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Fluctuation-dissipation relations of a tunnel junction driven by a quantum circuit

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We derive fluctuation-dissipation relations for a tunnel junction driven by a high impedance microwave resonator, displaying strong quantum fluctuations. We find that the fluctuation-dissipation relations derived for classical forces hold, provided the effect of the circuit's quantum fluctuations is incorporated into a modified non-linear $I(V)$ curve. We also demonstrate that all quantities measured under a coherent time dependent bias can be reconstructed from their dc counterpart with a photo-assisted tunneling relation. We confirm these predictions by implementing the circuit and measuring the dc current through the junction, its high frequency admittance and its current noise at the frequency of the resonator.

It is textbook knowledge that the fluctuations of any physical system held at equilibrium are proportional to its dissipative linear response [1, 2]. This universal fluctuation-dissipation theorem (FDT) is very useful: it allows predicting the fluctuations (noise) of any passive system from just a linear response measurement, which is technically simpler. Moreover, measuring both quantities implements a primary thermometer, and enables to reach metrological accuracy [3]. The FDT provides also a physical picture on the origin of macroscopic irreversibility: it shows that dissipation within a Hamiltonian system is made possible by the system's ability to dissolve the incoming excitation within its internal degrees of freedom through the system's fluctuations of both quantum and thermal origin. Turned into a rigorous formalism as in [4], such picture provides an efficient way to model dissipation in macroscopic quantum systems. Despite these successes, the standard FDT theorem relates the fluctuations of a quantum system to its response to a classical drive, ignoring the quantum fluctuations of the driving forces. In the particular case of a quantum electrical conductor connected to an electromagnetic environment, these quantum fluctuations are known to trigger inelastic electron tunneling, strongly modifying the charge transfer dynamics through the conductor, an effect known as Environmental or Dynamical Coulomb Blockade [5]. This raises the question of the existence of FDT relations for a quantum conductor coupled to an environment displaying strong quantum fluctuations. We address this question in this article, by deriving and testing experimentally stationary and time dependent fluctuation-dissipation relations for a normal tunnel junction biased through the elementary building block of the general description of any linear driving circuit, i.e. an harmonic oscillator, taking its quantum fluctuations into account. We find that once the effect of the biasing circuit's quantum fluctuations are incorporated into the non-linear $I(V)$ curve, the finite frequency current fluctuations and the admittance of the junction can still be recast in terms of the non-linear $I(V)$ curve,

extending the validity of expressions found for weakly coupled conductors driven by classical fields [6–8]. Our formalism also allows us to address finite time dependent drives, for which we show that the current fluctuations, the admittance and the dc current measured under a time dependent ac bias, have the same dependence and are all linked to their stationary value by photo-assisted tunneling relations [9–11]. The experiment consists in embedding a tunnel junction in a high impedance microwave resonator. The junction's dc conductance, its finite frequency admittance and the microwave power emitted by its current fluctuations are found in good agreement with predictions.

The circuit we consider theoretically is represented by Fig. 1-a) a tunnel junction of conductance G_T is embedded in an LC circuit, of resonant frequency $\nu_0 = 1/(2\pi\sqrt{LC})$, and characteristic impedance $Z_C = \sqrt{L/C}$, where C is the oscillator's capacitance, and L its inductance. We assume that the system is at temperature T and that the junction is DC biased at a voltage V , and an additional microwave source produces an RF voltage at the frequency of the resonator with a classical amplitude, i.e. neglecting quantum fluctuations, $V_{ac} \cos(2\pi\nu_0 t)$. We describe the resonator by a 'thermal coherent state' density matrix $\rho = D(i\alpha/2r)\rho_T D(i\alpha/2r)^\dagger$ [12], where $r = \pi\sqrt{L/C}/R_K$ ($R_K = h/e^2 = 25.8$ k Ω) characterizes the coupling between the oscillator and the tunnel junction, ρ_T is the usual density thermal density matrix and $D(i\alpha/2r) = \exp[i\alpha(a + a^\dagger)/2r]$ is the displacement operator associated to an amplitude $i\alpha/2r$, with $\alpha = eV_{ac}/h\nu_0$. We evaluate the average time dependent current and current fluctuations to first order in the tunnel coupling [13]. Taking the time average of the corresponding expressions, we find that the time average current $\overline{I(V_{dc}, \alpha)}$ and current fluctuation spectral density $\overline{S_I(V_{dc}, \alpha)}$ are obtained as copies of the same quantity measured under a stationary voltage bias, translated by the different harmonics of the irradiation frequency and weighted by Bessel functions, following photo-assisted tunneling relations [9–11] :

$$\bar{I}(V_{dc}, \alpha) = \sum_k J_k(\alpha)^2 I(V_{dc} - kh\nu_0/e) \quad (1)$$

$$\bar{S}_I(\nu, V_{dc}, \alpha) = \sum_k J_k(\alpha)^2 S_I(\nu, V_{dc} - kh\nu_0/e) \quad (2)$$

Of course, taking the time average one loses track of time dependence of these quantities. Such time dependence can be retained to calculate the Fourier transform of I and S_I , obtaining similar relations. The junction's conductance $\text{Re}[Y(V_{dc}, \nu, \alpha)]$ is derived by computing the time dependence of the current operator in response to an infinitesimal drive in a small impedance additional fictitious mode at frequency ν [13]. We find that $\text{Re}[Y(V_{dc}, \nu, \alpha)]$ obeys a photo-assisted tunneling formula analog to Eqs. 1-2 and that $\text{Re}[Y(V_{dc}, \nu, \alpha)]$ is given by a Rogovin-Scalapino like formula[6–8] :

$$\text{Re}[\bar{Y}(V_{dc}, \nu, \alpha)] = e \frac{\bar{I}(V_{dc} + h\nu/e, \alpha) - \bar{I}(V_{dc} - h\nu/e, \alpha)}{2h\nu} \quad (3)$$

while its imaginary part follows from Kramers-Kronig relations. The same can be done for the the noise spectral density, from which we recover the noise susceptibility derived in [14] for the symmetrized spectral density of the current fluctuations $((S_I(\nu, \nu_0) + S_I(-\nu, \nu_0))/2)$, using the Landauer-Büttiker formalism.

For $\alpha = 0$, the dc current $I(V)$ [5] and current noise [15] read:

$$I(V_{dc}) = \frac{G_T}{e} [\gamma * P(eV_{dc}) - \gamma * P(-eV_{dc})],$$

$$S_I(\nu, V_{dc}) = 2G_T [\gamma * P(eV_{dc} - h\nu) + \gamma * P(-h\nu - eV_{dc})],$$

where $\gamma * P(E) = \int d\varepsilon' \gamma(\varepsilon') P(E - \varepsilon')$ with $P(\varepsilon)$ the probability density for a tunneling electron to emit the energy ε in form of photons into the impedance [5], with $\gamma(\varepsilon) = \int d\varepsilon' f(\varepsilon') [1 - f(\varepsilon' + \varepsilon)] = \varepsilon / (1 - e^{-\varepsilon/k_B T})$, and with f the Fermi function. Combining these expressions with the conductance Eq.3, we obtain a Kubo-like relation:

$$\bar{S}_I(-\nu, V_{dc}, \alpha) - \bar{S}_I(\nu, V_{dc}, \alpha) = 2h\nu \text{Re} \bar{Y}(V_{dc}, \nu, \alpha). \quad (4)$$

Exploiting the detailed balance symmetry of $\gamma(E)$ and $P(E)$, we get:

$$\bar{S}_I(\nu, V_{dc}, \alpha) = \frac{e\bar{I}(V_{dc} - h\nu, \alpha)}{1 - e^{-\beta(eV_{dc} - h\nu)}} + \frac{e\bar{I}(V_{dc} + h\nu, \alpha)}{e^{-\beta(-eV_{dc} - h\nu)} - 1} \quad (5)$$

Therefore, the results found in the literature [6–8, 16] for tunnel elements driven by classical forces can be extended to quantum circuits, provided one incorporates the effect of the quantum fluctuations of the driving circuit into a "renormalized" non-linear $I(V)$ curve. Note that in the case of dc bias, a relation analog to Eq. 5

was derived in [17] for the symmetrized spectral density of the current fluctuations.

To validate the above predictions, we couple a high impedance tunnel junction to a microwave resonator with a tunable impedance $Z_C \sim 1 - 2 \text{ k}\Omega$, high enough to induce significant modifications of tunnel junction conductance and of its shot noise spectral density [15]. We then measure both the dc conductance, the finite frequency admittance and the finite frequency quantum shot noise of the junction as a function of its dc voltage bias and in the presence of an additional microwave tone inducing a coherent state in the resonator, using a cryogenic low noise amplifier. Our set-up allows the measurement of the excess power noise density, and thus gives only access to the emission noise spectral density, unlike on-chip quantum detectors such as SIS detectors which have allowed to measure both the emission and absorption noise of their linear environment [18]. Coupling two such detectors via a low impedance circuits, displaying negligible quantum fluctuations, has also allowed to measure their emission noise[18], found in agreement with Eq.5, their admittance [19], and to access their absorption noise [16].

The experimental set-up represented in Fig.1-b): a $100 \times 100 \text{ nm}^2$ tunnel junction with tunnel resistance $G_T^{-1} = 270 \text{ k}\Omega$ is embedded in an on-chip $\lambda/4$ coplanar resonator of resonant frequency ν_0 , whose inner conductor is made of an array of identical and equally spaced Al/AlOx/Al SQUIDs [15, 20]. The SQUIDs' Josephson inductance outranges the electroamgentic inductance by more than two orders of magnitude, increasing the resonator characteristic impedance Z_C above $1 \text{ k}\Omega$. Applying a flux through the SQUIDs and allows to decrease ν_0 while increasing Z_C . The 6 GHz maximum frequency of the resonator ensures $k_B T \ll h\nu_0$ at the 20 mK temperature of the experiment, so that thermal fluctuations do not blur Coulomb Blockade effects. We ensure that the SQUIDs behave as linear inductors by keeping the maximal total current through the junctions much lower than their critical current. In addition, a $30 \times 50 \times 0.3 \text{ }\mu\text{m}^3$ gold patch is inserted between the tunnel junction and the SQUID array in order to evacuate the Joule power dissipated at the tunnel junction via electron-phonon coupling. The chip is connected to the biasing and measurement circuits through a commercial 50Ω matched bias tee. The inductive (low frequency) path is used both to bias the sample through a cold $13 \text{ M}\Omega$ resistor, and to measure the dc voltage across the tunnel junction and its conductance $G(V_{dc}, \alpha)$. The capacitive (RF) path is connected to a 20 dB directional coupler, which allows us to (i) shine microwave power on the tunnel junction through the weakly coupled port while still (ii) measuring the microwave signals emitted/reflected by the sample via the well transmitted port, itself connected to a cryogenic amplifier with a $\sim 2.5 \text{ K}$ noise temperature in the 4-8 GHz bandwidth. Two cascaded circulators (only one being represented) divert the back-action noise of the ampli-

fier onto thermalized 50Ω matched loads, which re-emit to the sample a blackbody radiation only at the coldest temperature, ensuring a negligible parasitic photon occupation of the resonator at GHz frequencies.

The dc-conductance of the junction is obtained by driving a sinusoidal 5nA_{RMS} current modulation at 12 Hz through the low frequency port, and measuring the induced voltage drop across the sample via homodyne detection. At high bias voltage the tunnel resistance is $G_T^{-1} = 270\text{k}\Omega$. The $I(V_{dc})$ curve is then calculated by numerical integration of the differential conductance. The power emitted into the microwave detection chain by the tunnel junction at high bias, where Coulomb Blockade effects are negligible, so that $S_I(\nu) \simeq 2eI$ is in good agreement with our design parameters [20]. Moreover, the measured variations of the dc conductance with bias voltage, shown in Fig. 2, were found in good agreement with the $P(E)$ -theory prediction based on our microwave design of the resonator [15]. This good agreement allows us to use the designed impedance seen by the junction, $Z(\nu)$ to extract the admittance of the tunnel junction from the microwave signal reflected by the sample.

More specifically, we inject a small coherent tone delivered by a Vectorial Network Analyser (VNA) at the resonant frequency tuned to $\nu_0 = 4.1\text{GHz}$, into the resonator through the -20 dB port of the directional coupler. The reflected signal is then amplified and connected to the input port of the VNA. The ~ -140 dBm excitation signal amplitude yields a $\sim 1.4\mu\text{V}_{\text{RMS}}$ ac voltage on the sample, corresponding to $\alpha \simeq 0.1$, making photo-assisted tunneling negligible, thus assuring a linear response. Due to the finite ~ 15 dB directivity of the coupler, a coherent leak adds up to the signal, so that the total transmission coefficient can be expressed as $S_{\text{out,in}}(\nu_0) = G(\Gamma + F)$ where Γ stands for the sample reflection coefficient, F the coherent leak transmission, and G stands for the total gain of the chain (including the attenuation of the various microwave components). The gain G and the leakage coefficient F can be calibrated by two measurements of transmission $S_{\text{out,in}}$: (i) we first apply a large dc bias $eV \gg h\nu_0$ to the sample and assume that the corresponding junction admittance is given by the tunnel conductance, $\lim_{V \rightarrow \infty} Y(V, \nu_0) = G_T$ [13], and then (ii) detune the resonator frequency to $\sim 3.7\text{GHz}$, ensuring that $\Gamma \simeq -1$ [13]. We then measure $S_{21}(\nu_0)$ as a function of the bias voltage, from which we extract the variations of the finite frequency admittance. As shown by Fig. 2, the data are in good agreement with our theoretical predictions: the junction's conductance $\text{Re}[Y(V, \nu_0, \alpha = 0)]$ is well described Eq. 3 and its susceptance $\text{Im}[Y(V, \nu_0, \alpha = 0)]$ is negligible.

In a second experiment, we measure $\partial S_I(V, \nu_0)/\partial V$, the derivative of the shot noise spectral density at the resonant frequency with respect to the bias, using the technique reported in [15]: the output of the amplifying

chain is connected to a 180 MHz room temperature adjustable filter centered around ν_0 and to a quadratic detector whose output voltage is proportional to the noise power. We perform an homodyne detection of the variations of the system noise temperature induced by the 12 Hz modulation used to measure the conductance. Due to the impedance mismatch between the admittance of the sample and the impedance of the rf-detection chain $|Y(V, \nu)Z(\nu)| \ll 1$, the emitted power density reads $S_I(V, \nu) \text{Re} Z(\nu) / |1 + Y(V, \nu)Z(\nu)|^2$. The voltage dependence of the coupling coefficient arising from Coulomb blockade, $\sim 1\%$, can be neglected, so that we extract directly $\partial S_I(V, \nu_0)/\partial V$ from the noise temperature modulations. As the conductance, $\partial S_I(V, \nu_0)/\partial V$ exhibits step-like singularities for voltages $eV = nh\nu_0$, rounded by temperature and the fluctuation-dissipation like relation eq. 5 is in excellent agreement with experimental results.

For the photo-assisted experiments, a rf drive, ~ -120 dBm at $\nu_0 - 5$ MHz, is superposed to the VNA signal using a room temperature -10dB directional coupler, as shown by Fig. 2. The induced coherent state amplitude α at the input of the tunnel junction can be estimated from the independently calibrated attenuation of the feed line, and from the voltage divider $[1 + Z(\nu_0)Y(V, \nu_0)]^{-1}$. Furthermore, the experimental $G(V)$ data are well reproduced by the Tien-Gordon relation 1, shown in the right panel of Fig. 3, using α as a fitting parameter. The extracted value $\alpha = 1.15$, is found in agreement with the estimated value within 15% (1 dB). Note that since $Z(\nu_0)Y(V, \nu_0) \ll 1$, the variations of $Y(V, \nu_0)$ with V induce negligible variations of α with the dc bias. The deriving frequency is chosen close, but different than the VNA frequency, so that $Y(\nu \simeq \nu_0, \alpha, V_{dc})$ can still be measured. Here again, the Photo-assisted relation and hence Eq. 3 are well obeyed.

For the emission shot noise power measurement, we get rid of the driving tone parasitic signal by implementing a band rejection filter. We mix the total signal with a reference at the driving frequency, and low pass filter the down converted signal with a 80 MHz low pass filter, which is then fed to the quadratic detector. Thanks to the 1 MHz low frequency cut-off of the quadratic detector, its output is insensitive to the driving tone reflected signal. The results, shown in the right panel of Fig. 3 are found to follow the photo-assisted relation Eq. 2, so that our FDT relation Eq. 5 also holds for a time dependent bias.

In conclusion, we have shown theoretically and experimentally that in the presence of strong quantum fluctuations of the driving voltage, the finite frequency admittance and current fluctuations of a tunnel element follow fluctuation-dissipation relations derived for classical drives. This also holds in the presence of a time dependent bias, where photo-assisted tunneling expressions are also valid. Our derivation relies on the fact that no

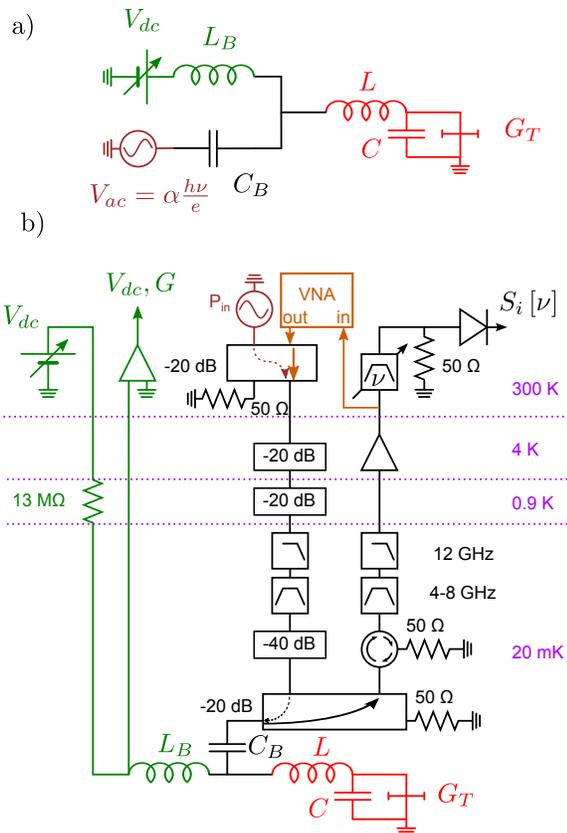


Figure 1. a) Model system: a small conductance tunnel junction G_T is embedded in a resonator of frequency $\nu_0 = 1/\sqrt{LC}$ and characteristic impedance $Z_C = \sqrt{L/C}$ high enough for its zero-point phase fluctuations to alter the junction's transport properties. The system is voltage connected to a dc (rf) voltage source V_{dc} (V_{ac}) through a large inductance (capacitance). b) Experimental set-up: A normal tunnel junction, cooled at 20 mK by a dilution refrigerator, is connected to a 50Ω -line through a high-impedance $\lambda/4$ -resonator, which inner conductor consists in a serial SQUID array. The resonant frequency ν_0 , can be tuned between 4 and 6 GHz by varying the magnetic field through the SQUIDS. The high impedance of the resonance ($Z_C \sim 1 - 2 \text{ k}\Omega$) is strong enough to induce Dynamical Coulomb Blockade non linearities in the tunnel junction transport properties. The resonator is connected to a bias-Tee, which inductive port allows to dc bias the junction and to measure its low frequency conductance. The RF port allows to shine microwave onto the resonator, and to measure the microwave signals emitted/reflected by the sample.

memory effect occur neither in the electromagnetic environment nor in the quantum conductor, and that both follow a detailed balance relation. Our experimental approach is very general and can be readily exploited to test fluctuation-dissipation relations for more complex quantum conductors, for instance by testing them well beyond the weak coupling limit in Quantum Point Contacts [15, 21–24], where DCB was recently demonstrated to bear a deep connection to the physics of impurities in

Luttinger liquids [25], or in quantum conductors having rich internal dynamics such as Quantum Dots [8, 26, 27].

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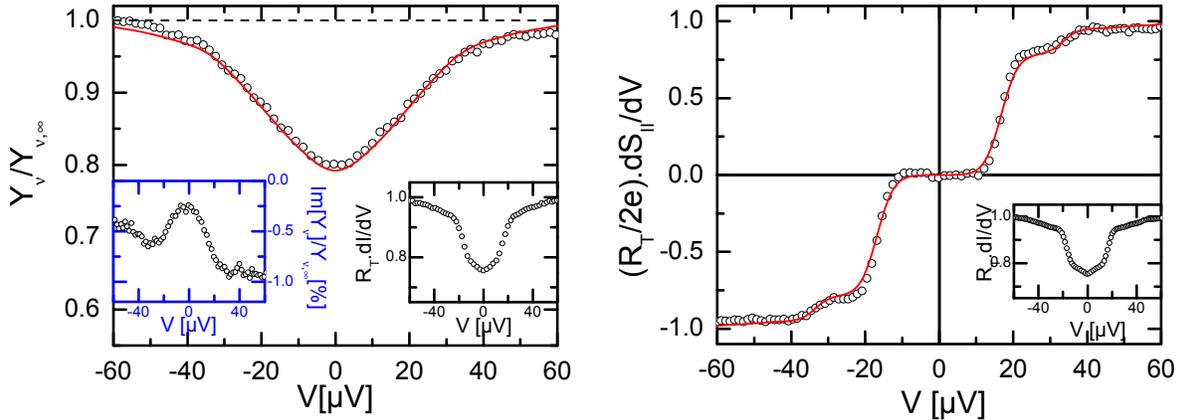


Figure 2. DC biased junction. (Black Insets) The dc-conductance is non-linear, its step-like features are characteristic of the DCB by a single mode, they are smoothed by temperature $T = 20$ mK. *Left panel:* (Black circles) The admittance is computed from reflection measurement at the frequency of the resonator $\nu_0 = 4.1$ GHz, as a function of the dc-voltage bias. (Blue Inset) Estimated susceptance of the sample. (Red curve) Theoretical curve computed from the $I(V)$ curve using equation 3. *Right panel:* (Black circles) Derivative of the noise with respect to dc bias, measured at resonant frequency, $\nu_0 = 4$ GHz, as a function of dc-voltage bias. Step-like singularities at $eV = \pm h\nu_0$ and $eV = \pm 2h\nu_0$ are characteristic of Coulomb blockade of shot noise with a single mode of high impedance. (Red curve) Curve computed from the dc-conductance using the equation 5.

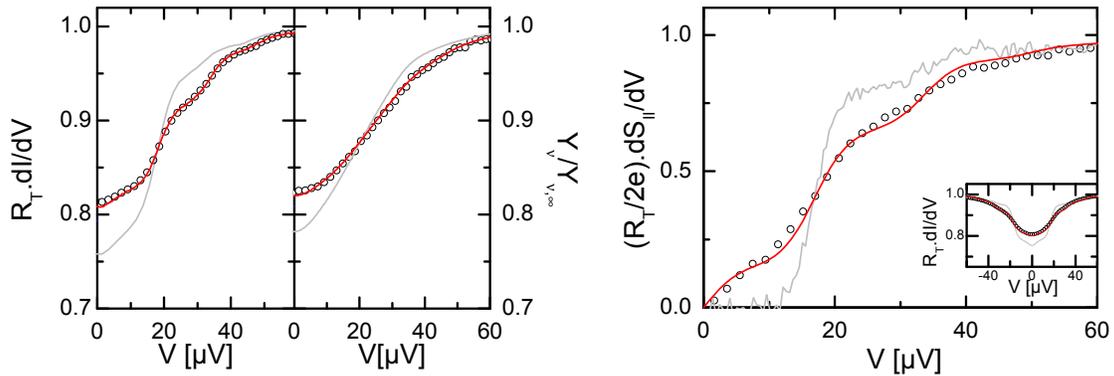


Figure 3. RF driven junction: Grey lines show the junction's conductance at low frequency (a), at the resonator's frequency $\nu_0 = 4.1$ GHz (b) and the derivative of the emission current noise with respect to bias voltage (c), measured as a function in the absence of RF drive. Black circle show the same data measured under RF irradiation ($\alpha = 1.15$). Red curves show the Photo-assisted tunneling predictions.

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