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ABSTRACT

While methods based on functional approaches for uncertainty quantification in physical models have reached maturity, multiscale stochastic models have recently been the focus of new numerical developments. Here we specifically take an interest in multiscale problems with numerous localized uncertainties at a micro level that can be associated with some variability in the operator or source terms, or even with some geometrical uncertainty. In order to handle the high dimensionality and the complexity that issue from such problems, a multiscale method based on patches has emerged as a relevant candidate for exploiting the localized side of uncertainties and has been extended to the stochastic framework in [1]. It proposes an efficient iterative global-local algorithm where the global problems at the macro level are made simple by introducing a fictitious patch that enables to define the (possibly coarse) global problem on a domain that contains no small scale geometrical details and that involves a deterministic operator. At the micro level, specific reformulations of local problems using fictitious domain methods [2] are introduced when the patch contains internal boundaries in order to formulate the local problem on a tensor product space. The global and local problems are solved using tensor based approximation methods [3] that allow the representation of high dimensional stochastic parametric solutions and at the same time make the stochastic methods non intrusive. In the present work, the approach is extended to problems with local non-linearities within the patches for which convergence properties are shown. We will also consider patches with variable positions which involve non conforming interfaces and for which rise questions of stability of approximation and optimal convergence with respect to the mesh.

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