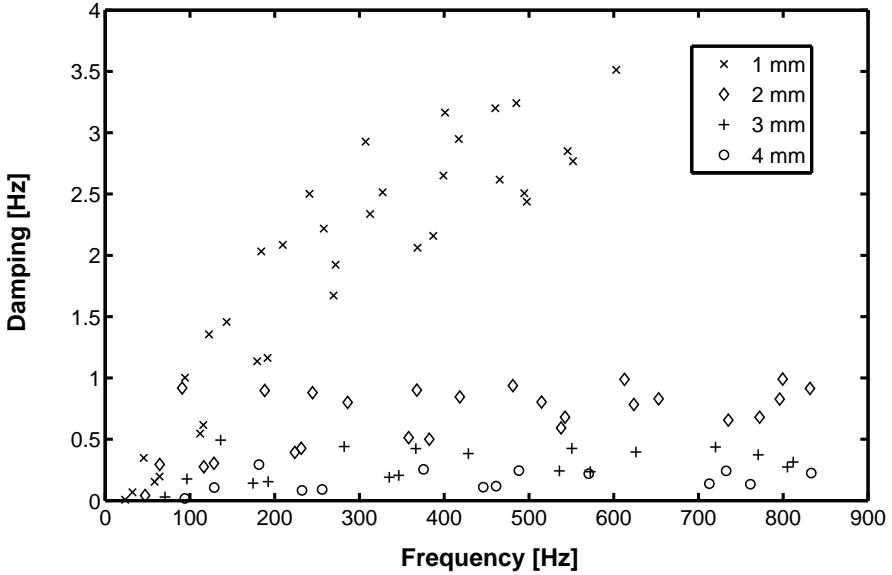


Figure 12: Thermoelastic damping computed for various plate thicknesses. First 35 modes in a free  $35\text{ cm} \times 40\text{ cm} \times 1\text{-}2\text{-}3\text{-}4\text{ mm}$  aluminum plate

478 be a constant function of frequency equal to  $Q^{-1} = 0.00037$ . This value is quite consistent  
 479 with the one given by Cremer et al. (1988), but it is below other values mentioned in the  
 480 literature.

481

## 482 5 Conclusions

483 A detailed analysis of the dissipation sources acting in a thin suspended aluminum plate  
 484 in the low frequency range (0-900 Hz) has been carried out in this paper. Air viscosity  
 485 and noise radiation have been modeled to account for the fluid influence on damping. Air  
 486 viscosity has been considered by combining Landau and Lifschitz stationary flow analysis  
 487 to a finite element capability, while acoustic radiation has been simulated with the same  
 488 capability and a boundary element program. A perturbation technique has finally been  
 489 implemented to observe how modes frequency and damping are shifted by both phenomena.  
 490 While the effect of air viscosity on the plate overall damping has been found negligible, the  
 491 low acoustic damping values increase with frequency and become substantial for a couple of  
 492 modes in the higher part of the considered frequency domain. Thermoelastic damping has  
 493 been computed by using analytical Fourier series of the temperature field, the same finite  
 494 element program and perturbation technique. Aluminum viscoelastic damping has been  
 495 identified by subtracting the computed values of the thermoelastic and acoustic damping  
 496 to the one measured with a contact-free modal analysis and the logarithmic decay method.  
 497 It is found almost constant over a broad range of frequencies when damping is expressed  
 498 as a loss factor or as the inverse of a quality factor. The general methodology proposed  
 499 in this paper, which consists in a systematic analysis of damping sources, thus provides an  
 500 efficient means of gaining insight into the dynamics of systems with very low damping such  
 501 as aluminum structures.

## 502 Acknowledgments

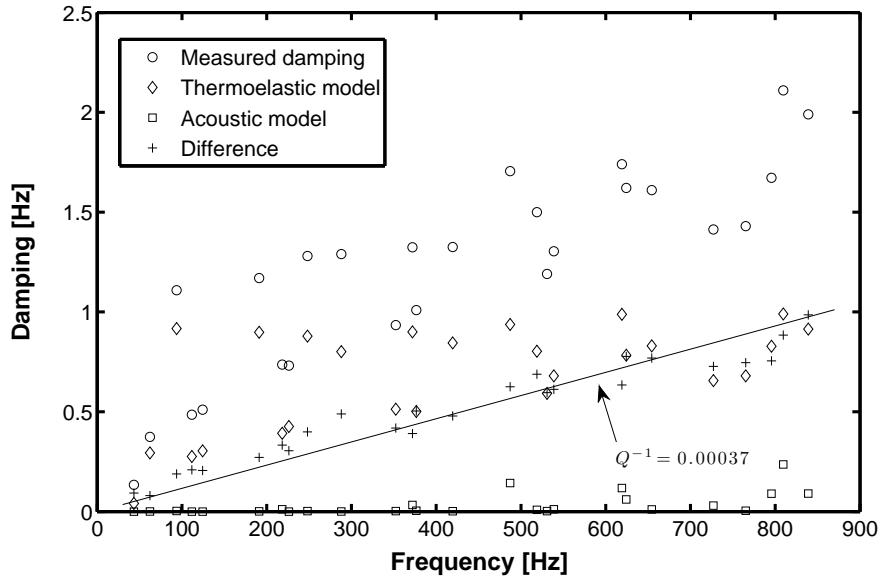


Figure 13: *Experimental/modeled thermoelastic damping. Free 35 cm × 40 cm × 2 mm aluminum plate*

503 This research received no specific grant from any funding agency in the public, commercial,  
504 or not-for-profit sectors.

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