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Abstract: This paper provides a game-theoretical analysis of the use by athletes of performance-enhancing drugs. We focus on a two-player game where players are heterogeneous and performances are subject to uncertainty. While the standard setup assumes these drugs increase maximum performances, we assume that they also increase the probability that a given athlete competes at his best possible level. This second effect drives the doping strategies alone, suggesting that focusing on the first effect leads to incorrect conclusions. Doping strategies are strategic complements for the top dog, whereas they are strategic substitutes for the underdog. We show that the top dog always dopes more than the underdog, and that the top dog will often prefer a world with doping than without it. We also argue that anti-doping tests may increase doping for the underdog, and that targeting such tests to the top dog provides incentive to dope for the underdog.

Keywords: Game theory; PED; Anti-doping legislation

1 Introduction

Doping in professional sports is a very interesting issue for game theorists: top agents are likely to be rational, financial incentives are very strong, and the gain function linking money prizes and wages to performances is very neat. The standard analysis is based on

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the prisoner's dilemma. It views athletes as victims of the doping system, so that the anti-doping fight should be led in the name of the athletes themselves, and increasing the doping costs through various policies should have strong effects on doping behaviors. Unfortunately, doping is very widespread and the anti-doping system as a whole seems very inefficient. Moreover, the top athletes do not seem particularly interested in the end of doping.

This paper provides a different view of the doping incentive. It mostly argues that the return to doping increases with athletes' talent and that the best athletes are happy to practice their sport in a doping environment. This view relies on the physiological effects of doping products and methods. Section 2 reviews the two main effects of performance-enhancing drugs (PED) such as listed by the World-Anti-Doping Code. As it is usually emphasized, they increase the maximum performances. In other words, dopers may become super selves. We refer to this effect as the *standard effect*. However, we also point out a much less emphasized effect of PED: they improve recovery, fatigue happens later with effort, the injury risk is reduced, and athletes receive more intensive training programs. Thus athletes become able to compete at their best possible level more frequently. We refer to this other effect as the *recovery effect*. We show that the introduction of the recovery effect in the model renews the analysis of doping conducts and so provides a new prism to understand both athletes' behaviors and the impacts of anti-doping efforts.

Section 3 considers a two-player game where athletes are heterogenous and performances are subject to uncertainty. Each player is in one of two states, good or bad, with some probability. Doping is continuous and comes at constant marginal cost. Doping increases both the maximum physiological level of the athletes (standard effect) and the probability to be in the good state (recovery effect). When the recovery effect is ignored, our model features the standard property whereby doping decreases players' payoffs. In particular, both players would be better off in a world without doping. When the recovery effect is introduced, the equilibrium of the corresponding game is unique if it exists, and it coincides with the equilibrium of the game in which the standard effect is ignored. Once the recovery effect is considered, the only role of the standard effect is to determine whether there is an equilibrium or not, but the characteristics of the equilibrium are entirely driven by the recovery effect.

Section 4 turns to the analysis of the game in which only the recovery effect exists. We show that doping strategies are strategic complements for the favorite, the top dog, whereas they are strategic substitutes for the other player, the underdog. Thus, the underdog's doping effort increases the top dog's marginal return to doping, whereas the top dog's effort reduces the underdog's incentive to dope. Consequently, the return to doping is higher for the top dog than for the underdog. The top dog is sure to win when he achieves his best performance, whereas, in the same situation, the underdog only wins when the top dog is in a bad state. Thus our model predicts that the top dog dopes more than the underdog.

Our key result is that for small doping costs, the top dog prefers a sport with doping than without it. As doping reduces instances when the best athletes underperform their best level, they have a higher probability of winning, and they fare better as a result. By contrast, the underdog always prefers the sport without doping possibilities. Not only winning is less likely, but he also pays the doping cost. This result explains why the most talented athletes do not want to fight the doping system: they actually benefit from it. This new result cannot occur when only the standard effect is considered because the winning probabilities remain unchanged in this case.

The recovery effect also renews the predicted impact of doping costs. When such costs increase for both players, the top dog's doping effort always decreases, whereas the underdog's may increase. This latter property hinges on strategic substitutability. When the top dog dopes less, the underdog has a higher chance to win, which raises his return to doping. In the same vein, targeted anti-doping controls have asymmetric effects depending on whether they target the top dog or the underdog. Because the top dog also has the highest incentive to dope, targeting him strongly reduces his doping effort. However, the underdog will dope more because of strategic substitutability. In contrast, targeting the underdog reduces his doping effort, and, because of strategic complementarity, it also reduces the top dog's.

1.1 Related Literature

Our paper takes for granted that high-level or professional athletes are rational agents who behave strategically. This view finds strong support in the literature on sports economics.

A first strand of papers test athletes' rationality in a non-strategic framework. Baskhar (2009) uses cricket players' decisions as whether to bat first or field first in order to assess the consistency of their decisions. Klaasen and Magnus (2009) study the optimal strategy of serving in tennis according to the speed of the first and second serves. Both papers find evidence that top athletes behave as the theory suggests they should do. That they are rational does not mean they are insensitive to stress. Apesteguia and Palacio-Huerta (2010) focus on soccer. They show evidence that the team kicking penalties in second position has less chances of winning the game, because of the pressure they are subject to.

A second strand of papers examine athletes' strategic interactions in sports competition. Malueg and Yates (2010) show that professional tennis players strategically adjust efforts during a best-of-three contest. Chiappori et al. (2002) develop a game-theoretic model of penalty kicks in soccer, in order to analyze on which side the kicker should shoot and on which side the goalkeeper should dive. They find that professional players behave accordingly to the predictions of their model. Walker and Wooders (2001) also argue that tennis professional players play mixed strategies when choosing whether to serve on the opponent's forehand or backhand.

There are already a number of papers applying game theory to the analysis of doping behaviors. Though these papers differ in focus, they share a common feature: all players would be better off in a sport without doping.

Bird and Wagner (1997) argue that one way to escape the prisoner's dilemma is to enforce social norms among athletes about drug use. Eber and Thepot (1999) conclude that doping should be fought by improving tests, decreasing the number of competitions in a season, by flattening the prize distribution and by enforcing prevention measures. Curry and Mongrain (2009) consider ways in which the prize structure can be manipulated in order to reduce monitoring costs. Berentsen et al. (2008) model the behavior of the controller. By allowing the loser to send a message to the controller, the authors show that at equilibrium both players reduce their doping probabilities, and welfare increases.

In Berentsen (2002), players differ in winning probability, and doping is more effective for the weaker player. The mixed-strategy equilibrium of this game can have different features according to the parameters of the model, but the interesting cases are such that the best players are more likely to dope and yet they are less likely to win the game. In Berentsen and Lengwiler (2004), players are heterogenous but their performances are not subject to uncertainty. Doping is a discrete choice, and the outcome of the competition when both players dope is the same as when they do not.

With respect to this literature, our model offers a framework where doping is actually more valuable to the best player. This is due to a specific micro scenario based on the recovery effect.

The closest paper is Krankel (2007). He studies a framework where doping is complementary to ability. However, doping is a binary strategy and Krankel focuses on the equilibrium where both players dope. Moreover he does not examine players' welfare. He actually examines a social welfare function that depends, among other things, on the sum of players' absolute performances, a proxy for the quality of the show. This social welfare function is far from the constant-sum game nature of the problem that we examine here.

2 Performance-Enhancing Drugs and their effects

Our analysis relies on the existence of two different effects of performance-enhancing drugs on athletes performances. This section documents the link between doping agents and methods and both the ability to perform better and to perform with increased regularity at the highest physiological level[1].

Consider an athlete who competes on a given day. Suppose that his performance can be summarized by a single number, the higher the number, the better the performance. For the sake of argument, this number is the mean speed during the ascent of Mont Ventoux. This speed is a random draw between two extreme values a and b. We argue that PED not only shift the upper bound b to the right, they also modify the distribution of performances towards better outcomes. On the one hand, PED allow the athletes to reach performances that are not compatible with their natural talent. Thus the max of the support of the distribution b increases. On the other hand, athletes are more likely to compete at their best level. The new distribution assigns more weight around the new upper bound.

For illustration purposes, the case of the Tour de France is interesting. The Tour is a three-week competition with two rest days. Thus what matters for riders is the distribution of the mean performance over the 19 days of the race. However, fatigue increases along the race, as the red cell count decreases with effort. Thus the distribution of one-day performances tends to shift leftwards with the number of days. By taking PED, riders can improve their best performance at each day of the competition but more importantly they can reduce tiredness and recovery so that the distribution of performances changes

[1]We thank Gerard Dine, hematologist and expert for the French Anti-Doping Agency, for his precious explanations.

much less over time.

The standard effect is well documented and undisputed. It comes from the fact that PED improve basic skills like strength or endurance. The second effect, however, is never mentioned as a key factor in understanding doping conducts. It comes from the fact that PED also improve recovery, reduce injury risks, reduce injury duration, reduce tiredness, allow for longer training periods etc. All of these effects reduce the odds of having a bad day, facilitate the repetition of excellent performances and are thus formalised as a shift of the performance distribution towards better outcomes.

Table 1 is based on the World Anti Doping Agency (WADA) list of prohibited products and methods. This list classifies doping products into six categories depending on their biological mechanisms. It also classifies two doping methods. In each case, we describe doping effects along the two dimensions: impact on maximum performance, and impact on the odds of having a bad day. Although doping effects differ across sports (for instance, improving endurance means improving max performance in cycling, whereas it means reducing the odds of a bad day in grass-court tennis), we try to stay as general as possible.

Table 1 displays a key message. Almost all doping agents and methods simultaneously increase maximum performances and reduce the odds of having a bad day. This is of course true of blood doping, i.e. EPO and transfusion methods. This is also the case of anabolic agents. Athletes who use such agents improve their strength above physiological possibilities. However, anabolic agents also improve recovery and reduce injury risks. Growth hormones as well as hormone and metabolic modulators have similar effects.

This broad classification distinguishes two main types of effects. However, there is some confusion between the two columns. For instance, someone who benefits from enhanced endurance also recovers faster from hard effort. Finally, some skills are more important in some sports than in others. Endurance sets the maximum performance for a tennis player who needs to kick the ball four hours in a five-set match in Roland-Garros. Now, in two-set matches in an indoor tournament, endurance mostly affects the ability to repeat great performances day after day.

The second effect is neglected most of the time. The case of Platelet Rich Plasma (PRP) is here interesting. A treatment of PRP consists in extracting some of the athlete's blood and to enrich it with platelets, which are a source of human growth factors. The blood is then reinjected into the athlete's body, and this blood manipulation helps the athlete recover faster from a muscle injury. Up to 2010, the side effects of such treatments

Category	Products	Max perf	Chances of bad day	Use frequency
S1 Anabolic agents	Exogenous anabolic androgenous steroids	Increases strength	Decreases injury risk	Not used much
	Endogenous anabolic androgenous steroids	Increases strength and tonus	Improves recovery reduces tiredness	In use
	Other anabolic agents (non-steroids)	Increases strength	Improves recovery	In use
S2 Peptide and growth hormones	Erythropoietin (EPO)	Increases endurance	Improves recovery reduces tiredness Allows for better training	In use
	Human Growth Hormone	Increases strength	Reduces injury risk and injury duration Allows for better training	In use
S3 Beta-2 agonist	Anti asthma treatments	Improves breathing (small doses) / Increases strength (high doses)	Improves recovery	In use (authorized at low concentrations)
S4 Hormone and metabolic modulators	Regulation factors	Improves HGH effects	Improves HGH effects	In use
	Insulins	Improves strength and Endurance	Improves recovery	In use
	Metabolic modulators (GW 15156 - AICAR)	Reduces weight/ Increases strength	Reduces tiredness	In use
S5 Masking agents	Diuretics, glycerol…			In use
M1 Manipulation of blood	Auto, hetero, Homotransfusion	Same as EPO	Same as EPO	In use
	Platelet Rich Plasma	None	Reduces injury duration	In use, prohibited until 2010.

Figure 1: WADA list of prohibited products, and their effects.

on athletes performances were unknown and the WADA consequently banned its use. However, since 2010 medical studies have shown that PRP has no effect on the maximal potential performance of an athlete. In consequence, the WADA authorised the use of PRP. That PRP can also be used in order to decrease the risk of injury, as well as the injury duration, has not been taken into account. But it is unambiguous that decreasing the risk of injury translates into a reduction of the chances of having a bad day for the athlete. The position of the WADA on PRP shows how the institutions focus mostly on the effects of drugs on maximal performance and underestimate the regularity of the performances as a potential target for drug users. We argue herein that this common analysis is misleading.

3 The model

There are two players who compete for a price, the value of which is normalized to 1. Each player i is characterized by a pair of possible performance levels (a_{ib}, a_{ig}), with $a_{ib} < a_{ig}$. When in a good state, player i achieves the performance level a_{ig}. When in a bad state s/he only achieves a_{ib}. Let a_i be the performance level realised by player i. Player 1 wins whenever $a_1 > a_2$. In case $a_1 = a_2$, the players share the price and both obtain 1/2.

Doping is a continuous variable with values between 0 (no doping) and 1 (maximal doping). It comes at marginal cost c and when player i uses doping effort d_i it has two effects: (i) it increases the upper performance a_{ig} by a quantity $a(d_i)$, where $a(.)$ is a non-decreasing function (We denote by $a_i(d_i)$ the quantity $a_{ig} + a(d_i)$); (ii) it enhances the probability of being in a good state by a quantity $h(d_i)$: without doping, each player is in a good state with probability 1/2. With doping, player i is in his good state with probability $1/2 + h(d_i)$. The function h satisfies the following assumption:

Hypothesis 3.1 *h is a continuous function on $[0,1]$, $\mathcal{C}^2$ on $]0,1]$ such that:*

(i) *h is strictly increasing on $[0,1]$, $h(0) = 0$, $h(1) = 1/2$*

(ii) *we have $h'(1) = 0$ and $\lim_{d \to 0^+} h'(d) = +\infty$.*

(iii) *h is strictly concave on $]0,1]$ and $h''(1) < 0$.*

Player i's expected payoff is given by

$$U_i(d_1, d_2) = \Pr(i \text{ wins} \mid d_1, d_2) + 1/2 \Pr(\text{draw} | d_1, d_2) - c.d_i. \tag{1}$$

3.1 The standard framework

We consider here there is no recovery effect, i.e. $h \equiv 0$. Thus doping only increases the maximum performance of each player. We start with the case where the two players are identical (i.e. $a_{1b} = a_{2b}$ and $a_{1g} = a_{2g}$) and they are facing binary choices $d = 0$ or $d = \bar{d}$. Then, we turn to the case where players are heterogeneous.

When players are identical, the payoffs are summarized by the following payment matrix

	0	$\bar{d}$
0	$\frac{1}{2}, \frac{1}{2}$	$\frac{3}{8}, \frac{5}{8} - c\bar{d}$
$\bar{d}$	$\frac{5}{8} - c\bar{d}, \frac{3}{8}$	$\frac{1}{2} - c\bar{d}, \frac{1}{2} - c\bar{d}$

One can easily check that for any c and $\bar{d}$ such that $c\bar{d} < 1/8$, the unique equilibrium is $(\bar{d}, \bar{d})$, which is the typical prisoner's dilemma situation. Notice that this easily extends when several doping levels are allowed, i.e. $d \in \{0, d_1, ..., d_k\}$. It can then be shown that whatever the value of c, there is always a unique equilibrium and it is symmetric. Further, any symmetric doping strategy can be an equilibrium for an appropriate value of c. Last, as the cost increases, the equilibrium level of doping decreases. From this we derive the following:

Proposition 1 (The standard analysis) *We have*

(i) *In equilibrium the winning probabilities are the same as those without doping, i.e.* $\Pr(i \text{ wins} \mid (d^*, d^*)) = \Pr(i \text{ wins} \mid (0,0))$ *for any equilibrium* (d^*, d^*)*. This implies that* $U_i(d^*, d^*) < U_i(0,0)$*;*

Consider two doping costs $\underline{c}$ *and* $\bar{c}$ *with* $\underline{c} < \bar{c}$ *and* $(\underline{d}, \underline{d})$ *and* $(\bar{d}, \bar{d})$ *the corresponding equilibria.*

(ii) *Doping decreases with the doping cost:* $\bar{d} < \underline{d}$*;*

(iii) *Players' welfare increases with the doping cost, that is* $U_i(\bar{d}, \bar{d}) > U_i(\underline{d}, \underline{d})$*.*

Part (i) states that doping should not be popular among athletes. In a world without doping, each player achieves the best performance with probability 1/2. The expected payoff is thus $U(0,0) = 1/2$. When doping is possible, players choose the same strategy (doping can be seen as a positional good) so that the odds of winning are unchanged although they bear a cost. Thus the possibility of doping reduces the welfare of homogenous athletes.

Part (ii) tells us that anti-doping regulations that increase doping costs should reduce doping efforts. That doping is illegal means athletes must be very cautious when they transport doping products. Similarly, that blood doping becomes easier to detect means athletes must contact better and more costly doctors.

Part (iii) goes a step further and shows that doping costs actually improve athletes' well-being. Parts (i) to (iii) suggest that anti-doping regulations should be supported by all athletes. Such regulations can be seen as a way to escape the prisoner's dilemma.

This proposition goes in line with the standard views about doping. However, Berentsen (2002) shows that the consideration of heterogeneity in players' type introduces drastic changes in the analysis. In particular, there is no non-trivial pure-strategy equilibrium, and the marginal cost of doping has asymmetric effects on the different players.

We stick to the discrete doping possibilities but we introduce heterogeneity in player type. Suppose that $a_{1g} > a_{2g}$ and $a_{1b} > a_{2b}$, whereas $a(\bar{d}) > a_{1g} - a_{2g}$. This means that the underdog can fill the gap with the top dog by doping, when the top dog stays clean. With player 1 standing for the top dog and player 2 for the underdog, the payment matrix is:

	0	$\bar{d}$
0	$\frac{3}{4}, \frac{1}{4}$	$\frac{1}{2}, \frac{1}{2} - c\bar{d}$
$\bar{d}$	$\frac{3}{4} - c\bar{d}, \frac{1}{4}$	$\frac{3}{4} - c\bar{d}, \frac{1}{4} - c\bar{d}$

By elimination of strictly dominant strategies, it comes that $(0,0)$ is the unique pure-strategy equilibrium if and only if $c\bar{d} > 1/4$. To find mixed-strategy equilibria, let $\gamma_i \in [0,1]$ be the probability that player i plays the doping strategy.

Proposition 2 (Player heterogeneity) *Let $a_{1g} > a_{2g}$, $a_{1b} > a_{2b}$, and $a(\bar{d}) > a_{1g} - a_{2g}$. We have*

(i) If $c\bar{d} > 1/4$, then the only pure-strategy equilibrium is $d_1^* = d_2^* = 0$;

(ii) If $c\bar{d} \leq 1/4$, then there is a unique mixed-strategy equilibrium. It is such that

- $\gamma_1^* = 1 - 4c\bar{d}$ *and* $\gamma_2^* = 4c\bar{d}$;
- $U_1(\gamma_1^*, \gamma_2^*) = 3/4 - c\bar{d} < U_1(0,0) = 3/4$ *and* $U_2(\gamma_1^*, \gamma_2^*) = 1/4 = U_2(0,0)$.

When the doping cost is sufficiently small, there is no pure-strategy equilibrium. The argument is simple: the underdog may choose to fill the natural performance gap by using PED. If this is profitable for him, then the other player will choose to dope as a response to the increase in his opponent's maximum performance. An arms race then takes place until the disadvantaged player goes back to non doping, because the required doping level is too high. As a response the best player will also quit doping and as they return to the $(0, 0)$ situation, the arms race starts over again.

Part (ii) details the mixed-strategy equilibrium. It has two important properties. First, the top dog's doping probability decreases with the marginal doping cost, whereas the underdog's increases with it. These asymmetric responses to changes in doping cost are strong departures from the homogeneous-player case examined above. The intuition behind this result is that doping acts as a strategic complement for player 1 while it acts as a strategic substitute for player 2. Indeed,

$$\frac{\partial^2 U_1(d_1, d_2)}{\partial d_1 \partial d_2} = -\frac{\partial^2 U_2(d_1, d_2)}{\partial d_1 \partial d_2} = \frac{1}{4}. \tag{2}$$

Therefore, an increase in the doping cost has a first direct effect which drives doping levels down. However, the reduction in the top dog's doping effort triggers an increase in the underdog's, due to this substitution effect. The complementary effect for player 1 is lower than the cost effect so that the overall effect is negative. Although this is not noticed in Berentsen (2002), this property drives the result according to which the top dog dopes more than the underdog in some parameterizations.

Second, the top dog is always worse off with doping than without it. The possibility of doping allows the underdog to increase the max performance above the top dog. The top dog is forced to play the doping strategy with positive probability to keep their advantage. Conversely, the underdog is indifferent between the two situations.

If doping is thought of as a continuous variable, the analysis again leads either to $(0, 0)$ as the unique equilibrium or to a no equilibrium situation. The $(0, 0)$ equilibrium exists whenever cost is too high, or when the differences in maximum performance levels are too large. In that case, the under dog is too far behind to engage in doping and stays at 0, and the top dog also stays at 0 as a best response. When the differences in maximum performance are small enough, then the under dog engages in doping and the arms race described earlier starts again, until the point where the under dog quits the race and goes back to non doping. As a best response the top dog also goes back to 0 and the cycle starts over again[2]. This shows that there exists no pure Nash equilibrium in this

[2] The threshold difference in maximum performances, when player 1 is better than player 2, is given by $a((4c)^{-1})$. Indeed, the highest possible doping level for player 2 is $\bar{d}$ such that $5/8 - c\bar{d} > 3/8$ so $\bar{d} < (4c)^{-1}$. When $a_1(0) - a_2(0) > a((4c)^{-1})$ then $(0, 0)$ is the unique equilibrium, otherwise there is no equilibrium.

case. However, there is a unique mixed strategy equilibrium, described in the following proposition:

Proposition 3 *Assume that $a(d) = d$ and call $\delta := a_{1g} - a_{2g}$. Then*

(i) *If $4c\delta \geq 1$, $(0,0)$ is the only Nash equilibrium in mixed strategies and the equilibrium payoff is $(3/4, 1/4)$;*

(ii) *if $4c\delta < 1$ there is a unique mixed Nash equilibrium (μ_1^*, μ_2^*), where the probability distributions μ_1^* and μ_2^* are given by*

$$\mu_1^*(0) = 4c\delta,\ \mu_1^*(]0, d_1]) = 4cd_1,\ \forall d_1 \in]0, \frac{1}{4c} - \delta];$$

$$\mu_2^*(0) = 4c\delta,\ \mu_2^*(]\delta, d_2]) = 4c(d_2 - \delta),\ \forall d_2 \in]\delta, \frac{1}{4c}].^3$$

and the equilibrium payoff is $(1/2 + c\delta, 1/4)$.

Introducing heterogeneity and/or continuity of the doping variable seriously affects the conclusions drawn from the prisoner's dilemma modeling. However, regardless of the model considered above, the athletes are always happier when the doping cost is high enough. Thus athletes have strong incentive to promote anti-doping legislation and enforce controls so that, at equilibrium, nobody dopes. This is not what is observed in reality and we believe that it points out that something is missing in the standard model.

Introducing the recovery effect (i.e. doping as a device for increasing the probability of being in a good state) totally changes the analysis.

3.2 Introducing the recovery effect

Assume, without loss of generality that $a_{2b} < a_{1b} < a_{2g} < a_{1g}$. The payoff for player 1 is:

$$\begin{cases} \frac{1}{2} + h(d_1) + (\frac{1}{2} - h(d_1))(\frac{1}{2} - h(d_2)) - c.d_1 & \text{if } a_1(d_1) > a_2(d_2) \\ \frac{1}{2}(\frac{1}{2} + h(d_1))(\frac{1}{2} + h(d_2)) + (\frac{1}{2} - h(d_2)) - c.d_1 & \text{if } a_1(d_1) = a_2(d_2) \\ \frac{1}{2} - h(d_2) - c.d_1 & \text{if } a_1(d_1) < a_2(d_2) \end{cases}$$

Consider for instance the first equation. If player 1's maximal performance, after doping, is higher than that of player 2, then player 1 will win whenever he realizes his

[3] In other terms, μ_1^* and μ_2^* are the sum of a dirac distribution in 0 and a uniform distribution.

high performance (this happens with probability $1/2 + h(d_1)$), and will win when he performs badly (with probability $1/2 - h(d_1)$) only if his opponent also performs badly (with probability $1/2 - h(d_2)$). Interpretation is similar for the two other cases.

We show that only two things can happen: first, there can be no equilibrium to the game. This is due to precisely the same reason as above, when players are very close one of the other in terms of maximum performance without doping. As the game is a winner-takes-all game, there is an arms race going on to rip off all the benefits.

Second, if there is an equilibrium (or several equilibria), it has to be also an equilibrium of the game in which the function $a(.)$ is set to 0, i.e. a game in which we consider that doping has no effect on the maximum performance of players.

Lemma 1 *Any equilibrium* (d_1^*, d_2^*) *of the doping game must be such that* $a_1(d_1^*) > a_2(d_2^*)$.

This lemma implies that if an equilibrium exists, the best player without doping will still be the best after doping. This allows us to say the following:

Proposition 4 *Any Nash equilibrium* (d_1^*, d_2^*) *of the doping game is also an equilibrium of the game with* $a(d) = 0$ *for all* d.

The consequence of this proposition is simple: once the recovery effect is introduced, it drives alone the characteristics of the equilibrium of the game, regardless of the existence of the standard effect. The only role for the increase of maximum performance is to determine whether there is an equilibrium or not. Therefore, focusing on the standard effect can only drive, at the best, to partial conclusions (no equilibrium) or to non valid conclusions (when the equilibrium exists). We now turn our attention to the interesting case: the analysis of the game with $a(d) = 0$ for all d and the examination of the effects of $h(.)$ on athletes behaviour.

4 Main results

Here, we focus on the recovery effect only. We assume that h satisfies Hypothesis 3.1.

For player 1, strategic interaction arises because s/he loses when in a bad state whereas player 2 is in a good state. If player 2 dopes more, s/he increases the probability of being in such a good state. This in turn raises player 1's marginal return to doping. Things are very different for player 2. S/he loses whenever player 1 is in a good state. Thus doping efforts are wasted when player 1 dopes to the grills.

We now present our main results. Player i's payoff is

$$U_i(d_i, d_{-i}) = \Pr(\text{player } i \text{ wins}|d_i, d_{-i}) - c.d_i. \tag{3}$$

Given the zero-sum game nature of athletic contests, players' doping actions always reduce the welfare of their opponents. However, they also affect the marginal return to their opponents' actions.

Call Br^i the best response map of player i: $Br^i(d_{-i}) := \text{Argmax}_{d_i} U_i(d_i, d_{-i})$. A *Nash equilibrium* is a fixed point of the map $Br := (Br^1, Br^2)$. It turns out that, for any M and any doping cost $c > 0$, there is a unique equilibrium d^*, where both players have "interior" doping efforts: $1 > d_1^* > 0$ and $1 > d_2^* > 0$.

Proposition 5 (Properties of doping efforts at equilibrium) *Given $c > 0$, we have that*

(*i*) *Player 1's best response function increases with player 2's doping effort, whereas player 2's best-response function decreases with player 1's doping effort. As a consequence, there is a unique Nash equilibrium $d^* = (d_1^*, d_2^*)$.*

(*ii*) *The top dog dopes more, that is $1 > d_1^* \geq d_2^* > 0$.*

Part (i).— d_2 acts as a strategic complement for player 1 while d_1 acts as a substitute for player 2. The complementarity effect is in line with the prisoner's dilemma story which is usually how people analyse doping. The substitution effect is one central piece of the model because it makes it different from a prisoner's dilemma and induces many of the interesting properties below. Strategic complementarity means that when the underdog dopes more, this provides incentive for the top dog to dope more as well. Strategic substitutability here acts as a deterrent for the underdog.

Part (ii).—The top dog has a stronger incentive to dope than the underdog. The top dog is sure to win when s/he achieves the highest performance level, whereas it is not the case for the underdog. Consequently, the return to doping is higher for the top dog. This conclusion departs from the standard view of doping. That the best athletes should also be the most doped creates a potential conflict of interest for the anti-doping authorities.

Increasing the severity of anti-doping measures may well hurt the sport's stars (see our comments on the welfare at equilibrium below). This may be very difficult to achieve in a context where the same institutions are in charge of promoting the sport and detecting dopers.

Proposition 6 (Welfare at equilibrium) *We have the following*

(i) *The underdog would fare better in a world without doping:* $U_2(d_1^*, d_2^*) < U_2(0,0)$*;*

(ii) *However, for* c *not too large,* $U_1(d_1^*, d_2^*) > U_1(0,0)$.

Doping vehicles a negative externality that generally costs to athletes. However, if the cost is not too high (i.e. the authorities are not very repressive), the top dog benefits a lot from blood doping. S/he can achieve his/her best performance more frequently, and so s/he can win more frequently than without doping.

That the best player's utility is greater with doping than without, in addition to the observation that $d_1^* > d_2^*$, explains an essential fact: it is difficult to fight against doping and to enhance the popularity of a sport at the same time. Because there is collusion between organisers (federations) and best players who need one another for their respective objectives. Therefore, the question of the independence of anti-doping institutions is important. Such conflict of interest seems at the heart of the Lance Armstrong's scandal. Trevor Tygart, head of the USADA, argues that the UCI covered Armstrong in one way or another. The AFLD repeatedly complains about its tumultuous relationships with the UCI or the ATP, which manages anti-doping tests in tennis.

This property explains why the best athletes do not want to fight against doping. Not only do they dope more than the others, but they are actually happy to practice their job in an environment where doping is possible. By contrast, the underdog is hurt by the doping system, regardless of the doping cost. His chances of winning go down and there is a doping cost to pay. This might also explain why the only voices that can be heard among sportsmen are those of poorly ranked individuals.

Proposition 7 (Doping efforts variations with respect to cost) *We have the following*

(i) $\lim_{c \to 0^+} d_1^* = 1$ *and* $\lim_{c \to 0^+} d_2^* = 0$;

(ii) *The top dog's doping effort decreases with the doping cost, that is* $d(d_1^*)/dc < 0$;

(iii) *The underdog's doping increases with the doping cost for small costs, that is* $d(d_2^*)/dc > 0$ *for* c *small enough.*

Part (i).— When the cost of doping is very low, everyone leans toward putting its doping effort at the maximum level. On the contrary, when the cost is high the competition will tend to a clean situation. In the limit case, we witness a drastic change in the behavior of the underdog, for small values of c. When the doping cost becomes very small, the top dog's doping effort is very close to one and he therefore becomes almost unbeatable, which makes doping pointless for the underdog – even at very small cost.

This result advertises against the view whereby free doping would lead to a level playing field. In short, everyone would dope and this would not affect the results of the competition. On the contrary, we suggest that the best atheletes would dope at maximum level, whereas the others would not even try. Competition results would be more polarized, with always the same winners, and a large differential between their performances and those of their opponents.

Parts (ii) and (iii).— Doping costs have an ambiguous impact on equilibrium doping efforts of the underdog. However, we know that his/her doping effort is increasing for small costs. This is due to the strategic substituability discussed above: as the top dog dopes less, the return to doping increases for the underdog.

Anti-doping agencies do not test all athletes with equal probability. There are two kinds of targeted testing. A first kind arose with the biological passport. The anti-doping authorities can intensify an athlete's testing following an unexpected change in his/her blood or hormone profile. We do not model this here and simply consider this is part of the marginal cost of doping. The second kind occurs when the anti-doping agency uses past information as a predictor of winning likelihood. Athletes are then tested because they were already successful. This type of targeted testing is popular because of two reasons. On the one hand, it reduces the extent of doping among the top athletes, thereby ensuring that the winner is not too dirty. On the other hand, it reduces the incentive to dope for all the others through the usual prisoner's dilemma effect. This second reason is not true in our model. We now explain this point.

Let c_i denote the marginal cost of doping for athlete i. We refer to overall doping as the sum $d_1 + d_2$.

Proposition 8 (Targeted tests) *We have*

(*i*) *The top dog's doping effort decreases with* c_1*, whereas the underdog's increases, that is* $d(d_1^*)/dc_1 < 0$ *and* $d(d_2^*)/dc_1 > 0$*;*

(*ii*) *Both players' doping efforts decrease with* c_2*, that is* $d(d_1^*)/dc_2 < 0$ *and* $d(d_2^*)/dc_2 < 0$*;*

(*iii*) *Overall doping decreases with* c_2*, whereas the impact of* c_1 *is ambiguous, that is* $d(d_1^* + d_2^*)/dc_2 < 0$ *and* $d(d_1^* + d_2^*)/dc_1 \lesseqgtr 0$*.*

Targeted doping involves strong redistributive effects between athletes. Targeting the best athletes reduces their doping investment. Because of strategic substitutability, this also increases the doping investment of their followers who now have a chance to win. Thus overall doping is ambiguously affected and competitions become more uncertain. Targeting the underdog decreases doping for both athletes. The reason is now due to strategic complementarity. The underdog is less threatning, which allows the top dog to reduce doping.

Anti-doping authorities willing to reduce overall doping should not target the top athletes. Random testing could yield better results because it would not provide the underdog with doping incentives.

5 Conclusion

This paper contributes to the economics of doping. It focuses on a simple but novel argument whereby performance-enhancing drugs (PED) allow the athletes to compete at their best level with higher frequency. They not only reach greater performances on average, but also such performances are concentrated around the best outcomes. This argument has important implications for the understanding of doping conducts by different athlete types. Namely, the return to doping increases with inner talent, which implies that the best athletes tend to dope more. Moreover, the best athletes derive large benefits from PED and their well-being would decrease if PED did not exist. This particular pattern by type and the nexus of strategic interactions between athletes renews policy suggestions. Anti-doping tests have contrasted effects on athlete types. Targeting tests to the expected winners of a competition for instance provides the outsiders with incentive to dope.

The issues covered here have applications beyond the doping case. They are involved each time there is a contest-like situation, cheating is feasible, and there is a clear ranking of the persons participating in the contest. Exam cheating for instance provides another illustration. For most of the students, the problem is to fail or pass the exam. The cheating technology bears costs (here mostly the risk of being discovered and its consequences) and gains (increased performance). The least able students have incentive to cheat: their failure probability is very high so there is not much to lose. However, for more talented students, the problem is to belong to the top of the grade distribution. They might be tempted to cheat as a result, whereas students of intermediate talent have no chance anyway to reach such grades. XXX conduct a survey on university students who answer different questions on their cheating practice. Other things equal, cheaters are over-represented among the less able and the most able students. Adapting the present framework to this situation, we predict that intensifying cheating monitoring may decrease cheating among the top students, whereas it may encourage cheating among their immediate followers.

6 Appendix

6.1 Proof of the results of Section 3

Proof of Proposition 3. Point $a)$ is obvious. We focus on proving $b)$. We call $Supp(\mu)$ the support of μ, i.e.

$$Supp(\mu) := \{d \in \mathbb{R}_+ : \ \mu(]d-\epsilon, d+\epsilon[) > 0 \ \forall \epsilon > 0\}$$

Recall that $Supp(\mu)$ is the smallest closed set F such that $\mu(F) = 1$.

Lemma 2 *Let $d \geq 0$. Then*

$$d \notin Supp(\mu_1^*) \Rightarrow d + \delta \notin Supp(\mu_2^*).$$

Proof. Pick $d \notin Supp(\mu_1^*)$ and $\epsilon > 0$ such that $\mu_1^*(]d-\epsilon, d+\epsilon[) = 0$. Then we claim that $\mu_2^*(]d+\delta-\epsilon/2, d+\delta+\epsilon/2[) = 0$. If this was not the case, then player 2 could deviate profitably by transferring the weight μ_2^* puts on $]d+\delta-\epsilon/2, d+\delta+\epsilon/2[$ to $\{d+\delta-\epsilon\}$. ■

Corollary 6.1 *Let $d \geq 0$. Then we have*

$$d \in Supp(\mu_1^*) \Longleftrightarrow d + \delta \in Supp(\mu_2^*).$$

Proof. A reverse argument in the previous proof gives

$$d + \delta \notin Supp(\mu_2^*) \Rightarrow d \notin Supp(\mu_1^*).$$

The corollary follows. ■

Lemma 3 *Let d_1 be such that $\mu_1^*(d_1) > 0$. Then*

$$\exists \epsilon > 0 : \ ]d_1 + \delta - \epsilon, d_1 + \delta[\subset (Supp(\mu_2^*))^c.$$

Similarly, let $d_2 > \delta$ be such that $\mu_2^*(d_2) > 0$. Then

$$\exists \epsilon > 0 : \ ]d_2 - \delta - \epsilon, d_2 - \delta[\subset (Supp(\mu_1^*))^c.$$

Proof. We prove the first point and the second follows by the same arguments. Assume that $\mu_1^*(d_1) > 0$ and, for any $\epsilon > 0$, there exists $d(\epsilon) \in]d_1 + \delta - \epsilon, d_1 + \delta[\cap Supp(\mu_2^*)$. Since $\mu_1^*(d_1) > 0$ there exists ϵ small enough so that deviating from $d_1 + \delta - \epsilon$ to $d_1 + \delta$ guarantees a strictly higher[4] payoff to player 2

[4]deviating from $d(\epsilon)$ to $d_1 + \delta$ costs her (at most) ϵc, but she increases her payoff by a positive quantity, which is independent of ϵ.

Corollary 6.2 *We have*

$$\mu_1^*(d_1) > 0 \Rightarrow d_1 = 0;\ \ \mu_2^*(d_2) > 0 \Rightarrow d_2 = 0.$$

Proof. For player 1, assume that $d_1 > 0$ is such that $\mu_1^*(d_1) > 0$. Then, by previous lemma, there exists $\epsilon > 0$ such that $\mu_2^*(]d_1 + \delta - \epsilon, d_1 + \delta[) = 0$. Consequently, player 1 can deviate profitably by transferring the weight that μ_1^* puts on d_1 to $d_1 - \epsilon/2$.

For player 2, the same argument states that, for any $d_2 > \delta$, we have $\mu_2^*(d_2) = 0$. Clearly, we have $\mu_2^*(]0, \delta[) = 0$. Consequently we just need to prove that $\mu_2^*(\delta) = 0$. Assume that $\mu_2^*(\delta) > 0$. If $\mu_1^*(0) > 0$ then player 2 can obtain a strictly better payoff by transferring the weight on δ to $\delta + \epsilon$. If $\mu_1^*(0) = 0$ then player 2 can deviate profitably by transferring the weight on δ to 0. ■

Lemma 4 *Call $[a_1, b_1]$ the smallest closed interval that contains Supp(μ_1^*). Then $[a_1, b_1] =$ Supp(μ_1^*). Call $[a_2, b_2]$ the smallest closed interval that contains Supp$(\mu_2^*) \setminus \{0\}$. Then $[a_2, b_2] =$ Supp$(\mu_2^*) \setminus \{0\}$.*

Moreover, $a_2 = a_1 + \delta$ and $b_2 = b_1 + \delta$.

Proof Since *Supp*(μ_1^*) is closed, we have $a_1, b_1 \in$ *Supp*(μ_1^*). Assume that there exists $c_1 \in]a_1, b_1[$ such that $c_1 \notin$ *Supp*(μ_1^*). Then $c_1 + \delta \notin$ *Supp*(μ_2^*). Now call $\underline{c_1} := \inf\{d > c_1 :\ d \in Supp(\mu_1^*)\}$, (resp. $\overline{c_1} := \sup\{d < c_1 :\ d \in Supp(\mu_1^*)\}$. By a basic property of a Nash equilibrium, player 1 must be indifferent between $\underline{c_1}$ and $\overline{c_1}$, against μ_2^*. However, by Corollary 6.1, we have $\mu_2^*(]\underline{c_1} + \delta, \overline{c_1} + \delta[) = 0$. Hence player 1 has a strictly higher payoff when he plays $\underline{c_1}$ than when he plays $\overline{c_1}$, a contradiction. Exactly the same argument proves the assertion concerning player 2.

At last, we derive the last claim from Corollary 6.1 ■

Corollary 6.3 *We have Supp$(\mu_1^*) = [0, b_1]$ and $\mu_1^*(0) > 0$. Also we have Supp$(\mu_2^*) = \{0\} \cup [\delta, b_1 + \delta]$.*

Proof. Assume that $\mu_1^*(a_1) = 0$. Then we have $U_2(\mu_1^*, 0) > U_2(\mu_1^*, a_1 + \delta)$, and $a_1 + \delta \in$ *Supp*(μ_2^*), which is a contradiction to the fact that μ^* is a Nash equilibrium. Consequently, $a_1 = 0$ and $\mu_1^*(a_1) > 0$.[5] ■

[5] recall that 0 is the only point μ_1^* can put a positive weight on.

We are now ready to prove the proposition. First, notice that, since $U_1(0,\mu_2^*) = U_1(b_1,\mu_2^*)$, we have

$$\frac{3}{4}\mu_2^*(0) + \frac{1}{2}(1-\mu_2^*(0)) = 3/4 - cb_1;$$

Hence $\frac{1}{4}(1-\mu_2^*(0)) = cb_1$

On the other hand, $\lim_{\epsilon\to 0^+} U_2(\mu_1^*, \delta+\epsilon) = U_2(\mu_1^*, b_1+\delta)$, i.e.

$$-c\delta + \frac{1}{2}\mu_1^*(0) + \frac{1}{4}(1-\mu_1^*(0)) = -c(b_1+\delta) + \frac{1}{2},$$

which gives $cb_1 = \frac{1}{4}(1-\mu_1^*(0))$. As a consequence, we have $\mu_2^*(0) = \mu_1^*(0) > 0$. Since $U_2(\mu_1^*, 0) = \frac{1}{4}$, we necessarily have $-c(\delta+b_1)+1/2 = 1/4$, i.e. $b_1 = \frac{1}{4c} - \delta$ and $\mu_1^*(0) = \mu_2^*(0) = 4c\delta$.

To see why the distributions μ_1^* and μ_2^* are uniform respectively on $]0,b_1]$ and $[\delta, b_1+\delta]$, notice that, for player 2, we must have $U_2(\mu_1^*,0) = U_2(\mu_1^*, d_2)$ for any $d_2 \in]\delta, b_1+\delta[$. Hence

$$\frac{1}{4} = \frac{1}{4} + \frac{1}{4}\mu_1^*([0, d_2-\delta]) - cd_2,$$

which means that $\mu_1^*(]0, d_2-\delta]) = 4c(d_2-\delta)$, for any $d_2 \in]\delta, b_1+\delta]$ and μ_1^* is uniform on $]0,b_1]$. Analogously, μ_2^* is uniform on $[\delta, b_1+\delta]$.

It is clear that no deviation is profitable for any player as, by construction of μ^*, we have

$$U_1(d_1,\mu_2^*) = c\delta + \frac{1}{2},\ \forall d_1 \in [0,b_1];\quad U_1(d_1,\mu_2^*) = \frac{3}{4} - cd_1 < \frac{1}{2} + c\delta,\ \forall d_1 > b_1.$$

Also

$$U_2(\mu_1^*, d_2) = \frac{1}{4},\ \forall d_2 \in \{0\} \cup]\delta, d_1+\delta];\quad U_2(\mu_1^*, d_2) = 1/2 - cd_2 < 1/4,\ \forall d_2 > b_1+\delta.$$

The proof is complete. ■

Proof of Lemma 1. By construction of the payoff function when $a_1(d_1) = a_2(d_2)$, we have

$$\lim_{\epsilon\to 0^+} U_1(d_1+\epsilon, d_2) > U_1(d_1,d_2),$$

which means that a profile (d_1,d_2) such that $a_1(d_1) = a_2(d_2)$ cannot be a Nash equilibrium. Next, if (d_1^*, d_2^*) is a Nash equilibrium such that $a_1(d_1^*) < a_2(d_2^*)$ then necessarily $d_1^* = 0$. Indeed,

$$U_1(d_1^*, d_2^*) = \frac{1}{2} - h(d_2^*) - c.d_1^* \tag{4}$$

which can only be sustained as an equilibrium if $d_1^* = 0$ This implies $d_2^* = \text{Argmax } U_2(0, \cdot)$ where

$$U_2(0, d_2) = \frac{1}{2} + h(d_2) - c.d_2 \tag{5}$$

so that $d_2^* = (h')^{-1}(c)$. Furthermore d_2^* is such that $a_2(d_2^*) > a_1(0)$, i.e. $d_2^* > a^{-1}(a_{1g} - a_{2g})$.

Also

$$U_2(0, d_2^*) > U_2(0, 0) \tag{6}$$

so that $\frac{1}{2} + h(d_2^*) > c.d_2^* + \frac{1}{4}$

We want to show that this cannot be an equilibrium. Assume player 2 chooses d_2^* and player 1 chooses d_2^* as well. Then $a_1(d_2^*) > a_2(d_2^*)$ and

$$U_1(d_2^*, d_2^*) - U_1(0, d_2^*) = \left(\frac{1}{2} + h(d_2^*)\right)\left(\frac{1}{2} + h(d_2^*)\right) - c.d_2^* \tag{7}$$

Using $\frac{1}{4} + h(d_2^*) > c.d_2^*$ we have

$$U_1(d_2^*, d_2^*) - U_1(0, d_2^*) > \left(\frac{1}{2} + h(d_2^*)\right)\left(\frac{1}{2} + h(d_2^*)\right) - \left(\frac{1}{4} + h(d_2^*)\right) = (h(d_2^*))^2 > 0 \tag{8}$$

■

Proof of Proposition 4. Call V_i the payoff functions in the game with $a(\cdot) \neq 0$ and U_i the payoffs in the game with $a(\cdot) = 0$. Player 1 is better off in the game with $a(\cdot) = 0$: $U_1(d_1, d_2) \geq V_1(d_1, d_2)$ and reversely for player 2: $U_2(d_1, d_2) \leq V_2(d_1, d_2)$. Moreover, if $a_1(d_1) > a_2(d_2)$ then $U_i(d_1, d_2) = V_i(d_1, d_2)$.

Let $d^* = (d_1^*, d_2^*)$ be a Nash equilibrium in the game with $a(\cdot) \neq 0$. By previous lemma, $a_1(d_1^*) > a_2(d_2^*)$ hence $U_i(d_1^*, d_2^*) = V_i(d_1^*, d_2^*)$. d_2^* is a best response to d_1^* in the game with $a(\cdot) \neq 0$. Consequently, for any d_2,

$$U_2(d_1^*, d_2^*) = V_2(d_1^*, d_2^*) \geq V_2(d_1^*, d_2) \geq U_2(d_1^*, d_2),$$

which means that d_2^* is a best response of player 2 against d_1^* in the game with $a(\cdot) = 0$.

On the other hand, d_1^* is a best response of player 1 against d_2^* in the game with $a(\cdot) \neq 0$. Let $\tilde{d}_1 := Br^1(d_2^*)$, the best response of player 1 against d_2^* in the game with $a(\cdot) = 0$. We have $\tilde{d}_1 > d_2^*$ (best responses of player 1 are always bigger than best responses of player 2; see the proof of point (ii) of Proposition 5 in the next section). Thus

$$V_1(\tilde{d}_1, d_2^*) = U_1(\tilde{d}_1, d_2^*) \geq U_1(d_1^*, d_2^*) = V_1(d_1^*, d_2^*) \geq V_1(\tilde{d}_1, d_2^*).$$

Finally $U_1(\tilde{d}_1, d_2^*) = U_1(d_1^*, d_2^*)$, and the proof is complete. ■

6.2 Proof of the results of Section 4

The probability of winning and losing of both players are given by

$$\Pr(\text{Player 1 wins}|d_1, d_2) = (1/2 + h(d_1)) + (1/2 - h(d_1))(1/2 - h(d_2)) \tag{9}$$

$$\Pr(\text{player 2 wins}|d_1, d_2) = \Pr(\text{player 1 loses}|d_1, d_2) = 1 - \Pr(\text{player 1 wins}|d_1, d_2) \tag{10}$$

which gives

$$U_1(d_1, d_2) = -cd_1 + \left(\frac{1}{2} + h(d_1)\right) + \left(\frac{1}{2} - h(d_1)\right)\left(\frac{1}{2} - h(d_2)\right)$$

and

$$U_2(d_1, d_2) = -cd_2 + \left(\frac{1}{2} - h(d_1)\right)\left(\frac{1}{2} + h(d_2)\right)$$

The map h' is strictly decreasing from $(0, 1]$ to $[0, +\infty)$. Hence the inverse function $(h')^{-1}$ is well defined and strictly decreasing from $[0, +\infty)$ to $(0, 1]$.

The best-response of each player results from the first-order condition:

$$Br^1(d_2) = (h')^{-1}\left(\frac{c}{1/2 + h(d_2)}\right),$$

and

$$Br^2(d_1) = (h')^{-1}\left(\frac{c}{1/2 - h(d_1)}\right).$$

Proof of Proposition 5. (i) Existence follows from Brouwer Theorem and the fact that $Br := (Br^1, Br^2)$ is continuous and maps $[0, 1]^2$ to itself. Uniqueness follows from the fact that Br^1 is increasing in d_2 and Br^2 is decreasing in d_1, since $(h')^{-1}$ is decreasing.

(ii) For any d_1, d_2,

$$h'^{-1}\left(\frac{c}{(1/2 + h(d_2))}\right) \geq h'^{-1}\left(\frac{c}{(1/2 - h(d_1))}\right),$$

which means that $Br^1(d_2) \geq Br^2(d_1)$, $\forall d_1, d_2$. We obtain point (ii) as a trivial consequence of this observation. ■

Proof of Proposition 6. Point (i) is a trivial consequence of

$$U_2(d_1^*(c), d_2^*(c)) \leq \frac{1}{4} - cd_2^*(c) = U_2(0, 0) - cd_2^*(c).$$

We have, omitting the dependency in c to shorten the equation,

$$\Delta_1(c) := U_1(d_1^*, d_2^*) - U_1(0, 0) = -cd_1^* + \frac{1}{2}(h(d_1^*) - h(d_2^*)) + h(d_1^*)h(d_2^*)$$

Hence,

$$\lim_{c \to 0} \Delta_1(c) = 1/4 > 0,$$

which proves (ii). ■

Before proving the next proposition, concerning the variations of the doping efforts, we make some preliminary observations. Notice that, for any $c > 0$, $(d_1^*(c), d_2^*(c))$ is the unique solution of the system

$$\begin{cases} h'(d_1)(1/2 + h(d_2)) - c &= 0 \\ h'(d_2)(1/2 - h(d_1)) - c &= 0 \end{cases}$$

By an application of the implicit function Theorem, for $c > 0$,

$$\frac{d(d_1^*)(c)}{dc} = \frac{1}{J}\left(h''(d_2^*(c))\left(\frac{1}{2} - h(d_1^*(c))\right) - h'(d_1^*(c))h'(d_2^*(c))\right),$$

and

$$\frac{d(d_2^*)(c)}{dc} = \frac{1}{J}\left(h''(d_1^*(c))\left(\frac{1}{2} + h(d_2^*(c))\right) + h'(d_1^*(c))h'(d_2^*(c))\right),$$

where J is positive:

$$J = h''(d_1^*(c))h''(d_2^*(c))\left(\frac{1}{2} - h(d_1^*(c))\right)\left(\frac{1}{2} + h(d_2^*(c))\right) + (h'(d_2^*(c))h'(d_1^*(c)))^2.$$

Proof of Proposition 7. The first limit in (i) is a straightforward consequence of the form of the best response maps. However, the last limit is less obvious. We have, for any $c > 0$,

$$h'(d_2^*(c)) = \frac{c}{1/2 - h(d_1^*(c))}$$

By concavity of h, we have

$$\frac{1}{2} - h(d_1^*(c)) \leq h'(d_1^*(c))\,(1 - d_1^*(c)) = \frac{c(1 - d_1^*(c)))}{1/2 + h(d_2^*(c))} \leq 2c(1 - d_1^*(c)).$$

Thus we obtain

$$h'(d_2^*(c)) \geq \frac{1}{2(1 - d_1^*(c))} \to_{c \to 0^+} +\infty,$$

which implies that

$$\lim_{c \to 0^+} d_2^*(c) = 0.$$

(ii) The expression of $\frac{d(d_1^*)(c)}{dc}$ given above proves this point, as the numerator in the expression of the derivative of $d_1^*(\cdot)$ is negative.

(iii) We can write the derivative of d_2^* in a more appropriate manner, omitting in the notations the dependency of the doping efforts in c:

$$\frac{d(d_2^*)(c)}{dc} = \frac{1}{D}\left(h''(d_1^*)\left(\frac{1}{2} - h(d_1^*)\right) + (h'(d_1^*))^2\right),$$

where D is positive:

$$D = h''(d_1^*)h''(d_2^*)\left(\frac{1}{2} - h(d_1^*)\right)^2 + (h'(d_1^*))^3 h'(d_2^*).$$

We study the sign of the numerator, as $c \to 0^+$:

$$h'(d) \sim_{d\to 1} h'(1) - h''(1)(1-d) = -h''(1)(1-d);$$

Moreover,

$$\frac{1}{2} - h(d) \sim_{d\to 1} -h'(1)(1-d) - \frac{h''(1)}{2}(1-d)^2 = -\frac{h''(1)}{2}(1-d)^2$$

Hence

$$h''(d)\left(\frac{1}{2} - h(d)\right) + h'(d)^2 \sim_{d\to 1} -\frac{h''(1)^2}{2}(1-d)^2 + (-h''(1)(1-d))^2 = \frac{h''(1)^2}{2}(1-d)^2.$$

This proves that

$$\frac{d(d_2^*)}{dc}(c) > 0,$$

for $c \in (0, b)$, where $b > 0$. ∎

Proof of Proposition 8. (i) Using the jacobian, we get

$$Sgn\left[\frac{d(d_1^*)}{dc_1}\right] = Sgn\left[h''(d_2)(\frac{1}{2} - h(d_1))\right] < 0, \tag{11}$$

$$Sgn\left[\frac{d(d_2^*)}{dc_1}\right] = Sgn[h'(d_1)h'(d_2)] > 0. \tag{12}$$

(ii) Moreover,

$$Sgn\left[\frac{d(d_1^*)}{dc_2}\right] = Sgn\left[-h'(d_1)h'(d_2)\right] < 0, \tag{13}$$

$$Sgn\left[\frac{d(d_2^*)}{dc_2}\right] = Sgn\left[h''(d_1)(\frac{1}{2} + h(d_2))\right] < 0. \tag{14}$$

(iii) Finally,

$$Sgn\left[\frac{d(d_1^* + d_2^*)}{dc_2}\right] < 0, \tag{15}$$

$$Sgn\left[\frac{d(d_1^* + d_2^*)}{dc_1}\right] = Sgn\left[h''(d_2)(\frac{1}{2} - h(d_1)) + h'(d_1)h'(d_2)\right]. \tag{16}$$

The latter sign depends on $h''(d_2)$ and thus it is ambiguous. ∎

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