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This study takes place in the context of extreme quantiles estimation by the block maxima method. We investigate the behaviour of the relative approximation error of a quantile estimator dedicated to the Gumbel maximum domain of attraction. Our work is based on a regular variation assumption on the first derivative of the logarithm of the inverse cumulative hazard rate function, introduced by de Valk (2016) [Approximation of high quantiles from intermediate quantiles. *Extremes* **19(4)**, 661-686].

Let us denote by $X_{m,m}$ the maximum of m iid observations from a distribution function F, where m is referred to as the block size. We focus on an extreme quantile associated with F defined by $x_{p_m} = F^{-1}(1-p_m) = H^{-1}(-\log p_m)$, where H is the cumulative hazard rate function and $p_m = m^{-\tau_m}$ with $\tau_m \geq 1$. Assume that $(X_{m,m} - b_m)/a_m$ converges to a Gumbel distribution for some normalizing constants $a_m > 0$ and $b_m \in \mathbb{R}$. The approximation $\tilde{x}_{p_m}$ of x_{p_m} by the block maxima method is given by

$$\tilde{x}_{p_m} = b_m - a_m \log(m p_m)$$

and the associated relative approximation error is

$$\epsilon_{app_m} = (x_{p_m} - \tilde{x}_{p_m})/x_{p_m}.$$

Our main result is :

$$\epsilon_{app_m} \overset{m\to+\infty}{\longrightarrow} 0 \iff (\tau_m - 1)^2 K_2(\log m) \overset{m\to+\infty}{\longrightarrow} 0,$$

where $K_2(t) = t^2 (H^{-1})''(t)/H^{-1}(t)$, $t > 0$. This result exhibits three families of distributions according to the limit of K_2 which can be either zero, a constant or infinite. We also provide a first order approximation of the relative approximation error when the latter converges towards zero. Our results are illustrated on simulated data.

Key Words: Extreme quantiles estimation, relative approximation error, asymptotic properties, regular variation.