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## Should a Non-Rival Public Good Always Be Provided Centrally?

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# Should a non-rival public good always be provided centrally?\*

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## Abstract

This paper discusses the problem of optimal design of a jurisdiction structure from the view point of a welfarist social planner when households with identical utility functions for non-rival public good and private consumption have private information about their contributive capacities. It shows that the superiority of a centralized provision of a non-rival public good over a federal one does not always hold. Specifically, when differences in households contributive capacities are large, it is better to provide the public good in several distinct jurisdictions rather than to pool these jurisdictions into a single one. In the specific case where households have logarithmic utilities, the paper provides a complete characterization of the optimal jurisdiction structure in the two-type case.

*“C’est pour unir les avantages divers qui résultent de la grandeur et de la petitesse des nations que le fédératif a été créé.”* (Alexis de Toqueville)

## 1 Introduction

In many countries, one finds significant regional variation in the bundles of public goods and

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taxes available to households. This is clearly the case in federal countries such as Canada and United States where the provinces and the states have the power to decide the provision of specific public goods (for instance education) and to collect taxes. But this phenomenon is also observed in "unitary" countries such as France or the UK where cities have specific powers in terms of public good production (for instance the financing of primary school infrastructure) and taxation (local taxes). This heterogeneity in the package of public goods and taxes offered to the citizens of a same country is sometimes perceived as the source of unacceptable inequalities. As a result, it is not uncommon to observe attempts made by central authorities to correct these inequalities by means of various cross-jurisdictions equalization payments schemes. But one may wonder why central authorities do not push further this equalizing logic by carrying themselves the task of providing their citizens with the *same* package of public goods and taxes instead of maintaining these distinct jurisdictions. As the recent North American episodes of city mergers in large agglomerations (Boston, Montreal, Toronto) illustrate, this centralizing solution is sometimes adopted. Yet, the decision to merge many cities into one large agglomeration that is responsible for providing the same public goods and taxes package to all its inhabitants has been received with great skepticism by many. Is this skepticism justified? Is there some argument that can justify cross-citizens heterogeneity in public goods and taxes packages from a normative standpoint or, to put it bluntly, *federalism*? These are the questions that are addressed in this paper.

These questions are not new. They were underlying the above quote from Alexis de Toqueville and were framed by Wallace Oates (1972) as follows:

"In the absence of cost-savings from the centralized provision of a [local public] good and of interjurisdictional externalities, the level of welfare will always be at least as high (and typically higher) if Pareto-efficient levels of consumption are provided in each jurisdiction than if any single, uniform level of consumption is maintained."<sup>1</sup>

In this paper, we formulate in a precise model Oates's and Toqueville's intuition that a federal provision of public goods in separated jurisdictions can be normatively better than a centralized provision "in the absence of cost-saving from the centralized provision". Specifically, we provide a model in which even when the cost-saving case in favour of a centralized provision of a public good is maximal - namely when the public good is non-rival - it can be optimal to organize its provision in a federal system when the relevant information needed to provide the public good is not available to the social planner.

The formal architecture of the model is as follows. There is a collection of households who have the same preferences for one public good and one private good. Each household has an exogenous pecuniary wealth that is unobserved by the central government. The public good can be provided *locally* in distinct jurisdictions organized through a federal system

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<sup>1</sup>A nice survey of the literature on fiscal federalism is provided by Oates (1999).

or *centrally* in one grand jurisdiction. In either case, public good provision is financed by taxes in such a way that the budget is balanced within the federation (but not necessarily within a given jurisdiction). The central government chooses the bundles of local public good provision and tax payment - one such a bundle for every jurisdiction - in such a way as to provide incentive for households to reveal their willingness to pay for the public good by their locational choice. As in Tiebout (1956), therefore, households “vote with their feet” (see also Wildasin (1987)). More specifically, the government chooses bundles of public goods and taxes that maximize a social welfare function under two constraints: 1) a preference revelation constraint (each household must prefer the combination assigned to it by the government to any other) and 2) a budget constraint (the taxes raised in all jurisdictions must be sufficient to finance public goods that are locally provided in all of them).

From a formal point of view, this problem is somewhat reminiscent of the classical Mirrlees optimal income taxation problem (see for instance Mirrlees (1971), Mirrlees (1976), Mirrlees (1986)) with leisure being replaced by a public good. Yet this “replacing” significantly modifies the nature of the problem. In the Mirrlees setting, since leisure is a purely private good - the fact that Bob has 24 hours of leisure *per* day cannot be used to improve Mary’s utility - there is no intrinsic benefit for the social planner to pool together households with different characteristics. Because of this, Mirrleesian optimal income tax schedules typically involve significant *separation* of workers with different characteristics in order to provide workers with the proper incentives to reveal their type. With public goods, the central planner does benefit from *pooling* together individuals with different characteristics since a given quantity of a non-rival public good may benefit many individuals at no extra cost. Because of this, the central planner must make a trade-off between the benefit of pooling individuals in order to reduce the cost of public good provision and the welfare cost associated to the provision of the same package of public good and taxation to heterogeneous households. The nature of this trade-off determines the optimal level of heterogeneity in public good and taxation packages for a community. It also determines the choice between a centralized provision of the public good within a single “grand” jurisdiction or its provision within separated jurisdictions between which the federal government organizes optimal equalization transfers (see e.g.; Boadway and Flatters (1982), Buchanan (1950), Flatters, Henderson, and Mieszcowski (1974) or Gravel and Poitevin (2006)).

While a few models have examined fiscal federalism issues under asymmetric information in the literature, including those of Aronsson and Blomquist (2008), Bordignon, Manasse, and Tabellini (2001), Breuillé and Gary-Bobo (2007), Cornes and Silva (2000) and Cornes and Silva (2002), none that we are aware of have considered the problem of choosing the appropriate jurisdiction structure – federal or centralized as it may be – for providing a non-rival public good.

The rest of the paper is organized as follows. In the next section, we set up the notation and examine in its full generality the problem of the optimal choice of a jurisdiction structure under asymmetric information. In section 3, we examine the problem in the (much) more specific setting where there are only two types of households, and where these households

have additively separable and symmetric preferences over the two goods. We provide a complete characterization of the solution of the problem in the important case where the social planner is utilitarian and would like, in the federal solution, to transfer wealth from the rich jurisdiction to the poor one. As shown in Gravel and Poitevin (2006), this implies indeed that the symmetric and additively separable utility function that represents the household's preference is in fact logarithmic with respect to both goods. Section 4 provides some conclusions.

## 2 The general structure of the problem

### 2.1 Notation

There are  $n \geq 2$  households taken from a finite set  $N$ . Household  $i$  has a monetary wealth  $w_i \in \mathbb{R}_+$  and consumes a public good ( $z$ ) and a private good ( $x$ ). Households are ordered by their wealth in such a way that  $w_i \geq w_{i+1}$  for  $i = 1, \dots, n-1$ . The public good is non-rival in consumption but is "excludable" in the sense that its consumption may be made contingent upon the fact of belonging to a specific jurisdiction. Specifically, exclusion can be made by partitioning the set  $N$  of households into pairwise disjoint sets  $N_j$  for  $j = 1, \dots, l$  for some number  $l \in \{1, \dots, n\}$  such that  $\bigcup_{j=1}^l N_j = N$ . Any set  $N_j$  of this partition is interpreted as a *jurisdiction* and the collection of  $l$ -such sets  $\{N_j\}_{j=1}^l$  is interpreted as a *jurisdiction structure*. The unique jurisdiction structure that is obtained if  $l = 1$  is the grand – or *centralized* – jurisdiction structure in which all households are pooled into one single jurisdiction. All other jurisdiction structures obtained for  $l > 1$  are referred to as "*federal*". An extreme form of federalism is the jurisdiction structure associated to the case where  $l = n$  (each household forms a jurisdiction on its own). An allocation of public and private goods for the jurisdiction structure  $J = \{N_j\}_{j=1}^l$  is defined as a list  $(z_1, \dots, z_l; x_1, \dots, x_n) \in \mathbb{R}_+^{l+n}$  with the interpretation that  $z_j$  is the consumption of public good in jurisdiction  $j$  (for  $j \in \{1, \dots, l\}$ ) and  $x_i$  is the consumption of private good by household  $i$  (for  $i \in N$ ). An allocation of public and private goods  $(z_1, \dots, z_l; x_1, \dots, x_n)$  for the jurisdiction structure  $J = \{N_j\}_{j=1}^l$  is *feasible* for that jurisdiction structure if it verifies the federation budget constraint:<sup>2</sup>

$$(1) \quad \sum_{j \in \{1, \dots, l\}} z_j + \sum_{i \in N} x_i \leq \sum_{i \in N} w_i.$$

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<sup>2</sup>We assume that the production of  $z$  units of the public good requires  $z$  units of the private good. This assumption is without loss of generality in our two goods model.

If  $(z_1, \dots, z_l; x_1, \dots, x_n)$  is an allocation of public and private goods that is feasible for the jurisdiction structure  $J = \{N_j\}_{j=1}^l$ , we denote by  $T_i = w_i - x_i$  the *tax paid* by household  $i$ . An equivalent reformulation of the budget constraint is of course that:

$$\sum_{j \in \{1, \dots, l\}} z_j \leq \sum_{i \in N} T_i$$

(taxes collected must be sufficient to finance the quantities of the public good provided to the citizens).

Households convert alternative combinations of private and public goods into utility by the same continuously differentiable, strictly increasing and concave utility function  $U : \mathbb{R}_+^2 \rightarrow \mathbb{R}$  (with image  $u$ ). The utility function is also assumed to be super-modular in the sense of satisfying  $U_{xz}(\bar{z}, \bar{x}) \geq 0$ . The second part of the paper also assumes, in addition to the above properties, that  $U$  is *additively separable* so that it can be written, for every bundle  $(\bar{z}, \bar{x}) \in \mathbb{R}_+^2$ , as:

$$U(\bar{z}, \bar{x}) = f(\bar{z}) + h(\bar{x}),$$

for some twice continuously differentiable increasing and concave real-valued functions  $f$  and  $h$  having both  $\mathbb{R}_+$  as domain. For further use, we denote by  $V$  ( $V : \mathbb{R}_{++}^3 \rightarrow \mathbb{R}$ ) the household's indirect utility function defined as usual by:

$$(2) \quad V(p_z, p_x, R) = \max_{z, x} U(z, x) \text{ subject to } p_z z + p_x x \leq R.$$

We also denote by  $z^M(p_z, p_x, R)$  and  $x^M(p_z, p_x, R)$  the (Marshallian) demands for public good and private consumption (respectively) when the prices for these two goods are  $p_z$  and  $p_x$  and when the wealth of the household is  $R$ . These Marshallian demands are defined, as usual, by the solution of program (2). Given the assumptions imposed on  $U$ , it can be seen easily that Marshallian demands and indirect utility are differentiable functions of prices and wealth that are both decreasing with respect to prices and increasing with respect to wealth (the two goods are normal if the households preferences are represented by a super-modular utility function). We denote by  $\mathcal{U}$  the class of all direct utility functions that satisfy all these properties and by  $\mathcal{U}_A$  the subset of  $\mathcal{U}$  consisting of those functions that are additively separable.

The criterion used by the social planner to compare alternative allocations of private and public goods from a social viewpoint is represented by a social evaluation function  $S : \mathbb{R}_+^{2n} \rightarrow \mathbb{R}$  (with the interpretation that  $S(x_1, z_1, \dots, x_n, z_n) \geq S(x'_1, z'_1, \dots, x'_n, z'_n)$  if and only if  $(x_1, z_1, \dots, x_n, z_n)$  is socially better than  $(x'_1, z'_1, \dots, x'_n, z'_n)$ ). We specifically assume that

the social criterion is *welfarist* and results from the aggregation of the households utilities by a Pareto-inclusive Schur-concave social welfare function so that, for every allocation  $(x_1, z_1, \dots, x_n, z_n) \in \mathbb{R}_+^{2n}$ , one can write  $S(x_1, z_1, \dots, x_n, z_n) = W(U(x_1, z_1), \dots, U(x_n, z_n))$  for some increasing and Schur-concave function  $W : \mathbb{R}^n \rightarrow \mathbb{R}$ . Most of the analysis will actually be conducted with the utilitarian objective (where  $W(u_1, \dots, u_n) = u_1 + \dots + u_n$  for every list  $(u_1, \dots, u_n)$  of utility levels).

## 2.2 The choice of a jurisdiction structure under public information

As is well-known from basic public economics, if households utilities and wealth are public information, the problem solved by the social planner is easy. Since the public good is non-rival in consumption, it is a waste to have different individuals consuming different quantities of the public good. For if  $(z_1, \dots, z_l; x_1, \dots, x_n)$  is a feasible allocation of private and public goods for a jurisdiction structure  $\{N_j\}_{j=1}^l$  with  $l > 1$ , one can improve everyone's utility by providing everyone with  $\bar{z} = \max_{h \in \{1, \dots, l\}} z_h$  units of the public good and  $x_i + (\sum_{j \in \{1, \dots, l\}} z_j - \bar{z})/n$  units of the private good. For this reason, the only jurisdiction structure that can be chosen by a Pareto sensitive social planner in a first-best environment is the grand jurisdiction structure associated to  $l = 1$ . In a first-best world, there is no dispute as to the superiority of a central provision of a non-rival public good over a federal one. There is, of course, a (distributive) need to have different households paying different taxes for the public good. More specifically, the welfarist social planner would choose, in the grand jurisdiction structure, a distribution of taxes that solves:

$$(3) \quad \max_{(T_1, \dots, T_n)} W(U(\sum_{i \in N} T_i, w_1 - T_1), \dots, U(\sum_{i \in N} T_i, w_n - T_n)).$$

Such a distribution of taxes – call it  $(T_1^*, \dots, T_n^*)$  – would satisfy the well-known Samuelson's condition that the sum of the households marginal rates of substitution between the private and the public goods equal 1. It is clear here that if  $W$  is Schur-concave, the central government would solve problem (3) by *equalizing* utility levels, an equalization which can only be achieved, given the same level of public good provided to all, by equalizing private consumptions. In this ideal *first-best* world, public and private good consumptions are perfectly equalized, and taxation is individualized by means of personalized Lindahl pricing.

## 2.3 The choice of a jurisdiction structure under private information

If the information on households' characteristics is private, the social planner is no longer able to levy *different* taxes on households who consume the same quantity of the public

good because these households are indistinguishable from its point of view. The social planner could, however, provide different households with different packages of taxes and public good levels based on, say, their place of residence, in order to make them reveal their willingness to pay. Households would then “vote with their feet” and choose to live at the place of residence offering them their favorite package of public spending and taxes, thus revealing their “type” to the social planner. Of course, separating households in different jurisdictions is costly because it requires the use of (much) more resources to provide different levels of the public good in different jurisdictions than what would be needed under a single jurisdiction with a single level of the non-rival public good for all households.

We consider herein the case where the private information concerns the household’s wealth – the only variable that differs across households – and where the social planner is utilitarian. While the wealth of a particular household is unknown to the social planner, we assume that the cumulative density of the wealth within the population (e.g., the number of households who have a wealth no greater than any real number) is known. With this knowledge, the planner chooses a jurisdiction structure  $\{N_j\}_{j=1}^l$  (for some  $l \in \{1, \dots, n\}$ ) and a feasible allocation of the private and the public good for that structure that maximizes its social objective, subject to the constraint that every household prefers the package of public good and tax in the jurisdiction to which it is assigned to any other. This problem is, in its full generality, complex, and proceeds in two steps.

In the *first* step, for a given jurisdiction structure  $\{N_j\}_{j=1}^l$  for some  $l \in \{1, \dots, n\}$ , the social planner solves the program:

$$(4) \quad \max_{z_1, T_1, \dots, z_l, T_l} \sum_{j=1}^l \sum_{i \in N_j} U(z_j, w_i - T_j,)$$

subject to the budget constraint:

$$(5) \quad \sum_{j=1}^l z_j \leq \sum_{j=1}^l \#N_j T_j$$

and, for every  $j \in \{1, \dots, l\}$ , and every  $i \in N_j$ , the  $l$  incentive-compatibility constraints:

$$(6) \quad U(z_j, w_i - T_j) \geq U(z_{j'}, w_i - T_{j'}) \text{ for all } j' \in \{1, \dots, l\}.$$

Let  $\Psi(\{N_j\}_{j=1}^l)$  denote the value of the objective function of the social planner at the solution of program (4) under the constraints (5) and (6). The *second* step of the central planner’s problem consists in choosing the jurisdiction structure  $\{N_j\}_{j=1}^l$  for some  $l \in \{1, \dots, n\}$  that maximizes the value of  $\Psi(\{N_j\}_{j=1}^l)$ . This second step is clearly a discrete problem since there is only a finite number of different possible partitions of  $N$  into jurisdiction structures.

A natural starting point for studying program (4) is to consider the centralized jurisdiction structure. Studying program (4) under the centralized jurisdiction structure is easy because there are no incentive constraints (6) to worry about. In that case program (4) writes (after substituting the budget constraint (5) satisfied at equality into the objective function):

$$(7) \quad \Psi(N) = \max_{T \in [0, w_n]} \sum_{i \in N} U(nT, w_i - T).$$

The necessary (and sufficient by concavity of  $U$ ) first-order condition for an interior solution  $T^*$  of this program can be written as:

$$(8) \quad n \frac{\bar{U}_z^*}{\bar{U}_x^*} = 1,$$

where  $\bar{U}_k^* = [\sum_{i \in N} U_k(nT^*, w_i - T^*)]/n$  for  $k = z, x$  is the average marginal utility of good  $k$  at the optimal choice. This condition looks somewhat like a Samuelson condition. It states that the optimal allocation of public good in the case where everybody is constrained to pay the same tax (because the government cannot distinguish between individuals) equalizes a sum of households' marginal rate of substitution between private and public goods to the marginal rate of transformation of 1. Yet the marginal rates of substitution involved in equation (8) are not the households' ones but are, instead, those of an abstract "representative individual" whose marginal rate of substitution is the ratio of the average marginal utility of the public good over the average marginal utility of the private good. Because of this, the centralized second-best solution is *a priori* different from the one associated to first-best optimality.

There is however an obvious case where condition (8) coincides with the standard Samuelson condition and where, as a result,  $\Psi(N)$  is the maximal sum of individual utilities that would obtain in the first-best world where the government had all the relevant information. This case is when the households' utility function is *quasi-linear* (in the private good) so that it writes, for every  $z$  and  $x$ , as  $U(z, x) = f(z) + x$  for some increasing and concave function  $f$ . Indeed with quasi-linear utility, condition (8) writes:

$$(9) \quad n \frac{\partial f(nT^*)}{\partial z} = 1,$$

which is nothing else than the Samuelson's condition associated with quasi-linear utility. Hence, with quasi-linear utility, it is possible to achieve a first-best allocation of private and public goods by pooling everybody in the same jurisdiction. Because of this, in the quasi-linear case, one has  $\Psi(N) \geq \Psi(\{N_j\}_{j=1}^l)$  for every jurisdiction structure  $\{N_j\}_{j=1}^l$  that the central planner could consider. In a quasi-linear world, it would *never* be optimal to create more than one jurisdiction for providing a non-rival public good.

In the following example, developed further in the second part of the paper, we show that this conclusion is highly dependent upon the quasi-linear assumption. If this assumption is relaxed, a federal jurisdiction structure can be better than the centralized one from a social welfare point of view.

**Example 1** *There are 100 households, 90 of which are “rich” - and have a wealth of 10 - and 10 of them are “poor” (and have a wealth of 1). Their common utility function is  $U(z, x) = \ln z + \ln x$ . The social planner uses a utilitarian social welfare function. If the central planner chooses the centralized jurisdiction structure, it solves*

$$\max_{T \in [0,1]} 100 \cdot \ln(100T) + 90 \cdot \ln(10 - T) + 10 \cdot \ln(1 - T).$$

*The necessary FOC for this program is:*

$$\frac{100}{T^*} - \frac{90}{10 - T^*} - \frac{10}{1 - T^*} = 0$$

*or, equivalently:*

$$T^* = 0.90108.$$

*This tax rate yields a production of 90.108 units of public good.*

*The social planner can do better than this by considering the jurisdiction structure where the 90 rich households belong to one jurisdiction with tax and public good provision  $T_R$  and  $z_R$ , and the poor in another with tax  $T_P$  and public good  $z_P$ . For instance, the social planner could select  $z_R = 409.5$ ,  $T_R = 5.45$ ,  $z_P = 45.5$  and  $T_P = -3.55$  (the poor receive a subsidy!!). Such an allocation is feasible for the federal jurisdiction structure because the budget constraint is satisfied, that is,  $z_R + z_P = 409.5 + 45.5 = 455 = n_R T_R + n_P T_P = 90 \times 5.45 - 10 \times 3.55 = 455$ . Moreover, any rich household prefers the bundle of public and private good  $(409.5, 4.55)$  obtained in the rich jurisdiction of the federation to the bundle  $(90.108, 9.09892)$  received with the central solution (indeed  $\ln 409.5 + \ln 4.55 \simeq 7.5301 > 6.7092 \simeq \ln 90.108 + \ln 9.09892$ ). Similarly, a poor household prefers its package of taxes and public good provision in the federal solution to that in the centralized one because  $\ln 45.5 + \ln 4.55 \simeq 5.3328 > 2.1876 \simeq \ln 90.108 + \ln 0.09892$ . Hence the federal jurisdiction structure is Pareto superior to the centralized one. Finally, it can be checked that the federal structure is “viable” in the sense that each of the two categories of households prefers its jurisdiction to that of the other. This is clear for the poor who could not even pay the tax of 5.45 that*

is charged in the rich jurisdiction. But that is also true for the rich who would get, were it to move to the poor jurisdiction, a utility of  $U(z^P, 10 - T_P) = \ln 45.5 + \ln 13.55 \simeq 6.4241$ , which is smaller than the utility of 7.5301 obtained in its own jurisdiction.

The question is thus: under which conditions is a centralized jurisdiction structure optimal? How can we characterize optimal jurisdiction structures in general contexts? These questions are difficult to answer in general. For one thing, the two-step program described above is difficult to solve because if the number of households with different wealth levels is large, then so is the number of jurisdiction structures that are to be compared.

Be it as it may, studying federal jurisdiction structure requires that constraints (6) be handled properly. It is easy to see that the preference of a household with wealth  $w_i$  in the space of all possible public good and tax packages is represented by the utility function  $\Phi^{w_i} : \mathbb{R}_+ \times \mathbb{R} \rightarrow \mathbb{R}$  defined by  $\Phi^{w_i}(\bar{z}, \bar{T}) = U(\bar{z}, w_i - \bar{T})$ . These preferences are convex, increasing in  $\bar{z}$  and decreasing in  $\bar{T}$ . Moreover, if the public good is normal (which is the case here thanks to the super-modularity assumption), these preferences satisfy the *single-crossing property* that  $\text{MRS}^{\Phi^{w_1}}(\bar{z}, \bar{T}) > \text{MRS}^{\Phi^{w_2}}(\bar{z}, \bar{T})$  at every  $(\bar{z}, \bar{T})$  where, for  $i = 1, 2$ ,  $\text{MRS}^{\Phi^{w_i}}(\bar{z}, \bar{T})$  is the slope, evaluated at  $(\bar{z}, \bar{T})$ , of the indifference curve of a wealth- $w_i$  household. It is defined by:

$$\text{MRS}^{\Phi^{w_i}}(\bar{z}, \bar{T}) = \frac{\partial U(\bar{z}, w_i - \bar{T}) / \partial z}{\partial U(\bar{z}, w_i - \bar{T}) / \partial x}.$$

Therefore, if the public good is normal, it is easy to show that  $\text{MRS}^{\Phi^{w_i}}(\bar{z}, \bar{T})$  is increasing with respect to  $w_i$  everywhere.

The indifference curves of two households with different wealth levels passing through some point  $c \in \mathbb{R}_+ \times \mathbb{R}$  are represented in Figure 1.

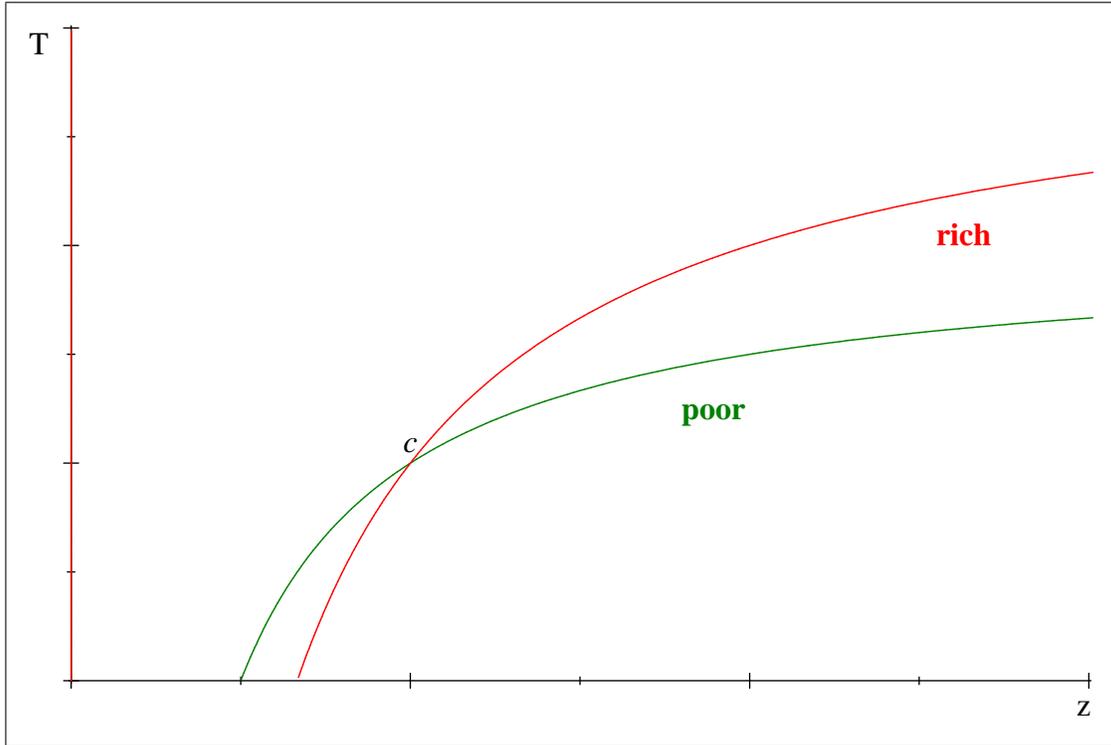


Figure 1: Two indifference curves in the  $(T, z)$  space.

This standard single-crossing property simplifies somehow program (4). Among other things, it guarantees that all jurisdictions of a given federation that satisfy incentive constraints (6) will be *consecutive* (e.g., see Greenberg (1983)) and, therefore, be such that for any two jurisdictions with distinct *per capita* wealth, the richest household of the poorer jurisdiction is weakly poorer than the poorest household of the richer one. Yet, significant as it is, this simplification leaves the analysis of program (4) in its full generality still quite difficult. One of the difficulty lies in handling the incentive constraints that may or may not be binding. Because of this complexity, we focus attention herein on the (much) simpler problem of characterizing the optimal jurisdiction structure when there are only two types of households.

### 3 The choice of a jurisdiction structure with two types of households

We assume accordingly that the  $n$  households can be split in two types:  $n_1$  households of type 1 (the "rich" with wealth  $w_1$ ) and  $n_2$  households of type 2 (the "poor" with wealth  $w_2$ ). In this setting the choice of the optimal jurisdiction structure made by the social planner amounts to comparing the value of its objective function in the centralized jurisdiction structure with that in the situation where the households are split into two jurisdictions: one inhabited by

the poor, and the other by the rich. It is clear that, beside this segregated federation in which all type-1 households live in a jurisdiction and all type-2 households live in another, there are no partition of the set of households into (more than one) disjoint subsets that can maximize a Pareto-inclusive objective while satisfying the incentive compatibility constraints.<sup>3</sup> We denote by  $(N_1, N_2)$  the federal jurisdiction structure in which the rich and the poor live in two distinct jurisdictions and by  $N$  the centralized jurisdiction structure in which all households live in the same jurisdiction.

### 3.1 The two-type case with general utility functions

Consider first the centralized jurisdiction structure. Let us examine the *utility possibility set* of this structure. The Pareto frontier of this set is easy to characterize. It is a curve lying between two points: One where each poor gets its “ideal utility” associated to its favorite tax and the rich gets the utility associated with the poor’s favorite tax and the other extreme situation where the rich gets its ideal utility and the poor gets the utility level associated to the fact of paying the most preferred tax of the rich.

The favorite tax  $T_2^*$  of the poor is the solution of the program:

$$\max_{T \in [0, w_2]} U(nT, w_2 - T).$$

It is immediate to see that this favorite tax is given by  $T_2^* = z^M(1/n, 1, w_2)/n$ . The poor’s favorite tax is therefore nothing else than the expenditure (expressed in units of the private good) that the poor would like to devote to the public good if the price of this public good was  $1/n$ . Hence the poor’s ideal utility level is  $V(1/n, 1, w_2)$ .

For the rich, things are a bit different because of the fact that, when deciding its favorite tax, it must take into account the ability to pay of the poor. The most preferred tax of the rich  $T_1^*$  is, in effect, the solution to the program:

$$\max_{T \in [0, w_2]} U(nT, w_1 - T).$$

There are two cases:

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<sup>3</sup>Suppose there are at least two jurisdictions with each of them inhabited by the two types of households. Denote by  $(z_j, T_j)$  the package of public good and tax prevailing in jurisdiction  $j$  (for  $j = 1, 2$ ). The constraints (6) of program (4) imply that:

$$U(z_1, w_i - T_1) = U(z_2, w_i - T_2)$$

for  $i = 1, 2$ . By the single crossing property, this can only hold if  $(z_1, T_1) = (z_2, T_2)$ . But if this is the case, then it is better for the social planner to merge the two jurisdictions into one, and to finance the (common) public good quantity by means of a lower tax levied on a larger number of households.

1.  $T_1^* = w_2$  (the rich chooses to tax all the poor's income) or
2.  $T_1^* = z^M(1/n, 1, w_1)/n$  (as for the poor, the most preferred tax of the rich is the expenditure that the rich would like to devote to the public good if it had the opportunity to purchase it at the Lindahl price of  $1/n$ .)

In short  $T_1^* = \min(z^M(1/n, 1, w_1)/n, w_2)$  and the rich's ideal utility level (denoted  $U_{SB1}^{1*}$  with the subscript standing for the "second-best 1-jurisdiction") is defined by:

$$U_{SB1}^{1*} = \begin{cases} V(1/n, 1, w_1) & \text{if } z^M(1/n, 1, w_1)/n \leq w_2 \\ U(nw_2, w_1 - w_2) & \text{otherwise.} \end{cases}$$

It is immediate to see that, if the public good is normal,  $T_1^* > T_2^*$  and that both

$$U_{SB1}^{1*} > U(z^M(1/n, 1, w_2), w_1 - T_2^*) > V(1/n, 1, w_2) > U(nT_1^*, w_2 - T_1^*).$$

Hence, the two extreme points ( $U(z^M(1/n, 1, w_2), w_1 - T_2^*), V(1/n, 1, w_2)$ ) and ( $U_{SB1}^{1*}, U(nT_1^*, w_2 - T_1^*)$ ) of the Pareto frontier will lie in the area where the utility of the rich is larger than that of the poor. It will therefore be difficult to be egalitarian in this second-best world with one jurisdiction. Between its extreme points ( $U(z^M(1/n, 1, w_2), w_1 - T_2^*), V(1/n, 1, w_2)$ ) and ( $U_{SB1}^{1*}, U(nT_1^*, w_2 - T_1^*)$ ), the Pareto frontier is defined by the function  $\Theta : \mathbb{R} \rightarrow \mathbb{R}$  with  $\Theta(u_1) = U(nT^{SB}(u_1), w_2 - T^{SB}(u_1))$  where the function  $T^{SB} : [U(z^M(1/n, 1, w_2), w_1 - T_2^*), U_{SB1}^{1*}] \rightarrow [0, T_1^*]$  is defined implicitly, for every  $u_1 \in [U(z^M(1/n, 1, w_2), w_1 - T_2^*), U_{SB1}^{1*}]$  by:

$$U(nT^{SB}(u_1), w_2 - T^{SB}(u_1)) = u_1.$$

It is easy to verify by usual implicit function arguments that the function  $\Theta$  is decreasing and concave on its domain.

This frontier is illustrated on Figure 2 in a situation where  $U_{SB1}^{1*} = V(1/n, 1, w_1)$  (the ideal tax of the rich is less than the poor's wealth).

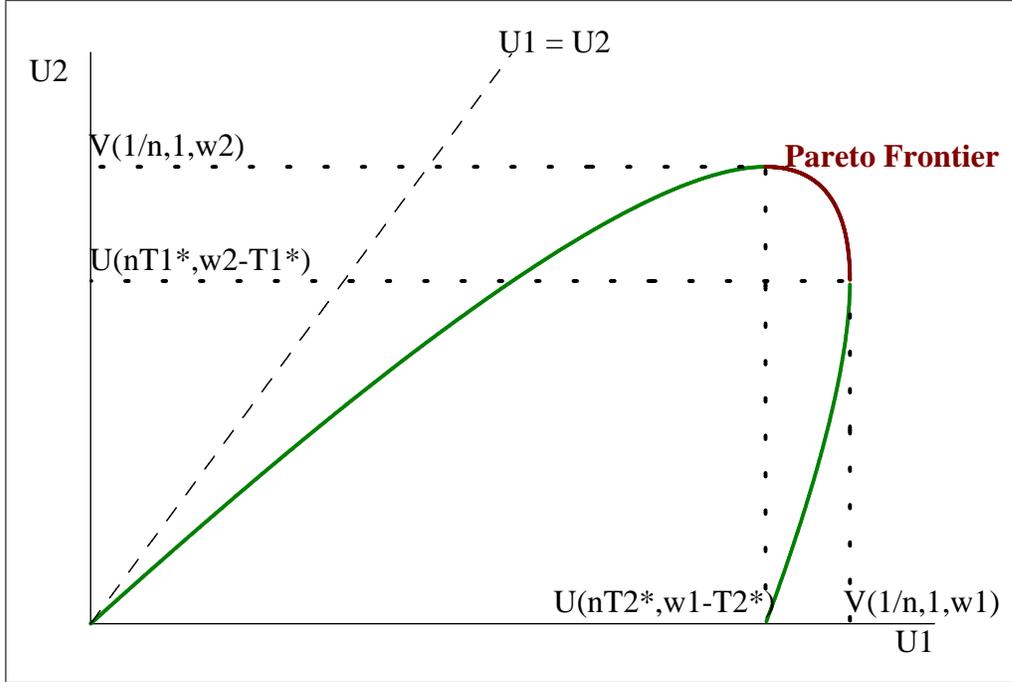


Figure 2: utility possibility set of a unitary jurisdiction structure.

Notice that  $T_2^*$  is the solution that would be selected by an extremely egalitarian leximin or a maximin social planner.

We now turn to the federal jurisdiction structure  $(N_1, N_2)$  in which the central government "separates" the two types into two different jurisdictions by solving program (4) for that case. A *first* possibility that can arise is that none of the two IC constraints (6) of this program binds. If this the case, one can solve program (4) by ignoring these constraints. This program, studied for an arbitrary number of jurisdictions by Gravel and Poitevin (2006), describes how a central planner would design optimal equalization payments in a federal system with an immobile population. Let  $B(n_1, n_2, w_1, w_2) = \{(z_1, T_1, z_2, T_2) \in \mathbb{R}^4 : n_1 T_1 + n_2 T_2 \geq z_1 + z_2 \text{ and } T_i \leq w_i \text{ for } i = 1, 2\}$  denote the set of two-jurisdiction packages of public goods and taxes that are feasible for a two-jurisdiction segregated federation notwithstanding the IC constraints. It is easy to establish the following lemma.

**Lemma 2** *Let  $U$  be a utility function in  $\mathcal{U}$ . Then  $(z_1^*, T_1^*, z_2^*, T_2^*)$  is Pareto-efficient in the set  $B(n_1, n_2, w_1, w_2)$  if and only if there exists  $s_i^* \in [-w_i, +\infty)$  satisfying  $z_i^* = z^M(1/n_i, 1, w_i + s_i^*)$  and  $T_i^* = w_i - x^M(1/n_i, 1, w_i + s_i^*)$  for  $i = 1, 2$ , and  $n_1 s_1^* + n_2 s_2^* = 0$ .*

**Proof.** Suppose first that there exists  $s_i^* \in [-w_i, +\infty)$  satisfying:

$$z_i^* = z^M(1/n_i, 1, w_i + s_i^*),$$

$$T_i^* = w_i - x^M(1/n_i, 1, w_i + s_i^*)$$

for  $i = 1, 2$  such that

$$n_1 s_1^* + n_2 s_2^* = 0.$$

Yet suppose that, contrary to the statement of the lemma,  $(z_1^*, T_1^*, z_2^*, T_2^*)$  is not Pareto-efficient in  $B(n_1, n_2, w_1, w_2)$ . This means that there exists some allocation  $(\hat{z}_1, \hat{T}_1, \hat{z}_2, \hat{T}_2)$  in  $B(n_1, n_2, w_1, w_2)$  such that  $U(\hat{z}_i, w_i - \hat{T}_i) \geq U(z_i^*, w_i - T_i^*)$  for  $i = 1, 2$  with at least one strict inequality. By standard revealed preference arguments, this implies that  $\hat{z}_i/n_i + (w_i - \hat{T}_i) \geq z_i^*/n_i + (w_i - T_i^*) = w_i + s_i^*$  for  $i = 1, 2$  with at least one strict inequality. Multiplying both sides of each of these inequalities by  $n_i$  and summing yields:

$$\begin{aligned} \hat{z}_1 - n_1 \hat{T}_1 + \hat{z}_2 - n_2 \hat{T}_2 &> n_1 s_1^* + n_2 s_2^* \\ &= 0. \end{aligned}$$

But this contradicts the assumption that  $(\hat{z}_1, \hat{T}_1, \hat{z}_2, \hat{T}_2)$  is in  $B(n_1, n_2, w_1, w_2)$ . Conversely, assume that  $(z_1^*, T_1^*, z_2^*, T_2^*)$  is Pareto-efficient in the set  $B(n_1, n_2, w_1, w_2)$ . Define then  $s_i^*$  by

$$(10) \quad s_i^* = \frac{z_i^*}{n_i} - T_i^*$$

for  $i = 1, 2$ . Since  $(z_1^*, T_1^*, z_2^*, T_2^*)$  is Pareto-efficient in  $B(n_1, n_2, w_1, w_2)$ , it satisfies  $n_1 T_1^* + n_2 T_2^* = z_1^* + z_2^*$  so that:

$$n_1 s_1^* + n_2 s_2^* = z_1^* + z_2^* - n_1 T_1^* - n_2 T_2^* = 0$$

as required. Let us now show that  $z_i^* = z^M(1/n_i, 1, w_i + s_i^*)$  and  $T_i^* = w_i - x^M(1/n_i, 1, w_i + s_i^*)$  for  $i = 1, 2$ . Thanks to (10), it is clear that, for  $i = 1, 2$ ,  $w_i + s_i^* = z_i^*/n_i + w_i - T_i^*$  so that the bundle of public and private good  $(z_i^*, w_i - T_i^*)$  satisfies the budget constraint defined by setting the price of public good at  $1/n_i$  and the household wealth at  $w_i + s_i^*$  (using private good as the numéraire). Suppose to the contrary that, for household 1 (say, but the argument works equally well for household 2 or for both types of households) one has  $(z_1^*, w_1 - T_1^*) \neq (z^M(1/n_1, 1, w_1 + s_1^*), x^M(1/n_1, 1, w_1 + s_1^*))$ . Since Marshallian demands are functions of the public good prices and wealth, this means that there exists some bundle

$(\hat{z}_1, \hat{x}_1)$  satisfying  $w_1 + s_1^* = \hat{z}_1/n_1 + \hat{x}_1$  such that  $U(\hat{z}_1, \hat{x}_1) > U(z_1^*, w_1 - T_1^*)$ . Defining  $\hat{T}_1$  by  $\hat{T}_1 = w_1 - \hat{x}_1$ , this means that:

$$n_1 w_1 + n_1 s_1^* + n_2 w_2 + n_2 s_2^* = \hat{z}_1 + n_1(w_1 - \hat{T}_1) + z_2^* + n_2(w_2 - T_2^*).$$

Because  $n_1 s_1^* + n_2 s_2^* = 0$ , this writes:

$$n_1 \hat{T}_1 + n_2 T_2^* = \hat{z}_1 + z_2^*.$$

But this means that the allocation  $(\hat{z}_1, \hat{T}_1, z_2^*, T_2^*) \in B(n_1, n_2, w_1, w_2)$  is Pareto superior to  $(z_1^*, T_1^*, z_2^*, T_2^*)$ , a contradiction. ■

This lemma says that any Pareto-efficient allocation of public goods and tax burdens in a two-jurisdiction federation – ignoring the incentive compatibility constraints – can be thought of as resulting from a two-step procedure: a *first* step in which a federal government selects a pair of *per capita net* equalization subsidies  $s_1$  and  $s_2$  (one such subsidy for every jurisdiction, with aggregate subsidies summing to 0) and a *second* step in which each household allocates its wealth – increased by the subsidy received – between public and private good expenditures assuming that it faces a (Lindahl) price of the public good given by the inverse of the population size of its jurisdiction of residence.

An efficient federal provision of public goods and tax burdens in this sense (that satisfies the incentive constraints (6)) is depicted on Figure 3.

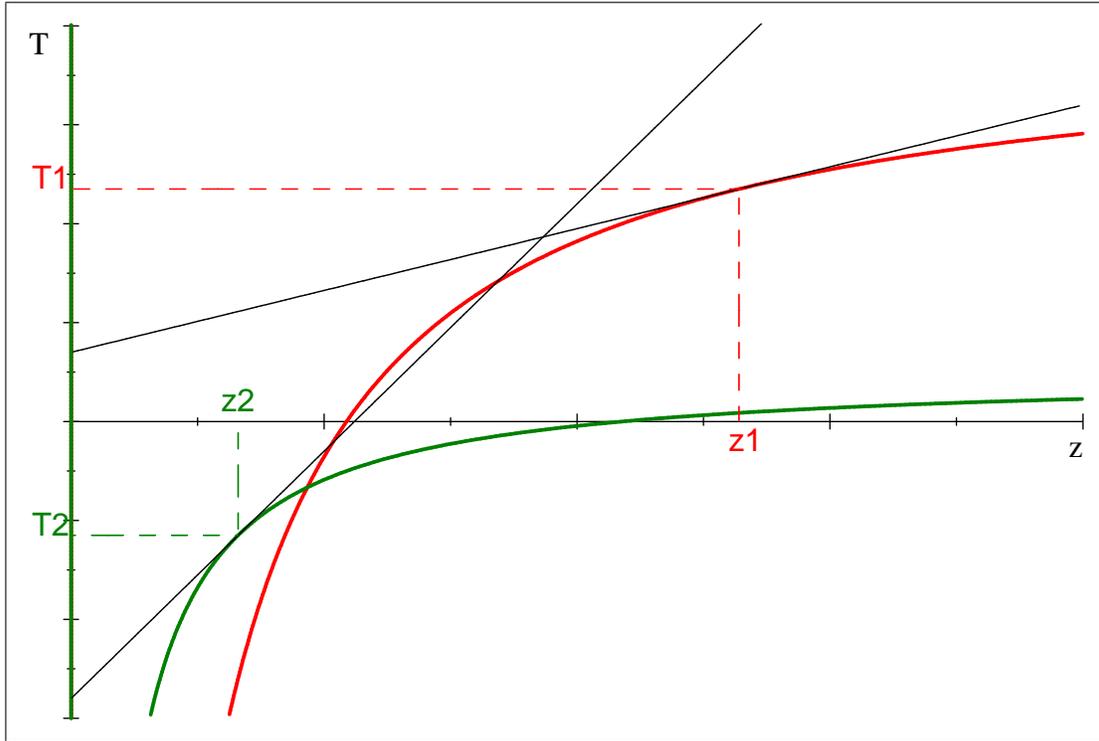


Figure 3: An efficient federal allocation that satisfies the incentive constraints.

A *second* possibility for the solution of program (4) is that at least one of the incentive constraints (6) binds. As it turns out, the handling of these constraints is somewhat facilitated by a proper understanding of the properties of the optimal subsidies of Lemma 2. As shown in Gravel and Poitevin (2006), it is possible that these subsidies chosen by a welfare-maximizing social planner be *regressive* in the sense of being increasing with respect to household wealth. That is, in the two-jurisdiction case, it is possible that the social planner chooses the subsidies in such a way that poor households transfer fiscal revenues to rich households. This regressivity may arise if the objective function of the social planner is not additively separable between individual wealth and the price of public good. If this objective function results from composition of an additively separable Bergson-Samuelson social welfare function with an individual indirect utility function, then a necessary and sufficient condition for avoiding this possible regressivity is for the indirect utility function to be itself additively separable between the price of the public good and the wealth of the household. In Gravel and Poitevin (2006), it was further shown that if the individual indirect utility function results from an additively separable individual direct utility functions, it can be additively separable between the price of the public good and the wealth if and only if the direct utility function is either quasi-linear or logarithmic with respect to the public good. As mentioned above, quasi-linear utility makes the problem studied in this paper trivial. Yet most equalization payments of federations that we are aware of are progressive. If we

want to limit our attention to additively separable utility functions while being sure that the subsidies chosen by the social planner be progressive, we are then forced to assume that the additively separable function is in fact logarithmic with respect to the public good. Beside its empirical plausibility, considering (optimally chosen) progressive *per capita* subsidies in federations with immobile households also simplifies the handling of incentive compatibility constraints in the study of the optimal jurisdiction structure. For it can be seen that, in the two-type case, if a pair of packages of local public goods and taxes that is Pareto efficient in the federation for a progressive subsidies scheme violates the incentive constraint of the poor, then the centralized jurisdiction structure Pareto dominates the federal one. We state this fact formally as follows.

**Lemma 3** *Let  $B(n_1, n_2, w_1, w_2) = \{(z_1, T_1, z_2, T_2) \in \mathbb{R}^4 : n_1 T_1 + n_2 T_2 \geq z_1 + z_2 \text{ and } T_i \leq w_i \text{ for } i = 1, 2\}$  and let  $(z_1^*, T_1^*, z_2^*, T_2^*)$  be efficient in  $B(n_1, n_2, w_1, w_2)$  with respect to a scheme of subsidies  $s_1$  and  $s_2$  as per Lemma 2 satisfying  $s_1 < 0 < s_2$ . Then, if  $U(z_2^*, w_2 - T_2^*) \leq U(z_1^*, w_2 - T_1^*)$  (incentive compatibility constraint of the poor is binding or violated),  $\Psi(N) > \Psi(N_1, N_2)$ .*

**Proof.** *Assume that  $(z_1^*, T_1^*, z_2^*, T_2^*)$  is efficient in  $B(n_1, n_2, w_1, w_2)$  with respect to a scheme of subsidies  $s_1$  and  $s_2$  as per Lemma 2 satisfying  $s_1 < 0 < s_2$ , and is such that:*

$$(11) \quad U(z_2^*, w_2 - T_2^*) \leq U(z_1^*, w_2 - T_1^*).$$

*Because the per capita subsidies  $s_1$  and  $s_2$  that support this Pareto-efficient federal provision of public good as per Lemma 2 are progressive,  $s_1 = z_1^*/n_1 - T_1^* < 0$  so that  $T_1^* > 0$  (since  $z_1^* > 0$ ). Consider then merging jurisdictions 1 and 2 and providing every one with  $z_1^*$  unit of public good while asking a tax payment of  $T_1^*$ . If it was feasible, such a centralized provision of the public good would weakly Pareto dominate the federal provision associated with  $(z_1^*, T_1^*, z_2^*, T_2^*)$  because type 1 individual would be indifferent and, because of inequality (11), type 2 households would be weakly better off. Let us show that the centralized provision  $(z_1^*, T_1^*)$  is feasible. This amounts to showing that:*

$$(n_1 + n_2)T_1^* \geq z_1^*.$$

*Since  $(z_1^*, T_1^*, z_2^*, T_2^*)$  is feasible and efficient in  $B(n_1, n_2, w_1, w_2)$ , one has:*

$$\begin{aligned} n_1 T_1^* + n_2 T_2^* &= z_1^* + z_2^* \\ &\iff \\ n_1 T_1^* + n_2 T_2^* - z_2^* &= z_1^* \end{aligned}$$

which implies, given that  $n_2T_2^* - z_2^* = n_2s_2 > 0$ , that  $n_1T_1^* > z_1^*$  and, therefore, that  $(n_1 + n_2)T_1^* > z_1^*$  (since  $T_1^* > 0$ ). Because this inequality is strict, it is actually possible to increase public good provision for all without increasing taxes, that is, it is possible to increase the utility level of all. ■

Simple as it is, this lemma facilitates the analysis of the two-type case by restricting attention, in the study of the optimal federal provision of public good, to the incentive compatibility constraint of type-1 households. Of course, the progressivity of the equalization subsidies that supports an efficient federal provision of public goods as per Lemma 2 is crucial for Lemma 3. Yet, if we take this progressivity as granted, the only relevant possibility other than that when no IC is binding when analyzing program (4) in the two-type case is to solve:

$$(12) \quad \begin{aligned} \max_{z_1, T_1, z_2, T_2} \quad & n_1U(z_1, w_1 - T_1) + n_2U(z_2, w_2 - T_2) \\ \text{s.t.} \quad & z_1 + z_2 \leq n_1T_1 + n_2T_2 \\ & U(z_1, w_1 - T_1) \geq U(z_2, w_1 - T_2). \end{aligned}$$

Two general remarks can be made about this program. A first one is that its solution entails the well-known “no distortion at the top” property that the bundle of public and private goods consumed by a rich household in a federal system where its incentive constraint is binding is the bundle it would like to consume in such a federal system without incentive constraint if it was facing a price of public good of  $1/n_1$  and had a wealth of  $w_1 - s$  for some *per capita* subsidy  $s$  transferred to the poor jurisdiction. A formal way to write this (the proof of which is left to the reader) is as follows.

**Lemma 4** *Let  $U$  be a utility function in  $\mathcal{U}$ . Then  $(\hat{z}_1, \hat{T}_1, \hat{z}_2, \hat{T}_2)$  is the solution of program (12) if and only if  $\hat{z}_1 = z^M(1/n_1, 1, w_1 - \hat{s})$  and  $\hat{T}_1 = w_1 - x^M(1/n_1, 1, w_1 - \hat{s})$  for the *per capita* subsidy  $\hat{s}$  that solves, along with  $(\hat{z}_2, \hat{T}_2)$ , the program:*

$$(13) \quad \begin{aligned} \max_{s, T_2, z_2} \quad & n_1V(1/n_1, 1, w_1 - s) + n_2U(z_2, w_2 - T_2) \\ \text{s.t.} \quad & n_1s + n_2T_2 \geq z_2 \\ & V(1/n_1, 1, w_1 - s) \geq U(z_2, w_1 - T_2). \end{aligned}$$

The second remark concerns the relation between the *per capita* subsidy given by rich households to poor ones when the incentive constraint of the rich household binds (Lemma 4) and the subsidy when this incentive constraint does not bind (Lemma 2). As shown in the following lemma, the *per capita* subsidy given by the rich jurisdiction to the poor one will

always be *smaller* when the incentive constraint binds than when the incentive constraint does not bind. As in the Mirrleesian optimal taxation literature, giving incentives to the rich to stay in their jurisdiction somewhat mitigates the equalizing propensity of the social planner as compared to what it would do if the incentives of the rich were not constraining.

**Lemma 5** *Let  $U$  be a utility function in  $\mathcal{U}$  and  $s^*$  be the solution of the program:*

$$(14) \quad \max_s n_1 V(1/n_1, 1, w_1 - s) + n_2 V(1/n_2, 1, w_2 + n_1 s/n_2).$$

*Let  $(\hat{s}, \hat{z}_2, \hat{T}_2)$  be the solution of program (13). Then  $\hat{s} \leq s^*$  and  $\hat{z}_2 \leq z_2^M(1/n_2, 1, w_2 + n_1 s^*/n_2)$ .*

**Proof.** *The proof proceeds as follows. First, we show that:*

$$V_R(1/n_1, 1, w_1 - s^*) > \frac{\partial U(\tilde{z}_2, w_2 - \tilde{T}_2)}{\partial x},$$

*where  $(\tilde{z}_2, \tilde{T}_2)$  denote the solution of program (13) when the subsidy  $s$  is constrained to take the value  $s^*$ . We then show that, at the solution of program (13), we must have*

$$V_R(1/n_1, 1, w_1 - \hat{s}) < \frac{\partial U(\hat{z}_2, w_2 - \hat{T}_2)}{\partial x}.$$

*Finally, we show that in order to obtain this latter inequality, it must be the case that  $\hat{s} \leq s^*$ . It is then easy to show that  $\hat{z}_2 \leq z_2^M(1/n_2, 1, w_2 + n_1 s^*/n_2)$ .*

*We know that the solution of program (14),  $s^*$ , is such that:*

$$V_R(1/n_1, 1, w_1 - s^*) = V_R(1/n_2, 1, w_2 + n_1 s^*/n_2),$$

*( $s^*$  equalizes the marginal utility of income across households). Suppose that we fix  $s^*$  and that we solve program (13) by choosing  $z_2$  and  $T_2$  given  $s^*$ . The allocation of the rich household does not change since the subsidy  $s$  does not change. The allocation of the poor household is now implicitly characterized by the two constraints of program (13) that will be binding at the solution of the program (the proof of this is left to the reader):*

$$\begin{aligned} n_1 s^* + n_2 \tilde{T}_2 &= \tilde{z}_2 \\ V(1/n_1, 1, w_1 - s^*) &= U(\tilde{z}_2, w_1 - \tilde{T}_2). \end{aligned}$$

In general, there are two allocations that simultaneously satisfy these two equations. However, only one also satisfies the (unwritten) incentive constraint for the poor household. At this allocation, we have that:

$$\tilde{z}_2 < z_2^M(1/n_2, 1, w_2 + n_1s^*/n_2)$$

and that:

$$\tilde{T}_2 < T_2^M = \frac{z_2^M(1/n_2, 1, w_2 + n_1s^*/n_2) - n_1s^*}{n_2}.$$

Moreover, we know that:

$$V_R(1/n_2, 1, w_2 + n_1s^*/n_2) = \frac{\partial U(z_2^*, w_2 - T_2^*)}{\partial x}.$$

Let us now show that:

$$\frac{\partial U(z_2^*, w_2 - T_2^*)}{\partial x} > \frac{\partial U(\tilde{z}_2, w_2 - \tilde{T}_2)}{\partial x},$$

that is, let us show that the marginal utility of income of the poor household decreases when we move from the allocation  $(z_2^*, T_2^*)$  to the allocation  $(\tilde{z}_2, \tilde{T}_2)$ .

We first note that we have

$$\begin{aligned} n_1s^* + n_2T_2^* &= z_2^* \\ n_1s^* + n_2\tilde{T}_2 &= \tilde{z}_2, \end{aligned}$$

that is, both allocations are on the same "budget line" for the poor household. Let  $U_x(n_1s^* + n_2T_2, w_2 - T_2)$  be the partial derivative of  $U$  with respect to its second argument. Notice that, since  $U_{xz} \geq 0$  by super-modularity, one has:

$$(15) \quad \frac{\partial U_x(n_1s^* + n_2T_2, w_2 - T_2)}{\partial T_2} = n_2U_{xz} - U_{xx} > 0.$$

Notice also that, as we move from  $(z_2^*, T_2^*)$  to  $(\tilde{z}_2, \tilde{T}_2)$  along the budget line,  $T_2$  decreases. Combined with inequality (15), this implies that  $U_x$  decreases as well along the budget line. Hence, one has:

$$V_R(1/n_1, 1, w_1 - s^*) > \frac{\partial U(\tilde{z}_2, w_2 - \tilde{T}_2)}{\partial x}.$$

We now show that, at the solution of program (13), we have:

$$V_R(1/n_1, 1, w_1 - \widehat{s}) < \frac{\partial U(\widehat{z}_2, w_2 - \widehat{T}_2)}{\partial x}.$$

We do this by writing down the the first-order conditions of program (13):

$$\begin{aligned} T_2 & : -n_2 U_x(\widehat{z}_2, w_2 - \widehat{T}_2) + \mu \cdot n_2 + \lambda \cdot U_x(\widehat{z}_2, w_1 - \widehat{T}_2) = 0 \\ z_2 & : n_2 U_z(\widehat{z}_2, w_2 - \widehat{T}_2) - \mu - \lambda \cdot U_z(\widehat{z}_2, w_1 - \widehat{T}_2) = 0 \\ s & : -n_1 V_R(1/n_1, 1, w_1 - \widehat{s}) + n_1 \cdot \mu - \lambda \cdot V_R(1/n_1, 1, w_1 - \widehat{s}) = 0 \end{aligned}$$

where  $\mu$  and  $\lambda$  are the multipliers of the two constraints. Substituting for the value of  $\mu$  drawn from the first condition into the third one yields (after some straightforward manipulations):

$$n_1(U_x(\widehat{z}_2, w_2 - \widehat{T}_2) - V_R(1/n_1, 1, w_1 - \widehat{s})) - \lambda \left( \frac{n_1}{n_2} U_x(\widehat{z}_2, w_1 - \widehat{T}_2) + V_R(1/n_1, 1, w_1 - \widehat{s}) \right) = 0.$$

This can only hold if the first term is positive. Hence,

$$V_R(1/n_1, 1, w_1 - \widehat{s}) < \frac{\partial U(\widehat{z}_2, w_2 - \widehat{T}_2)}{\partial x}.$$

Since the above inequality is reversed at  $s^*$ , the final step of the proof requires showing that  $s$  must decrease for this inequality to be satisfied. Consider first the effect of  $s$  on  $V_R(1/n_1, 1, w_1 - s)$ . A standard comparative statics exercise shows that, if  $U$  is concave, then  $V$  is concave in income. This implies that  $V_{Rs}(1/n_1, 1, w_1 - s) > 0$ . Hence, if  $s$  decreases, the marginal utility of income of the rich household decreases.

Consider now the effect of  $s$  on  $U_x(z_2, w_2 - T_2)$  where  $(z_2, T_2)$  are the solution of program (13) for a given  $s$ . We know that  $(z_2, T_2)$  is at the intersection of the two constraints of program (13). Inserting the budget constraint into the incentive constraint yields

$$(16) \quad V(1/n_1, 1, w_1 - s) - U(n_1 s + n_2 T_2, w_1 - T_2) = 0.$$

This implicitly defines  $T_2$  as a function of  $s$ . We want to evaluate the effect of  $s$  on  $U_x(n_1 s + n_2 T_2, w_2 - T_2)$  when  $T_2$  is characterized by the condition (16). We have

$$(17) \quad \frac{dU_x(n_1 s + n_2 T_2, w_2 - T_2)}{ds} = U_{xz} \left[ n_1 + n_2 \frac{dT_2}{ds} \right] - U_{xx} \frac{dT_2}{ds},$$

where

$$\frac{dT_2}{ds} = -\frac{-V_R - n_1 U_z^1}{-n_2 U_z^1 + U_x^1} < 0$$

is calculated along condition (16) and  $U^1 = U(n_1 s + n_2 T_2, w_1 - T_2)$  (as opposed to  $U = U(n_1 s + n_2 T_2, w_2 - T_2)$ ). It is easy to show that the denominator of  $dT_2/ds$  is negative because of the single-crossing property and the fact that constraint (16) is binding. If it was positive, then the solution would not be optimal since it would be possible to increase the utility of the poor household without decreasing that of the rich household while satisfying all constraints. This implies that  $dT_2/ds < 0$ .

We now evaluate the sign of the coefficient of  $U_{xz}$  in (17).

$$\left[ n_1 + n_2 \frac{dT_2}{ds} \right] = \frac{-n_1 n_2 U_z^1 + n_1 U_x^1 + n_2 V_R + n_2 n_1 U_z^1}{-n_2 U_z^1 + U_x^1} = \frac{n_1 U_x^1 + n_2 V_R}{-n_2 U_z^1 + U_x^1} < 0.$$

These sign calculations imply that  $dU_x/ds < 0$  in equation (17). Hence, if  $s$  decreases, the marginal utility of income of the poor household increases.

We have shown that, at  $s^*$ , the marginal utility of the rich household is higher than that of the poor. We have also shown that it must be lower at the optimal separating allocation. To lower the marginal utility of the rich and increase that of the poor (to reach the optimal separating allocation), we have to decrease  $s$  (starting from  $s^*$ ). Hence, it must be the case that  $\hat{s} \leq s^*$ .

It is easy to show that  $\hat{z}_2 \leq z_2^M(1/n_2, 1, w_2 + n_1 s^*/n_2)$ . Since the public good is normal, one must have  $z_2^M(1/n_2, 1, w_2 + n_1 \hat{s}/n_2) \leq z_2^M(1/n_2, 1, w_2 + n_1 s^*/n_2)$ . Hence, handling the incentive constraints of the rich households impose a distortion on the consumption of public good of the poor that is that  $\hat{z}_2 \leq z_2^M(1/n_2, 1, w_2 + n_1 \hat{s}/n_2)$ . ■

The reason for the fact that the subsidy from the rich to the poor in a federation is smaller when the incentive constraint binds than when it does not is clear. When the incentive constraint does not bind, the subsidy serves to some extent the purpose of equalizing the marginal utility of income of both types of households. It acts therefore as a redistributing device. When the incentive compatibility constraint is binding, the central planner must cool down its redistributive propensity and must leave rich households with a higher income and a lower marginal utility of income than that of the poor. This is achieved with a lower subsidy. Note that the cost of redistribution is in terms of distortion to the amount of public good consumed by the poor household. If the two goods are normal, this means that the rich household will consume more of the public good when the constraint is binding than

when it is not. We can show that the rich household will increase its tax bill slightly so that the increase in income due to the decrease in subsidy will be shared between the two goods. As for the poor household, it will consume less of the public good. In general, it is not possible to determine whether the taxes on the poor household are larger or smaller when the incentive constraint binds as compared to the situation where it does not bind. On the one hand, less resources are needed since less public good is provided. On the other hand, the subsidy from the rich households is smaller so that more taxes may need to be raised.

### 3.2 The two-type case with logarithmic utility functions

In what follows, we compare the federal and the centralized provisions of the public good with private information in the two-type case from the view point of a utilitarian social planner when the household's utility function writes:

$$(18) \quad U(z, x) = \ln z + \ln x.$$

This case, which assumes a logarithmic form also with respect to the private good, and which treats the two goods symmetrically, is (much) more specific than the mere assumption that utility be logarithmic with respect to the public good that is required by the assumption that the subsidy from the rich household to the poor household in a federal system without the incentive constraint be progressive (see Gravel and Poitevin (2006)). Yet, it is tractable and provides a representative example of the kinds of situations that are covered when the household's utility is additively separable and when the social planner wishes to solve program (4) without the IC constraints by choosing a progressive subsidy scheme. Another advantage of this specification is that, for the most part, it makes the choice between a federal and centralized structure depending upon only *two* parameters: the ratio of the high wealth over the small one (the interquartile) and the ratio of the number of rich over the number of poor (demographic ratio). As will be seen, this two-dimensional representation is quite useful for identifying the set of parameters that determine the social planner's preference for the federal structure over the centralized one. Among other things, it enables a nice two-dimensional geometric depiction of the situation.

We start by finding the provision of public good and tax that the planner would choose in the centralized jurisdiction structure as per program (7). The first order condition of program (7) writes:

$$n_1 U_z(z^{sb1}, w_1 - T^{sb1}) + n_2 U_z(z^{sb1}, w_2 - T^{sb1}) = \frac{n_1 U_c(z^{sb1}, w_1 - T^{sb1}) + n_2 U_c(z^{sb1}, w_2 - T^{sb1})}{n}.$$

Applying this to the logarithmic utility function defined by (18) yields (after lengthy manip-

ulations, done with the precious help of *Mathematica*):<sup>4</sup>

$$z^{sb1} = \frac{n_1 w_1 + n_2 w_2 + 2n_1 w_2 + 2n_2 w_1 - g(n_1, n_2, w_1, w_2)}{4},$$

$$T^{sb1} = \frac{z^{sb1}}{(n_1 + n_2)},$$

where:

$$g(n_1, n_2, w_1, w_2) = (-8(n_1 + n_2)^2 w_1 w_2 + (n_2(2w_1 + w_2) + n_1(w_1 + 2w_2))^2)^{1/2},$$

and where the superscript *sb1* refers to the **1**-jurisdiction **second-best** allocation.

We now consider the optimal allocation of public good and taxes that the utilitarian social planner would choose in the federal system in which the two types of households are separated into two distinct jurisdictions. Again, this amounts to analyzing program (4) for the specific logarithmic utility function of equation (18).

As discussed above, we start this analysis by first considering the case where the two incentive constraints are satisfied and where the social planner solves:

$$(19) \quad \max_{z_1, T_1, z_2, T_2} n_1[\ln z_1 + \ln(w_1 - T_1)] + n_2[\ln z_2 + \ln(w_2 - T_2)] \quad \text{s.t.} \quad z_1 + z_2 \leq n_1 T_1 + n_2 T_2.$$

It is not difficult to show that the solution of program (19) is:

$$z_1^{fb2} = \frac{n_1(n_1 w_1 + n_2 w_2)}{2(n_1 + n_2)},$$

$$z_2^{fb2} = \frac{n_2(n_1 w_1 + n_2 w_2)}{2(n_1 + n_2)},$$

$$T_1^{fb2} = \frac{n_1 w_1 - n_2 w_2 + 2n_2 w_1}{2(n_1 + n_2)},$$

$$T_2^{fb2} = \frac{n_2 w_2 - n_1 w_1 + 2n_1 w_2}{2(n_1 + n_2)},$$

where the superscript *fb2* refers to the **2**-jurisdiction **first-best** allocation in which incentive constraints are ignored. Notice that this solution implies that private good consumption – equal to the Marshallian demand for the private good (Lemma 2) – is equalized across jurisdictions because the household's wealth (net of the subsidy received) is equalized across jurisdictions and, thanks to the (Cobb-Douglas) structure of the households' preferences, Marshallian demands for the private good do not depend upon the price of the public good. Notice that the equalization of wealth achieved here results from the fact that the indirect

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<sup>4</sup>All computations performed on *Mathematica* are available upon request.

utility function is additively separable in the wealth and the price of the public good. However, the quantity of the public good consumed in the two jurisdictions differs because the population size (equal to the inverse of the price of the public good) differs. Note that the level of public good in one jurisdiction is increasing in that jurisdiction's population and decreasing in the other jurisdiction's population.

We can compute the (aggregate) subsidy as per Lemma 2 which, given the progressivity associated with these preferences, comes from the rich (type 1) to the poor (type 2). This subsidy, denoted  $s^{fb2}$ , is nothing else than the difference between tax revenues and public good expenditure in the rich jurisdiction:

$$(20) \quad s^{fb2} \equiv n_1 T_1^{fb2} - z_1^{fb2} = \frac{n_1 n_2 (w_1 - w_2)}{n_1 + n_2}.$$

The subsidy is (thanks to progressivity) monotonically increasing with respect to the wealth difference between the two types of households. The subsidy is also increasing with respect to the population size of each jurisdiction. When population increases in one jurisdiction, the demand for public good provision increases. For the assumed utility function, the increase in public good is exactly financed by the increase in population so that aggregate taxes do not have to increase. When  $n_2$  increases, the per capita subsidy to the poor decreases so that the marginal utility of income for the poor increases. To restore the equality of marginal utility of income across jurisdictions, the subsidy has to increase. When  $n_1$  increases, the per capita subsidy from the rich decreases so that their marginal utility of income has increased. The subsidy then has to increase for optimal redistribution.

We now consider the incentive compatibility constraints. Since we do not need to worry about the incentive constraint of the poor type in so far as the comparison of the federal and the centralized provision of the public good is concerned thanks to Lemma 3, we restrict our attention to the program:

$$(21) \quad \begin{aligned} & \max_{z_1, T_1, z_2, T_2} \quad n_1 [\ln(w_1 - T_1) + \ln z_1] + n_2 [\ln(w_2 - T_2) + \ln(z_2)] \\ & \text{s.t.} \quad \sum_{j=1}^2 z_j \leq \sum_{j=1}^2 n_j T_j \\ & \quad \quad \ln(w_1 - T_1) + \ln z_1 \geq \ln(w_1 - T_2) + \ln z_2. \end{aligned}$$

If the incentive constraint of the rich does not bind at the solution of program (21), then the solution of this program is precisely that of program (19) above. Hence, we start by evaluating the incentive constraint of the rich at the solution of program (19). We can show (again with the help of *Mathematica*) that:

$$\ln(w_1 - T_1^{fb2}) + \ln z_1^{fb2} \geq \ln(w_1 - T_2^{fb2}) + \ln z_2^{fb2}$$

is equivalent to:

$$(22) \quad \frac{n_1^2 w_1 + n_2^2 (w_2 - 2w_1) + 3n_1 n_2 (w_2 - w_1)}{n_2 (3n_1 w_1 + 2n_2 w_1 - 2n_1 w_2 - n_2 w_2)} \geq 0.$$

Since the denominator is positive, only the numerator matters for the sign of the expression. Fortunately, as mentioned above, it is possible to restrict the number of parameters by replacing  $n_1$  by  $a * n_2$  and  $w_1$  by  $b * w_2$  where:

$$a = n_1/n_2$$

is the demographic ratio (of the number of rich over the number of poor) and

$$b = w_1/w_2$$

is the interquartile ratio (the ratio of the highest income over the smallest income in this two-type world). By assumption  $a > 0$  and  $b > 1$ . Condition (22) can then be rewritten as:

$$n_2^2 w_2 (1 - 3a(b - 1) - 2b + a^2 b) \geq 0,$$

or, equivalently,

$$(23) \quad \text{IC}_1(a, b) \equiv a^2 b - 3(b - 1)a + 1 - 2b \geq 0.$$

When inequality (23) holds, the incentive constraint for the rich household is satisfied at the  $fb2$  allocation. While, thanks to Lemma 3, this is not needed for the comparison of the federal and the centralized structure, we can also write down the inequality that corresponds to the incentive constraint of the poor. This inequality can be shown to be equivalent to:

$$(24) \quad \text{IC}_2(a, b) \equiv a^2(b - 2) + 3(b - 1)a + 1 \geq 0.$$

Figure 4 provides a graphical representation of these two incentive constraints in the  $(a, b)$ -space.

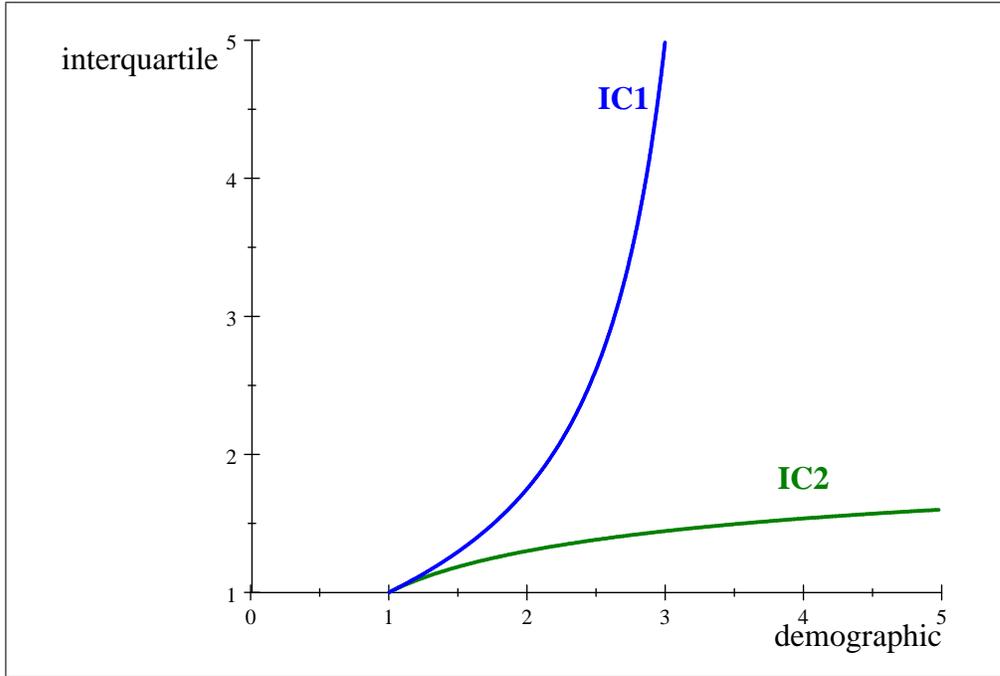


Figure 4: Incentive constraints in the  $(a, b)$  space.

As can be seen, the curves  $IC_1$  and  $IC_2$  intersect at  $a = b = 1$ , which is to be expected. When  $b = 1$ , all households have the same income. Hence, if they live in two separated jurisdictions, they will only differ by their consumption of the public good, which depends on the population of their jurisdiction. When  $a = 1$ , both jurisdictions have the same population, hence the same consumption of the public good. Both incentive constraints are therefore satisfied at this point.

For all values of  $a < 1$ , the incentive constraint of the rich household is violated at the solution of program (19). That is, rich households always have an incentive to move from their jurisdiction and to go to that of the poor when there are more poor than rich. Indeed, when the number of rich is lower than that of the poor, the rich face a larger price of public good and, as a result, consume less public good than the poor. Since the poor also pay less tax than the rich, the rich then have incentive to join the poor jurisdiction. In the realistic case where the number of rich is smaller than that of the poor, the social planner will face a binding incentive constraint of the rich if it chooses a federal structure.

We can now compare the centralized and the federal jurisdiction structures allowing, in the later case, for the incentive constraint to bind or not.

The first comparison we make is between the centralized and the federal structures when the incentive constraint of the rich is satisfied at allocation  $fb2$ .

The social welfare at the second-best allocation with one jurisdiction is:

$$(25) \quad SW^{sb1} = n_1 U(z^{sb1}, w_1 - T^{sb1}) + n_2 U(z^{sb1}, w_2 - T^{sb1}).$$

while the social welfare at the two-jurisdiction federal allocation without incentive constraints is:

$$(26) \quad SW^{fb2} = n_1 U(z_1^{fb2}, w_1 - T_1^{fb2}) + n_2 U(z_2^{fb2}, w_2 - T_2^{fb2}).$$

After some manipulations (again performed by *Mathematica*), we can show that:

$$SW^{fb2} - SW^{sb1} \propto 4^{1+a} a^a (1 + ab)^{2+2a} - (1 + a)^{1+a} (1 + 2b + a(2 + b) - h(a, b))^{1+a} \times \\ (3 + 2a - 2b - ab + h(a, b)) (-1 - 2a + 2b + 3ab + h(a, b))^a$$

where  $a = n_1/n_2$ ,  $b = w_1/w_2$ , and:

$$h(a, b) = \sqrt{-8(1 + a)^2 b + (1 + 2b + a(2 + b))^2}.$$

The sign of  $SW^{fb2} - SW^{sb1}$  is the same as the sign of the expression on the right-hand side. This is quite a messy expression, but it does depend only upon the demographic and the interquartile ratios. This expression is plotted (in red color) on the  $(a, b)$ -space on Figure 5.

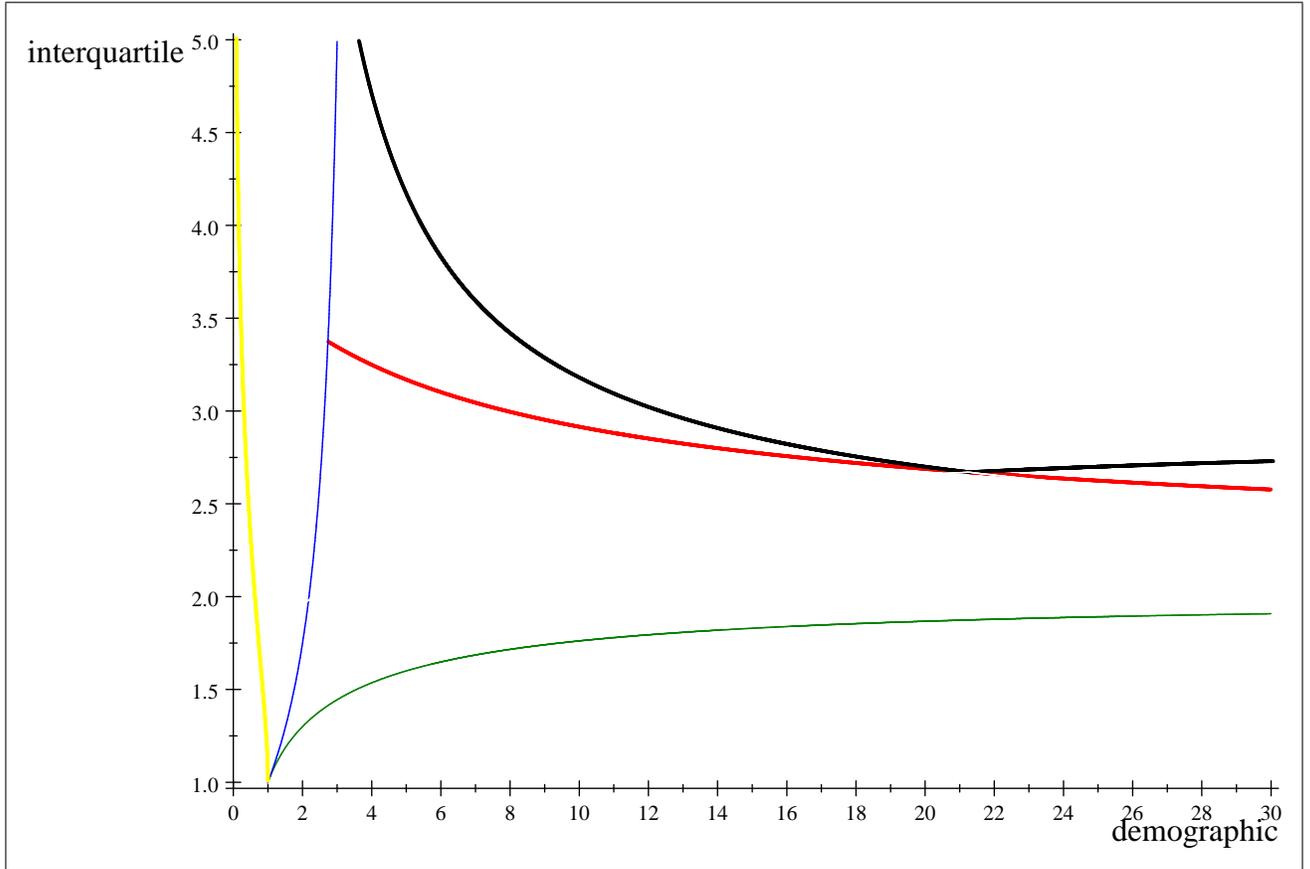


Figure 5: Distributional parameters that determine the choice of the optimal jurisdiction structure.

Over the region where both incentive constraints (in blue and in green as in Figure 4) are satisfied – that is, when the allocation  $fb2$  is the solution to the second-best problem – the set of combinations of values for the parameters  $(a, b)$  that lie above the red curve are those for which the utilitarian social planner prefers the federal structure to the centralized one. As can be seen, this preference arises when income disparities, as measured by the interquartile ratio  $w_1/w_2$ , are relatively large (so that there is a strong motive for income redistribution and adjustment of the private and public good provision to the heterogeneity of households willingness to pay for the public good). Analogously, the larger is the number of rich (relative to the number of poor), the more likely it is that the federal solution be favored by the utilitarian planner. In addition, Figure 5 illustrates the result of Lemma 3 since the red curve on the North East of which the federal solution dominates the centralized one never intercepts the boundary of the zone where the incentive constraint of the poor (in blue) is violated.

Figure 5 also uses the Pareto criterion to compare the two structures. The area above

the black curve indicates indeed the set of parameters values for which the federal structure chosen by the utilitarian social planner is unanimously preferred to the centralized solution chosen by this very same social planner. The area above the black curve is simply the set of all parameter value for which:

$$U(z^{sb1}, w_1 - T^{sb1}) \leq U(z_1^{fb2}, w_1 - T_1^{fb2})$$

$$U(z^{sb1}, w_2 - T^{sb1}) \leq U(z_2^{fb2}, w_2 - T_2^{fb2}),$$

so that both types of households prefer the *fb2* allocation to the *sb1* for all parameter values.

Unfortunately, the two-dimensional feature of the analysis in  $(a, b)$ -space is lost when the incentive constraint of the rich is binding at the solution of program (21). For we cannot plot a frontier at the “north-west” of the IC1 curve in Figure 5 that separates the set of combinations of demographic and interquartile ratios for which a federal jurisdiction is preferable to a centralized one from a social welfare view point. In this case, the choice between the two structures depends upon the four parameters  $(n_1, n_2, w_1, w_2)$ , and not only upon the ratios  $a = n_1/n_2$  and  $b = w_1/w_2$ .

We can nonetheless state results about the set of values of the parameters that would lead the utilitarian social planner to favor the federal system over the centralized one. The key to this part of the analysis is the study of the subsidy given by the rich to the poor mentioned in Lemma 4. As shown in Lemma 5, this subsidy is smaller when the IC constraint of the rich is binding at the optimal allocation in a federal structure than when it is not binding. As it turns out, it is even possible for this subsidy be negative when the IC constraint of the rich is binding.

Indeed, consider fixing  $s$  in the program (21) and, for such a subsidy, solving the program for  $z_2$  and  $T_2$  using the two constraints. Denote this partial solution  $z_2(s)$  and  $T_2(s)$ . Now, the optimal subsidy  $\hat{s}$  solves:

$$(27) \quad \max_s n_1 V(1/n_1, 1, w_1 - s) + n_2 U(z_2(s), w_2 - T_2(s)).$$

and satisfies therefore the first order condition:

$$-n_1 V_R(1/n_1, 1, w_1 - \hat{s}) + n_2 \left( \frac{1}{z_2(s)} \frac{dz_2(\hat{s})}{ds} - \frac{1}{w_2 - T_2(s)} \frac{dT_2(\hat{s})}{ds} \right) = 0.$$

There are values of  $(w_1, w_2, n_1, n_2)$  for which this condition is solved for  $\hat{s} = 0$ . It is not difficult (thanks to *Mathematica*) to see that the set of parameters for which this happens depends only upon  $a$  and  $b$ . This set is in fact described by the equality:

$$(28) \quad b = \frac{-\sqrt{1-a} - 2a}{-2a + a\sqrt{1-a}}.$$

On Figure 5, we have plotted (in yellow) the curve described by equation (28). It is easy to see that  $b$  goes to infinity when  $a$  becomes negligible. For all values of  $a$  and  $b$  to the left (right) of this line, the optimal subsidy is negative (positive). When  $a$  is very small, there are very few rich households. Furthermore, when  $a$  is small, the implicit price for the public good for the rich is very large relative to that for the poor. Hence, the amount of public good provided for the rich is small. This implies that the optimal subsidy from the rich to the poor must be negative in order to prevent the rich from moving to the poor jurisdiction.

As it turns out, if the optimal subsidy chosen by the utilitarian social planner for the federal solution is 0, then it is always better to choose the centralized provision. We state this formally as follows.

**Lemma 6** *Suppose the solution to program (27) is  $\hat{s} = 0$ . Then the utilitarian social planner prefers the centralized jurisdiction structure to the federal one.*

**Proof.** *Assume that*

$$b = \frac{-\sqrt{1-a} - 2a}{-2a + a\sqrt{1-a}}.$$

Call the solution to program (21)  $SW^{sb2}$ .

$$SW^{sb2} = n_1 U(\hat{z}_1, w_1 - \hat{T}_1) + n_2 U(\hat{z}_2, w_2 - \hat{T}_2),$$

where  $\hat{z}_1 = n_1 w_1 / 2$ ,  $\hat{T}_1 = w_1 / 2$ ,  $\hat{z}_2 = z_2(0)$ ,  $\hat{T}_2 = T_2(0)$ . When we replace  $w_1$  by  $bw_2$  and  $n_1$  by  $an_2$ , we can show that

$$\begin{aligned} \frac{SW^{sb2}}{n_2} &= -a \log[4an_2] + 2a \log \left[ -\frac{(2a+\sqrt{1-a})}{\sqrt{1-a}-2} n_2 w_2 \right] + \log \left[ \frac{-1+\sqrt{1-a}-a}{2a(\sqrt{1-a}-2)} w_2 \right] + \\ &\quad \log \left[ -\frac{(-3+3\sqrt{1-a}+a)(2a+\sqrt{1-a})}{2a(\sqrt{1-a}-2)^2} n_2 w_2 \right] \end{aligned}$$

and that

$$\begin{aligned} \frac{SW^{sb1}}{n_2} &= a \log \left[ \frac{(\sqrt{-a^3-2(\sqrt{1-a}-1)a^2+(2\sqrt{1-a}-1)a+1+a\sqrt{1-a}+a+\sqrt{1-a}})w_2}{2a(2-\sqrt{1-a})} \right] + \\ &\quad \log \left[ \frac{(\sqrt{-a^3-2(\sqrt{1-a}-1)a^2+(2\sqrt{1-a}-1)a+1-a\sqrt{1-a}+a-\sqrt{1-a}})w_2}{2a(2-\sqrt{1-a})} \right] + \\ &\quad (a+1) \log \left[ \frac{-(a+1)(\sqrt{-a^3-2(\sqrt{1-a}-1)a^2+(2\sqrt{1-a}-1)a+1+a\sqrt{1-a}-3a-\sqrt{1-a}})n_2 w_2}{2a(2-\sqrt{1-a})} \right]. \end{aligned}$$

Using Mathematica, we compute the difference  $SW^{sb2}/n_2 - SW^{sb1}/n_2$ . We can show that the resulting expression is independent of  $w_2$  and  $n_2$ . We then set without loss of generality

$w_2 = n_2 = 1$ . This implies that the difference in welfare only depends on the ratio  $a$ . We can plot it over the interval  $a \in ]0, 1]$ . The graph demonstrates that the expression is negative over this domain of  $a$ . Consequently, the one-jurisdiction structure yields higher social welfare than the two-jurisdiction structure does. ■

When the optimal subsidy is zero in the two-jurisdiction structure, there is no redistribution from the rich to the poor. Furthermore, there is duplication in the production of the public good. It is then socially optimal to have only one jurisdiction. The preference for a federal jurisdiction structure over a centralized one can only appear when it is optimal for the central planner in a federal structure to redistribute tax revenue across jurisdictions.

The optimal subsidy can also be examined along the  $IC_1$  curve, where the incentive constraint of the rich is weakly binding. Along this curve, the allocation  $sb2$  coincides with the allocation  $fb2$ . The optimal subsidy is then  $\hat{s} = s^{fb2}$ . As illustrated in Figure 5, there exists a point on the curve above which the two-jurisdiction structure socially dominates the one-jurisdiction structure. This occurs for a value of  $b$  that is sufficiently large.

From these two observations, one concludes that the region of social indifference between the federal and the centralized structures is located somewhere between the line for which  $\hat{s} = 0$  and that for  $IC_1$ . This suggests that for any given demographic ratio  $a$  however small, there exists a high enough interquartile ratio  $b$  above which the social planner will always favour a federal structure over a centralized one. Unfortunately, Figure 5 does not provide us with any clue about whether or not this intuition is correct because what happens between the  $IC_1$  curve and the yellow curve does not depend only upon the ratios  $a$  and  $b$ . It depends upon the four parameters  $(w_1, w_2, n_1, n_2)$ . However, as shown in the following lemma, we can prove indirectly that this intuition is correct.

**Lemma 7** *For any demographic ratio  $a$  such that the constraint  $IC_1$  is binding, there exists a value for the interquartile ratio  $b$  at which the utilitarian social planner prefers the two-jurisdiction structure.*

**Proof.** *One needs to compute the difference  $SW^{sb2} - SW^{sb1}$ . The difficulty lies in characterizing the optimal level of subsidy  $\hat{s}$  in the allocation  $sb2$  where the  $IC_1$  constraints binds. Instead of doing so, we compute this difference using the subsidy  $s^{fb2}$ . Since this subsidy is suboptimal when  $IC_1$  is binds, this computation underestimates the difference  $SW^{sb2} - SW^{sb1}$ .*

*Using Mathematica, we get*

$$SW_f^{sb2}/n_2 - SW^{sb1}/n_2 = 2a \log \left[ \frac{an_2w_2(ab+1)}{a+1} \right] - a \log[4an_2] + \log \left[ \frac{n_2w_2 \left( \sqrt{a^3(-b^2)+a^2(4b^2-6b+1)+a(4b^2-2b-1)+b^2+a-b+2} \right) \left( \sqrt{a^3(-b^2)+a^2(4b^2-6b+1)+a(4b^2-2b-1)+b^2-2ab+a-b} \right)}{(a+1) \left( -\sqrt{(a(b+2)+2b+1)^2-8b(a+1)^2+ab-2a+2b-3} \right)} \right] -$$

$$(a + 1) \log \left[ \frac{1}{4} n_2 w_2 \left( -\sqrt{(a(b+2) + 2b + 1)^2 - 8b(a+1)^2 + ab + 2a + 2b + 1} \right) \right] - a \log \left[ \frac{w_2 \left( \sqrt{(a(b+2) + 2b + 1)^2 - 8b(a+1)^2 + 3ab - 2a + 2b - 1} \right)}{4(a+1)} \right]$$

where  $SW_f^{sb2}$  refers to the second-best social welfare with the suboptimal subsidy  $s^{fb2}$ . We can show that the sign of this expression only depends on the ratios  $a$  and  $b$ , that is, it is independent of  $n_2$  and  $w_2$ . We then fix the ratio  $a$  and take the limit of  $SW_f^{sb2} - SW^{sb1}$  when  $b$  goes to infinity. We can show that this difference converges to infinity. This implies that, for any demographic ratio  $a$ , there exists a high enough interquartile ratio  $b$  such that the two-jurisdiction structure is socially preferable. ■

## 4 Conclusion

The conclusion of this paper holds in one sentence that can be stated, after Wallace Oates (1972)'s quotation recalled above, as follows. *Even when the "cost-savings from the centralized provision of a public good" is maximal, the level of (utilitarian) welfare may be at least as high (and typically higher) if Pareto-efficient levels of consumption are provided in each jurisdiction than if any single, uniform level of consumption is maintained.* In the model considered in this paper, we have indeed considered the most extreme form of "cost-savings from a centralized provision of a public good" that one can imagine: that of a non-rival public good with no congestion. We have shown in such a setting that if the public authority is imperfectly informed about the willingness to pay of its citizens, it may find optimal to organize the provision of the public good into several distinct jurisdictions rather than in a single one, even at the high cost of provision that results from the replication in several jurisdictions of the cost of providing the very same non-rival public good. The reason for this preference comes from the information that the social planner obtains from having individuals "choosing" their jurisdiction of residence and, therefore, expressing their preferences for their favorite tax and public good packages. In a single "uniform level of consumption" centralized jurisdiction structure, individuals must all pay the same taxes and consume the same amount of public good because under private information, the public authority is incapable to individualize those. In a federal structure, the central government may achieve a better targeting of the packages to the tastes of its citizens. Our analysis shows that the benefit of this better targeting may outweigh the cost of unnecessarily replicating the provision of a non-rival public good into several jurisdictions. We have shown more specifically that the superiority of a federal provision of a public good over a centralized one is all the more likely as the heterogeneity in the population is large. Using the somewhat specific case of a two-type population of unequally wealthy households, we have shown in particular that federal provision tends to dominate the centralized one when the wealth differences between the rich and the poor is large, and when the ratio of the rich over the poor is also large. As the fraction of rich people in the population becomes small, the case in favor of a federal

solution vanishes even though, for any fraction of rich in the population however how small, it is always possible to find a sufficiently large wealth discrepancy between the rich and the poor that would make federal provision preferable to the centralized one.

While we believe this analysis, and the strong case that it makes in favor of a federal provision of public good, to be of some interest, we are aware of many of its limitations. For one thing, we have limited our attention to households who differ only in contributive capacities (wealth), and who have the same preferences for the private and the public good. An alternative would have been to consider the case of households with the same wealth, but with different tastes for the public good. We conjecture that one would obtain very similar conclusions to the one obtained here in this case. A more realistic, but analytically much more challenging, situation would have been that where households differ both in their wealth and their preferences.

Another obvious limitation is the restriction of a large part of our analysis to a two-type setting. Yet the difficulty of the problem of the optimal choice of a jurisdiction structure under private information was already significant. Imagine for instance a three-type setting. Then, one would need to consider a large number of possibilities: a centralized solution with all households' types pooled into a unique jurisdiction, a "completely decentralized" setting where each type forms a jurisdiction on its own, and two "mixed federal" structures (one where the rich are pooled with the "middle" in one jurisdiction and the poor are left alone and the other where the rich stay alone and the "middle" and the poor form a jurisdiction). Thanks to the single crossing property, these are the only jurisdiction structures that would satisfy the incentive compatibility constraints. But the analysis of all of these cases, with all the varying possibilities for the incentive constraints to bind or not, would have taken us somewhat too far for a first pass on the subject. But we plan to explore more fully this  $k$ -type problem in our future work.

## References

- ARONSSON, T., AND S. BLOMQUIST (2008): "Redistribution and Provision of Public Goods in an Economic Federation," *Journal of Public Economic Theory*, 10, 125–143.
- BOADWAY, R., AND F. FLATTERS (1982): "Efficiency and equalization payments in a federal system of government: A synthesis and extension of recent results," *Canadian Journal of Economics*, 15, 613–633.
- BORDIGNON, M., P. MANASSE, AND G. TABELLINI (2001): "Optimal Redistribution under Asymmetric Information," *American Economic Review*, 91, 709–723.
- BREUILLÉ, M.-L., AND R. J. GARY-BOBO (2007): "Sharing Budgetary Austerity under

- Free Mobility and Asymmetric Information : an Optimal Regulation Approach to Fiscal Federalism,” *Journal of Public Economics*, 91, 1177–1196.
- BUCHANAN, J. M. (1950): “Federalism and Fiscal Equity,” *American Economic Review*, 40, 583–599.
- CORNES, R. C., AND E. C. SILVA (2000): “Local Public Goods, Risk Sharing and Private Information in Federal Systems,” *Journal of Urban Economics*, 47, 39–60.
- (2002): “Local Public Goods, Interegional Transfers and Private Information,” *European Economic Review*, 46, 329–356.
- FLATTERS, F., V. HENDERSON, AND P. MIESZCOWSKI (1974): “Public goods, efficiency, and regional fiscal equalization,” *Journal of Public Economics*, 3, 99–112.
- GRAVEL, N., AND M. POITEVIN (2006): “The Progressivity of Equalization Payments in Federations,” *Journal of Public Economics*, 90, 1725–1743.
- GREENBERG, J. (1983): “Local Public Goods with Mobility: Existence and Optimality of a General Equilibrium,” *Journal of Economic Theory*, 16, 137–150.
- MIRRELEES, J. A. (1971): “An Exploration in the Theory of Optimal Income Taxation,” *Review of Economic Studies*, 38, 175–208.
- (1976): “Optimal Tax Theory: A Synthesis,” *Journal of Public Economics*, 6, 327–358.
- (1986): “The Theory of Optimal Income Taxation,” in *Handbook of Mathematical Economics*, ed. by K. J. Arrow, and M. D. Intriligator. North Holland, Amsterdam.
- OATES, W. E. (1972): *Fiscal Federalism*. Harcourt Brace Jovanovich, New York.
- (1999): “An Essay on Fiscal Federalism,” *Journal of the Economic Literature*, 37, 1120–1149.
- TIEBOUT, C. M. (1956): “A Pure Theory of Local Expenditures,” *Journal of Political Economy*, 64, 416–424.
- WILDASIN, A. (1987): “Theoretical Analysis of Local Public Economics,” in *Handbook of Regional and Urban Economics*, ed. by E. Mills. North Holland.