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Abstract: A robust control for a stepper motor with no position nor velocity sensors and only needing current and voltage measurements is designed. Second order sliding mode based observers are realized to estimate both rotor angular position and velocity. Moreover, a robust control law, which is also based on second order sliding modes and which uses the estimates of the observer, is designed. The stability of the observer based control loop is discussed. The results obtained in simulations indicate the usefulness and the robustness of the method.

Keywords: stepping motor, observer, control, stability, robustness, sliding mode.

1. INTRODUCTION

Stepper motors are widely used in industry and in consumer goods. In order to design a robust control law for such a motor, the knowledge of the angular position and velocity of the motor is required. However, in many applications, the implementation of mechanical sensors measuring these signals is not desirable. This may be due to the space they require, the wires, their fragility, and also because of non negligible additional cost. For all these reasons, during the last years there has been an increasing interest in designing observers and control laws for the stepper motor which rely on measurements of electrical variables only.

One can find work in the literature which deals with the design of a control supposing that the rotor position is known and using a speed observer (Chiasson et al. [1993], Defoort et al. [2009], Nollet et al. [2008]). Recently, some other work deals with "sensorless" control based on different velocity and position estimation methods, as in Chen et al. [2000], Poulain et al. [2008], and Xu et al. [2007].

The subject of the present work is resumed in Fig. 1. An observer based on second order sliding modes is presented, which provides accurate estimates of the velocity and the position of the stepper motor and uses electric variables measurements only. Then, a robust control law using the estimated variables, based on second order sliding modes and on the notion of flatness (see Fliess et al. [1992]) is designed in order to achieve trajectory tracking. The performance of this control scheme is illustrated by simulation results showing the robustness of the method, even in the presence of perturbation and noise.

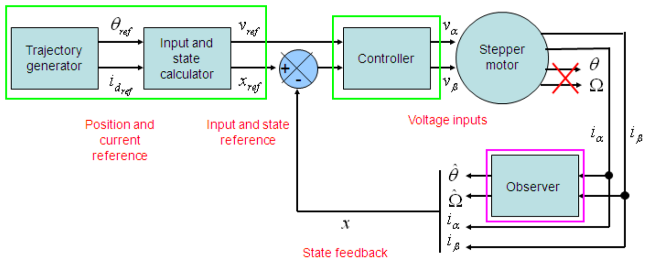

Fig. 1. Sensorless control scheme for the stepper motor

The paper is organized as follows. In Section 2, the different models of the stepper motor that will be used for the observation and the control are displayed. The notion of higher order sliding modes is recalled in Section 3. In Section 4, observers are presented, and in Section 5, a control law based on those observers is analyzed. The results obtained are illustrated by simulations.

2. STEPPER MOTOR MODELS

2.1 Model in the fixed frame (α - β)

The following model will be used to design the observer:

$$\begin{aligned}
\frac{di_\alpha}{dt} &= \frac{1}{L}(v_\alpha - Ri_\alpha + K\Omega \sin N\theta) \\
\frac{di_\beta}{dt} &= \frac{1}{L}(v_\beta - Ri_\beta - K\Omega \cos N\theta) \\
\frac{d\Omega}{dt} &= \frac{1}{J}(K(i_\beta \cos N\theta - i_\alpha \sin N\theta) - f_v\Omega - \tau_L) \\
\frac{d\theta}{dt} &= \Omega
\end{aligned} \tag{1}$$

in which i_α and i_β are the coil currents, v_α and v_β are the voltage inputs, θ is the rotor position, and Ω its velocity. The parameter R represents the resistance, L the inductance, N the rotor number of teeth, K the motor torque constant, J the rotor moment of inertia, f_v the coefficient of friction, and τ_L the torque load, which will be assumed to be an unknown perturbation.

Only the voltage inputs and the stator currents will be assumed to be measured. Note that the stepper motor without mechanical sensors is observable except for zero velocities.

2.2 Model in the moving frame (d - q)

From the model presented above, one can derive a model in a moving frame, linked to the rotor, thanks to a simple rotation:

$$\begin{array}{l} [i_d, i_q]^T = M_p\,[i_\alpha, i_\beta]^T \\ [v_d, v_q]^T = M_p\,[v_\alpha, v_\beta]^T \end{array} \text{ with } M_p = \begin{pmatrix} \cos N\theta & \sin N\theta \\ -\sin N\theta & \cos N\theta \end{pmatrix}.$$

Using this change of coordinates, the system (1) becomes

$$\begin{aligned} \frac{di_d}{dt} &= \frac{1}{L}(v_d - Ri_d + NL\Omega i_q) \\ \frac{di_q}{dt} &= \frac{1}{L}(v_q - Ri_q - NL\Omega i_d - K\Omega) \\ \frac{d\Omega}{dt} &= \frac{1}{J}(Ki_q - f_v\Omega - \tau_L) \\ \frac{d\theta}{dt} &= \Omega. \end{aligned} \tag{2}$$

This model will be used to design the control law in Section 5.

3. HIGHER ORDER SLIDING MODE

Sliding modes (see Perruquetti et al. [2002]) are often used to design robust nonlinear observers or control laws. The aim of sliding mode control is, by means of a discontinuous control, to robustly constrain the system to reach and stay, after a finite time, on a sliding surface where the resulting behavior has some prescribed dynamics. The sliding surface is defined by the vanishing of a corresponding sliding variable s and its successive time derivatives up to a certain order, i.e. the r^{th} order sliding set

$$\mathcal{S}_r = \left\{ x \in \mathbb{R}^n : s(x) = \dot{s}(x) = \cdots = s^{(r-1)}(x) = 0 \right\}.$$

A control law leading to such a behavior is called a r^{th} order ideal sliding mode algorithm with respect to s. Higher order sliding modes, that are characterized by a discontinuous control acting on the r^{th}, $r > 1$, time derivatives of the sliding vector (instead of the first time derivative in classical sliding mode, $r = 1$), can reduce the chattering phenomenon while preserving the robustness properties.

A wide range of applications using sliding modes for either observation or control, as in mechanics, robotics, or electric machines, can be found in the literature (see for example Bartolini et al. [2003], Butt et al. [2008], Canale et al. [2008], Defoort et al. [2008], Drakunov et al. [2005] Floquet et al. [2003], Martinez et al. [2008], Pisano et al. [2008], Riachy et al. [2008]). In what follows, higher order sliding mode observers will be designed because they allow one to obtain estimates in finite time without introducing a low-pass filter (which is not the case with a first order sliding mode, for which it is necessary to filter the high frequency oscillations). Especially, the super twisting algorithm (see Levant [2001]) will be used, which is a second order sliding mode defined by

$$u_{st}(s) = u_1(s) + u_2(s) \tag{3}$$

with

$$\begin{cases} \dot{u}_1(s) = -\alpha \operatorname{sign}(s) \\ u_2(s) = -\lambda |s|^{\frac{1}{2}} \operatorname{sign}(s) \end{cases} \tag{4}$$

where s is the sliding variable, and where α and λ are two parameters which have to satisfy sufficient conditions (see Levant [2001]) depending on the system and the chosen sliding variable in order to ensure the finite time convergence towards the second order sliding set

$$S_2 = \{x \in \mathbb{R}^n : s(x) = \dot{s}(x) = 0\}. \tag{5}$$

4. POSITION AND VELOCITY OBSERVER BASED ON SECOND ORDER SLIDING MODES

In order to design the observer, consider the two electric equations of model (1), and replace the unknown terms by an output injection based on the second order sliding mode algorithm described above. The observer dynamics is described by

$$\begin{aligned} \frac{d\hat{i}_\alpha}{dt} &= \frac{1}{L}[v_\alpha - Ri_\alpha] - \chi_1 \\ \frac{d\hat{i}_\beta}{dt} &= \frac{1}{L}[v_\beta - Ri_\beta] - \chi_2 \end{aligned} \tag{6}$$

The sliding variables are chosen as the current estimation errors:

$$\begin{aligned} \varepsilon_1 &= i_\alpha - \hat{i}_\alpha \\ \varepsilon_2 &= i_\beta - \hat{i}_\beta. \end{aligned}$$

and the output injections χ_1 and χ_2 are functions of ε_1 and ε_2 given by the super twisting algorithm (3):

$$\begin{aligned} \chi_1 &= u_{st}(\varepsilon_1) \\ \chi_2 &= u_{st}(\varepsilon_2). \end{aligned} \tag{7}$$

The observation error dynamics is obtained using (1) and (6):

$$\begin{aligned} \dot{\varepsilon}_1 &= \frac{K}{L}\Omega \sin N\theta + \chi_1 \\ \dot{\varepsilon}_2 &= -\frac{K}{L}\Omega \cos N\theta + \chi_2. \end{aligned} \tag{8}$$

In order to satisfy the conditions given in Levant [2001] that insure the finite time convergence of the estimation errors onto the sliding set defined in (5), one needs to set the parameters α and λ from both super twisting algorithms described in (3) and (4):

$$\alpha > C_0 \text{ and } \lambda^2 > 4C_0\frac{\alpha + C_0}{\alpha - C_0},$$

with

$$C_0 = \frac{K}{L}(|\dot{\Omega}|_{max} + N|\Omega^2|_{max}).$$

With these conditions, one has $\varepsilon_1 = \dot{\varepsilon}_1 = 0$ and $\varepsilon_2 = \dot{\varepsilon}_2 = 0$ after a finite time. This yields

$$\chi_1 = -\frac{K}{L}\Omega \sin N\theta$$
$$\chi_2 = \frac{K}{L}\Omega \cos N\theta. \tag{9}$$

From the two latter equations, estimates of the rotor position θ and velocity Ω are given by

$$\hat{\Omega} = \pm\frac{L}{K}\sqrt{\chi_1^2+\chi_2^2}$$
$$\hat{\theta} = \begin{cases} \hat{\theta}_0 \text{ if } \hat{\Omega}=0, \\ -\frac{1}{N}\arctan\left(\frac{\chi_1}{\chi_2}\right) \text{ if } \hat{\Omega}\chi_2 > 0, \\ \frac{1}{N}\arctan\left(\frac{\chi_1}{\chi_2}\right) \text{ otherwise} \end{cases} \tag{10}$$

where $\hat{\theta}_0$ is the latest calculated value of $\hat{\theta}$ before $\hat{\Omega}=0$.

Two problems caused by the non-uniqueness of the solutions of the electrical system of (1) can be pointed out. The first is that the sign of the velocity is undetermined, since if (θ, Ω) is a solution of (9), $(\theta \pm \frac{\pi}{N}, -\Omega)$ is a solution too. This also means that if the estimate of the sign of the velocity is not correct, the position estimate will be shifted by $\pm\frac{\pi}{N}$. The second problem is that the value of the position estimate is only obtained modulo $\frac{2\pi}{N}$ because $\hat{\theta}$ is calculated according to (10) within the interval $]-\frac{\pi}{N}, \frac{\pi}{N}[$.

Assuming the sign of the velocity is known, the observer can achieve good performance even in presence of perturbations. However, the two problems mentioned above are still to be solved.

The idea for solving the sign issue is to study the evolution of the position. To this end, the estimate $\hat{\theta}$ of the position is used and another super twisting algorithm based observer is introduced:

$$\frac{d\hat{\hat{\theta}}}{dt} = \chi_3(\hat{\theta} - \hat{\hat{\theta}}) \tag{11}$$

where χ_3 is given by the super twisting algorithm.

Writing $\varepsilon_3 = \hat{\theta} - \hat{\hat{\theta}}$ as the observation error, the error dynamics is:

$$\dot{\varepsilon}_3 = \frac{d\hat{\theta}}{dt} - \chi_3(\varepsilon_3).$$

Setting the parameters for this other super twisting algorithm

$$\alpha > C_0' \text{ and } \lambda^2 > 4C_0'\frac{\alpha + C_0'}{\alpha - C_0'},$$

with

$$C_0' = \left|\frac{d\hat{\Omega}}{dt}\right|_{max}$$

implies the finite time convergence of the observer towards the second order sliding set $\{\varepsilon_3 = \dot{\varepsilon}_3 = 0\}$ and thus: $\hat{\hat{\theta}} = \hat{\theta}$ and $\hat{\hat{\Omega}} = \chi_3(\varepsilon_3) = \frac{d\hat{\theta}}{dt}$.

It has been seen that one has $\hat{\theta} = \theta \pm \frac{\pi}{N}$ if the estimate of the velocity sign is wrong. In this case, the time derivative of $\hat{\theta}$ is equal to Ω. Then, the output of the third observer $\hat{\hat{\Omega}}$ converges towards the actual value of the velocity Ω. As a consequence, the sign of the velocity estimate $\hat{\hat{\Omega}}$ is fully determined, except locally around possible discontinuity points of $\hat{\theta}$, for which the sliding mode could be destroyed. Those discontinuity points can create estimation errors for this observer. In order to solve this problem, one can use an algorithm to detect these discontinuity points on the sliding variable $\varepsilon_3 = \hat{\theta} - \hat{\hat{\theta}}$ (see Fig. 2).

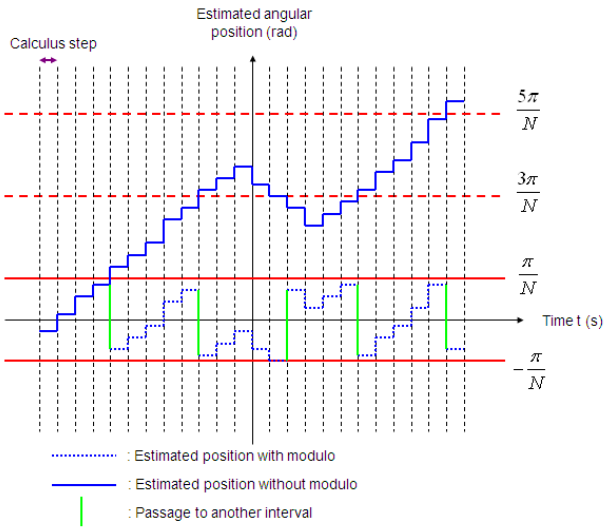

Fig. 2. Discontinuity point detection and position reconstruction algorithm

Such an algorithm to detect the discontinuity points is by construction sensitive to measurement noise: a "jump" caused by the noise could be interpreted as a passage to another interval by the algorithm. However, the super twisting algorithm used for the position observation in the observer (11) plays the role of a filter and provides a continuous position estimate $\hat{\hat{\theta}}$. The results obtained in simulation for a system with large perturbations ($\tau_L = \frac{\tau_h}{2}\sin(6\pi t)$, with τ_h the motor holding torque) and measurement noise (white noise - up to 5% of the nominal current) highlight the interest of the method (see Fig. 3).

5. SENSORLESS CONTROL

In this section, a robust control law without any mechanical sensor is designed using the estimate obtained with the observers defined above.

The control objective is to accurately track a given trajectory. First, note that the system (2) is flat (see Fliess et al. [1992]) with $z = [\theta, i_d]^T$ as a flat output. This implies that all the system states and inputs can be expressed as functions of a flat output z and a finite number of its time derivatives. Thanks to this property, one can easily define a reference trajectory (denoted Γ_r), satisfying the nominal system dynamics without load torque. The dynamics of the tracking error

$$e = [i_d - i_{d_r}, i_q - i_{q_r}, \Omega - \Omega_r, \theta - \theta_r]^T = [e_1, e_2, e_3, e_4]^T$$

is then given by

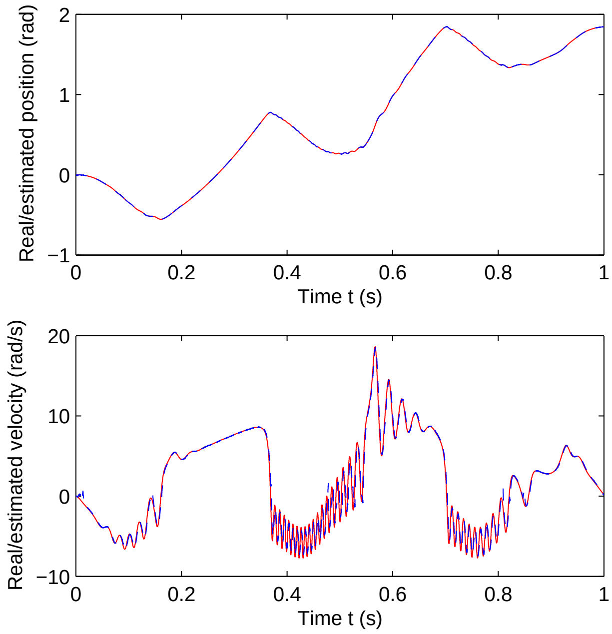

Fig. 3. Real (in red) and estimated (in blue) values of θ and Ω

$$
\begin{aligned}
\dot{e}_1 &= \frac{1}{L}(\bar{v}_d - Re_1 + NL(e_3e_2 + e_3 i_{q_r} + e_2\Omega_r)) \\
\dot{e}_2 &= \frac{1}{L}(\bar{v}_q - Re_2 - NL(e_3e_1 + e_3 i_{d_r} + e_1\Omega_r) - Ke_3) \\
\dot{e}_3 &= \frac{1}{J}(Ke_2 - f_v e_3 - \tau_L) \\
\dot{e}_4 &= e_3
\end{aligned}
\tag{12}
$$

with $\bar{v}_d = v_d - v_{d_r}$ and $\bar{v}_q = v_q - v_{q_r}$ the new system inputs.

5.1 State feedback control

Before designing the sensorless control law, let us recall a second order sliding mode control described in Nollet et al. [2008] that requires the knowledge of the full state.

Two sliding variables are defined as

$$S_\theta = ke_4 + \dot{e}_4 \text{ and } S_{i_d} = e_1, \text{ for } k > 0.$$

It can be seen that the sliding motion $S_\theta = S_{i_d} = 0$ implies the vanishing of the position tracking error e_4 and the direct current tracking error e_1.

Because the system has relative degree 2 with respect to S_θ, one has to compute the second time derivative of S_θ in order to derive the control law used for the tracking of the position:

$$\ddot{S}_\theta = \frac{K}{JL}\bar{v}_q + \mu_\theta(e, \Gamma_r) - \left(\frac{k}{J} - \frac{f_v}{J^2}\right)\tau_L - \frac{1}{J}\frac{d\tau_L}{dt}$$

where

$$
\begin{aligned}
\mu_\theta(e, \Gamma_r) = {} & \frac{k}{J}(Ke_2 - f_v e_3) - \frac{f_v}{J^2}(Ke_2 - f_v e_3) \\
& - \frac{K}{JL}(Re_2 + NL(e_3e_1 + e_3 i_{d_r} + e_1\Omega_r) + Ke_3).
\end{aligned}
$$

Then, a control law based on the sampled twisting algorithm (see Levant [1993]) is considered (it is well suited to systems with relative degree 2):

$$\frac{K}{JL}\bar{v}_q = -\mu_\theta(e, \Gamma_r) + u_{te}(S_\theta, \Delta S_\theta) \tag{13}$$

$$u_{te}(S, \Delta S) = \begin{cases} -\lambda_m \mathrm{sign}(S) \text{ if } S\Delta S \le 0 \\ -\lambda_M \mathrm{sign}(S) \text{ if } S\Delta S > 0 \end{cases} \tag{14}$$

where ΔS is the variation of S during a sampling period.

To guarantee the finite time convergence in finite time on the surface $S_\theta = 0$, λ_m and λ_M have to verify

$$
\begin{aligned}
\lambda_m &> \left| \left(\frac{k}{J} - \frac{f_v}{J^2}\right)\tau_L - \frac{1}{J}\frac{d\tau_L}{dt} \right|_{max} \\
\lambda_M &> \lambda_m + 2\left| \left(\frac{k}{J} - \frac{f_v}{J^2}\right)\tau_L - \frac{1}{J}\frac{d\tau_L}{dt} \right|_{max}
\end{aligned}
\tag{15}
$$

Because the system has relative degree 1 with respect to S_{i_d}, the first time derivative of S_{i_d} is calculated in order to derive the control law used for the tracking of the direct current:

$$\dot{S}_{i_d} = \dot{e}_1 = \frac{1}{L}\bar{v}_d + \mu_{i_d}(e, \Gamma_r) \tag{16}$$

where

$$\mu_{i_d}(e, \Gamma_r) = \frac{1}{L}(-Re_1 + NL(e_3e_2 + e_3 i_{q_r} + e_2\Omega_r)).$$

The super twisting algorithm (well suited to systems with relative degree 1) is then used to design the control law

$$\frac{1}{L}\bar{v}_d = -\mu_{i_d}(e, \Gamma_r) + u_{st}(S_{i_d}). \tag{17}$$

Setting the parameters $\alpha > 0$ and $\lambda > 0$ of this super twisting algorithm ensures the finite time convergence towards the surface $S_{i_d} = 0$.

Therefore, the designed control law guarentees the exponential convergence of the tracking error for a system in which the whole state is measured. It is also shown (see Nollet et al. [2008]) that the system is robust with respect to perturbations and parametric uncertainties.

5.2 Observer based control

Suppose now that only the electrical variables are measured and derive a sensorless control law based on the state feedback one described in Section 5.1. Denote the observation error vector as

$$\varepsilon = [i_\alpha - \hat{i}_\alpha, i_\beta - \hat{i}_\beta, \hat{\theta} - \hat{\hat{\theta}}]^T = [\varepsilon_1, \varepsilon_2, \varepsilon_3]^T.$$

Then, the dynamics of the whole system state, composed of the tracking error and the observation errors

$$\Xi = [e_1, e_2, e_3, e_4, \varepsilon_1, \varepsilon_2, \varepsilon_3]^T$$

is given by

$$
\dot{\Xi} = \begin{pmatrix}
-\frac{R}{L}e_1 + N(e_3e_2 + e_3 i_{q_r} + e_2\Omega_r) \\
-\frac{Re_2 + Ke_3}{L} - N(e_3e_1 + e_3 i_{d_r} + e_1\Omega_r) \\
\frac{1}{J}(Ke_2 - f_v e_3 - \tau_L) \\
e_3 \\
\frac{K}{L}(e_3 + \Omega_r)\sin(N(e_4 + \theta_r)) \\
-\frac{K}{L}(e_3 + \Omega_r)\cos(N(e_4 + \theta_r)) \\
e_3 + \Omega_r
\end{pmatrix}
+ \begin{pmatrix}
\frac{1}{L}\bar{v}_d \\
\frac{1}{L}\bar{v}_q \\
0 \\
0 \\
\chi_1(\varepsilon_1) \\
\chi_2(\varepsilon_2) \\
\chi_3(\varepsilon_3)
\end{pmatrix}.
\tag{18}
$$

Consider the control given in (13) and (17). The values of the control inputs $\bar{v}_q$ and $\bar{v}_d$ have to be adapted to the sensorless case. For this, the following error between the estimated values of the state and the reference values is introduced:

$$\xi = [\hat{i}_d - i_{d_r}, \hat{i}_q - i_{q_r}, \hat{\hat{\Omega}} - \Omega_r, \hat{\hat{\theta}} - \theta_r]^T = [\xi_1, \xi_2, \xi_3, \xi_4]^T.$$

Then, one has

$$\begin{aligned} \frac{K}{JL}\bar{v}_q &= -\hat{\mu}_\theta(\xi, \Gamma_r) + u_{te}(\hat{S}_\theta, \Delta\hat{S}_\theta) \\ \frac{1}{L}\bar{v}_d &= -\hat{\mu}_{i_d}(\xi, \Gamma_r) + u_{st}(\hat{S}_{i_d}) \end{aligned} \quad (19)$$

where $\hat{S}_\theta$ and $\hat{S}_{i_d}$ are the new sliding variables (similar to S_θ and S_{i_d})

$$\begin{cases} \hat{S}_\theta = k\xi_4 + \dot{\xi}_4 \\ \hat{S}_{i_d} = \xi_1 \end{cases}, k > 0$$

and where $\hat{\mu}_\theta(\xi, \Gamma_r)$ and $\hat{\mu}_{i_d}(\xi, \Gamma_r)$ are given by

$$\begin{aligned} \hat{\mu}_\theta(\xi, \Gamma_r) = &\ \frac{k}{J}(K\xi_2 - f_v\xi_3) - \frac{f_v}{J^2}(K\xi_2 - f_v\xi_3) \\ &- \frac{K}{JL}(R\xi_2 + NL(\xi_3\xi_1 + \xi_3 i_{d_r} + \xi_1\Omega_r) + K\xi_3) \\ \hat{\mu}_{i_d}(\xi, \Gamma_r) = &\ \frac{1}{L}(-R\xi_1 + NL(\xi_3\xi_2 + \xi_3 i_{q_r} + \xi_2\Omega_r)). \end{aligned}$$

The superposition principle cannot be used to prove the closed-loop system stability because the system is nonlinear. Therefore, the convergence of both systems (observer/control), considered separately, does not imply the convergence of the whole system.

In order to prove the exponential stability of the closed-loop system, it will first be shown that the state is bounded in finite time. Then, since the designed observers converge in finite time, after a finite time the system will behave exactly like the system controlled with the state feedback described in the subsection 5.1 and will hence be exponentially stable.

Let us prove that the system (18) is bounded in finite time. To do this, write ξ as a function of the system state Ξ, the reference trajectory Γ_r, and the control inputs $\chi_1(\varepsilon_1)$, $\chi_2(\varepsilon_2)$, $\chi_3(\varepsilon_3)$, $u_{te}(\hat{S}_\theta, \Delta\hat{S}_\theta)$ and $u_{st}(\hat{S}_{i_d})$. One has

$$\begin{pmatrix} \xi_1 \\ \xi_2 \end{pmatrix} = \begin{pmatrix} \hat{i}_d \\ \hat{i}_q \end{pmatrix} - \begin{pmatrix} i_{d_r} \\ i_{q_r} \end{pmatrix}.$$

Since one also has

$$\begin{pmatrix} \hat{i}_d \\ \hat{i}_q \end{pmatrix} = \begin{pmatrix} \cos(N\hat{\theta}) & \sin(N\hat{\theta}) \\ -\sin(N\hat{\theta}) & \cos(N\hat{\theta}) \end{pmatrix} \begin{pmatrix} \hat{i}_\alpha \\ \hat{i}_\beta \end{pmatrix}$$

with

$$\hat{\theta} = -\frac{1}{N}\arctan\left(\frac{\chi_1}{\chi_2}\right), \quad \begin{cases} \hat{i}_\alpha = i_\alpha - \varepsilon_1 \\ \hat{i}_\beta = i_\beta - \varepsilon_2 \end{cases}$$

as well as

$$\begin{pmatrix} i_\alpha \\ i_\beta \end{pmatrix} = \begin{pmatrix} \cos(N(e_4 + \theta_r)) & -\sin(N(e_4 + \theta_r)) \\ \sin(N(e_4 + \theta_r)) & \cos(N(e_4 + \theta_r)) \end{pmatrix} \begin{pmatrix} i_d \\ i_q \end{pmatrix}$$

and

$$\begin{pmatrix} i_d \\ i_q \end{pmatrix} = \begin{pmatrix} e_1 + i_{d_r} \\ e_2 + i_{q_r} \end{pmatrix}$$

then one obtains

$$\begin{pmatrix} \xi_1 \\ \xi_2 \end{pmatrix} = \Phi(\chi_1, \chi_2)\left(\Psi(e_4 + \theta_r)\begin{pmatrix} e_1 + i_{d_r} \\ e_2 + i_{q_r} \end{pmatrix} - \begin{pmatrix} \varepsilon_1 \\ \varepsilon_2 \end{pmatrix}\right) - \begin{pmatrix} i_{d_r} \\ i_{q_r} \end{pmatrix} \quad (20)$$

with

$$\Phi(\chi_1, \chi_2) = \begin{pmatrix} \frac{1}{\sqrt{1 + \left(\frac{\chi_1}{\chi_2}\right)^2}} & -\frac{\frac{\chi_1}{\chi_2}}{\sqrt{1 + \left(\frac{\chi_1}{\chi_2}\right)^2}} \\ \frac{\frac{\chi_1}{\chi_2}}{\sqrt{1 + \left(\frac{\chi_1}{\chi_2}\right)^2}} & \frac{1}{\sqrt{1 + \left(\frac{\chi_1}{\chi_2}\right)^2}} \end{pmatrix}$$

and

$$\Psi(e_4 + \theta_r) = \begin{pmatrix} \cos(N(e_4 + \theta_r)) & -\sin(N(e_4 + \theta_r)) \\ \sin(N(e_4 + \theta_r)) & \cos(N(e_4 + \theta_r)) \end{pmatrix}.$$

The variable ξ_3 can also be expressed as

$$\xi_3 = \hat{\hat{\Omega}} - \Omega_r = \chi_3 - \Omega_r. \quad (21)$$

After substituting in the system (18) the equations of the controls $\bar{v}_d$ and $\bar{v}_q$ (19) with the expressions (20) and (21) of the errors between the estimated values and the reference values, one gets a model depending only on the state and the control inputs:

$$\dot{\Xi} = F(\Xi, \Gamma_r, \chi_1, \chi_2, \chi_3, u_{te}(\hat{S}_\theta, \Delta\hat{S}_\theta), u_{st}(\hat{S}_{i_d})).$$

Without loss of generality, it can be assumed that the chosen reference trajectory Γ_r is bounded. Then one has, using (20) and (21)

$$\begin{cases} \xi_1 = O(e_1 + e_2 + \varepsilon_1) \\ \xi_2 = O(e_1 + e_2 + \varepsilon_2) \\ \xi_3 = O(\varepsilon_3). \end{cases} \quad (22)$$

Since the currents i_d and i_q are saturated, as well as their estimated values $\hat{i}_d$ and $\hat{i}_q$, the tracking errors of the currents, e_1 and e_2, as well as the estimation errors ε_1 and ε_2 are bounded. Hence, ξ_1 and ξ_2 are also bounded according to (22).

It can be seen that the sampled twisting control input (14) is also bounded, as well as the super twisting algorithms (7), (11) and (17) since they are continuous functions with values in a compact set.

As a consequence, the following inequality can be obtained:

$$\|\dot{\Xi}\| \leq Q\|\Xi\| + \bar{g} \quad (23)$$

where Q and $\bar{g}$ are positive constants.

Then, by integrating (23), one gets

$$\|\Xi(t)\| \leq \|\Xi(0)\| + \int_0^t (Q\|\Xi(\tau)\| + \bar{g})d\tau$$

and after applying Grönwall's lemma, one obtains

$$\|\Xi(t)\| \leq \|\Xi(0)\| \exp(Ct) + \frac{\bar{g}}{Q}\exp(Ct - 1)$$

where C is a positive constant.

Thus, the trajectories of the whole system state Ξ are bounded in finite time, which concludes the proof of exponential stability of the closed-loop system (18)-(19).

In Fig. 4 are reported simulation results obtained with large perturbations ($\tau_L = \frac{2}{3}\tau_h \sin(6\pi t)$, with τ_h the motor holding torque), measurement noise (white noise - up to 5% of the nominal current) and parametric errors (5% on parameters R, L, K, J and f_v).

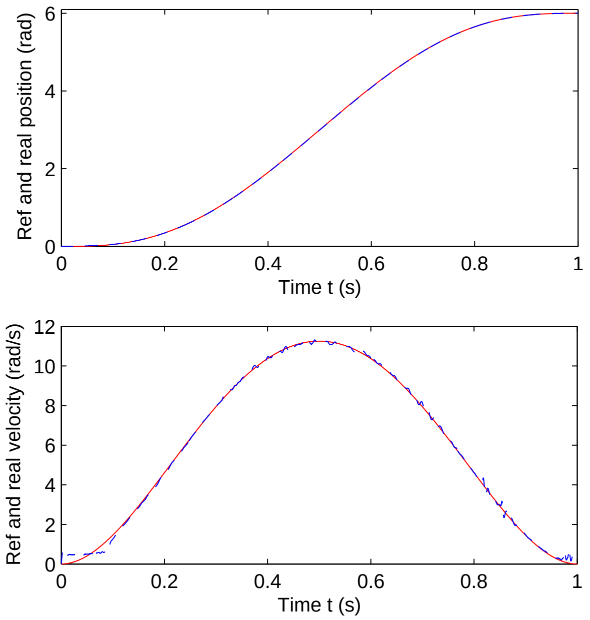

Fig. 4. Reference (in red) and real (in blue) values of θ and Ω with measurement noise and parametric errors

It can be seen in Fig. 4 that good tracking of both position and velocity is achieved, even in the presence of perturbations, noise and parameter uncertainties.

6. CONCLUSION

Higher order sliding mode observers have been designed to estimate the mechanical state of a stepper motor. Then, a robust sensorless control law using those observers has been presented. The results obtained in simulation for the observation scheme as well as the control are convincing and show the potential of the method.

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