Network Non-Neutrality through Preferential Signaling

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***Abstract*—One of the central issues in the debate on network neutrality has been whether one should allow or prevent preferential treatment by an internet service provider (ISP) of traffic according to its origin. This raised the question of whether to allow an ISP to have exclusive agreement with a content provider (CP). In this paper we consider discrimination in the opposite direction. We study the impact that a CP can have on the benefits of several competing ISPs by sharing private information concerning the demand for its content. More precisely, we consider ISPs that compete over access to one common CP. Each ISP selects the price that it charges its subscribers for accessing the content. The CP is assumed to have private information about demand for its content, and in particular, about the inverse demand function corresponding to the content. The competing ISPs are assumed to have knowledge on only the statistical distribution of these functions. We derive in this paper models for studying the impact that the CP can have on the utilities of the ISPs by favoring one of them by exclusively revealing its private information. We also consider the case where CP can charge ISPs for providing such information. We propose two mechanisms based on *weighted proportional fairness* for payment between ISPs and CP. Finally, we compare the social utility resulting from these mechanisms with the optimal social utility by introducing a performance metric termed as *price of partial bargaining*.**

***Index Terms*—Net neutrality, Game theory, Nonneutral network, Pricing, Nash bargaining solution**

I. Introduction

The past few years have seen much public debate and legislation initiatives concerning access to the global Internet. Some of the central issues concerned the possibility of discrimination of packets by service providers according to their source or destination, or the protocol used. A discrimination of a packet can occur when preferential treatment is offered to it either in terms of the quality of service it receives, or in terms of the cost to transfer it. Much of this debate took part in anticipation of the legislation over "Net Neutrality", and several public consultations were launched in 2010 (e.g. in the USA, in France and in the E.U.). Network neutrality asserts that packets should not be discriminated. Two of the important issues concerning discrimination of traffic are whether (i) an internet service provider (ISP) may or may not request payment from a content provider (CP) in order to allow it reach its end users, and (ii) whether or not an ISP can have an exclusive agreement with a given CP resulting in a vertical monopoly. Indeed, for Hahn and Wallsten [1], net neutrality "usually means that broadband service providers charge consumers only once for internet access, do not favor one content provider over another, and do not charge content providers for sending information over broadband lines to end users".

The network neutrality legislation will determine much of the socio-economic role of the Internet in the future. The Internet has already had a huge impact not only on economy, but also on the exercise of socio-cultural freedom. Directive 2002/22/EC of the European Union, as amended by the Directive 2009/136/EC, established internet access as a universal service[1]. However, internet is a conglomeration of several profit making entities. Interaction among these entities is largely governed by economic interests, and their decisions can adversely impact the socio-economic role of the Internet. Thus, it is necessary to understand the interplay between various agents involved, and the knowledge gained can be used in enabling laws that benefits society and its economic development.

This paper pursues a line of research that we have been carrying on for modeling exclusive agreements between service and content providers and study their economic impact. Such agreements are often called "vertical monopolies". In some branches of industry, steps have been taken against vertical monopolies. As a result, several railway companies in Europe had to split the railway infrastructure activity from the transportation activity. However, in the telecommunication industry impact of vertical monopolies is not yet clear. The international community is still debating the laws to regulate, or not to regulate, interaction between various agents in the Internet.

In this paper we study another form of nonneutrality[2] resulting from vertical monopolies that arises when a CP provides private information to an ISP. The private information could be popularity of its content, profiles of users interested in different types of content, traffic characterization, usage pattern, etc. We assume that the CP's private information is related to the demand generated through the ISPs. If CPs can share this private information with an ISP, then that ISP can adopt a more efficient pricing policy than its competitors. For example, recent acquisition of Dailymotion by France Télécom

[1] A universal service has been defined by the EU, as a service guaranteed by the government to all end users, regardless of their geographical location, at reasonable quality and reliability, and at affordable prices that do not depend on the location.

[2] The traditional net-neutrality discussion is about ISPs discriminating CPs by giving them preferential treatment.

(an ISP) enables it to have exclusive information about demand for its video content. We derive game theoretic models that enable to compute the impact of such discrimination on the utility of the ISPs. We model the interaction between the ISPs and a CP as a game, where the CP can share its private information through signals. We also look at the possibility of ISPs paying CP for access to its private information and study mechanism to decide these payments.

Related Work: We have used in the past game theoretical models to study two aspects of vertical monopolies. In [2] and [3], we studied the impact of collusion between an ISP and a CP by jointly determining the price each one charges. We evaluated the impact of such collusion, both on the colliding companies as well as on the benefits of other ISPs and CPs. In [4] we studied the impact that an ISP can have by proposing preferential quality of service or cheaper prices for accessing a CP with which it has an exclusive agreement. We refer the reader to [5] for a survey on net neutrality debate. In [6], the authors study a signaling game between high quality and low quality firms in a Bertrand oligopoly [7]. The quality of each firm is a private information which is signaled to others by the price set on their products. In [8], the authors propose a metric called *price of collusion* to study impact of collusion. In [9], similar definitions are proposed to consider several other scenarios. The authors in [10] study cooperation in routing games using Nash bargaining solution concept. They study degradation in network performance by introducing a metric called *price of selfishness*. Nash bargaining solution is also used in [11] to study contracts in nonneutral networks.

Our contributions: In this paper, we propose a simple model with one CP and several ISPs to analyze a network with vertical monopolies.

- We first consider the neutral network where the CP shares private information with all the ISPs or none of them. We compare this case with a nonneutral network where the CP colludes with one of the ISPs, say ISP_1, and provides signal only to ISP_1. We show that an ISP receiving signal improves its monetary gains, while CP may not.
- We then consider a case where the CP charges the ISPs for sharing private information. We show that the colluding pair, i.e., the CP and ISP_1 that obtains signal on payment, may not always gain. We characterize the price, that colluding ISP pays to the CP, that results in collusion beneficial to the colluding pair.
- We then propose mechanisms based on weighted proportional fairness criteria for deciding payments that the colluding ISP makes to the CP to obtain signal. We compare the social utilities induced by these mechanisms with the optimal social utility by introducing a metric termed as *price of partial bargaining*.

The paper is organized as follows: In Section II, we introduce the model and set up the notations. In Section III, we consider the neutral network in which the CP provides signals either to all the ISPs or to none of them. In section V, we study the competition assuming the demand generated through ISPs is linear in the user price. Section IV studies a nonneutral behavior in which the CP colludes with one of the ISPs. In Section VI, we allow the CP to charge the colluding ISP for providing the signals. In Section VII, we consider two mechanisms to determine the payment between the colluding pair. In Section VIII, we propose a new metric to compare social utilities induced by these mechanism. Finally, we end with conclusions in Section IX. Proofs not provided in this paper are available in our arxiv paper with the same title.

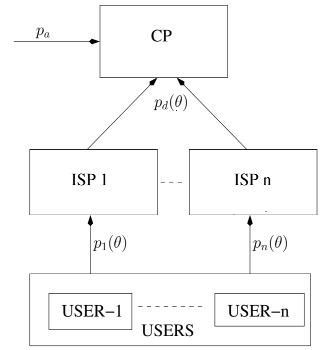

Figure 1. Monetary flow in a nonneutral network.

II. Model

Consider n competing internet service providers (ISPs), namely $\text{ISP}_i, \quad i = 1, 2 \cdots, n$, that provide access to a common content provider (CP). Each ISP determines the price (per unit of content) that it charges its subscribers. In our model we consider single CP as few players, like YouTube, Netflix, account for a significant amount of traffic generated in the Internet. The demand generated by the subscribers of ISP_i depends on the price of all ISPs as well as on some parameter θ reflecting private information of the CP. We assume that θ takes values in some discrete space Θ. The model is show in Figure 1. We summarize the parameters of the model in Table II.

Parameter	Description
θ	indicator of private information of the CP (signal).
$p_i(\theta)$	Price per unit demand charged by ISP_i to its users; this can be a function of θ.
$d_i(\cdot)$	Demand generated by ISP_i. It is a function of the price set by all the ISPs and θ.
p_a	Advertising revenue per unit demand, earned by the CP. This satisfies $p_a \geq 0$.
p_d	Price per unit demand paid by the ISPs to the CP for providing signals.
U_{ISP_i}	The revenue or utility of the ISP_i.
U_{CP}	The revenue or utility of the CP.
γ	Bargaining power of the ISPs with respect to the CP. This satisfies $0 < \gamma < 1$.
n	Number of ISPs.

Let the vector $\mathbf{p}(\theta) := (p_1(\theta), p_2(\theta), \cdots, p_n(\theta))$ denote the price set by all the ISPs when the signal is θ. We write the demand generated by the subscribers of ISP_i as

$$d_i(\theta, \mathbf{p}(\theta)) = d_i(\theta, p_i(\theta), \mathbf{p}_{-i}(\theta)) \quad i = 1, 2 \cdots, n \text{ where}$$

$$\mathbf{p}_{-i}(\theta) = (p_1(\theta), p_2(\theta), \cdots, p_{i-1}(\theta), p_{i+1}(\theta), \cdots, p_n(\theta)).$$

We shall assume that for each θ and i the demand functions are twice differentiable and satisfy the following monotonicity properties

$$\frac{\partial d_i(\theta, \mathbf{p}(\theta))}{\partial p_i} < 0 \text{ and } \frac{\partial d_i(\theta, \mathbf{p}(\theta))}{\partial p_j} > 0 \text{ for } j \neq i. \tag{1}$$

These conditions imply that if ISP_i increases the access price p_i then subscribers of ISP_i can shift to other ISPs, decreasing the demand generated through ISP_i while increasing that generated from the other ISPs. Above conditions are common in modeling demand functions in a price competition [12]. The CP is assumed to have knowledge on the exact value of θ. The probability $P(\theta)$ that the private information is of type θ is a common knowledge to all ISPs.

The utility of ISP_i is assumed to have the form

$$U_{\text{ISP}_i}(\theta, p_i(\theta), \mathbf{p}_{-i}(\theta)) = p_i(\theta) d_i(\theta, p_i(\theta), \mathbf{p}_{-i}(\theta)).$$

The CP earns a fixed advertisement revenue of p_a per unit demand. The total revenue earned by the CP depends on the effective demand generated by all the ISPs. Utility of the CP is given by

$$U_{\text{CP}}(\theta, \mathbf{p}(\theta)) = \sum_{i=1}^{n} d_i(\theta, \mathbf{p}(\theta)) p_a.$$

If ISP_i does not know the actual signal θ then it can set the price knowing only the distribution of θ. In this case we denote the price by simply p_i (doesn't depend on particular realization of Θ). With some abuse of notation, we denote the utilities of ISP_i as $U_{\text{ISP}_i}(\cdot)$ in both the cases. It should be clear from the context if an ISP obtains signals or not. In the current setting CP acts as a passive player. It can only provide signals to the ISPs, but does not control any prices. Its revenue is influenced by the prices set by the ISPs. Again, with some abuse of notation we denote the utility of CP as U_{CP} in all the cases.

The demand functions defined above are quiet general. To study the price competition between the ISPs we further assume that the demand function $d_i(\theta, \mathbf{p}(\theta))$ is supermodular for each i and θ, and satisfy 'dominant diagonal' property. For a twice differentiable function supermodularity property is equivalent to the condition

Assumption 1 (Supermodularity, [13]):

$$\frac{\partial^2 d_i(\theta, \mathbf{p})}{\partial p_i \partial p_j} \geq 0 \quad j \neq i.$$

The dominant diagonal property is defined for all $\gamma, \theta \in \Theta$ as

Assumption 2 (dominant diagonal):

$$\sum_{j=1}^{n} d_i(\theta, \mathbf{p}) \frac{\partial^2 d_i(\gamma, \mathbf{p})}{\partial p_i \partial p_j} - \frac{\partial d_i(\theta, \mathbf{p})}{\partial p_i} \frac{\partial d_i(\gamma, \mathbf{p})}{\partial p_j} \leq 0.$$

For simplicity, we also assume that the price charged by ISP_i is bounded, say by $p_i^{\max}$ for all θ, such that demand from for all the ISPs is positive. Also, the price sensitivity of the subscribers is the same for all the ISPs. If ISP_i increases access charges while the others maintain their price, then a fraction of the subscribers move from ISP_i to other ISPs without assigning preference to any particular ISP. Thus demand function of all the ISPs is symmetric.

In the next two sections we study price competition in neutral and nonneutral networks. We define utility and objectives of all the players and compare their revenues in each cases.

III. Neutral behavior

In this section we study price competition in a neutral network. In the neutral regime the CP does not discriminate between the ISPs: It shares private information about its content with all the ISPs or none of them. We study these two cases separately, and analyze the impact of having the information on the expected utility of each ISP and the CP at equilibrium.

A. No information

We first consider the neutral behavior in which no information is shared with the ISPs. The ISPs set their prices knowing only distribution $P(\theta)$. Recall that in this case we denoted the price charged by ISP_i as p_i. The objective of ISP_i is to set p_i that maximizes its expected utility, i.e.,

$$E[U_{\text{ISP}_i}] := E_\theta[U_{\text{ISP}_i}(\theta, p_i, \mathbf{p}_{-i})] = \int P(d\theta) p_i d_i(\theta, p_i, \mathbf{p}_{-i}).$$

where the operator $E_\theta[\cdot]$ denotes expectation with respect to the random signal θ.

B. Full information

Let us consider the case where the CP gives signals to all the ISPs, i.e., all the ISPs are given θ. We also assume that the signal is sent to all the ISPs simultaneously. Note that the CP providing signals to all the ISPs is a non-discriminatory act. Hence we consider this case under neutral regime.

ISPs can use knowledge of θ to set the price charged form their users. The objective of ISP_i is to maximize its expected utility given by

$$\begin{aligned} E[U_{\text{ISP}_i}] &= E[U_{\text{ISP}_i}(\theta, p_i(\theta), \mathbf{p}_{-i}(\theta))] \\ &= \int P(d\theta) U_i(\theta, p_i(\theta), \mathbf{p}_{-i}(\theta)). \end{aligned}$$

Note that if any vector $(p_i(\theta))_{\theta \in \Theta}$ maximizes the expected utility for a given $\{p_{-i}(\theta) : \theta \in \Theta\}$, then $p_i(\theta)$ also maximizes $U_{\text{ISP}_i}(\theta, \mathbf{p}(\theta))$ for each θ. Thus the objective of each player is to maximize $U_{\text{ISP}_i}(\theta, p_i(\theta), \mathbf{p}_{-i}(\theta))$ for each value of θ. In this case strategy of each ISP_i is to choose a pricing function on Θ, i.e., $p_i : \Theta \to [0 \; p_i^{\max}]$.

Theorem 1: Assume that the demand function are supermodular and satisfy the dominant diagonal property for all the ISPs and θ. Then the price competition in the neutral regime has the following properties:

- When all the ISPs obtain the signals, equilibrium exists and unique.
- When none of the ISPs obtain the signals, equilibrium exists and unique. □

The proof follows by verifying that the expected utilities are supermodular and satisfy dominant diagonal condition.

IV. Nonneutral behavior

In a nonneutral network the CP can discriminate between the ISPs by giving preferential treatment to, or making an exclusive agreement with, one of the ISPs. In this subsection we assume that CP shares information with one of the ISPs. Without loss of generality we assume that the CP shares information with ISP_1 through signals. Then ISP_1 can set access price knowing the signal, whereas the other ISPs does so knowing only the distribution. In this case we say that CP and ISP_1 are in collusion, and refer to them as colluding pair. The utilities of ISP_1 and other ISPs are given, respectively, as follows:

$$\begin{aligned} U_{\text{ISP}_1}(\theta, p_1(\theta), p_{-1}) &= d_i(\theta, p_1(\theta), p_{-1}) p_1(\theta) \\ U_{\text{ISP}_j}(\theta, p_j, p_{-j}) &= d_j(\theta, p_j, p_{-j}) p_j \; j = 2, \cdots, n. \end{aligned}$$

In the above utilities we write $p_1(\cdot)$ as a function of the θ, whereas $p_j, \; j = 2, 3, \cdots, n$ are constants chosen knowing only distribution of θ. The objective of ISP_1 and $\text{ISP}_j, \; j = 2, 3, \cdots, n$ is to maximize their expected utilities given, respectively, as follow

$$\begin{aligned} E[U_{\text{ISP}_1}] &= \int P(d\theta) U_1(\theta, p_1(\theta), p_{-1}), \\ E[U_{\text{ISP}_j}] &= \int P(d\theta) U_j(\theta, p_j, p_{-j}). \end{aligned}$$

Theorem 2: Assume that the demand functions are supermodular and satisfy diagonal dominance property for each ISP and θ. Assume that the CP provides information only to ISP_1. Then equilibrium exists in the nonneutral regime and is unique. □

After establishing existence of equilibrium prices in both neutral and nonneutral regimes it is interesting to compare the utilities in both the regimes to see the impact of sharing private information on revenues of the ISPs. Though it appears that an ISP receiving signal should obtain higher revenue compared to an ISP without signal, this observation is not true in general: One can construct simple examples where a player with more information can gain or loose even when equilibrium is unique. For example, see [14]. Also, the authors in the same paper give conditions on the feasible payoff sets that ensures higher utilities at equilibrium for a more informed player. However, these conditions are not easy to verify in a game with continuous strategy space.

There are several demand functions which are log-supermodular and satisfy monotonicity and diagonal property. For example, Linear, Logit, Constant Elasticity of Substitution (CES), Cobb-Douglas, etc [12]. By appropriately choosing the term that depends on the signal θ, we can use any of them to model demand functions. In the following we restrict our attention to linear demand function to study the impact of signaling. Linear demand functions are often used to model demand functions in economic literatures [12] due to simplicity of analysis.

V. Linear demand function

For given price vector $\mathbf{p}(\theta)$, assume that demand generated through ISP_i is given by

$$d_i(\theta, \mathbf{p}(\theta)) = D_i(\theta) - \alpha p_i(\theta) + \beta \sum_{j \neq i} p_j. \tag{2}$$

where $\alpha > 0$ and $\beta > 0$, and $D_i(\theta)$ denotes the demand generated through ISP_i when private information of the CP corresponds to θ and all the ISPs give free access to the CP . For simplicity, we assume that all the ISPs are equally competitive and set $D_i(\theta) = D(\theta)$, i.e., if all the ISPs offer free subscription then the demand generated through each ISP is the same. Also, the users are assumed to be equally sensitive to the price set by each ISP, thus we take α and β to be the same in the demand function of each ISP. Note that linear demand function is supermodular and satisfies the dominant diagonal property if $\alpha > (n-1)\beta$. We assume this relation holds in the sequel.

Due to simple structure of linear demand function one can compute the equilibrium prices and equilibrium utilities in both the neutral regime and nonneutral regime explicitly and compare.

Theorem 3: Assume that demand function is (2) for each ISP and $\alpha > (n-1)\beta$. Then, in the neutral regime, ISPs obtain higher revenue when all of them receive signals, compared to the case where none of them receive signals. In the nonneutral regime,

1) the colluding ISP obtains higher revenue at equilibrium than the noncolluding ISPs.
2) the revenue of the colluding ISP further improves if the noncolluding ISPs also receive signals.
3) the revenue of the noncolluding ISPs remains the same as in the neutral regime where none of the ISPs receive signals (no information).

Finally, the revenue of the CP at equilibrium is the same in all the cases. □

As we note from the above theorem, if ISPs receive signal they only earn higher revenues. However, this increase in ISPs revenue is not because of increased demand, but due to the optimal choice of subscription prices. In particular, as shown in the proof, the demand generated through each ISP remains the same at equilibrium, irrespective of whether it receives signal or not. Thus, the CP does not gain anything by sharing its private information as its revenue depends only on the total demand generated. Hence the CP has an incentive to charge the ISPs for sharing its private information.

In the rest of the paper we restrict our attention to the case with just two ISPs for ease of exposition. However, it is not very restrictive as end users often face a duopoly ISP market.

VI. Signaling with Side Payments

In this section we assume that CP charges ISPs for providing signaling information. We assume that if an ISP receives signal from the CP it pays a fixed price of p_d per unit demand generated by its subscribers. We refer to p_d as side payment. We consider the nonneutral regime where ISP_1 gets preferential treatment from the CP. Our aim is to study the impact of side payment on the equilibrium utilities of the colluding pair (ISP_1 and CP) and on the non-colluding ISP_2. In particular, we will be interested in characterizing the values of side payment that makes the collusion with the CP beneficial to ISP_1 and vice versa.

A. Nonneutral network with pricing

We define the utility of the ISP_1, ISP_2 and CP respectively as follows:

$$\begin{aligned} U_{\text{ISP}_1}(\theta, p_1(\theta), p_2) &= (D_0(\theta) - \alpha p_1(\theta) + \beta p_2)(p_1(\theta) - p_d) \\ U_{\text{ISP}_2}(\theta, p_1(\theta), p_2) &= (D_0(\theta) - \alpha p_2 + \beta p_1(\theta))p_2 \\ U_{CP}(\theta, p_1(\theta), p_2) &= (D_0(\theta) - \alpha p_1(\theta) + \beta p_2)(p_a + p_d) \\ &+ (D_0(\theta) - \alpha p_2 + \beta p_1(\theta))p_a \end{aligned}$$

The CP informs the value of p_d to ISP_1 while they enter in to the agreement. Thus CP acts as a passive player, providing signals, which in turn affects the demand generated by the subscribers of the ISPs. We proceed to analyze the game between ISP_1 and ISP_2. The objective of each ISP is to maximize its expected utility.

Proposition 1: In the collusion assume that the CP imposes price on ISP_1 for sharing private information rather than giving it for free. Then, equilibrium revenue of ISP_1 decreases, whereas that of the noncolluding ISP increases. □

Thus pricing in the nonneutral network have a positive externality on the non-colluding ISP. This behavior can be explained as follows: When ISP_1 is charged, it too charges its subscribers higher to compensate for the extra payment it makes to the CP (see proof of Prop. 1). Whereas ISP_2 does not need to increase its access fee, and also, some of the users of ISP_1 shift to ISP_2. This increases demand generated through ISP_2, thus improving the revenue of the non-colluding ISP. Unlike for the non-colluding ISP_2, revenue of the colluding pair may or may not improve. It depends on the value of p_d. The following theorem characterizes its range.

Theorem 4: Assume that in a collusion CP provides signal to ISP_1 by charging a price p_d per unit demand. Also assume that the distribution of θ is such that $Var(D(\theta)) \leq E[D(\theta)]$. Then,

1) ISP_1 has an incentive to collude with the CP if and only if

$$p_d \leq \frac{(2\alpha+\beta)E[D(\theta)]}{2\alpha^2-\beta^2}\left\{1-\sqrt{1-\frac{(2\alpha-\beta)^2 Var(D(\theta))}{4\alpha^2 E^2[D(\theta)]}}\right\}.$$

2) The CP has an incentive to collude if and only if

$$p_d \leq \frac{E[D(\theta)](4\alpha^2-\beta^2)}{(2\alpha-\beta)(2\alpha^2-\beta^2)} - \frac{(2\alpha^2-\beta^2-\alpha\beta)}{2\alpha^2-\beta^2}p_a \quad (3)$$

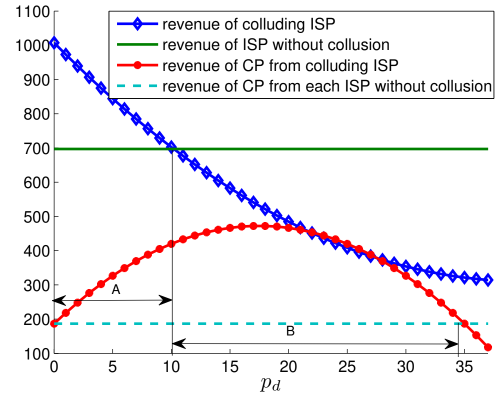

Figure 2. Utilities of colluding ISP and CP.

Further, the colluding ISP obtains higher revenue than the non-colluding ISP if and only if

$$p_d \leq \frac{(2\alpha+\beta)E[D(\theta)]}{2\alpha^2-\beta^2+\alpha\beta} \times\left\{1-\sqrt{1-\frac{(2\alpha-\beta)^2 Var(D(\theta))(2\alpha^2-\beta^2+\alpha\beta)}{4\alpha^2E^2[D(\theta)](2\alpha^2-\beta^2-\alpha\beta)}}\right\}.$$

□

In Figure 2, we plot the utilities of colluding ISP and CP as a function of p_d. In generating the plot we used the following parameters. The signal θ takes three values: high (H), medium (M) and low (L), which corresponds to demands $D(H) = 200, D(M) = 50, D(L) = 20$. The distribution of Θ is taken as $\Pr(\Theta = H) = 0.1, \Pr(\Theta = M) = 0.6, \Pr(\Theta = L) = 0.3$. The other parameters are $\alpha = 2$, $\beta = 1$ and $p_a = 5$.

As shown in the figure, if the side payment lies in the region marked A, then the colluding ISP obtains higher revenue at equilibrium. If it is charged a price outside the region A, then collusion with the CP is not beneficial to any ISP. For the CP, it appears that higher side payment will increase its revenues. But this is not the case. If ISP has to pay higher price to CP its demand goes down, which in turn reduces the revenue of the CP. Thus it is not beneficial for the CP to charge high prices from ISPs. Indeed, if CP charges price beyond region B, governed by (3), it will not improve its revenue. The collusion between CP and ISP is profitable to both if and only if side payment lies in the region A. Also, Note that in the region A, though both ISP and CP benefit, CP can obtain higher revenues by increasing the side payment but at the cost of reducing the revenue of ISP_1. Thus it becomes important to decide how the side payments should be set so that all the players remain satisfied. In the next section we look for mechanisms to address this issue.

VII. Mechanisms for setting side payments

In this section we look for mechanisms that take into account the bargaining power (weight) of each player. As in

the previous section, ISP_1 is in collusion with CP, in which CP shares private information with ISP_1 on payment. We assume that both ISP_1 and CP decide payment in presence of an arbitrator, and refer to the process as *bargaining*. Arbitrator can be a regulating authority, or a disinterested third party who aims to set a side payment that maximizes, in a sense made precise below, the revenues earned by the colluding pair. We consider the following two game models.
The timing for the first game is as follows.

- ISP_1 and the CP bargain over the payment p_d.
- ISP_1 and ISP_2 set the access price. The prices are set simultaneously .
- The subscribers react to the prices and set the demand generated through each ISP.

In the second game, timing is as follows:

- ISP_1 and ISP_2 set their access price simultaneously.
- ISP_1 and the CP then bargain over the payment p_d.
- The subscribers react to the prices and set the demand generated through each ISP.

The first game arises when the private information of the CP changes over a slower time-scale making the agreement between ISP_1-CP last for longer duration, whereas the ISPs vary access fee over a comparatively faster time-scale. The second game arises in cases where private information of the CP varies over a faster time-scale making the ISP_1-CP to renegotiate the side payments often, whereas the ISPs price to its subscribers varies over a slower time-scale. We analyze both models via backward induction and identify the equilibria. In the sequel, we refer to the first game as *pre bargaining* game, and the second one as *post bargaining* game.

In deciding side payment the arbitrator takes into account only revenue of CP earned by traffic generated through ISP_1. For a given $\{p_1(\theta), \theta \in \Theta\}$ and p_2, the arbitrator decides side payment p_d from ISP_1 to the CP based on weighted proportionally fair allocation criteria given as follows:

$$p_d^* \in \arg\max_{p^d} E[U_{\text{ISP}_1}]^{\gamma} E[U_{\text{CP}}]^{1-\gamma}. \tag{4}$$

The parameter $\gamma \in [0,\ 1]$ determines the bargaining power of ISP_1 with respect to the CP.

If we take $\gamma = 1/2$, then the above maximization is equivalent to that of product of the utilities of ISP_1 and the CP. This is then the standard proportional fair allocation [15] and is based on *Nash bargaining solution* which is known to satisfy set of four axioms [16]. We note the method discussed in this section is a modified version of the standard Nash bargaining solution, abusing terminology we continue to refer to it as bargaining. We may imagine that the bargaining is done by another player, the regulator, whose (log) utility equals

$$\overline{U}_{\text{regulator}} := \gamma \overline{U}_{\text{ISP}_1} + (1-\gamma)\overline{U}_{\text{CP}}, \tag{5}$$

where $\overline{U}_{\text{ISP}_1} = \log E[U_{\text{ISP}_1}]$ and $\overline{U}_{\text{CP}} = \log E[U_{\text{CP}}]$.

Recall that the strategy of ISP_1 is to choose price vector $\{p_1(\theta), \theta \in \Theta\}$ and that of ISP_2 is to choose p_2 knowing only distribution of Θ, and utilities of ISP_1, ISP_2 and CP are defined respectively as follows:

$$\begin{aligned} E[U_{\text{ISP}_1}] &= E[(D(\theta) - \alpha p_1(\theta) + \beta p_2)(p_1(\theta) - p_d)] \\ E[U_{\text{ISP}_2}] &= E[(D(\theta) - \alpha p_2 + \beta p_1(\theta))]p_2 \\ E[U_{\text{CP}}] &= E[(D(\theta) - \alpha p_1(\theta) + \beta p_2)](p_a + p_d) \\ &\quad + E[(D(\theta) - \alpha p_2 + \beta p_1(\theta))]p_a. \end{aligned}$$

We now return to our game model where the colluding pair decides side payment in presence of the arbitrator.. In both games the CP is a passive player. In the first game, ISP_1 and the CP bargain over side payment and then the ISPs set their price competitively. In the second, ISP_1 choose price knowing that he will bargain with the CP subsequently. Our aim is to compare the expected utilities of each player as a function of γ, and study how the bargaining power influences the players' preference for the bargaining modes, i.e, pre bargaining or post bargaining.

A. Pre bargaining

At the beginning, ISP_1 bargain with the CP and decide side payments. In this bargaining process we take into account the bargaining power of each player. Once the side payment is set, the game is between the two ISPs alone who set their prices competitively to maximize their revenue.
We computed the equilibrium utilities of players in the proof of Proposition 1 given as follows:

$$E[U_{\text{ISP}_1}] = \alpha E\left[\left(\frac{D(\theta)}{2\alpha} + \frac{\beta E[D(\theta)]}{2\alpha(2\alpha-\beta)} - \frac{p_d(2\alpha^2-\beta^2)}{4\alpha^2-\beta^2}\right)^2\right] \tag{6}$$

$$E[U_{\text{ISP}_2}] = \alpha\left(\frac{E[D(\theta)]}{(2\alpha-\beta)} + \frac{\alpha\beta}{4\alpha^2-\beta^2}p_d\right)^2 \tag{7}$$

$$\begin{aligned} E[U_{\text{CP}}] &= \frac{2\alpha E[D(\theta)]p_a}{2\alpha-\beta} - \alpha\frac{(2\alpha^2-\beta^2)}{4\alpha^2-\beta^2}p_d^2 \\ &+ \left(\frac{\alpha E[D(\theta)]}{2\alpha-\beta} - \frac{\alpha(2\alpha^2-\beta^2-\alpha\beta)p_a}{4\alpha^2-\beta^2}\right)p_d. \end{aligned} \tag{8}$$

Utility of the CP given above consists of revenue generated from traffic of both the ISPs. The portion that comes from ISP_1 is given by

$$\alpha E\left[\left(\frac{D(\theta)}{2\alpha} + \frac{\beta E[D(\theta)]}{2\alpha(2\alpha-\beta)} - \frac{p_d(2\alpha^2-\beta^2)}{4\alpha^2-\beta^2}\right)\right]$$

Then the optimization problem of the arbitrator is given by

$$\begin{aligned} &\arg\max_{p_d} \alpha E\left[\left(\frac{D(\theta)}{2\alpha} + \frac{\beta E[D(\theta)]}{2\alpha(2\alpha-\beta)} - \frac{p_d(2\alpha^2-\beta^2)}{4\alpha^2-\beta^2}\right)^2\right]^{\gamma} \times \\ &E\left[\left(\frac{D(\theta)}{2\alpha} + \frac{\beta E[D(\theta)]}{2\alpha(2\alpha-\beta)} - \frac{p_d(2\alpha^2-\beta^2)}{4\alpha^2-\beta^2}\right)\right]^{1-\gamma}(p_a+p_d)^{1-\gamma}. \end{aligned} \tag{9}$$

Taking logarithm of the objective function and differentiating with respect to p_d it is easy to verify that the above optimization problem has unique solution.

B. Post Bargaining

In the second game, ISPs set price competitively knowing that the arbitrator will decide side payment between ISP_1 and CP according to (4).

As in the pre bargaining case, we analyze this game in the reverse order, i.e., we first look at the side payment set by the arbitrator as a function of ISP prices, and then study the competition between the ISPs. Note that the demand function $d_2(\theta, p_1(\theta), p_2)$ does not depend on p_d. Then the side payment set by the arbitrator for a given $\{p_1(\theta), \theta \in \Theta\}$ and p_2 is such that

$$p_d^* \in \arg\max_{p_d} E[d_1(\theta, p_1(\theta), p_2)(p_1(\theta) - p_d))]^\gamma \times E[d_1(\theta, p_1(\theta), p_2)(p_a + p_d))]^{1-\gamma}.$$

Lemma 1: For a given $\{p_1(\theta), \theta \in \Theta\}$ and p_2 the arbitrator sets side payment as

$$p_d^* = \frac{E[d_1(\theta, p_1(\theta), p_2)(p_1(\theta))]}{E[d_1(\theta, p_1(\theta), p_2)]} - \frac{E[d_1(\theta, p_1(\theta), p_2)p_a]}{E[d_1(\theta, p_1(\theta), p_2)]}.$$

□

Substituting this expression of p_d^* in the utility of ISP_1 and CP, the modified utilities are as follows:

$$\overline{U}_{\text{ISP}_1} = \gamma E[d_1(\theta, p_1(\theta), p_2)(p_1(\theta) + p_a)]$$
$$\overline{U}_{\text{CP}_1} = (1-\gamma) E[d_1(\theta, p_1(\theta), p_2)(p_1(\theta) + p_a)]$$

Note that in the above expressions $E[d_1(\theta, p_1(\theta), p_2)(p_1(\theta) + p_a)]$ is total revenue earned by both ISP_1 and CP from the traffic generated through ISP_1. In the post bargaining they shares this total revenue in proportion to their bargaining power. We proceed to analyze the game between ISPs with the modified utility of ISP_1.

Proposition 2: In the post bargaining game with the modified utilities, the equilibrium utilities are as follows:

$$E[U_{\text{ISP}_1}] = \gamma\alpha E\left[\left(\frac{D(\theta)}{2\alpha} + \frac{\beta E[D(\theta)]}{2\alpha(2\alpha-\beta)} + \frac{(2\alpha^2-\beta^2)p_a}{4\alpha^2-\beta^2}\right)^2\right]$$

$$E[U_{\text{ISP}_2}] = \alpha\left(\frac{E[D(\theta)]}{2\alpha-\beta} - \frac{\alpha\beta p_a}{4\alpha^2-\beta^2}\right)^2$$

$$E[U_{\text{CP}}] = \alpha\left(\frac{E[D(\theta)]}{2\alpha-\beta} - \frac{\alpha\beta p_a}{4\alpha^2-\beta^2}\right)p_a + (1-\gamma)\alpha E\left[\left(\frac{D(\theta)}{2\alpha} + \frac{\beta E[D(\theta)]}{2\alpha(2\alpha-\beta)} + \frac{(2\alpha^2-\beta^2)p_a}{4\alpha^2-\beta^2}\right)^2\right].$$

□

We can now compare the expected utilities of the players in the two games obtained in Proposition 2 and equations (6)-(8). First note that the expression for the utility of ISP_1 and the CP in the pre bargaining game is similar to that in the post bargaining game with p_d replaced by $-p_a$. Also, as seen from (9) the side payment set in the pre bargaining game satisfies $p_d > -p_a$. This gives the impression that ISP_1 prefers the post bargaining mechanism in setting side payment. However, note the multiplicative factor γ in the utility of ISP_1 in the post bargaining game. When the bargaining power of ISP_1 is small, it gets only a small fraction of the joint revenue it earns with the CP. Thus ISP_1 prefers pre bargaining to decide side payment, whereas CP_1 prefers post bargaining. As the bargaining power of the ISP_1 increases, CP gets only a smaller fraction of the total revenue earned, and it prefers post bargaining. We plot the expected utility of ISP_1 and CP in both the game models as a function of γ in Figure 3. In generating the plots we used the same parameters as in Figure 2 with $\beta = 1.5$. As seen, there is a threshold on the bargaining power, marked as point a in the figure, below which CP_1 prefers pre bargaining and above post bargaining.

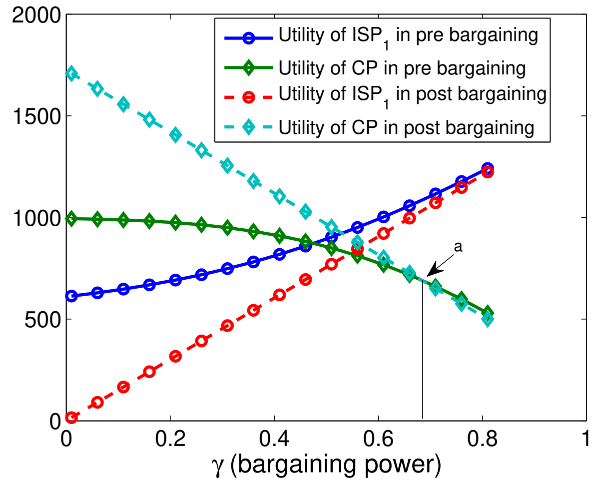

Figure 3. ISP and CP utility in two game models $\alpha = 2, \beta = 1.5$

VIII. Price of Partial Bargaining

In the previous section we studied mechanisms to decide the payment based on weighted proportionally fair criteria. Another natural choice is to set a side payment such that the sum of the utility of all the players is maximized at equilibrium. Let $\tilde{p}_d$ denote this side payment, i.e.,

$$\tilde{p}_d \in \arg\max_{p_d} \quad E[U_{\text{ISP}_1}] + E[U_{\text{ISP}_2}] + E[U_{\text{CP}}],$$

where the utilities are computed at the equilibrium prices of the players. We denote the expected sum of equilibrium utilities calculated at $\tilde{p}_d$ as $\tilde{U}$. Recall that we denoted the side payment obtained in weighted proportional fairness solution as p_d^* in (4). Let the expected sum of the equilibrium utilities calculated at the weighted proportional fairness solution be denoted as $\overline{U}$. We will be interested in studying how good $\overline{U}$ is compared to $\tilde{U}$.

In this section we do not take into account the bargaining power of each player. We shall be interested in simply the product of utilities (i.e., without the exponents in (4)). A more interesting analysis would be to compare optimal α-fair social equilibrium utility, interpreting fairness factor as the bargaining power. However, we will not pursue this thought in this work.

In [10], the authors proposed a new measure called *Price of Selfishness* (PoS) to compare the optimal social utility with

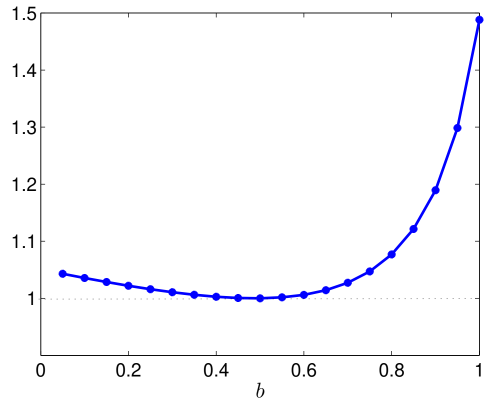

Figure 4. Price of partial Bargaining $\alpha = 2$

the social utility obtained at the Nash bargaining solution. However, their definition of PoS is not suitable in our setting to compare $\tilde{U}$ and $\overline{U}$. This is because, in [10] the problem is defined in a cooperative context in which the regulator determines the actions taken by all the players, i.e, $p_1(\theta)$ and p_2, and also the value of p_d that maximizes the product of the utilities of all the players. In our case the problem is not fully cooperative. Bargaining is restricted to the parameter p_d alone. The other parameters are set through competition. Thus in our model bargaining is over a subset of the parameters. We therefore propose an alternative metric called *Price of Partial Bargaining* (PoPB), which we define as

$$PoPB = \frac{\tilde{U}}{\overline{\overline{U}}}.$$

We will next compute the PoPB in the nonneutral regime analyzed in the previous section where CP shares private information with ISP_1 on payment. In the pre bargaining game, side payment $\tilde{p}_d$ is the maximizer of sum of utilities given by (6)-(8) and p_d^* is obtained from (9). The resulting optimal values and the corresponding utilities are cubersome to manupulate. We plot the PoPB in Figure 4 as a function of $\tau = \beta/\alpha$ fixing $\alpha = 2$. From the figure we note that when τ is close to 1, the PoPB is large. When τ is close to 1, the demand generated from each ISP is equally sensitive to price set by the competing ISPs, in this case pre bargaining leads to poor social utility. However, when τ is close to $1/2$, PoPB is close to one. This implies that when demand generated from an ISP half as much sensitive as to its own price then the resulting social utility in pre bargaining is close to optimal.When τ is close to zero, again the pre bargaining results in poor equilibrium social utility.

We note that our definition of PoPB is not appropriate for the post bargaining game. In this game, first optimal side payment is evaluated for a given price of ISPs, and then the equilibrium prices are computed. But the similar process of evaluating social utility becomes independent of side payment.

IX. Conclusions

In this paper we studied preferential treatment of ISPs by CPs through collusions. We modeled a nonneutral behavior in which a CP shares private about its content through signals. We showed that the CP may not benefit sharing its private information, whereas ISPs always benefit receiving signals. If the CP charges the ISPs to share its private information, both the CP and the ISP in collusion may lose, whereas the ISPs which do not receive signals may gain.

We also studied two mechanisms based on weighted proportional fairness criteria to set the price (side payments) that ISP pays to the CP for providing signals. In deciding this side payments we took into account the bargaining power of the players. We noted that the bargaining power influences players preference for the mechanisms. We also introduced a new performance measure to compare the social utility at equilibrium with the optimal social utility when some parameters are agreed through bargaining and others are set competitively.

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APPENDICES

A. Proof of Theorem 1

Proof: We begin with the neutral regime with no information. Taking the logarithm of the utility of ISP_i we get

$$\log E[U_{\text{ISP}_i}] = \log E[d_i(\theta, p_i, \mathbf{p}_{-i})] + \log p_i$$

Using Assumption 1 and monotonicity properties of $d_i(\theta, \mathbf{p})$ given in (1), it is easy to verify that $\log E[d_i(\theta, p_i, \mathbf{p}_{-i})]$ satisfies supermodular property, i.e,

$$\frac{\partial^2 \log E[d_i(\theta, p_i, \mathbf{p}_{-i})]}{\partial p_i \partial p_j} \geq 0 \text{ for } j \neq i.$$

Then existence of equilibrium follows from Topkis's theorem [13].

Using the dominant diagonal property it is easy to verify that for all $i = 1, 2, \cdots, n$

$$\sum_{j=1}^{n} \frac{\partial^2 \log E d_i(\theta, \mathbf{p})}{\partial p_i \partial p_j} \leq 0.$$

Then uniqueness of equilibria follows from [13].

Now consider the case with full information. We first note that the demand function for each ISP is separable in θ. Thus, given θ, ISP_i sets a price that maximizes $U_{\text{ISP}}(\theta, p_i(\theta), \mathbf{p}_{-i}(\theta))$ independent of what other ISPs set when the signal is different from θ. Hence we can restrict the study of price competition between the ISP for a given θ. It can be easily verified that $d_i(\theta, p(\theta))$ is log-supermodular. Also, by setting $\theta = \gamma$ in Assumption II the condition

$$\sum_{j=1}^{n} \frac{\partial^2 \log d_(\theta, p(\theta))}{\partial p_i \partial p_j} \leq 0$$

holds for all θ and i. Then existence and uniqueness follows from Topkis's theorem [13].

■

B. Proof of Theorem 3

Proof: For a given θ the utility of each ISP is quadratic in price. We compute the equilibrium prices by simply solving the best response. A straight forward calculations results in the following equilibrium utilities when the CP does not give signal to any of the ISPs:

$$E[U_{\text{ISP}_1}] = E[U_{\text{ISP}_2}] = \frac{\alpha (E[D(\theta)])^2}{(2\alpha - \beta)^2}, \tag{10}$$

$$E[U_{\text{CP}}(\theta, p_1^*, p_2^*)] = \frac{2\alpha E[D(\theta)]}{2\alpha - \beta} p_a. \tag{11}$$

Similar calculation results in the following utilities of when the CP gives signal to both the ISPs.

$$E[U_{\text{ISP}_1}] = E[U_{\text{ISP}_2}] = \frac{\alpha}{(2\alpha - \beta)^2} E[D^2(\theta)], \tag{12}$$

$$E[U_{\text{CP}}(\theta, p_1^*(\theta), p_2^*(\theta))] = \frac{2\alpha E[D(\theta)]}{2\alpha - \beta} p_a. \tag{13}$$

Subtracting the expected utility of the ISP_1 in (10) from (12) we have

$$\begin{aligned} &\frac{\alpha}{(2\alpha - \beta)^2}(E[D^2(\theta)] - E^2[(\theta)]) \\ &= \frac{\alpha}{(2\alpha - \beta)^2} Var(D(\theta)) \geq 0. \end{aligned}$$

Where $Var(D(\theta))$ denotes the variance of the random variable $D(\theta)$. Now assume that CP colludes with ISP_1 and shares private information only it. Then, the expected utility of ISP_2 and CP at equilibrium can be computed, respectively, as follows:

$$E[U_{\text{ISP}_1}] = \frac{E[D^2(\theta)]}{4\alpha} + \frac{\beta E^2[D(\theta)](4\alpha - \beta)}{4\alpha(2\alpha - \beta)^2}. \tag{14}$$

$$E[U_{\text{ISP}_2}] = \frac{\alpha E^2[D(\theta)]}{(2\alpha - \beta)^2}, \tag{15}$$

$$E[U_{\text{CP}}] = 2\alpha \frac{E[D(\theta)]}{(2\alpha - \beta)}. \tag{16}$$

We now compare the performance of ISP that receives the signaling information with the ISP which do not have this information. To prove the first claim in the nonneutral regime we compare the expected utility in (14) with the expected utility in (10) obtained when both the ISPs do not get signaling information. A simple manipulations yields that (14) is larger than (10) if and only if $E[D^2(\theta)] \geq E^2[D(\theta)]$, which always holds.

To prove the second claim we compare the expected utility in (14) with the expected utility in (12) obtained when both the ISPs get signaling information. Again, a simple manipulation shows that (12) is larger than (14) if and only if $E[D^2(\theta)] \geq E^2[D(\theta)]$ which holds always.

The third claim holds by comparing utility of the non-colluding ISP in (12) and (15).

Finally, the last claim follows by noting that expected utility of the CP given by in the three cases reo(15) and (10) are the same. ■

C. Proof of Proposition 1

Proof: Best response of ISP_1 for a given value of θ and p_2 is

$$p_1(\theta) = \frac{D(\theta) + \beta p_2 + \alpha p_d}{2\alpha}. \tag{17}$$

Similarly, the best response of ISP_2 for a given strategy profile $\{p_1(\theta) : \theta \in \Theta\}$

$$p_2 = \frac{E[D(\theta + \beta p(\theta))]}{2\alpha}.$$

Solving the above best response equations simultaneously, the equilibrium prices are given by

$$\begin{aligned} p_1^*(\theta) &= \frac{D(\theta)}{2\alpha} + \frac{\beta E[D(\theta)]}{2\alpha(2\alpha - \beta)} + \frac{2\alpha^2 p_d}{4\alpha^2 - \beta^2}, \\ p_2 &= \frac{E[D(\theta)]}{2\alpha - \beta} + \frac{\alpha\beta}{4\alpha^2 - \beta^2}. \end{aligned}$$

Substituting these prices in the utility functions and taking expectation we obtain the equilibrium utility for each player as follows:

$$E[U_{\text{ISP}_1}]= \alpha E\left[\left(\frac{D(\theta)}{2\alpha} + \frac{\beta E[D(\theta)]}{2\alpha(2\alpha-\beta)} - \frac{p_d(2\alpha^2-\beta^2)}{4\alpha^2-\beta^2}\right)^2\right]$$

$$E[U_{\text{ISP}_2}] = \alpha\left(\frac{E[D(\theta)]}{(2\alpha-\beta)} + \frac{\alpha\beta}{4\alpha^2-\beta^2}p_d\right)^2$$

$$\begin{aligned} E[U_{\text{CP}}] &= \frac{2\alpha E[D(\theta)]}{2\alpha-\beta} - \alpha\frac{(2\alpha^2-\beta^2)}{4\alpha^2-\beta^2}p_d^2 \\ &+ \left(\frac{\alpha E[D(\theta)]}{2\alpha-\beta} - \frac{\alpha(2\alpha^2-\beta^2-\alpha\beta)p_a}{4\alpha^2-\beta^2}\right)p_d. \end{aligned}$$

The claim follows by comparing the above utilities of the ISPs with the utilities given in (14)-(15) which corresponds to the case when both the ISPs receive signals. ■

D. Proof of Theorem 4

Proof: Collusion with the CP is beneficial to ISP_1 if it can get higher expected utility compared to the case when it does not enter into any agreement. This happens if the expected utility, given in (6), is larger than that given in (10). Subtracting (10) from (6) and simplifying, we get the following quadratic equation in p_d.

$$f(p_d) := p_d^2 - \frac{2E[D](2\alpha+\beta)p_d}{2\alpha^2-\beta^2} + \frac{(4\alpha^2-\beta^2)^2Var(D(\theta))}{(2\alpha^2-\beta^2)4\alpha^2}.$$

The roots of this quadratic equation are

$$\frac{(2\alpha+\beta)E[D(\theta)]}{2\alpha^2-\beta^2}\left\{1 \pm \sqrt{1 - \frac{(2\alpha-\beta)^2Var(D(\theta))}{4\alpha^2E^2[D(\theta)]}}\right\}.$$

Let x_1 and x_2 denote the smaller and larger root respectively. Note that $f(p_d)$ is a concave function in p_d. It takes non negative values outside the interval $[x_1,\ x_2]$. It is easy to verify that for $p_d \geq x_2$ revenue obtained by the colluding ISP is negative. Thus the claim follows by noting that $f(p_d)$ is nonnegative for $p_d \leq x_1$.

Similarly, collusion with ISP_1 is beneficial to CP if it can get higher expected utility compared to the case when it does not enter into any agreement. This happens if the expected utility for CP in collusion, given in (8), is larger than that given in (11). Subtracting (11) from (8) and simplifying, we get the following quadratic equation in p_d.

$$g(p_d) := p_d\left(\frac{E[D(\theta)]}{2\alpha-\beta} - \frac{(2\alpha^2-\beta^2-\alpha\beta)p_a}{4\alpha^2-\beta^2} - \frac{2\alpha^2-\beta^2}{4\alpha^2-\beta^2}p_d\right).$$

Now the claim follows by noting that $g(p_d)$ is positive if and only if p_d satisfies the relation (3).

The last claim can be verified in a similar way by comparing expected utility of ISP_1 and ISP_2 given in (6) and (7) respectively. ■